Labor market policy instruments and the role of economic turbulence

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Abstract

Times of high unemployment always inspire debates on the role of labor market policy and its optimal implementation. This paper uses a dynamic model of search unemployment and bilateral wage bargaining, rich enough to analyze a set of policy instruments with respect to their employment and welfare effects: payroll, output and firing taxes as well as wage, hiring and recruitment subsidies. It is shown that in presence of unemployment benefits a first best implementation using only firing taxes - in the spirit of Blanchard and Tirole (2008) - is not possible if job acceptance is endogenous. However, the socially optimal allocation can be implemented - even if the Hosios-condition does not hold - using a mix of the aforementioned instruments unless firms are liquidity constrained. In a second exercise, the model is extended to allow for worker transitions between skill classes reflecting economic turbulence. It is shown that the effectiveness of intergroup redistribution schemes as suggested by Mortensen and Pissarides (2003) is considerably reduced in the presence of economic turbulence. Instead of redistribution from high to low-skilled workers or from firing firms to unemployed workers, the paper identifies a scheme involving redistribution from firing to hiring firms to be optimal.

Keywords: search and matching, employment subsidies, economic turbulence

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1 Introduction

Times of high unemployment always inspire debates on the role of labor market policy and most of the time lead to a variety of policy advice. The motivation of this work stems from the on-going controversy about optimal policy instruments for Germany that experienced a dramatic increase in unemployment during the years 2000 to 2005. While people agree that especially the rate of low-skilled unemployment is excessively high and current policy is suboptimal and leaves room for improvement, opinions on what should be done are mostly at odds with each other. Take as an example debate the Sinn et al. (2006) versus Brown et al. (2007). While the former prefer wage subsidies targeted at persons with low abilities, the latter favor hiring subsidies for long-term unemployed workers with low skills. Other empirical studies suggest that the effectiveness of both subsidies is limited. Bonin et al. (2002) find that wage subsidies for low-skilled workers in order to decrease disincentives do not appear to work very effectively and that such a policy is likely to be too costly. Boockmann et al. (2007) draw their conclusions from legal changes in the eligibility of German workers to EGZ[1] which they use as natural experiments. They find that eligibility to this subsidy did not change the transition rates from unemployment to employment significantly. However, named empirical studies provide limited conclusion concerning the macroeconomic effects of hiring and wage subsidies as they cannot directly measure the effect of the subsidies in question, because they have never been implemented Germany-wide.

Cahuc and Le Barbanchon (2010), in their study on counseling, very well notice that micro evaluations neglecting crowding out, adverse spill over effects on non-targeted persons, and other equilibrium effects can lead to misguided policy advice. A similar point is made by Van der Linden (2005) who endogenizes job search effort and wages in his evaluation. In addition, the sort of studies mentioned above tends to lack a thorough evaluation of the cost side. Therefore, we base our analysis on a model of equilibrium unemployment and our conclusions rely on theoretical reasoning and numerical simulations.

[1] *'Eingliederungszuschuss'* (EGZ), is a form of a hiring subsidy especially targeted at disadvantaged groups, like elderly or handicapped people.

Dynamic search and matching models have been widely used to evaluate employment subsidies ever since the influential paper of Mortensen and Pissarides (1994). However, the conclusions so far are mixed. While Bovenberg et al. (2000) and Cardullo and Van der Linden (2006) argue that wage subsidies can substantially reduce unemployment, Boone and van Ours (2004) find no such effect. The majority of these simulation studies focuses on the effect of wage subsidies or in-work benefits on unemployment. One exception is Yashiv (2004) who compares wage versus hiring subsidies in a partial equilibrium approach with endogenous job finding but constant separation rates. The results of his empirical simulations using Israeli data suggest that hiring subsidies should be preferred. A more theoretical treatment - probably most closely related to this paper - is provided by Mortensen and Pissarides (2003) (henceforth MP). First, they derive conditions on the policy instruments for the implementation of the welfare optimizing first-best solution. In an extension of the model they find evidence that a wage subsidy for low-skilled workers cross-financed by high-skilled workers is an effective measure to reduce overall unemployment. We will contribute to those two findings. First, MP do not allow for a revenue generating firing tax as modeled in Blanchard and Tirole (2008) (henceforth BT), who - in a static setup - find that the distortions of unemployment benefits can be exactly offset by a firing tax. We will allow for this policy instrument in our dynamic framework where job acceptance and destruction margins are explicitly modeled and might be distorted by such a tax. It will be shown that taking job acceptance into account - which is not done in MP - will alter the first-best implementation by requiring an additional policy instrument. In this extended MP-style intragroup model, we characterize two possible implementations of the social optimum, one utilizing hiring and the other wage subsidies. Both schemes are limited: the first, which involves redistribution from firing to hiring firms, is limited if firms are liquidity constrained. The wage subsidy implementation of the first-best cannot work if the replacement ratio is too large.

Building on the intragroup model we introduce a second skill class to allow for intergroup redistribution. We will characterize optimal policy with and without the presence of economic turbulence. We will argue that in a world of economic turbulence a cross-financed wage subsidy scheme as suggested by

MP might be considerably less efficient[2]. They assume skill classes to operate in complete juxtaposition, except for the connection via the government's budget constraint, which underestimates the adverse effect on high-skilled workers. In order to model the interaction of different skill groups we will follow the idea of economic turbulence proposed by Ljungqvist and Sargent (1998), where unemployed workers will lose their skills in the course of time.

The outline of the paper is as follows. First a simple intragroup model is developed to analytically derive the effects of the considered subsidies and different forms of taxes on the equilibrium variables. The next part deals with efficiency aspects where the optimal policy mix, implementing the social planner's solution, is derived. Section 3 extends the intragroup to an intergroup model and introduces economic turbulence in form of state-dependent transitions between skill classes. We will derive analytic results as far as possible and then also discuss some simulation results.

2 A simple intragroup model

The model is based on the standard Mortensen and Pissarides (1994)-framework with search frictions and is closely related to MP but extended by endogenous acceptance (the $\underline{z}$-margin). In this section we consider only one skill class.

There are two types of rational, forward looking agents: workers and firms, each subject to a dichotomous action set. A firm can post a vacancy or not, while a worker can accept or reject a wage contract, which is derived via Nash bargaining, once it arrives. Labor force L is comprised of atomistic risk neutral workers. There is a sufficiently large number of risk neutral firms that can enter the labor market without restrictions. Each firm can employ at most one worker and is subject to a per period net flow cost c if it posts a vacancy. Labor is the only production factor and technology is Ricardian. Once a worker has accepted a job offer she will inelastically supply one unit of labor. Unemployed workers are matched to a vacancy at a rate depending on current

[2]While Oskamp and Snower (2008) draw similar conclusions based on their numerical simulations, our work has a considerably more theory-focused approach especially concerning the question of how an optimal scheme should look like.

labor market conditions, reflected by labor market tightness θ. This can be interpreted as meeting for a job interview. Only then the agents will find out how well suited an applicant is for the specific job. This is modeled as drawing a job-specific productivity z from a known distribution $G(z)$. Upon the realization of the draw the parties will decide whether to enter the relationship, i.e. $z > \underline{z}$, or not, i.e. $z < \underline{z}$, where $\underline{z}$ denotes the reservation productivity of accepting a new job. Technically, this is the main difference compared to MP who assume that every job is created at maximum idiosyncratic productivity, trivializing the acceptance decision because job offers are rejected with probability zero. In an alternative interpretation this relates to Hall (2005) who also allows for less qualified persons to apply[3]. Further, it is assumed that an idiosyncratic job-specific productivity shock arrives with probability π^n conditional on being employed. Again, the worker can decide whether to accept the offer and continue working, i.e. $z > \underline{\hat{z}}$, or to reject implying job destruction, i.e. $z < \underline{\hat{z}}$. $\underline{\hat{z}}$ denotes the reservation productivity of continuing an existing job. Hence, there are three decision margins reflected by three decision variables: job creation (θ), job acceptance ($\underline{z}$) and job destruction ($\underline{\hat{z}}$).

Three different subsidies will be analyzed: a lump-sum wage subsidy (D), a one-time hiring subsidy to the firm (H), and a recruitment subsidy (R) which is merely a reduction in vacancy posting costs. On the other hand we will analyze three distortionary taxes, namely: firing taxes (F), output taxes (τ), and payroll or wage taxes (t). In the beginning all subsidies and taxes will be balanced by a non-distortionary lump-sum consumption tax/subsidy (T). This assumption will be relaxed when looking for the optimal policy mix. Analytically, the model can be described as follows[4].

As usual for this kind of framework an aggregate matching function $m(u, v)$, which maps the stock of unemployed (u) and the stock of vacancies (v) into the flow of new matches (m) is assumed to be homogeneous of degree one with elasticity w.r.t. u of $0 < \eta < 1$. Defining labor market tightness as $\theta \equiv \frac{v}{u}$

[3]In contrast to our analysis, Hall (2005) assumes that the qualification of an applicant is not completely revealed to the employer in the first meeting. This can only be resolved if the employer decides to costly evaluate the application.

[4]The notation is based on Pissarides (2000). A description of all used variables can be found in appendix section G.

results in the matching probability functions (2.1) and (2.2) for firms and workers, respectively.

$$\text{prob. of a match for the firm:} \qquad \frac{m(u,v)}{v} = q(\theta) \tag{2.1}$$

$$\text{prob. of a match for the worker:} \qquad \frac{m(u,v)}{u} = \theta q(\theta) \tag{2.2}$$

with $q'(\cdot) < 0$ and $q''(\cdot) < 0$. Further define: $q^f = q(\theta)(1 - G(\underline{z}))$ and $q^w = \theta q(\theta)(1 - G(\underline{z}))$ as the joint probabilities of matching and accepting.

A worker can be either employed (e) or unemployed (u), that is we abstract from transitions into and out of labor force, hence $e + u = L$. Each state is associated with a specific capital value, U for being unemployed and $W(z)$ or $\hat{W}(z)$ for becoming or being employed, respectively. In general, the hat-notation always indicates that the worker or the firm has been in the same state before the arrival of a shock. Or put differently, 'without hat' can be referred to as the initial or 'outside' value while 'with hat' denotes the continuation or 'inside' value after a shock arrived. Given the assumption of perfect capital markets, where r denotes the exogenous interest rate, we can write both asset equations of working as follows:

$$rW(z) = (1-t)w(z) - T + \pi^n \left[(1 - G(\hat{\underline{z}}))\, \hat{W}^{\hat{e}} + G(\hat{\underline{z}})U - W(z) \right] \tag{2.3}$$

$$r\hat{W}(z) = (1-t)\hat{w}(z) - T + \pi^n \left[(1 - G(\hat{\underline{z}}))\, \hat{W}^{\hat{e}} + G(\hat{\underline{z}})U - \hat{W}(z) \right] \tag{2.4}$$

A just recently employed worker's felicity equals after tax wage income $(1-t)w(z) - T$. When a shock arrives he loses $W(z)$ and gains U if the new productivity draw z is lower than the cut-off $\hat{\underline{z}}$, hence with probability $G(\hat{\underline{z}})$. With probability $(1 - G(\hat{\underline{z}}))$ he gets $\hat{W}^{\hat{e}}$, which denotes the conditional expectation[5] of the value of being employed. The asset value of being unemployed is given by

$$rU = \mu - T + q^w \left(W^e - U \right) \tag{2.5}$$

where μ denotes the value of leisure which is composed of unemployment

[5]The conditional expectation of some random variable $X(z)$ w.r.t. $\hat{\underline{z}}$ is defined as $E(X(z)|z > \hat{\underline{z}}) = X^{\hat{e}} = \int_{\hat{\underline{z}}}^{\infty} \frac{X(\tilde{z})}{1-G(\hat{\underline{z}})}\, dG(\tilde{z})$. Note the difference in notation compared to $E(X(z)|z > \underline{z}) = X^e$.

compensation b and home production h in a linear way, $\mu = b + h$. Turning to the firms' side the asset value of a vacancy can be written as

$$rV = -c + q^f \left(J^e - V + H\right), \quad \text{where} \quad c = C - R \tag{2.6}$$

Two subsidies enter this relationship. In case of an accepted match the firm has to give up the value of a vacancy V but gets the expected value of a job for the firm J^e plus a hiring subsidy H. The gross flow costs of maintaining a vacancy C minus the recruitment subsidy R give the net costs c. As free entry is imposed and V is decreasing in θ, in equilibrium V is driven down to zero which will pin down θ, hence:

$$V = 0 \Rightarrow \theta \tag{2.7}$$

The asset values of a job are given in (2.8) and (2.9):

$$rJ(z) = (1-\tau)z - w(z) + D + \pi^n \left[(1 - G(\underline{\hat{z}}))\hat{J}^{\hat{e}} - G(\underline{\hat{z}})F - J(z)\right] \tag{2.8}$$

$$r\hat{J}(z) = (1-\tau)z - \hat{w}(z) + D + \pi^n \left[(1 - G(\underline{\hat{z}}))\hat{J}^{\hat{e}} - G(\underline{\hat{z}})F - \hat{J}(z)\right] \tag{2.9}$$

In the current period a firm receives after tax production $(1-\tau)z$ minus wage rate $w(z)$ plus a wage subsidy D[6]. In case of a separation, which occurs with probability π^n and the probability of $z < \underline{\hat{z}}$, a firm has to pay a firing tax F. Observe that given the wage determination explained below a firm and a worker will always mutually agree to destroy or create a job, i.e. both sides have the same reservation productivities. Hence, the notions of a 'firing' and 'separation' tax are equivalent. The reservation productivities are pinned down by the following conditions

$$J(\underline{z}) + H = 0 \Rightarrow \underline{z} \tag{2.10}$$

$$\hat{J}(\underline{\hat{z}}) + F = 0 \Rightarrow \underline{\hat{z}} \tag{2.11}$$

The first relation states that after meeting for an interview and observing

[6] Note that with Nash bargaining it does not matter economically whether the wage subsidy is given to the worker or the firm but the interpretation of w changes. In our setting w and $(1-t)w$ are interpreted as gross and net wages received by worker already including all subsidies.

the match specific productivity z, a job will only be generated if the value of a job including the one-time hiring subsidy is non-negative. The second condition reflects that a firm will only want to continue a job if its value covers at least the firing tax. Wages are determined via Nash bargaining and are renegotiated every time a shock arrives. The result of maximizing the Nash product[7] w.r.t. to the wage rate, where the weight ω can be interpreted as the worker's bargaining power, is given by the following optimality conditions:

$$(1-\omega)\left(W(z)-U\right)=\omega(1-t)\left(J(z)+H\right) \tag{2.12}$$

$$(1-\omega)\left(\hat{W}(z)-U\right)=\omega(1-t)\left(\hat{J}(z)+F\right) \tag{2.13}$$

The equilibrium 'outside' and 'inside' wage rates can then be solved for explicitly[8]:

$$w(z)=(1-\omega)\frac{\mu}{1-t}+\omega((1-\tau)z+D+c\theta+rH)-\omega\pi^{n}(F-H) \tag{2.14}$$

$$\hat{w}(z)=(1-\omega)\frac{\mu}{1-t}+\omega((1-\tau)z+D+c\theta+rF) \tag{2.15}$$

Observe that the 'inside' and 'outside' wage distributions are directly related to the productivity distribution $G(\cdot)$ if z is larger than the respective cut-off. A wage subsidy D will increase both wage schedules by the share the worker can claim in the process of bargaining, ωD. While a recruitment subsidy R, which is included in c, decreases both wages to the same extent, they respond differently to a hiring subsidy H and a firing tax F. A hiring subsidy will increase the outside wage of a worker while it does not affect the inside wage as the subsidy is already sunk by then. A firing tax will abate outside wages as firms are more cautious about hiring workers because they eventually have to pay F. In contrast, inside wages will be inflated by F because firms are more willing to hold on to workers once they are employed.

At last, in equilibrium the government's budget constraint has to hold. For now we will assume that a non-distortionary consumption tax/subsidy T is

[7]The Nash product is a simple Cobb-Douglas function assumed to be homogeneous of degree one, with the difference of inside and outside options for worker and firm, respectively, as arguments.

[8]See appendix section C for derivation.

used to balance the budget. In section 2.2 where the optimal policy mix is discussed we will abandon this assumption.

$$\begin{aligned} -TL = (L-u)\bar{w}t + (L-u)\bar{z}\tau + (L-u)\pi^n G(\underline{\hat{z}})F \\ - uq^w H - \theta u R - (L-u)D - ub \end{aligned} \tag{2.16}$$

where $\bar{w}$ and $\bar{z}$ denote average wage and productivity, respectively. The first line represents tax income from the lump sum tax, the wage tax, the output tax and the firing tax. The second gives expenditure on hiring, recruitment, and wage subsidies as well as unemployment benefits.

2.1 Equilibrium

The equilibrium vector $\langle u, \theta, \underline{z}, \underline{\hat{z}} \rangle$ is pinned down by the four equations (2.17) to (2.20)[9]. Equilibrium is partly recursive, i.e. only (2.17) and (2.18), henceforth referred to as the JD-JC system, have to be solved simultaneously for θ and $\underline{\hat{z}}$ after inserting (2.19). The job creation (JC) curve, which is derived from the free entry condition, equates expected gain and cost of a vacancy

$$JC \quad : \quad (1-\omega)\left(\frac{(z^e - \underline{\hat{z}})(1-\tau)}{\pi^n + r} - F + H\right) - \frac{c}{q^f} = 0 \tag{2.17}$$

The first term is the expected gain of job creation for a firm, i.e. the firm's after tax share of excess output discounted by $\pi^n + r$. The gain is additionally raised or lowered depending on whether the hiring subsidy H exceeds the firing tax F, or vice versa. The second term reflects the expected costs of job creation, i.e. the net flow cost c times the average duration of a vacancy $1/q^f$.

$$\begin{aligned} JD \quad : \quad (1-\tau)\underline{\hat{z}} + D + \frac{\pi^n(1-\tau)}{\pi^n + r}\int_{\underline{\hat{z}}}^{\infty} (\tilde{z} - \underline{\hat{z}})\, dG(\tilde{z}) \\ - \frac{\mu}{1-t} + rF - \frac{\omega}{1-\omega} c\theta = 0 \end{aligned} \tag{2.18}$$

The first line of the job destruction (JD) condition, which represents the 'inside' cut-off condition, gives the joint inside value of a job, i.e. the after tax reservation product plus a wage subsidy D and the option value of keeping a

[9]See appendix section C for a detailed derivation of (2.17)-(2.20).

worker as her productivity might change. The second line can be interpreted as the joint outside value, which increases in μ and θ, as both raise the worker's outside option, and decreases in F. The analytic relationship of the 'outside' to the 'inside' productivity cut-off is novel because the job acceptance margin is taken into account in this paper.

$$\underline{z} = \hat{\underline{z}} + \frac{(\pi^n + r)}{(1-\tau)}(F - H) \tag{2.19}$$

Observe that both cut-offs coincide in a policy free environment where $F = H = 0$. A hiring subsidy H will put a wedge between those cut-offs in a way that agents' will more easily accept than destroy a job ($\underline{z} < \hat{\underline{z}}$). A firing tax F will have the opposite consequence, $\underline{z} > \hat{\underline{z}}$. Having derived all three decision variables θ, $\hat{\underline{z}}$, and $\underline{z}$, we can compute unemployment u. Just insert in the typical Beveridge curve (2.20), which is derived by setting the change in u in $\dot{u} = (L-u)\pi^n G(\hat{\underline{z}}) - uq^w$ to zero.

$$u = \frac{\pi^n G(\hat{\underline{z}})}{\pi^n G(\hat{\underline{z}}) + q^w} \cdot L \tag{2.20}$$

The JD-JC diagram

As mentioned, the recursion of the system reduces the problem to solving only two equations simultaneously. Therefore, we can conveniently analyze comparative statics in the JD-JC diagram[10], drawn in the θ-$\hat{\underline{z}}$-space (see Pissarides (2000)). The JC-curve is sloping downward because firms will post fewer vacancies the higher $\hat{\underline{z}}$, as average duration of a job decreases in $\hat{\underline{z}}$. The JD-curve slopes upward because workers will want to terminate jobs more easily, implying higher $\hat{\underline{z}}$, the higher θ, as their outside options increase in labor market tightness. Hence, the curves will intersect at most one time, as illustrated by figure 2.1, which makes the equilibrium unique in case of existence.

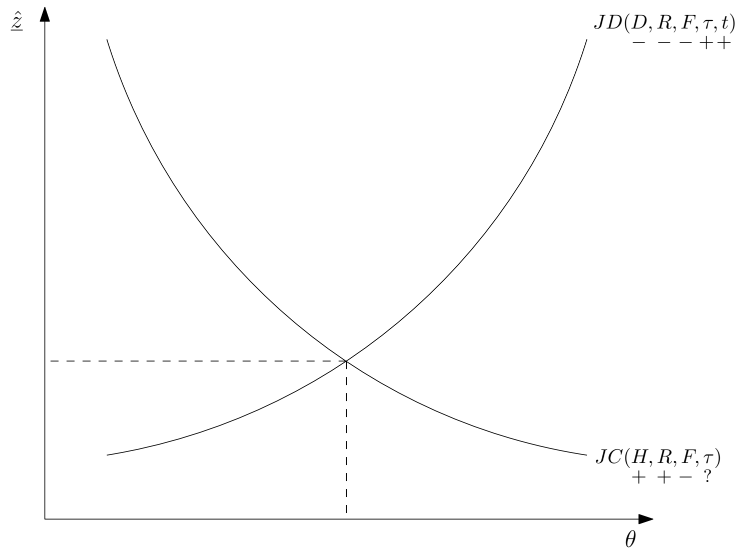
Policy effects

We will now address the effects of uncompensated (or compensated by a non-distortionary consumption tax/subsidy T) changes in our policy instruments[11].

[10] See appendix section F.1 for more details.

[11] See appendix section F.2 for the analytic derivation.

Figure 2.1: The job creation, job destruction diagram



A wage subsidy D has no effect on the JC-curve but shifts out the JD-curve. The intuition behind the shift is the following. For the same θ, which translates into the value of the outside option, a worker receiving a wage subsidy will hold on to a job for lower realizations of productivity. Hence, equilibrium labor market tightness θ will go up, the reservation productivities $\underline{z} = \hat{\underline{z}}$ will fall, leading to more job creation, more acceptance and less destruction. Therefore, unemployment will unambiguously decrease. A hiring subsidy H works quite differently. While there is no effect on the JD- curve the JC-curve will shift outward. This raises labor market tightness and consequently job creation as well as job destruction. In relation to job destruction, job acceptance is boosted, i.e. $\underline{z} < \hat{\underline{z}}$. Whether job acceptance rises or falls in absolute terms is ambiguous. Proposition 2.1 states a condition for the direction of the absolute effect.

Proposition 2.1. (Hiring subsidy and job acceptance) *A hiring subsidy can lead to more or less job acceptance. Assume for simplicity that* $t = \tau = 0$. *Whenever* $\nabla^{-1}\omega c(1-\Omega) < \pi^n + r$ *the effect of* H *on* $\underline{z}$ *will be negative, leading to more job acceptance.*

Proof. Differentiating (2.19) w.r.t. H gives $\frac{\partial \underline{z}}{\partial H} = \frac{\partial \hat{\underline{z}}}{\partial H} - (\pi^n + r)$. Inserting for $\frac{\partial \hat{\underline{z}}}{\partial H}$ derived by using the implicit function theorem and rearranging completes the proof[12]. ∎

[12] ∇ denotes the determinant of the JD-JC system and will always be positive. See

A firing tax F has very similar but inverted effects compared to a hiring subsidy H. While the JC-curve moves inward to the same extent additionally also JD-curve shifts outward if $r > 0$. One can show that the JC-curves moves more strongly which implies that θ will fall unambiguously. The job destruction margin ($\hat{\underline{z}}$) will fall, while the absolute effect on job acceptance ($\underline{z} > \hat{\underline{z}}$) is again ambiguous. We can make the following statements about the relationship of F and H.

Proposition 2.2. (Hiring - firing policy equivalence) *Let $r \to 0$. Then a firing tax (F) and a negative hiring subsidy ($-H$) have exactly the same effect.*

Proof. This follows directly from (2.17), (2.18), and (2.19). ∎

Corollary 2.1. (Hiring - firing policy equivalence 2) *Let $r \to 0$. Then $F = H$ has no effect on the equilibrium outcome.*

Proof. This follows directly from (2.17), (2.18), and (2.19). ∎

Proposition 2.3. (The $H = F$ scheme) *Let $r > 0$. Then $F = H$ has no effect on the job creation curve but shifts the job destruction curve outward, leading to more job creation and acceptance, less job destruction and consequently reduced unemployment.*

Proof. This follows directly from (2.17), (2.18), and (2.19). ∎

The positive effect can be explained as follows. A $F = H$ scheme can be compared to an interest free loan to the firm, as it gets H at the beginning of a job and eventually pays back the same amount without user costs. The gain is therefore reflected in the rF-term in the job destruction condition (2.18).

A recruitment subsidy affects both curves. Both shift outward leading to an increase in labor market tightness, but the JC curve moves more strongly implying more job destruction. In contrast to H and F the job acceptance margin is not additionally distorted, hence $\underline{z} = \hat{\underline{z}}$. Compared to a hiring subsidy which is only paid if a job is created, a recruitment subsidy is received by the firm irrespective of whether a match occurs or not. The main consequence is that a hiring subsidy will partly go to the worker, while

appendix section F.1 for details.

the latter subsidy is already sunk in the wage bargaining. All effects are summarized in table 2.1. Uncompensated changes in the policy instruments naturally do not reveal implications about their costs, and hence the relative effectiveness. In the next section we will therefore close the government's budget constraint using only distortive ways of revenue generation. The social planner's solution will serve as a benchmark to assess efficiency.

Table 2.1: Summary of the policy effects on equilibrium variables

	θ	$\hat{\underline{z}}$	$\underline{z}^*$	u
μ	$-$	$+$	$=$	$+$
R	$+$	$+$	$=$	?
D	$+$	$-$	$=$	$-$
H	$+$	$+$	$<$	?
F	$-$	$-$	$>$	?
τ	$-$	?[13]	$=$	?[13]
t	$-$	$+$	$=$	$+$

* This column gives the effect on $\underline{z}$ in relation to $\hat{\underline{z}}$.

2.2 Efficiency, welfare and the optimal policy mix

Efficiency can be distorted in many ways. We will consider two possibilities: first, the typical search externality that comes from the way workers and firms are matched. Second and more important for our analysis, we will consider a firing externality in the spirit of BT stemming from the requirement to finance unemployment benefits in a distortionary way. In order to analyze these inefficiencies we compute the solution to the social planner's problem, which is given by the following three reduced equations for socially optimal job creation, job destruction, and job acceptance[14]:

$$(1-\eta)\frac{z^e - \hat{\underline{z}}}{\pi^n + r} - \frac{C}{q^f} = 0 \tag{2.21}$$

$$\hat{\underline{z}} + \frac{\pi^n}{\pi^n + r}\int_{\hat{\underline{z}}}^{\infty} (\tilde{z} - \hat{\underline{z}})\, dG(\tilde{z}) - h - \frac{\eta}{1-\eta} C\theta = 0 \tag{2.22}$$

[13] The sufficient and necessary condition for $\hat{\underline{z}}$ to increase (assuming $F = H$ and $\omega = \eta$ for simplicity) is: $[\pi^n + r + q^w G(\underline{z})]\, \underline{z} > [q(\theta) - \pi^n]\,(r + \pi^n)\Gamma$. Hence, it is also sufficient to raise u.

[14] See appendix section D for derivation.

$$\underline{z} = \hat{\underline{z}} \tag{2.23}$$

Comparing those relations with the decentralized equilibrium equations (2.17)-(2.19) in a policy free world, i.e. $b = F = \tau = t = D = H = R = 0$, reveals that they coincide if and only if $\omega = \eta$ (Hosios (1990)). From now on we will follow the Ramsey approach and assume that unemployment compensation $b > 0$ is exogenously given[15], reflecting the assumption that institutions are rigid (even in the long run), and have to be financed with the least possible distortions using our instruments, except a consumption tax. Can the first-best allocation still be implemented? Subtracting (2.21)-(2.23) from (2.17)-(2.19) gives the conditions that the policies in question have to fulfill to restore efficiency.

$$\frac{z^e - \hat{\underline{z}}}{\pi^n + r}\left[(1-\omega)(1-\tau) - (1-\eta)\right] + \frac{R}{q^f} = (1-\omega)(H - F) \tag{2.24}$$

$$-\tau\left(\hat{\underline{z}} + \pi^n \Gamma\right) - \frac{b + th}{1-t} + D + rF - C\theta\left[\frac{\omega}{1-\omega} - \frac{\eta}{1-\eta}\right] + \frac{\omega}{1-\omega} R\theta = 0 \tag{2.25}$$

$$F = H \tag{2.26}$$

where $\Gamma \equiv \frac{1}{r+\pi^n}\int_{\hat{\underline{z}}}^{\infty}\left(\tilde{z} - \hat{\underline{z}}\right) dG(\tilde{z})$. In addition, the government's budget constraint[16] must be met without consumption taxation, i.e. $T = 0$ in:

$$-TL = (L-u)\bar{w}t + (L-u)\bar{z}\tau + q^w u(F - H) - \theta u R - (L-u)D - ub \tag{2.27}$$

In what follows we characterize two alternative implementations of the first-best solution, one involving hiring and the other using wage subsidies. We depict the limitations to both schemes.

Let us first assume that the search externalities do not distort the equilibrium, i.e. $\omega = \eta$. Inserting (2.26) in (2.24) reveals that output taxation and recruiting cost subsidization are not required for efficiency, hence: $\tau = R = 0$.

[15] This can be motivated by a welfare optimizing government that sets $b > 0$ in case of workers' risk-aversion that is not explicitly modeled here.

[16] Note that in equilibrium the number of outflows $\pi^n G(\underline{z})(L-u)$ is equal to the inflows $q^w u$. Hence, $F = H$ is budget neutral in equilibrium. One should keep in mind that the introduction of a $F = H$ scheme shifts in the JD-curve leading to more outflow out of and less inflow into unemployment. Hence, during transition the outlay on H will exceed the revenue generated by F. For simplicity, we will assume that these costs of transition are financed using a non-distortionary consumption tax.

Unemployment benefits then have to be financed via the wage tax $t = \frac{b}{\bar{w}}\frac{u}{L-u} > 0$, which is chosen to fulfill (2.27). As a compensated firing tax, $F = H$, is budget neutral, we can set F in order to fulfill (2.25), hence $F = \frac{b+th}{(1-t)r} > 0$.

Proposition 2.4. *In case of unemployment compensation $b > 0$ and $\omega = \eta$ it is possible to implement the socially optimal allocation and balance the budget using a wage tax, $t > 0$, a firing tax and a hiring subsidy, $F = H > 0$.*

Observe the difference compared to BT. In their framework the optimal policy consists of zero wage taxes and a firing tax to finance unemployment benefits and offset the involved distortions. Here, a firing tax will distort the acceptance margin unless a firing tax is fully compensated by a hiring subsidy. As both instruments together are budget neutral a firing tax cannot be used for financing unemployment compensation. Instead of the redistribution from the firms to the workers as in BT, we require redistribution from employed to unemployed workers and from firing to hiring firms.

Now consider the case where $\omega \neq \eta$. Observe that at least one of the two policy instruments τ or R, is needed to satisfy equation (2.24). First we focus on output taxation, hence setting $R = 0$. The efficient output tax rate is then given by $\tau = 1 - \frac{1-\eta}{1-\omega}$ which is smaller than zero i.e. a subsidy if $\omega > \eta$ and positive if $\omega < \eta$. Therefore, the budget-solving wage tax rate will be higher ($\omega > \eta$) or smaller ($\omega < \eta$) compared to the benchmark tax rate where the Hosios condition holds. Again F is set to fulfill (2.25) and therefore the implementation of the first-best solution is complete. Note that the case $F < 0$ cannot be ruled out now. Instead of τ one could alternatively use $R = \frac{z^e - \hat{z}}{\pi^n + r}(\omega - \eta)q^f$ by the same argument.

Proposition 2.5. *In case of unemployment compensation $b > 0$ and $\omega \neq \eta$ it is possible to implement the social optimal allocation and balance budget using a wage tax t, a firing tax and a hiring subsidy, $F = H$, and at least one of the following two instruments: output tax (τ) and recruitment subsidy (R).*

MP do not explicitly consider the case of $\omega \neq \eta$ but it is easy to see that their job creation curve can be moved to optimum just by adjusting $H \neq F$ accordingly. In our case this is not possible as $H = F$ is always required to offset the distortions on the job acceptance margin. Hence, the job creation curve can only be shifted by additional instruments, such as output taxation τ or a recruitment subsidy R.

The above implementations might require the firing tax to be of considerable magnitude. This will certainly be an issue when firms are liquidity constrained, e.g. $F \leq F_{max}$ (see BT) which will eventually prevent the implementation of the social planner's optimum. This becomes even more severe in the following extension. One can assume that F only partly improves government's budget, say by F_{tax} as a fixed part $F_{cost} = F - F_{tax}$ reflects sunk firing costs, e.g. the administrative costs of a lay-off, etc. Obviously, $F = H$ is no longer budget neutral, implying that the wage tax t has to rise to close the budget constraint and $F = H$ have to be even higher to undo the additional distortion of the increased wage tax. Hence, it is more likely to hit F_{max}.

Note that a wage subsidy D is not required for achieving efficiency but provides an alternative implementation. For simplicity assume again that $\omega = \eta$ and that we set $F = H = \tau = R = 0$. The wage subsidy D, in addition to unemployment compensation, is financed via a wage tax t, ergo $D = \bar{w}t - \frac{u}{L-u}b$. The job destruction curve will coincide with its social optimal counterpart if and only if $\frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{L-u}b = \bar{w}t$. For $h > 0$ and $u > 0$ we can derive a necessary condition for the replacement ratio, namely $\frac{b}{\bar{w}} < \frac{1}{4}$, a condition hardly met in any OECD economy. The contrapositive reads:

Proposition 2.6. *In case of unemployment compensation $b > 0$ and $\omega = \eta$ it is **not** possible to implement the social optimal allocation and balance budget using only a wage tax t and a wage subsidy D, if the replacement ratio is higher than 25%.*

Proof. $\frac{b}{1-t} + \frac{ht}{1-t} + \frac{u}{1-u}b = \bar{w}t \overset{h>0 \wedge u>0}{\Longrightarrow} \frac{b}{1-t} < \bar{w}t \Rightarrow \frac{b}{\bar{w}} < t(1-t) \Rightarrow \frac{b}{\bar{w}} < \max_t t(1-t) \Leftrightarrow \frac{b}{\bar{w}} < \frac{1}{4}$ ∎

3 An intergroup model with economic turbulence

So far, we focused on intragroup redistribution. Allowing for intergroup redistribution enriches the model considerably because it enables us to evaluate more realistic policies. This comes at the cost of losing analytic tractability as we have to partly rely on numerical computations. In such a simulation MP find that a wage subsidy targeted at low-skilled and financed by high-skilled workers works quite well in bringing down overall unemployment[17]. Besides the connection via the government's budget constraint, they assume the two skill classes to operate in complete juxtaposition. The issue that "targeting is likely to damage the quality and quantity of labor supply" (Bovenberg et al. (2000)) is therefore hardly addressed. The aim of this section is to show how the optimal policy mix is altered by the presence of economic turbulence and we find that a scheme as proposed by MP might be considerably less effective in such an environment. The idea that increased economic turbulence affects labor market outcomes is related to Ljungqvist and Sargent (1998), who assume that unemployed workers lose their skills in the course of time[18] as they can not keep up to date with new production technologies[19]. In a broader interpretation, these new production techniques and requirements emerge as a result of ongoing restructuring from manufacturing to services, spread of new information technologies, internationalization of production, etc. which all lead to expeditious changes in the economic environment, and render previous ways of production obsolete. Hence, a worker who is only familiar with those outdated techniques is less productive when confronted with state-of-the-art production technology.

It will be shown that subsidizing low-skilled workers in the presence of economic turbulence amplifies the discouraging effect for high-skilled workers beyond the mere consequence of coping with a higher tax rate. The dynamic equilibrium model we will employ for this exercise is designed to fit German labor market data. The key differences compared to the simple intragroup model described above follow from the introduction of a second skill class with the property that the productivity distribution function of the high-

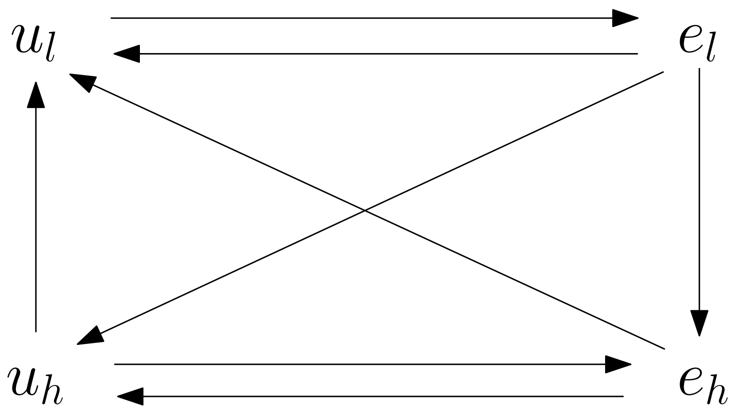
[17]For the 'European' calibration they find that a 20% wage subsidy decreases low-skilled unemployment from 16.2% to 7.6% while the unemployment rate of high-skilled workers rises from 4.5% to only 4.9%.

[18]Empirical evidence for skill loss upon separation or during unemployment, which is often approximated by the difference btween the old wage and the reemployment wage, is widely documented. See for example Fallick (1996) for the U.S. and Burda and Mertens (2001) for Germany.

[19]This strand of the literature was initially concerned with persistence in unemployment, see Pissarides (1992), Ljungqvist and Sargent (1998, 2004), and Den Haan et al. (2005).

skilled (h) first-order stochastically dominates the cdf of the low-skilled (l), i.e. $G_h(z) \leq G_l(z)\ \forall z$. For simplicity we assume that the skill of a specific worker can be observed by firms or the government at any time. Introducing economic turbulence is modeled as follows. High-skilled workers lose their skills conditional on job loss and during unemployment with probability π^l, which means that they can only draw from $G_l(\cdot)$ when they are matched again. Low-skilled workers, on the other hand, receive a skill upgrade during employment, reflecting 'learning-on-the-job', with probability π^h, which allows them to draw a new productivity from $G_h(\cdot)$ instead of $G_l(\cdot)$. Hence, the skill distribution is endogenous. An individual can be in 4 different states, employed with high or low skills and unemployed with high or low skills, where we assume that total labor force is normalized to 1, hence: $e_l + u_l + e_h + u_h = L_l + L_h = 1$. Transitions between these states are illustrated by figure 3.1 and are formally reported in appendix section A.1. Note that in addition we now allow for exogenous, productivity unrelated, separation at a rate π^x, which does not provide additional analytic insight, but is important to quantitatively match the model to the data. Besides the productivity distribution we allow high

Figure 3.1: The transition flows



and low-skilled workers to differ in other dimensions, like the matching technologies, as well. Differences are indicated by the subscript $j \in \{h, l\}$. All the assumptions of the intragroup model still apply unless stated otherwise. Hence, the models are nested, i.e. the intragroup model is a special case of the intergroup model with $\pi^h = \pi^l = 0$ and dropped skill indices. The asset value of unemployment for the low-skilled is the same as before, while high-skilled workers lose U_h in case they are not matched with probability π^l

and only get U_l instead.

$$rU_l = \mu_l - T + q_l^w \left(W_l^e - U_l\right) \tag{3.1}$$

$$rU_h = \mu_h - T + q_h^w \left(W_h^e - U_h\right) + (1 - q_h^w)\pi^l \left(U_l - U_h\right) \tag{3.2}$$

The value of working for high-skilled workers only differs by the outside value $\bar{U} \equiv \pi^l U_l + (1 - \pi^l)U_h$ and the additional term reflecting the possibility of an exogenous separation.

$$\begin{aligned} rW_h(z) =& (1 - t_h)w_h(z) - T + \pi^x \left[\bar{U} - W_h(z)\right] \\ &+ \pi^n \left[(1 - G_h(\underline{\hat{z}}_h))\hat{W}_h^{\hat{e}} + G_h(\underline{\hat{z}}_h)\bar{U} - W_h(z)\right] \end{aligned} \tag{3.3}$$

The value of working as a low-skilled is constructed in a similar way with the additional possibility of a skill upgrade, which is reflected in the last line of (3.4).

$$\begin{aligned} rW_l(z) =& (1 - t_l)w_l(z) - T + \pi^x \left[U_l - W_l(z)\right] \\ &+ \pi^n \left[(1 - G_l(\underline{\hat{z}}_l))\hat{W}_l^{\hat{e}} + G_l(\underline{\hat{z}}_l)U_l - W_l(z)\right] \\ &+ \pi^h \left[(1 - G_h(\underline{\hat{z}}_h))\hat{W}_h^{\hat{e}} + G_h(\underline{\hat{z}}_h)\bar{U} - W_l(z)\right] \end{aligned} \tag{3.4}$$

Note that the inside asset values ($\hat{W}_h(z)$ and $\hat{W}_l(z)$) are set up analogously and are not reported in the text but only in the appendix section B for the sake of completeness. We turn to the firms' side. As the skill of the workers can be perfectly observed, firms are able to discriminate and specifically post a vacancy for high or low-skilled workers. A firm will enter the labor market that generates higher returns. We further assume that it can reassess this decision every period. Let us therefore define $V^m \equiv \max\{V_h, V_l\}$. The values of posting vacancies in the high and the low-skilled market, respectively, are given as:

$$rV_h = -c_h + q_h^f \left(J_h^e + H_h - V_h\right) + (1 - q_h^f)\left(V^m - V_h\right), \text{ with } c_h \equiv C_h - R_h \tag{3.5}$$

$$rV_l = -c_l + q_l^f \left(J_l^e + H_l - V_l\right) + (1 - q_l^f)\left(V^m - V_l\right), \text{ with } c_l \equiv C_l - R_l \tag{3.6}$$

Employing a high-skilled worker yields a per period return of rJ_h similar to before, while J_l again accounts for the possibility of a skill upgrade.

$$\begin{aligned} rJ_h(z) =& (1-\tau_h)z - w_h(z) + D_h + \pi^x\left[(V^m - F_h) - J_h(z)\right] \\ &+\pi^n\left[(1-G_h(\underline{\hat{z}}_h))\hat{J}_h^{\hat{e}} + G_h(\underline{\hat{z}}_h)(V^m - F_h) - J_h(z)\right] \end{aligned} \tag{3.7}$$

$$\begin{aligned} rJ_l(z) =& (1-\tau_l)z - w_l(z) + D_l + \pi^x\left[(V^m - F_l) - J_l(z)\right] \\ &+\pi^n\left[(1-G_l(\underline{\hat{z}}_l))\hat{J}_l^{\hat{e}} + G_l(\underline{\hat{z}}_l)(V^m - F_l) - J_l(z)\right] \\ &+\pi^h\left[(1-G_h(\underline{\hat{z}}_h))\hat{J}_h^{\hat{e}} + G_h(\underline{\hat{z}}_h)(V^m - F_h) - J_l(z)\right] \end{aligned} \tag{3.8}$$

Wages are again determined by Nash bargaining. Let us first define $\tilde{r} = r + \pi^l$, $r_h = r + \pi^x + \pi^n$,and $r_l = r_h + \pi^h$. Inside wages for both skill groups are then given by[20]

$$\begin{aligned} \hat{w}_h(z) &= \frac{1-\omega}{1-t_h}\left(\frac{(1-\pi^l)r}{\tilde{r}}\mu_h + \frac{\pi^l(1+r)}{\tilde{r}}\mu_l\right) + \omega\left[(1-\tau_h)z + D_h + rF_h\right] \\ &+ \omega\left[\frac{(1-\pi^l)r}{\tilde{r}}c_h\theta_h + \frac{1-t_l}{1-t_h}\frac{\pi^l(1+r)}{\tilde{r}}c_l\theta_l\right] \end{aligned} \tag{3.9}$$

$$\begin{aligned} \hat{w}_l(z) &= -\frac{1-\omega}{1-t_l}\pi^h(1-\pi^l)\left[\frac{\mu_h-\mu_l}{\tilde{r}} + \frac{\omega}{1-\omega}\frac{(1-t_h)c_h\theta_h - (1-t_l)c_l\theta_l}{\tilde{r}}\right] \\ &+ \omega\left[(1-\tau_l)z + D_l - \pi^h(F_h - F_l) + rF_l + c_l\theta_l\right] + \frac{1-\omega}{1-t_l}\mu_l \\ &+ \omega\pi^h(1-G_h(\underline{\hat{z}}_h))\frac{t_h - t_l}{1-t_l}\left[\frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h\Sigma\right] \end{aligned} \tag{3.10}$$

The outside wages are simply $w_j(z) = \hat{w}_j(z) - r_j\omega(F_j - H_j)$. Wages now do not only depend on their 'own' endogenous variables but also on those of the other skill group. Observe how $\pi^h = \pi^l = 0$, as well as $\pi^x = 0$, because we abstracted from exogenous separation before, reduces the wage equations to the intragroup expressions (2.14) and (2.15). It is most important to understand how the variables of the other skill class influence wages. Note the difference in the direct spillover effects. While an increase in θ_h or μ_h reduces $\hat{w}_l$, $\hat{w}_h$ is inflated if θ_l or μ_l rise, i.e. $\frac{\partial \hat{w}_h}{\partial \theta_l}, \frac{\partial \hat{w}_h}{\partial \mu_l}, -\frac{\partial \hat{w}_l}{\partial \theta_h}, -\frac{\partial \hat{w}_l}{\partial \mu_h} > 0$. High-skilled workers can bargain a higher wage as their fall back option, which includes

[20]See appendix section B for a detailed derivation and the definition of Σ and $\widetilde{\widetilde{\omega}}_h$. Note that $\Sigma = 0$ if $F_h = H_h$.

that they eventually become low-skilled, increases. By contrast, low-skilled workers will bargain a lower wage because working as a low-skilled includes the increased option value of becoming high-skilled. We will now characterize the equilibrium.

3.1 Equilibrium

The equilibrium vector $\langle u_h, u_l, e_h, e_l, \theta_h, \theta_l, \underline{z}_h, \underline{z}_l, \hat{\underline{z}}_h, \hat{\underline{z}}_l \rangle$ is pinned down by equations (3.11)-(3.16) and the steady state flow equations (A.1). In comparison to the intragroup model, the job creation conditions hardly change

$$JC_h \quad : \quad (1-\omega)\left(\frac{(z_h^e - \hat{\underline{z}}_h)(1-\tau_h)}{r_h} - F_h + H_h\right) = \frac{c_h}{q_h^f} \tag{3.11}$$

$$JC_l \quad : \quad (1-\omega)\left(\frac{(z_l^e - \hat{\underline{z}}_l)(1-\tau_l)}{r_l} - F_l + H_l\right) = \frac{c_l}{q_l^f} \tag{3.12}$$

The job destruction conditions are now more involved. After defining $\Gamma_j \equiv \frac{1}{r_j}\int_{\hat{\underline{z}}_j}^{\infty}(\tilde{z} - \hat{\underline{z}}_j)\, dG_j(\tilde{z})$, they read

$$\begin{aligned} JD_h \quad : \quad & (1-\tau_h)\hat{\underline{z}}_h - \hat{w}_h(\hat{\underline{z}}_h) + D_h + rF_h \\ & +(1-\omega)\pi^n(1-\tau_h)\Gamma_h = 0 \end{aligned} \tag{3.13}$$

$$\begin{aligned} JD_l \quad : \quad & (1-\tau_l)\hat{\underline{z}}_l - \hat{w}_l(\hat{\underline{z}}_l) + D_l + rF_l - \pi^h(F_h - F_l) \\ & +(1-\omega)\pi^n(1-\tau_l)\Gamma_l + (1-\omega)\pi^h(1-\tau_h)\Gamma_h = 0 \end{aligned} \tag{3.14}$$

The relationship of the cut-off productivities is given by

$$\underline{z}_h = \hat{\underline{z}}_h + \frac{r_h}{1-\tau_h}(F_h - H_h) \tag{3.15}$$

$$\underline{z}_l = \hat{\underline{z}}_l + \frac{r_l}{1-\tau_l}(F_l - H_l) \tag{3.16}$$

Analogously to before equilibrium is partly recursive. After inserting (3.15)-(3.16) in (3.11)-(3.12) in order to compute z_j^e, one can solve the remaining JD_j-JC_j system of 4 equations for θ_h, θ_l, $\hat{\underline{z}}_h$, and $\hat{\underline{z}}_l$. Knowing θ_j, $\hat{\underline{z}}_j$, and $\underline{z}_j$ enables us to solve for e_j and u_j using (A.1). We will skip the exercise of analyzing uncompensated changes in policy instruments, also because convenient comparative statics in a two dimensional diagram are not possible anymore, and turn directly to efficiency issues.

3.2 Efficiency, welfare and the optimal policy mix

As before we start out by computing the solution to the social planner's problem, which is documented in appendix section E. Again, efficiency in a policy-free world is guaranteed if and only if $\omega = \eta$. Hence, the Hosios (1990)-condition generalizes to the complex intergroup model. We use the same Ramsey approach as before, i.e. b_h and b_l are exogenously given and have to be financed with the least possible distortion. We do not allow for non-distortive consumption taxes. As the implementation of the first-best should be feasible we are bound to the following budget constraint that allows for intergroup redistribution

$$0 = GB_h + GB_l \tag{3.17}$$

$$\begin{aligned} GB_h =& e_h \left[\bar{w}_h t_h + \bar{z}_h \tau_h - D_h\right] - u_h b_h - \theta_h u_h R_h \\ & -q_h^w u_h H_h + (e_h \pi^n + e_l \pi^h) G_h(\hat{\underline{z}}_h) F_h + e_h \pi^x F_h \end{aligned} \tag{3.18}$$

$$\begin{aligned} GB_l =& e_l \left[\bar{w}_l t_l + \bar{z}_l \tau_l - D_l\right] - u_l b_l - \theta_l u_l R_l \\ & -q_l^w u_l H_l + e_l \pi^n G_l(\hat{\underline{z}}_l) F_l + e_l \pi^x F_l \end{aligned} \tag{3.19}$$

Many insights from the intragroup model generalize to the extended model. First, $F_j = H_j$ is a necessary condition for efficiency. Second, if $\omega = \eta$ we do not require output taxation τ_j or a recruiting subsidy R_j. If $\omega \neq \eta$ we need at least one of those instruments. These are important guidelines for finding an implementation of the first-best for the complex intergroup model which is a non-trivial task because of several complications. First, even $b_j > 0$ and $b_{i \neq j} = 0$ requires both wage tax rates to be non-zero. Second, a $F_j = H_j$-scheme is not completely budget neutral anymore. Hence, a first-best implementation using the $F_j = H_j$-scheme is only possible if the adjustments in t_j required for the budget constraint to hold can be compensated by the increase in F_j in the job destruction conditions.

3.3 Simulation

Although the theoretical treatment has given us a lot of insights, in order to get a feeling for magnitude and to resolve some ambiguities we will perform some numerical simulations[21]. The first task is to find a reasonable calibration for the model to fit German labor market characteristics. We specify the functional forms of $q_j(\cdot)$ and $G_j(\cdot)$ following MP, Den Haan et al. (2005), or Ljungqvist and Sargent (1998, 2004)

$$q_j(\theta_j) = A_j \theta_j^{-\eta} \tag{3.20}$$

and a uniform distribution on the interval $[\underline{x}, \bar{x}]$

$$G_j(x) = \frac{x - \underline{x}_j}{\bar{x}_j - \underline{x}_j} \tag{3.21}$$

A period is chosen to be a month. Targeting an interest rate of 5 % p.a. results in $r = 0.0041$ or $\beta = 0.9959$. Nashbargaining is chosen to be symmetric as done by many authors[22]. Estimates for the elasticity of the matching function vary between 0.45 (Fahr and Sunde (2001)) and 0.7 (Burda and Wyplosz (1994)). For simplicity, we abstract from inefficiencies generated by search externalities for the moment. Hence, we set $\eta = 0.5$ in order to fulfill the Hosios (1990)-condition. The expectations of the two productivity distributions were chosen to be $E(z_l) = 1$ and $E(z_h) = 1.35$. As data for per worker productivity broken down into skill classes is not available, gross wage was used as a proxy by assuming that productivities and wages are proportional. During 2002 - 2006 a white collar worker earned approximately 1.35 as much as a blue collar worker (Statistisches Bundesamt (2007)). Variances were set such that the model's wage predictions result in a wage ratio of approximately 1 : 1.35. This implies $Var(z_l) = \frac{1^2}{12}$ and $Var(z_h) = \frac{1.3^2}{12}$. A crucial choice is the value of leisure μ_j. We use the following approach to disentangle home production h and replacement income b_j. As mentioned earlier we impose linearity in the value of leisure, hence $\mu_j = h + b_j$. We further assume that there is no skill specific difference in the value of home

[21] The simulations were performed using MATLAB. The code is available on my website `http://sites.google.com/site/schusterphilip/`.

[22] See Hall and Milgrom (2008) for an additional motivation of setting $\omega = 0.5$.

production. In line with the results of the OECD tax-benefit calculator we target replacement ratios of $\frac{b_h}{w_h} = 0.6$ and $\frac{b_l}{w_l} = 0.65$. Costain and Reiter (2008) estimate the semi-elasticity of unemployment with respect to benefits $\frac{d\ln u}{db} \approx 2$. Given the linearity assumption we have $\frac{d\ln u}{db} = \frac{d\ln u}{dh}$. Based on that we set a common $h = 0.25$ such that we come close to the Costain and Reiter (2008)-target, $\frac{d\ln u}{dh} = 2.2$. This implies total replacement ratios of $\frac{\mu_h}{w_h} = 0.77$ and $\frac{\mu_l}{w_l} = 0.87$. Hence, our calibration addresses the argument of Hagedorn and Manovskii (2008) that the value of non-work is substantially high, but at the same time produces a realistic responsiveness of unemployment to changes in benefits[23]. In order to finance the expenditure on b_h and b_l we set $t_h = 0.065$ and $t_l = 0.05$, which reflects progression in the existing tax system.

The transition probabilities π^l and π^h are chosen in order to replicate the empirical skill distribution. We use the following targets based on the publicly available statistics provided by Bundesagentur für Arbeit (2008). Among the unemployed the ratio of blue to white collar workers is approximately 60 %, hence $\frac{u_l}{u} \approx 0.6$. We further target $\frac{e_l}{e} \approx 0.2$, where the low-skilled are measured as workers with no professional education and apprentices. Given an unemployment rate of $u = 0.1$, this gives a skill composition of the labor force of $\frac{L_l}{L} \approx 0.25$. We set $\pi^h = 0.01$, which implies that it takes on average 8 years and 4 months to become high-skilled, conditional on no job loss. A skill loss occurs after 1 year and 10 months on average, i.e. $\pi^l = 0.05$. Those values are in line with the choices of Den Haan et al. (2005) and Ljungqvist and Sargent (2004). These papers and MP also inspire the choice for the rate at which new shocks arrive, i.e. $\pi^n = 0.02$. As we do not interprete the average duration of a vacancy or the number of vacancies but just target the duration of unemployment we are free to choose $C_h = 1.509$ and $C_l = 0.274$ in order to normalize $\theta_h = \theta_l = 1$[24]. The probability of an exogenous split $\pi^x = 0.00668$ and the scaling factors[25] $A_h = 0.563$ and $A_l = 0.148$ are set to replicate an

[23]In terms of productivity, $\frac{\mu_h}{av.prod_h} = 0.717$ and $\frac{\mu_l}{av.prod_l} = 0.814$ our calibration is also close to the corresponding value of 0.71 derived by Hall and Milgrom (2008) for the U.S. using a completely different calibration approach relying on estimates of the Frisch elasticity.

[24]This normalization is more thoroughly described in Shimer (2005).

[25]The chosen ratio of A_h to A_l is admittedly a little bit arbitrary but could be fixed if average duration of unemployment is known for each skill class seperately. For the time

unemployment rate of $u = 0.1$ and average duration of unemployment of 9 months (long term averages for 1998-2007, Bundesagentur für Arbeit (2008)).

Table G.1 summarizes the calibration choices and results for the decentralized economy, which serves as our benchmark.

Insert table G.1 here.

In contrast, table G.2 shows the results of the social optimum. Unemployment is at 4 % compared to 10 %, while average duration of unemployment should optimally be 3 months instead of 9. Comparing the endogenous decision variables we observe two things. First, reservation productivities for accepting and destructing jobs are inefficiently high, especially for the low-skilled who reject almost every second offer instead of one out of four which would be optimal. Second, job creation is inefficiently low. Again, this is more servere for low-skilled workers where market tightness is about one forth of what it should be.

Insert table G.2 here.

first-best implementation

Let us now address possible implementations of the social optimum. We have learnt from the previous sections that in case of $\omega = \eta$ we do not require output taxation τ_j or a recruiting subsidy R_j. Further, given proposition 2.6 for the intragroup model and the high empirical replacement ratios an implementation relying on wage subsidies does not seem to be very promising. Hence, we try to implement the corresponding intergroup variant of the policy scheme suggested in proposition 2.4. We proceed as follows. First we set $H_j = F_j$. As $\omega = \eta$ the job creation conditions coincide with their social optimal counterparts. Given $H_j = F_j$ and b_j we can now compute the tax rates t_j that satisfy the two optimal job destruction conditions simultaneously. All the possible pairs of $H_h = F_h$ and $H_l = F_l$ that satisfy the budget constraint, i.e. set the budget surplus to 0, represent an implementation of the first-best. Figure G.1 illustrates these social optimal combinations.

Insert figure G.1 here.

being average duration of unemployment was set to 3 and a half months for high-skilled and a little bit more than 1 year for low-skilled.

Moving along the optimal isoline does not only change the combination of $H_h = F_h$ and $H_l = F_l$ but also the corresponding optimal tax rates as shown by table G.3. The higher $H_h = F_h$ the higher t_h has to be compared to t_l.

INSERT TABLE G.3 HERE.

The striking result is that such schemes involve tremendously high firing taxes and hiring subsidies. To get a feeling for magnitude: the lowest possible value for $H_l = F_l$ is still more than 100 times larger than the monthly wage of a low-skilled in our benchmark case.

Cross-financed wage subsidy schemes

In this section we argue that a wage subsidy scheme for low-skilled financed by high-skilled as suggested by MP is considerably less suitable to reduce unemployment when economic turbulence is taken into account. To have a reference point we first replicate the MP result in our model when turbulence is switched off, i.e. $\pi^h = \pi^l$. Hence, the skill composition of the labor force is not endogenous anymore but exogenously fixed, i.e. $L_h = 0.7398$. To replicate our targets for unemployment, its duration and composition we have to recalibrate some of the remaining transition probabilities[26]. We then rerun the MP experiment by increasing D_l stepwise from 0 to 0.5. This is done in an uncompensated way and also if financed by the high-skilled workers. Table G.4 summarizes the results.

INSERT TABLE G.4 HERE.

As in MP a low wage subsidy scheme seems to be very effective in reducing overall unemployment, which can be brought down to 7.02 % for $D_l = 0.66$ in the tax compensated scenario. However, when we take economic turbulence into account, the results reverse. It is striking that even in the uncompensated case, i.e. the subsidy given away for free, total unemployment will increase with D_l. Two effects, one boosting u_h and the other dampening the reduction in u_l come into play. In a first direct effect a rise in D_l increases the value of working as low-skilled (W_l) and consequently the value of being

[26] In detail, $A_h = 2.23$, $A_l = 0.28$, $\pi^n = 0.01$, $\pi_h^x = 0.0065$, and $\pi_l^x = 0.0185$. In addition, as wages slightly differ we have to set the tax rates (keeping the relative ratio constant) to $t_h = 0.054$ and $t_l = 0.046$. Again, we choose $C_h = 0.571$ and $C_l = 0.178$ in order to normalize $\theta_h = \theta_l = 1$.

unemployed (U_l) with low skills. That is where the mechanism stops in the non-turbulence framework. In our case additional indirect effects start to work. As U_l increases so does the fall back option of the high-skilled workers ($\bar{U}$), which will raise the reservation productivities, inflate wages and therefore reduce vacancy creation for high-skilled workers. Consequently, u_h has to rise. In a third round, as the value of being high-skilled drops this feeds back in a negative way to the low-skilled workers as the motive of accepting a low wage job in order to eventually become high-skilled diminishes. A direct consequence is that the skill composition in the labor force in shifted towards low-skilled workers. This is the reason why such a scheme can lead to a break down of the equilbrium even for small values of D_l if the subsidy is financed through t_h as the high-skill tax base shrinks. In conclusion, a low wage subsidy is good to increase low-skill employment, but is less effective in reducing low-skill unemployment, let alone total unemployment.

The implementation of the first-best was described above and although it is unlikely that such an implementation is feasible if firms are liquidity constrained to some extent, the lesson to be learnt is the following. Instead of a redistribution from high to low-skilled workers (MP) or from firing firms to unemployed workers (BT) the results suggest that an effective policy should involve redistribution from firing to hiring firms. In a broader sense this idea translates also to policy scenarios where not welfare but unemployment is of concern. Consider - just as an example - the following scheme, where we use R instead of H. We set firing taxes uniformly to $F_h = F_l = 10$, which amounts to approximately 7 months of average wage. The revenue generated from that is used to finance b_h and b_l (i.e. $t_h = t_l = 0$) and the 'leftovers' are spent on recruitment subsidies, $R_h = R_l = 0.24$. Such a scheme, which also represents some form of redistribution from firing to hiring firms, while leaving welfare practically unchanged, reduces overall unemployment to 6.3% and average duration of unemployment to 5.7 months. The reduction is mostly due to the drop in low-skilled unemployment, which more than halves in absolut numbers. In contrast to the MP-scheme the suggested policy does not subsidize low-skill employment and hence does not imply an impairment of the skill-composition.

4 Conclusion

A dynamic model of equilibrium unemployment and bilateral wage bargaining was used to analyze welfare and employment effects of different labor market policy instruments. We apply a Ramsey approach and try to find a solution to the problem of financing exogenously fixed unemployment benefits using a given set of instruments with the least possible distortions. In their static framework, Blanchard and Tirole (2008) derive an optimal policy, where benefits should be completely financed through firing taxes. This idea does not completely translate to our dynamic set up. In any case a firing tax has to be compensated one-for-one by a hiring subsidy to prevent distortions along the job acceptance margin. This redistribution from firing to hiring firms works like an interest free loan when looking at the life cycle of a firm-worker match. The saved expenditures on user costs can be used to undo the distortions from a wage tax, which is used to finance unemployment benefits, along the job destruction margin. In an extension we consider a model of two skill classes and the possibility of intragroup redistribution. Mortensen and Pissarides (2003) argue that a policy consisting of wage subsidies for low-skilled workers which are cross-financed by high-skilled workers can considerably reduce unemployment. We find that this result is completely overthrown if we add economic turbulence, modeled as state-dependent transitions between skill classes in the spirit of Ljungqvist and Sargent (1998), to our framework. The discouraging effect for high-skilled workers is amplified beyond the mere consequence of coping with a higher tax rate, as their fall back option in which they eventually lose their skills, increases which leads to a deterioration of the skill-composition. Again we identify a firing tax - hiring subsidy scheme to be optimal. In conclusion, the paper argues that instead of redistribution from high to low-skilled workers (Mortensen and Pissarides (2003)) or from firing firms to unemployed workers (Blanchard and Tirole (2008)), a scheme involving redistribution from firing to hiring firms should be preferred.

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Appendix

A Laws of motion

$$
\begin{aligned}
\dot{u}_h =& e_h \left[\pi^x + \pi^n G_h(\underline{\hat{z}}_h)\right](1-\pi^l) + e_l \pi^h G_h(\underline{\hat{z}}_h)(1-\pi^l) \\
& - u_h \left[(1-q_h^w)\pi^l + q_h^w\right] \\
\dot{e}_l =& (1-e_h-e_l-u_h)q_l^w - e_l \left[\pi^x + \pi^n G_l(\underline{\hat{z}}_l) + \pi^h\right] \\
\dot{e}_h =& e_l \pi^h (1 - G_h(\underline{\hat{z}}_h)) + u_h q_h^w - e_h \left[\pi^x + \pi^n G_h(\underline{\hat{z}}_h)\right] \\
\dot{y}_h =& - y_h(\pi^x + \pi^n) + e_h \pi^n \widetilde{G}_h(\underline{\hat{z}}_h) + e_l \pi^h \widetilde{G}_h(\underline{\hat{z}}_h) \\
& + u_h \theta_h q_h(\theta_h) \widetilde{G}_h(\underline{z}_h) \\
\dot{y}_l =& - y_l(\pi^x + \pi^n + \pi^h) + e_l \pi^n \widetilde{G}_l(\underline{\hat{z}}_l) \\
& + (1-u_h-e_l-e_h)\theta_l q_l(\theta_l) \widetilde{G}_l(\underline{z}_l)
\end{aligned}
\tag{A.1}
$$

Equilibrium states are derived by setting the left hand sides to zero.

B Unreported value functions and Nash bargaining

Unreported value functions:

$$
\begin{aligned}
r\hat{W}_h(z) =& (1-t_h)\hat{w}_h(z) - T + \pi^x \left[\bar{U} - \hat{W}_h(z)\right] \\
& + \pi^n \left[(1 - G_h(\underline{\hat{z}}_h))\hat{W}_h^{\hat{e}} + G_h(\underline{\hat{z}}_h)\bar{U} - \hat{W}_h(z)\right]
\end{aligned}
\tag{B.1}
$$

$$
\begin{aligned}
r\hat{W}_l(z) =& (1-t_l)\hat{w}_l(z) - T + \pi^x \left[U_l - \hat{W}_l(z)\right] \\
& + \pi^n \left[(1 - G_l(\underline{\hat{z}}_l))\hat{W}_l^{\hat{e}} + G_l(\underline{\hat{z}}_l)U_l - \hat{W}_l(z)\right] \\
& + \pi^h \left[(1 - G_h(\underline{\hat{z}}_h))\hat{W}_h^{\hat{e}} + G_h(\underline{\hat{z}}_h)\bar{U} - \hat{W}_l(z)\right]
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
r\hat{J}_h(z) =& (1-\tau_h)z - \hat{w}_h(z) + D_h + \pi^x \left[(V^m - F_h) - \hat{J}_h(z)\right] \\
& + \pi^n \left[(1 - G_h(\underline{\hat{z}}_h))\hat{J}_h^{\hat{e}} + G_h(\underline{\hat{z}}_h)(V^m - F_h) - \hat{J}_h(z)\right]
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
r\hat{J}_l(z) =&(1-\tau_l)z - \hat{w}_l(z) + D_l + \pi^x \left[(V^m - F_l) - \hat{J}_l(z)\right] \\
&+\pi^n \left[(1-G_l(\hat{\underline{z}}_l))\hat{J}_l^{\hat{e}} + G_l(\hat{\underline{z}}_l)(V^m - F_l) - \hat{J}_l(z)\right] \\
&+\pi^h \left[(1-G_h(\hat{\underline{z}}_h))\hat{J}_h^{\hat{e}} + G_h(\hat{\underline{z}}_h)(V^m - F_h) - \hat{J}_l(z)\right]
\end{aligned} \tag{B.4}
$$

Nash bargaining implies:

$$
W_h - \bar{U} = \frac{\omega(1-t_h)}{1-\omega}(J_h + H_h) \text{ and } \hat{W}_h - \bar{U} = \frac{\omega(1-t_h)}{1-\omega}(\hat{J}_h + F_h) \tag{B.5}
$$

$$
W_l - U_l = \frac{\omega(1-t_l)}{1-\omega}(J_l + H_l) \text{ and } \hat{W}_l - U_l = \frac{\omega(1-t_l)}{1-\omega}(\hat{J}_l + F_l) \tag{B.6}
$$

or

$$
W_h - \bar{U} = \widetilde{\omega}_h s_h \text{ and } \hat{W}_h - \bar{U} = \tilde{\omega}_h \hat{s}_h \tag{B.7}
$$

$$
W_l - U_l = \widetilde{\omega}_l s_l \text{ and } \hat{W}_l - U_l = \tilde{\omega}_l \hat{s}_l \tag{B.8}
$$

$$
J_j + H_j = \widetilde{\widetilde{\omega}}_j s_j \text{ and } \hat{J}_j + F_j = \widetilde{\widetilde{\omega}}_j \hat{s}_j \tag{B.9}
$$

where

$$
\widetilde{\omega}_j \equiv \frac{\omega(1-t_j)}{1-\omega t_j} \text{ and } \widetilde{\widetilde{\omega}}_j \equiv \frac{1-\omega}{1-\omega t_j} \text{ and } (1-\omega)\widetilde{\omega}_j = \omega(1-t_j)\widetilde{\widetilde{\omega}}_j
$$

C Derivation of the equilibrium conditions

This section formally derives the equilibrium conditions for the intergroup model. As the model is nested, the conditions for the simple intragroup model can be found by dropping the skill index and setting $\pi^h = \pi^l = 0$. Equilibrium is determined by the free entry conditions (C.1) and the cut-off conditions (C.2).

$$
V_l = V_h = 0 \quad \Rightarrow \quad V^m = 0 \quad \Rightarrow \quad J_h^e = \frac{c_h}{q_h^f} - H_h \text{ and } J_l^e = \frac{c_l}{q_l^f} - H_l \tag{C.1}
$$

$$
\hat{J}_j(\hat{\underline{z}}_j) + F_j = 0 \Rightarrow \hat{\underline{z}}_j \text{ and } J_j(\underline{z}_j) + H_j = 0 \Rightarrow \underline{z}_j \tag{C.2}
$$

Take conditional expectation of (B.6), insert in (3.1) and eliminate J_l^e using the free entry condition (C.1) to get

$$
rU_l = \mu_l - T + \frac{\omega(1-t_l)}{1-\omega} c_l \theta_l \tag{C.3}
$$

Proceeding analogously for U_h results in

$$rU_h = \mu_h - T + \frac{\omega(1-t_h)}{1-\omega} c_h\theta_h - \pi^l(U_h - U_l) \tag{C.4}$$

We use C.4 and C.3 to solve for the difference in the values of unemployment

$$U_h - U_l = \frac{\mu_h - \mu_l}{\tilde{r}} + \frac{\omega}{1-\omega}\frac{(1-t_h)c_h\theta_h - (1-t_l)c_l\theta_l}{\tilde{r}} \tag{C.5}$$

Wages:
To get the wage equations proceed as follows. Multiplying (B.5) by r and rearranging gives $\omega(1-t_h)r\hat{J}_h(z)-(1-\omega)r\hat{W}_h(z) = -(1-\omega)r\bar{U}-\omega(1-t_h)rH_h$. Replace $r\hat{W}_h(z)$ and $r\hat{J}_h(z)$ by (B.1) and (B.3). Most of the remaining values cancel out after eliminating them using the FOCs from the Nash bargaining (B.7)-(B.9), and their conditional expectations. Solving for $\hat{w}_h(z)$ gives

$$\hat{w}_h(z) = \omega\left[(1-\tau_h)z + D_h + rF_h\right] + \frac{1-\omega}{1-t_h}\left[T + r\bar{U}\right] \tag{C.6}$$

Eliminating the remaining values of being unemployed, realizing that $r\bar{U} = r(1-\pi^l)\left[U_h - U_l\right] + rU_l$, results in

$$\begin{aligned} \hat{w}_h(z) &= \frac{1-\omega}{1-t_h}\left(\frac{(1-\pi^l)r}{\tilde{r}}\mu_h + \frac{\pi^l(1+r)}{\tilde{r}}\mu_l\right) + \omega\left[(1-\tau_h)z + D_h + rF_h\right] \\ &+ \omega\left[\frac{(1-\pi^l)r}{\tilde{r}}c_h\theta_h + \frac{1-t_l}{1-t_h}\frac{\pi^l(1+r)}{\tilde{r}}c_l\theta_l\right] \end{aligned} \tag{C.7}$$

The derivation of the outside wage works analogously and results in $w_h(z) = \hat{w}_h(z) - r_h\omega(F_h - H_h)$ given

$$w_h(z) = \omega\left[(1-\tau_h)z + D_h - (\pi^x + \pi^n)F_h + r_hH_h\right] + \frac{1-\omega}{1-t_h}\left[T + r\bar{U}\right] \tag{C.8}$$

We proceed the same way to get $\hat{w}_l(z)$ and $w_l(z)$.

$$\begin{aligned} \hat{w}_l(z) =& \omega\left[(1-\tau_l)z + D_l - \pi^h(F_h - F_l) + rF_l\right] \\ &+ \frac{1-\omega}{1-t_l}\left[T + rU_l - \pi^h(1-\pi^l)(U_h - U_l)\right] \\ &+ \omega\pi^h(1 - G_h(\underset{\sim}{\hat{z}}_h))\frac{t_h - t_l}{1-t_l}\widetilde{\widetilde{\omega}}_h\hat{s}_h^{\hat{e}} \end{aligned} \tag{C.9}$$

Note that in case of $t_l \neq t_h$, $\widetilde{\widetilde{\omega}}_h \hat{s}_h^{\hat{e}}$ does not drop out and is replaced by $\frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h \Sigma$. See below for the derivation. Eliminating the values of being unemployed gives

$$\begin{aligned} \hat{w}_l(z) &= -\frac{1-\omega}{1-t_l}\pi^h(1-\pi^l)\left[\frac{\mu_h - \mu_l}{\tilde{r}} + \frac{\omega}{1-\omega}\frac{(1-t_h)c_h\theta_h - (1-t_l)c_l\theta_l}{\tilde{r}}\right] \\ &+ \omega\left[(1-\tau_l)z + D_l - \pi^h(F_h - F_l) + rF_l + c_l\theta_l\right] + \frac{1-\omega}{1-t_l}\mu_l \\ &+ \omega\pi^h(1-G_h(\hat{\underline{z}}_h))\frac{t_h - t_l}{1-t_l}\left[\frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h\Sigma\right] \end{aligned} \quad \text{(C.10)}$$

Similar to before the outside wage is given by $w_l(z) = \hat{w}_l(z) - r_l\omega(F_l - H_l)$ knowing that

$$\begin{aligned} w_l(z) =& \omega\left[(1-\tau_l)z + D_l - \pi^h F_h - (\pi^x + \pi^n)F_l + r_l H_l\right] \\ &+ \frac{1-\omega}{1-t_l}\left[T + rU_l - \pi^h(1-\pi^l)(U_h - U_l)\right] \\ &+ \omega\pi^h(1-G_h(\hat{\underline{z}}_h))\frac{t_h - t_l}{1-t_l}\widetilde{\widetilde{\omega}}\hat{s}_h^{\hat{e}} \end{aligned} \quad \text{(C.11)}$$

For the derivation of Σ we start out by noting that the surplus functions are linear of form $s_h(z) = s_h^0 + s_h^1 z$, and further

$$\hat{s}_h(z) = s_h(z) + (1-\omega t_h)(F_h - H_h) = s_h^0 + s_h^1 z + (1-\omega t_h)(F_h - H_h) \quad \text{(C.12)}$$

as will be established below. Taking conditional expectation gives

$$\begin{aligned} \hat{s}_h^{\hat{e}} &= \int_{\hat{\underline{z}}_h}^{\infty}\left[\frac{s_h^0 + s_h^1\tilde{z} + (1-\omega t_h)(F_h - H_h)}{1 - G_h(\hat{\underline{z}}_h)}\right]dG_h(\tilde{z}) \\ &= s_h^0 + (1-\omega t_h)(F_h - H_h) + s^1\frac{\widetilde{G}(\hat{\underline{z}}_h)}{1 - G_h(\hat{\underline{z}}_h)} \end{aligned} \quad \text{(C.13)}$$

Where the partial expectation is defined as $\widetilde{G}_j(x) = \int_x^\infty \tilde{z}dG_j(\tilde{z})$. Taking conditional expectation of $s_h(z) = s_h^0 + s_h^1 z$, eliminating s_h^0 by using (C.13) and inserting for s_h^1 establishes $\hat{s}_h^{\hat{e}} = s_h^e + \Sigma$. Combine (C.1) and (B.9) to get

$s_h^e = \frac{c_h}{q_h^f}\frac{1}{\widetilde{\widetilde{\omega}}_h}$ which gives $\widetilde{\widetilde{\omega}}_h \hat{s}_h^{\hat{e}} = \frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h \Sigma$, with

$$\Sigma = \underbrace{\frac{(1-\tau_h)(1-\omega t_h)}{r_h}}_{s_h^1}\left[\frac{\widetilde{G}_h(\hat{\underline{z}}_h)}{1-G_h(\hat{\underline{z}}_h)} - \frac{\widetilde{G}_h(\underline{z}_h)}{1-G_h(\underline{z}_h)}\right]\underbrace{+(1-\omega t_h)(F_h - H_h)}_{\hat{s}_h(z)-s_h(z)}$$

Note that that $\Sigma = 0$ if $F_h = H_h$ because it also implies $\underline{z}_h = \hat{\underline{z}}_h$ as we will prove below.

Job creation conditions:
The job creation conditions are derived as follows. Subtract (B.3) and (B.4) evaluated at $\hat{\underline{z}}_h$ and $\hat{\underline{z}}_l$, respectively, from (3.7) and (3.8) and replace $\hat{J}_j(\hat{\underline{z}})$ by $-F_j$ using (C.2). Taking conditional expectation w.r.t. $\underline{z}_j$ and replacing J_j^e using (C.1) gives the **job creation curves**:

$$JC_h \quad : \quad (1-\omega)\left(\frac{(z_h^e - \hat{\underline{z}}_h)(1-\tau_h)}{r_h} - F_h + H_h\right) = \frac{c_h}{q_h^f} \qquad \text{(C.14)}$$

$$JC_l \quad : \quad (1-\omega)\left(\frac{(z_l^e - \hat{\underline{z}}_l)(1-\tau_l)}{r_l} - F_l + H_l\right) = \frac{c_l}{q_l^f} \qquad \text{(C.15)}$$

Job destruction conditions:
First define: $\Gamma_j \equiv \frac{1}{r_j}\int_{\hat{\underline{z}}_j}^{\infty}(\tilde{z} - \hat{\underline{z}}_j)\, dG_j(\tilde{z})$. Subtract (B.3) and (B.4) evaluated at $\hat{\underline{z}}_h$ and $\hat{\underline{z}}_l$, respectively, from themselves and eliminate $\hat{J}_j(\hat{\underline{z}})$ by $-F_j$ again using (C.2). Use the conditional expectation w.r.t. $\hat{\underline{z}}_j$ of the resulting expressions $\hat{J}_h(z)$ and $\hat{J}_l(z)$ to eliminate $\hat{J}_j^{\hat{e}}$ in (B.3) and (B.4). Evaluate again at $\hat{\underline{z}}_j$ and make use of (C.2) to arrive at

$$\begin{aligned} JD_h \quad : \quad & (1-\tau_h)\hat{\underline{z}}_h - \hat{w}_h(\hat{\underline{z}}_h) + D_h + rF_h \\ & +(1-\omega)\pi^n(1-\tau_h)\Gamma_h = 0 \end{aligned} \qquad \text{(C.16)}$$

$$\begin{aligned} JD_l \quad : \quad & (1-\tau_l)\hat{\underline{z}}_l - \hat{w}_l(\hat{\underline{z}}_h) + D_l + rF_l - \pi^h(F_h - F_l) \\ & +(1-\omega)\pi^n(1-\tau_l)\Gamma_l + (1-\omega)\pi^h(1-\tau_h)\Gamma_h = 0 \end{aligned} \qquad \text{(C.17)}$$

Eliminating the wages and diving by $(1-\omega)$ then gives the final **job de-**

struction curves:

$$(1-\tau_h)\underline{\hat{z}}_h + D_h + rF_h - \frac{1}{1-t_h}\left[\frac{(1-\pi^l)r}{\tilde{r}}\mu_h + \frac{\pi^l(1+r)}{\tilde{r}}\mu_l\right] - \frac{\omega}{1-\omega}\left[\frac{(1-\pi^l)r}{\tilde{r}}c_h\theta_h + \frac{1-t_l}{1-t_h}\frac{\pi^l(1+r)}{\tilde{r}}c_l\theta_l\right] + \pi^n(1-\tau_h)\Gamma_h = 0 \tag{C.18}$$

$$\begin{aligned}&(1-\tau_l)\underline{\hat{z}}_l + D_l + rF_l - \pi^h(F_h - F_l) - \frac{\mu_l}{1-t_l} - \frac{\omega}{1-\omega}c_l\theta_l \\ &+ \frac{\pi^h(1-\pi^l)}{1-t_l}\frac{\mu_h-\mu_l}{\tilde{r}} + \pi^h(1-\pi^l)\frac{\omega}{1-\omega}\left[\frac{1-t_h}{1-t_l}\frac{c_h\theta_h}{\tilde{r}} - \frac{c_l\theta_l}{\tilde{r}}\right] \\ &- \pi^h(1-G_h(\underline{\hat{z}}_h))\frac{\omega}{1-\omega}\frac{t_h-t_l}{1-t_l}\left[\frac{c_h}{q_h^f} + \widetilde{\widetilde{\omega}}_h\Sigma\right] \\ &+ \pi^n(1-\tau_l)\Gamma_l + \pi^h(1-\tau_h)\Gamma_h = 0\end{aligned} \tag{C.19}$$

Cut-off relationships:

The relation between the reservation productivities $\underline{z}_j$ and $\underline{\hat{z}}_j$ stems from a simple observation. The cut-off conditions in (C.2) in combination with (B.5) and (B.6) imply that firms and workers will always mutually agree on creating and destroying jobs. Hence, $\underline{z}$ and $\underline{\hat{z}}$ set the joint surpluses to 0. The surpluses in equilibrium are given by

$$s_h(z) = W_h(z)+J_h(z)-\bar{U}+H_h \quad \text{and} \quad s_l(z) = W_l(z)+J_l(z)-U_l+H_l \tag{C.20}$$

$$\hat{s}_h(z) = \hat{W}_h(z)+\hat{J}_h(z)-\bar{U}+F_h \quad \text{and} \quad \hat{s}_l(z) = \hat{W}_l(z)+\hat{J}_l(z)-U_l+F_l \tag{C.21}$$

Observe that for the same z the difference between the surplus functions is given by: $s_j(z)-\hat{s}_j(z) = \frac{-t_j}{r_j}(w_j(z)-\hat{w}_j(z))+H_j-F_j = -(1-\omega t_j)(F_j-H_j)$, which is independent of z. Hence, the surplus functions have the following linear structure

$$s_j(z) = s_j^0 + s_j^1 z - (1-\omega t_j)(F_j - H_j) \tag{C.22}$$

$$\hat{s}_j(z) = s_j^0 + s_j^1 z \tag{C.23}$$

From (3.3), (3.7), (3.4), and (3.8) we infer that $s_j^1 = \frac{(1-\tau_j)(1-\omega t_j)}{r_j}$. The cut-offs solving $s_j(\underline{z}) = 0$ and $s_j(\underline{\hat{z}}) = 0$ are therefore given by

$$\underline{z}_j = -\frac{s_j^0 - (1-\omega t_j)(F_j - H_j)}{s_j^1} \tag{C.24}$$

$$\underline{\hat{z}}_j = -\frac{s_j^0}{s_j^1} \tag{C.25}$$

Hence, the **relationship of the cut-offs** can be written as

$$\underline{z}_h = \underline{\hat{z}}_h + \frac{r_h}{1-\tau_h}(F_h - H_h) \tag{C.26}$$

$$\underline{z}_l = \underline{\hat{z}}_l + \frac{r_l}{1-\tau_l}(F_l - H_l) \tag{C.27}$$

D Derivation of the social planner's optimum in the simple intragroup model

The constrained social optimum is derived by maximizing the social welfare function $\Theta(\cdot)$ subject to the matching constraints and the evolution of total production y, hence:

$$\max_{\{\underline{z},\underline{\hat{z}},\theta\}} \Theta = \max_{\{\underline{z},\underline{\hat{z}},\theta\}} \int_0^\infty e^{-rt}(y + uh - C\theta u)dt \tag{D.1}$$

subject to:

$$\dot{u} = \pi^n G(\underline{\hat{z}})(L-u) - q^w u \tag{D.2}$$

$$\dot{y} = u\theta q(\theta) \int_{\underline{z}}^\infty \tilde{z}\, dG(\tilde{z}) + (L-u)\pi^n \int_{\underline{\hat{z}}}^\infty \tilde{z}\, dG(\tilde{z}) - \pi^n y \tag{D.3}$$

We set up the present-value Hamiltonian

$$\begin{aligned}\mathcal{H} = {} & e^{-rt}(y + uh - C\theta u) + \lambda_1 \left[\pi^n G(\underline{\hat{z}})(L-u) - q^w u\right] \\ & + \lambda_2 \left[u\theta q(\theta) \int_{\underline{z}}^\infty \tilde{z}\, dG(\tilde{z}) + (L-u)\pi^n \int_{\underline{\hat{z}}}^\infty \tilde{z}\, dG(\tilde{z}) - \pi^n y\right]\end{aligned} \tag{D.4}$$

The optimality conditions, i.e. $\frac{\partial \mathcal{H}}{\partial \underline{z}} = 0$, $\frac{\partial \mathcal{H}}{\partial \underline{\hat{z}}} = 0$, $\frac{\partial \mathcal{H}}{\partial \theta} = 0$, $\frac{\partial \mathcal{H}}{\partial u} = -\dot{\lambda}_1$, $\frac{\partial \mathcal{H}}{\partial y} = -\dot{\lambda}_2$, imply (D.5) to (D.9):

$$\lambda_1 - \underline{z}\lambda_2 = 0 \tag{D.5}$$

$$\lambda_1 - \underline{\hat{z}}\lambda_2 = 0 \tag{D.6}$$

From (D.5) and (D.6) we infer that the cut-off productivities irrespective of whether one arrives at or has already been in a job coincide, i.e. $\underline{z} = \underline{\hat{z}}$. From now on we will just use $\underline{z}$. Before stating the remaining first order conditions define $\widetilde{G}(\underline{z}) \equiv \int_{\underline{z}}^{\infty} \tilde{z}\, dG(\tilde{z})$ and $\Gamma \equiv \frac{1}{r+\pi^n} \int_{\underline{z}}^{\infty} (\tilde{z} - \underline{z})\, dG(\tilde{z})$:

$$-e^{-rt}C - \lambda_1(1-\eta)q(\theta)(1-G(\underline{z})) + \lambda_2(1-\eta)q(\theta)\widetilde{G}(\underline{z}) = 0 \tag{D.7}$$

$$-e^{-rt}(h - C\theta) - \lambda_1\left[\pi^n G(\underline{z}) + q^w\right] + \lambda_2\left[\theta q(\theta) - \pi^n\right]\widetilde{G}(\underline{z}) = -\dot{\lambda}_1 \tag{D.8}$$

$$e^{-rt} - \pi^n \lambda_2 = -\dot{\lambda}_2 \tag{D.9}$$

Eliminating λ_1 in (D.7) using (D.5) gives

$$-e^{-rt}C + \lambda_2(1-\eta)q(\theta)(r+\pi^n)\Gamma = 0 \tag{D.10}$$

which implies the following relationships for λ_1 and λ_2:

$$\lambda_1 = \frac{e^{-rt}C\underline{z}}{(1-\eta)q(\theta)(r+\pi^n)\Gamma} \quad \text{and} \quad \lambda_2 = \frac{e^{-rt}C}{(1-\eta)q(\theta)(r+\pi^n)\Gamma} \tag{D.11}$$

Differentiating (D.10) w.r.t. t and subtracting (D.10) again results in the following relations:

$$\dot{\lambda}_2 = -\lambda_2 r \quad \text{and consequently} \quad \dot{\lambda}_1 = -\lambda_1 r \tag{D.12}$$

Inserting for λ_2 and $\dot{\lambda}_2$ in (D.9) and rearranging gives the reduced optimality condition that has a similar structure compared the job creation condition:

$$(1-\eta)\frac{z^e - \underline{z}}{\pi^n + r} - \frac{C}{q^f} = 0 \tag{D.13}$$

To derive the last reduced optimality condition, i.e. the job destruction condition counterpart, we eliminate λ_1, λ_2 and $\dot{\lambda}_1$ in (D.8) and rearrange:

$$\underline{z} - h + \pi^n\Gamma - \frac{\eta}{1-\eta}C\theta = 0 \tag{D.14}$$

E Derivation of the social planner's optimum in the intergroup model

Again we maximize discounted social welfare

$$\int_0^{\infty} e^{-rt}\left[y_h + y_l + (u_h + u_l)h - u_h C_h \theta_h - u_l C \theta_l\right] dt \tag{E.1}$$

where $u_l = (1 - u_h - e_h - e_l)$, subject to the evolution of the employment states $\dot{u}_h$, $\dot{e}_l$, $\dot{e}_h$ and of total production $\dot{y}_h$ and $\dot{y}_l$ as given by (A.1), over the choice variables $\underline{z}_j$, $\hat{\underline{z}}_j$ and θ_j. We set up the present-value Hamiltonian

$$\begin{aligned}\mathcal{H} = e^{-rt}\left[y_h + y_l + (1 - e_h - e_l)h - u_h C_h \theta_h - (1 - u_h - e_h - e_l)C\theta_l\right] \\ + \lambda_1 \dot{u}_h + \lambda_2 \dot{e}_l + \lambda_3 \dot{e}_h + \lambda_4 \dot{y}_h + \lambda_5 \dot{y}_l\end{aligned} \tag{E.2}$$

The optimality conditions $\frac{\partial \mathcal{H}}{\partial \underline{z}_j} = 0$, $\frac{\partial \mathcal{H}}{\partial \hat{\underline{z}}_j} = 0$ imply

$$\lambda_1(1 - \pi^l) - \lambda_3 - \underline{z}_h \lambda_4 = 0 \quad \text{and} \quad \lambda_1(1 - \pi^l) - \lambda_3 - \hat{\underline{z}}_h \lambda_4 = 0 \tag{E.3}$$

$$\lambda_2 + \underline{z}_l \lambda_5 = 0 \quad \text{and} \quad \lambda_2 + \hat{\underline{z}}_l \lambda_5 = 0 \tag{E.4}$$

Hence, reservation productivities have to coincide again, i.e. $\underline{z}_j = \hat{\underline{z}}_j$. For simplicity will will just use $\underline{z}_j$ from now on. Define $\widetilde{G}_j(\underline{z}_j) \equiv \int_{\underline{z}_j}^{\infty} \tilde{z}\, dG_j(\tilde{z})$ and $\Gamma_j \equiv \frac{1}{r_j} \int_{\underline{z}_j}^{\infty} \left(\tilde{z} - \underline{z}_j\right) dG_j(\tilde{z})$ and note their relationship $r_j \Gamma_j = \widetilde{G}_j(\underline{z}_j) - \underline{z}_j(1 - G_j(\underline{z}_j))$ which will be used frequently in what follows. Next, we set $\frac{\partial \mathcal{H}}{\partial \theta_h} = 0$ and eliminate $\lambda_1(1 - \pi^l) - \lambda_3$ using (E.3) to get

$$- e^{-rt} C_h + \lambda_4 r_h \Gamma_h (1 - \eta) q_h(\theta_h) = 0 \tag{E.5}$$

which solved for λ_4 implies

$$\lambda_4 = \frac{e^{-rt} C_h}{(1 - \eta) q_h(\theta_h) r_h \Gamma_h} \quad \text{and} \quad \dot{\lambda}_4 = -r\lambda_4 \tag{E.6}$$

Inserting again in (E.5) gives

$$\begin{aligned}\lambda_1(1 - \pi^l) - \lambda_3 &= \frac{e^{-rt} C_h \underline{z}_h}{(1 - \eta) q_h(\theta_h) r_h \Gamma_h} \quad \text{and} \\ \dot{\lambda}_1(1 - \pi^l) - \dot{\lambda}_3 &= -r\left(\lambda_1(1 - \pi^l) - \lambda_3\right)\end{aligned} \tag{E.7}$$

Proceeding analogously for θ_l implies

$$\lambda_2 = \frac{e^{-rt} C_l}{(1 - \eta) q_l(\theta_l) r_l \Gamma_l} \quad \text{and} \quad \dot{\lambda}_2 = -r\lambda_2 \tag{E.8}$$

$$\lambda_5 = \frac{e^{-rt} C_l \underline{z}_l}{(1 - \eta) q_l(\theta_l) r_l \Gamma_l} \quad \text{and} \quad \dot{\lambda}_5 = -r\lambda_5 \tag{E.9}$$

The optimality condition for y_h reads $e^{-rt} - \lambda_4(\pi^x + \pi^n) = -\dot{\lambda}_4$. We eliminate λ_4 and $\dot{\lambda}_4$ to get the optimal job creation condition for high-skilled jobs

$$(1-\eta)q_h(\theta_h)\Gamma_h = C_h \quad \text{or} \quad (1-\eta)\left(\frac{z_h^e - \hat{\underline{z}}_h}{r_h}\right) = \frac{C_h}{q_h^f} \tag{E.10}$$

Similarly, transforming $\frac{\partial \mathcal{H}}{\partial y_l} = e^{-rt} - \lambda_5(\pi^x + \pi^n + \pi^h) = -\dot{\lambda}_5$ gives the optimal low-skill job creation condition

$$(1-\eta)q_l(\theta_l)\Gamma_l = C_l \quad \text{or} \quad (1-\eta)\left(\frac{z_l^e - \hat{\underline{z}}_l}{r_l}\right) = \frac{C_l}{q_l^f} \tag{E.11}$$

Combine those two conditions with our expressions for the co-states to get

$$\lambda_1(1-\pi^l) - \lambda_3 = \frac{e^{-rt}\underline{z}_h}{r_h}, \lambda_2 = \frac{e^{-rt}\underline{z}_l}{r_l}, \lambda_4 = \frac{e^{-rt}}{r_h}, \lambda_5 = \frac{e^{-rt}}{r_l} \tag{E.12}$$

Compute $\frac{\partial \mathcal{H}}{\partial e_l} = -\dot{\lambda}_2$, eliminate all known co-states and transform to get

$$\underline{z}_l - h - \frac{\eta}{1-\eta}C_l\theta_l + \pi^h\Gamma_h + \pi^n\Gamma_l + \frac{\lambda_1}{e^{-rt}}(1-\pi^l)\pi^h = 0 \tag{E.13}$$

Note that this equation implies that $\dot{\lambda}_1 = -r\lambda_1$ and consequently $\dot{\lambda}_3 = -r\lambda_3$. Next, we calculate $\frac{\partial \mathcal{H}}{\partial u_h} = -\dot{\lambda}_1 = r\lambda_1$ which gives

$$\lambda_1\tilde{r} = -e^{-rt}\left[C_h\theta_h - C_l\theta_l\right] + e^{-rt}\theta_h q_h(\theta_h)\Gamma_h - e^{-rt}\theta_l q_l(\theta_l)\Gamma_l \tag{E.14}$$

Use the job creation conditions (E.10) and (E.11) to eliminate $q_j(\theta_j)\Gamma_j$ by $\frac{C_j}{1-\eta}$ and rearrange to arrive at

$$\frac{\lambda_1}{e^{-rt}} = \frac{\eta}{1-\eta}\left[\frac{C_h\theta_h - C_l\theta_l}{\tilde{r}}\right] \tag{E.15}$$

Insert this expression in (E.13) to derive the optimal job destruction condition for low-skilled workers

$$\hat{\underline{z}}_l - h - \frac{\eta}{1-\eta}C_l\theta_l + \pi^h(1-\pi^l)\frac{\eta}{1-\eta}\left[\frac{C_h\theta_h - C_l\theta_l}{\tilde{r}}\right] + \pi^n\Gamma_l + \pi^h\Gamma_h = 0 \tag{E.16}$$

Compute $\frac{\partial \mathcal{H}}{\partial e_h} = -\dot{\lambda}_3$ and eliminate $-\dot{\lambda}_3$ using $\lambda_3 = e^{-rt}(1-\pi^l)\frac{\eta}{1-\eta}\left[\frac{C_h\theta_h - C_l\theta_l}{\tilde{r}}\right]$. Rearranging reveals the optimal job destruction condition for high-skilled workers

$$\underline{\hat{z}}_h - h - \frac{\eta}{1-\eta}\left[\frac{(1-\pi^l)r}{\tilde{r}}C_h\theta_h + \frac{\pi^l(1+r)}{\tilde{r}}C_l\theta_l\right] + \pi^n\Gamma_h = 0 \qquad \text{(E.17)}$$

Observe how $\pi^l = \pi^h = 0$ make the conditions collapse to their intragroup forms as derived in appendix section D.

F More comparative statics for the intragroup model

F.1 JD-JC diagram

Note that the determinant of the Jacobian of the JD-JC system is always positive, as $JD_\theta \equiv \frac{\partial JD}{\partial \theta} < 0$, $JD_{\underline{\hat{z}}} \equiv \frac{\partial JD}{\partial \underline{\hat{z}}} > 0$, $JC_\theta \equiv \frac{\partial JC}{\partial \theta} < 0$, and $JC_{\underline{\hat{z}}} \equiv \frac{\partial JC}{\partial \underline{\hat{z}}} < 0$, i.e. $Det(JDJC) = JD_\theta JC_{\underline{\hat{z}}} - JC_\theta JD_{\underline{\hat{z}}} \equiv \nabla > 0$. The elements of the inverse of the Jacobian of $JDJC$ system have the following signs:

$$(Jac_{JDJC})^{-1} = \nabla^{-1}\begin{pmatrix} JC_{\underline{\hat{z}}} & -JD_{\underline{\hat{z}}} \\ -JC_\theta & JD_\theta \end{pmatrix} = \begin{pmatrix} - & - \\ + & - \end{pmatrix}$$

To prove that the JD-curve slopes upward and the JC-curve is downward sloping proceed as follows. Total differentiation of the JD-curve w.r.t. $\underline{\hat{z}}$ and θ gives

$$\frac{(1-\tau)(1-G(\underline{\hat{z}}))\pi^n + (1-\tau)r}{\pi^n + r}d\underline{\hat{z}} = \frac{\omega c}{1-\omega}d\theta, \text{ hence}$$

$$\frac{d\theta}{d\underline{\hat{z}}}|_{\text{JD}} > 0, \text{ the JD curve is increasing.}$$

Before deriving the slope of the JC-curve, let us define $\frac{\partial z^e}{\partial \underline{z}} = \frac{g(\underline{z})(z^e - \underline{z})}{1-G(\underline{z})} \equiv \Omega$.

Assumption F.1. $\Omega < 1$. *This is true in any case for some distributions (e.g. uniform, normal,...) and very likely to be true for others (e.g. log-normal, with sufficiently small variance)*[27].

[27] It is easy to analytically show that for the uniform distribution $\Omega = 1/2, \forall \underline{\hat{z}}$. Statements about the other distributions are based on numerical simulations.

Again, total differentiation reveals that:

$$\left[\frac{(1-\tau)(1-\omega)}{\pi^n + r}(\Omega - 1) - \frac{c}{(q^f)^2}g(\underline{z})q(\theta)\right] d\hat{\underline{z}} = \frac{\eta c}{q^w}d\theta,$$

$$\frac{d\theta}{d\hat{\underline{z}}}|_{\text{JC}} < 0, \text{ the JC curve is decreasing.}$$

F.2 Policy effects

If total effects are not mentioned, it means that they are ambiguous.

Wage subsidy (D)

$$\frac{d\theta}{dD}|_{\text{JD}} = \frac{1-\omega}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dD}|_{\text{JC}} = 0$$

Effect: The JD curve shifts outward. The JC curve does not move. $\theta \uparrow, \hat{\underline{z}} = \underline{z} \downarrow$, $u \downarrow$.

Hiring subsidy (H)

$$\frac{d\theta}{dH}|_{\text{JD}} = 0 \quad \text{and} \quad \frac{d\theta}{dH}|_{\text{JC}} = -\frac{q^w(1-\omega)\,[\Omega - 1]}{\eta c} > 0$$

Effect: The JD curve does not move. The JC curve shifts outward. $\theta \uparrow$, $\hat{\underline{z}} \uparrow$, $\underline{z} < \hat{\underline{z}}$. To determine the effect on the direction of $\underline{z}$ see proposition 2.1.

Recruitment subsidy (R)

$$\frac{d\theta}{dR}|_{\text{JD}} = \frac{\theta}{c} > 0 \text{ and } \frac{d\theta}{dR}|_{\text{JC}} = \frac{\theta}{\eta c} > 0$$

Effect: The JD and the JC curves shift outward. $\theta \uparrow$. As the JC curve moves stronger we have that: $\hat{\underline{z}} = \underline{z} \uparrow$.

Firing tax (F)

$$\frac{d\theta}{dF}|_{\text{JD}} = \frac{(1-\omega)r}{\omega c} > 0 \quad \text{and} \quad \frac{d\theta}{dF}|_{\text{JC}} = \frac{q^w(1-\omega)\,[\Omega - 1]}{\eta c} < 0$$

Effect: The JD shifts outward and the JC curve shift inward. $\hat{\underline{z}} \downarrow, \underline{z} > \hat{\underline{z}}$. Using the implicit function theorem one can show that $\theta \downarrow$.

Output taxes (τ)

$$\frac{d\theta}{d\tau}|_{\text{JD}} = -\frac{(1-\omega)\left[(z^{\hat{e}} - \underline{\hat{z}})(1 - G(\underline{\hat{z}}))\pi^n + (\pi^n + r)\underline{\hat{z}}\right]}{\omega c(\pi^n + r)} < 0$$

$$\frac{d\theta}{d\tau}|_{\text{JC}} = -\frac{q^w(1-\omega)\left[(z^e - \underline{\hat{z}})(1-\tau) - \Omega(F - H)(\pi^n + r)\right]}{(1-\tau)(\pi^n + r)\eta c}$$

This expression is smaller than 0, i.e. the JC shifts inward, whenever $F = H$. The bigger F in comparison to H, the smaller the inward shift.

Effect: The JD and the JC curves shift inward. $\theta \downarrow$.

Wage taxes (t)

$$\frac{d\theta}{dt}|_{\text{JD}} = -\frac{\mu(1-\omega)}{(1-t)^2 \omega c} < 0 \quad \text{and} \quad \frac{d\theta}{dt}|_{\text{JC}} = 0$$

Effect: The JD curve shifts inward. The JC does not move, implying $\theta \downarrow, \underline{\hat{z}} \uparrow$.

G Tables and figures

Table G.1: Decentralized economy, benchmark

			Parameters				
β	0.996	b_h	0.950	H_h	0.000	$\underline{x}_h$	0.700
r	0.004	b_l	0.750	H_l	0.000	$\bar{x}_h$	2.000
ω	0.500	C_h	1.509	F_h	0.000	$\underline{x}_l$	0.500
η	0.500	C_l	0.274	F_l	0.000	$\bar{x}_l$	1.500
π^x	0.007	R_h	0.000	D_h	0.000	A_h	0.563
π^n	0.020	R_l	0.000	D_l	0.000	A_l	0.148
π^h	0.010	τ_h	0.000	t_h	0.065	h	0.250
π^l	0.050	τ_l	0.000	t_l	0.050		
			Results				
type	θ_j	$\underline{z}_j$	$\underline{\hat{z}}_j$	L_j	e_j	u_j	duration
h:	1.000	1.345	1.345	0.740	0.702	0.038	3.524
l:	1.000	0.950	0.950	0.260	0.198	0.062	12.296
total:	-	-	-	1.000	0.900	0.100	9.000
type	$G_j(\underline{z}_j)$	av. prod	av. w_j	av. $\hat{w}_j$	repl.	tot. repl.	welfare
h:	0.496	1.673	1.562	1.562	0.608	0.768	1.108
l:	0.450	1.225	1.151	1.151	0.651	0.869	0.210
total:	-	1.574	1.472	1.472	-	-	1.318

Note: 'repl.' gives the replacement ratio, i.e. $\frac{b_j}{\hat{w}_j}$. 'tot. repl.' is $\frac{\mu_j}{\hat{w}_j}$. 'av.' denotes average. 'welfare' is per period in steady state. Other variables as in the paper.

Table G.2: Social planner's solution

			Parameters				
β	0.996	b_h	-	H_h	-	$\underline{x}_h$	0.700
r	0.004	b_l	-	H_l	-	$\bar{x}_h$	2.000
ω	-	C_h	1.509	F_h	-	$\underline{x}_l$	0.500
η	0.500	C_l	0.274	F_l	-	$\bar{x}_l$	1.500
π^x	0.007	R_h	-	D_h	-	A_h	0.563
π^n	0.020	R_l	-	D_l	-	A_l	0.148
π^h	0.010	τ_h	-	t_h	-	h	0.250
π^l	0.050	τ_l	-	t_l	-		
			Results				
type	θ_j	$\underline{z}_j$	$\hat{\underline{z}}_j$	L_j	e_j	u_j	duration
h:	1.821	1.239	1.239	0.850	0.824	0.026	2.248
l:	3.677	0.738	0.738	0.150	0.137	0.014	4.630
total:	-	-	-	1.000	0.961	0.039	3.066
type	$G_j(\underline{z}_j)$	av. prod	av. w_j	av. $\hat{w}_j$	repl.	tot. repl.	welfare
h:	0.415	1.620	-	-	-	-	1.257
l:	0.238	1.119	-	-	-	-	0.136
total:	-	1.548	-	-	-	-	1.392

Note: see table G.1.

Table G.3: Possible implementations of the first-best

$H_h = F_h$	$H_l = F_l$	t_h	t_l
0	131.2	-0.062	0.738
20	123.4	-0.046	0.668
40	122.4	-0.033	0.594
60	125.6	-0.022	0.519
80	131.5	-0.014	0.443
100	139.2	-0.007	0.365
120	148.3	-0.001	0.287
140	158.3	0.004	0.208
160	169.0	0.008	0.129
180	180.3	0.012	0.050
200	192.0	0.016	-0.029

Figure G.1: Budget surplus for efficient tax rates and corresponding $H_l = F_l$ and $H_h = F_h$ combinations

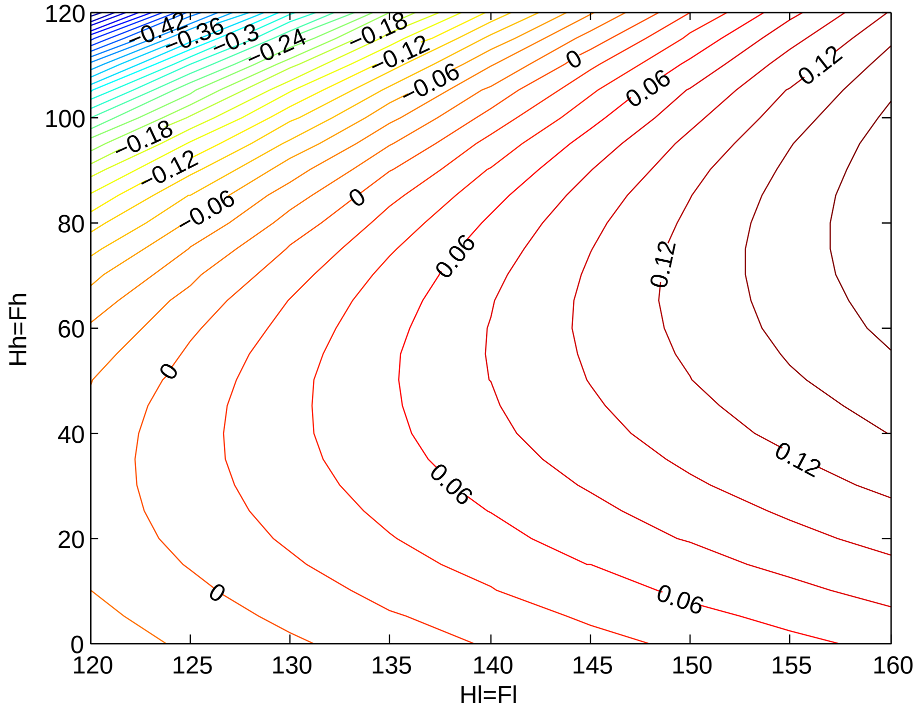

Table G.4: Effect of a low wage subsidy on unemployment rates

	D_l change uncompensated					
	no turbulence			turbulence		
D_l	u_h	u_l	u	u_h	u_l	u
0.0	5.08	23.98	10.00	5.08	23.98	10.00
0.1	5.08	19.29	8.78	5.80	22.09	10.39
0.2	5.08	16.18	7.97	6.74	20.40	10.95
0.3	5.08	13.95	7.39	7.99	18.90	11.71
0.4	5.08	12.30	6.96	9.68	17.54	12.68
0.5	5.08	10.99	6.62	12.10	16.30	13.91
	D_l change compensated by t_h					
	no turbulence			turbulence		
D_l	u_h	u_l	u	u_h	u_l	u
0.0	5.08	23.98	10.00	5.08	23.98	10.00
0.1	5.15	19.29	8.83	6.63	23.69	11.98
0.2	5.26	16.18	8.10	-	-	-
0.3	5.39	13.95	7.62	-	-	-
0.4	5.56	12.30	7.31	-	-	-
0.5	5.76	10.99	7.12	-	-	-

Note: Unemployment rates are computed in percent relative to L_j. '-' denotes break down of equilibrium.

Table G.5: Variable names

$j = l, h$	subscript indicating the skill type
$\hat{}$	hat notation refers to 'inside'-variables
$\bar{z}$	average productivity
$\bar{w}$	average wage
ω	bargaining weight for the worker
$G(z)$	cdf for productivity draws
X^e	conditional expectation of some random variable X w.r.t. $\underline{z}$
$X^{\hat{e}}$	conditional expectation of some random variable X w.r.t. $\hat{\underline{z}}$
β	discount factor, $\beta = \frac{1}{1+r}$
Ω	derivative of z^e w.r.t. $\underline{z}$
∇	determinant of the JD-JC system
F	firing taxes
H	hiring subsidy
h	home production
μ	instantaneous value of leisure ($\mu = b + h$)
r	interest rate, $r = \frac{(1-\beta)}{\beta}$
L	labor force
θ	labor market tightness
T	lump sum tax
e	mass of employed people
u	mass of unemployed people
$\widetilde{G}(z)$	partial expectation of productivity
$g(z)$	pdf for productivity draws
π^l	prob. of downgrade
π^x	prob. of exogenous separation
q^f	prob. of filling a vacancy
q^w	prob. of finding and accepting a job
$q(\theta)$	prob. of match for the firm
$\theta q(\theta)$	prob. of match for the worker
π^n	prob. of new productivity draw
π^h	prob. of upgrade
z	productivity
R	recruitment subsidy
$\underline{z}$	reservation productivity, 'outside'
$\hat{\underline{z}}$	reservation productivity, 'inside'
Θ	social welfare
$s(z)$	surplus function
y	total production
τ	output tax rate
b	unemployment compensation
C	vacancy creation costs (gross)
c	vacancy creation costs (net of subsidies, i.e. $c = C - R$)
U	value of a being unemployed
$J(z)$	value of an employment for the firm
$W(z)$	value of an employment for the worker
V	value of a vacancy
D	wage subsidy (lump-sum)
t	wage tax rate
η	weight in the matching function