Earnings Dynamics and Firm-Level Shocks

- Preliminary and Incomplete*-

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Abstract

This paper uses a newly set up matched employer-employee data set from Sweden to test whether firm-level productivity shocks are transmitted to workers' wages. To disentangle shocks from endogenous responses to shocks, selection into employment and job-to-job mobility are explicitly modeled. Instead of a structural model that requires strong assumptions on both pay setting and the structure of production, we estimate a flexible empirical model that allows for a rich stochastic structure of the income process both at the individual and the match level. Our setting is consistent with many theoretical arguments in the literature. Thus our paper is the empirical opening investigation into an agenda that will lead to richer structural models taking the firm side more seriously.

Keywords: Income process, Wage dynamics, Firm dynamics

JEL-codes: H51, H55, I18, J26

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1 Introduction

There has been an increased interest in understanding pay policies of firms, and in particular the extent to which firm-level productivity shocks are transmitted to workers' wages. Such departures from perfect competition and the law of one price have been motivated by the developments in search theory starting by the seminal models of Burdett and Mortensen (1998) and Mortensen and Pissarides (1994). In a competitive labor market, workers only bare the risk of shocks to their productivity, but in the presence of search frictions, there are multiple sources of risk distinct from workers' productivity shocks.¹ While the theoretical justifications for departures from the law of one price are compelling, the empirical evidence is not quite there.

First, most equilibrium search models that have been estimated on empirical data assume no productivity shocks, see Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay and Robin (2006). An exception is the model of Lise, Meghir and Robin (2013) which allows for the effect of firm-level productivity shocks in a context of a model with productive complementarities. However, their model is estimated on individual-level data and hence cannot measure productivity shocks directly but infers them from the structure of the model.

The recent availability of matched employer-employee data gives rise to major new opportunities in this direction. In particular, these new data sources allow for an extended analysis of the role of the firm for the dynamics of earnings. An additional source of risk is shocks to firm-level productivity. Yet most papers using matched worker-firm data have focused on sorting and firm-worker heterogeneity rather than the dynamics of shocks, see Abowd, Kramarz and Margolis (1999). An exception is the paper by Guiso, Pistaferri and Schivardi (2005) who estimate the amount of income insurance provided by firms using

 $^{^{1}\}mathrm{Low},$ Meghir and Pistaferri (2010) illustrate the importance of such distinctions for the welfare effects of risk.

a matched employer-employee data set from Italy. However, their approach is limited by the fact that they ignore job-to-job mobility and transitions between employment and unemployment. Such transitions may well hide the impact of firm-level shocks on wages because a worker may quit or switch jobs instead of suffering too large a pay cut.

In this paper, we propose a framework for introducing the firm in empirical models of the dynamic income process. In particular, the proposed model allows for studying the extent to which firm-level productivity shocks are transmitted to wages. The key innovation is that we account both for job-to-job transitions and for transitions between employment and unemployment to capture the role of job mobility and labor force participation in hiding the impact of shocks. The model allows for a rich stochastic structure of the income process, both at the individual and the firm-worker match level, and is consistent with most findings in the literature. The proposed approach is not structural in the sense that we do not specify a model that defines the way pay setting is carried out. While an equilibrium model defines clearly the way that shocks are transmitted, a tractable model comes with a number of strong assumptions both on the form of contracting and on the structure of production.² Thus our reduced form approach allows us to investigate the transmission of shocks with greater flexibility.

In a related paper, Low, Meghir and Pistaferri (2010) find that making job mobility and employment choices endogenous reduces the estimated variance of the permanent shock compared to earlier studies. In their model firms are represented as a fixed matched heterogeneity effect. However because they do not observe firms they are not able to measure the impact of shocks to firms separately from worker productivity shocks. A related paper is Altonji, Smith and Vidangos (2013), who specify a model of employment, hours, wages and

 $^{^{2}}$ See Lamadon, Lise, Meghir and Robin (2013) for an effort in this direction.

earnings in order to distinguish different sources of risk. Selection into employment and between jobs is modeled in a similar way as in Low, Meghir and Pistaferri (2010). Yet none of these studies considers the role of firm-level shocks for earnings dynamics, which is the main contribution of the present paper.

The paper proceeds as follows. Section 2 presents the model of the income process. Section 3 introduces the dataset and presents descriptive statistics. Section 4 presents the estimation and identification strategy. Section 5 shows the main results for the two-stage estimation procedure. Section 6 concludes.

2 The Model

Wages We consider a quarterly model of wages, participation and job mobility. Log wages of individual i aged a at calendar time t who started to work at firm j at the age of a_0 is given by

$$\ln w_{i,j(a_0),a,t} = x'_{i,a,t}\gamma + P_{i,a,t} + \varepsilon_{i,a,t} + v_{i,j(a_0),a,t},\tag{1}$$

where x are observable worker characteristics, $\epsilon \sim N(0, \sigma_{\epsilon}^2)$ is an iid transitory productivity shock³ and P_a is permanent productivity at age a specified as

$$P_{i,a,t} = P_{i,a-1,t-1} + \zeta_{i,a,t} = P_{i,0,t-a}^{init} + \sum_{s=1}^{a} \zeta_{i,s,t-a+s}$$
(2)

$$P^{init} \sim N\left(0, \sigma_P^2\right), \, \zeta \sim N\left(0, \sigma_{\zeta}^2\right).$$
 (3)

³Note that we assume no measurement error because we will use high quality administrative data for estimation. Meghir and Pistaferri (2004) point out the inability to disentangle the variance of the transitory shock, the variance of the measurement error and the parameters of the transitory process in a similar setting. The distinction has economic implications, however, since measurement error is pure noise while transitory shocks reflect uncertainty that may give rise to economic responses. The authors suggest two ways of handling this issue, by obtaining bounds for the unidentified variances or by using an external estimate of the measurement error to recover the variance of the transitory shock.

This random walk specification is in line with the research by MaCurdy (1982), Abowd and Card (1989) and Meghir and Pistaferri (2004) who reject the existence of an individual-specific random growth term⁴ that was proposed in early work by Lillard and Weiss (1979) for example.

The firm enters the model through the match-specific productivity $\nu_{i,j(a_0),a,t}$ of worker *i* at firm $j(a_0)$ at tenure $a - a_0$ and time *t* that follows a law of motion

$$v_{i,j(a_0),a,t} = \begin{cases} v_{i,j(a_0),a,t}^p + v_{i,j(a_0),a,t}^t & \text{if } J_{i,a,t} = 0\\ v_{i,j(a),a,t}^{init} & \text{if } J_{i,a,t} = 1 \end{cases}$$
(4)

where

$$v_{i,j(a_0),a,t}^p = \begin{cases} v_{i,j(a_0),a_0,t-1}^{init} + \kappa^p \xi_{j(a_0),t}^p + \psi_{i,j(a_0),a,t}^p & \text{if } a = a_0 + 1\\ v_{i,j(a_0),a-1,t-1}^p + \kappa^p \xi_{j(a_0),t}^p + \psi_{i,j(a_0),a,t}^p & \text{if } a > a_0 + 1 \end{cases}$$

$$(5)$$

$$v_{i,j(a_0),a,t}^{\iota} = \kappa^{\iota} \xi_{j(a_0),t}^{\iota} + \psi_{i,j(a_0),a,t}^{\iota}$$
(6)

$$v_{i,j(a),a,t}^{init} = \tau_{j(a)} + \psi_{i,j(a),a,t}^{init}.$$
(7)

The initial match value of a job $v_{i,j(a),a,t}^{init}$ is affected by firm characteristics $\tau_{j(a)}$ and an idiosyncratic match component $\psi_{i,j(a),a,t}^{init} \sim N\left(0, \sigma_{\psi^{init}}^2\right)$. In subsequent periods, the match effect consists of two components to distinguish the role of permanent and transitory firm-level shocks following Guiso, Pistaferri and Schivardi (2005). The first component is a random walk that is updated each period based on transmission of permanent firm-level shocks ξ^p and permanent match-specific shocks $\psi^p \sim N\left(0, \sigma_{\psi^p}^2\right)$. The second component is a transitory

 $^{^4}$ Guvenen (2007) points out that the power of the tests used in these papers might be low since most panel data sets used in the literature follow individuals for a limited period of time or are subject to substantial attrition, which implies that higher-order auto covariances are estimated on a small sample of individuals. He further shows that the random growth model is consistent with the observed pattern of increased earnings variance over the life cycle if one allows for learning about individual-specific growth rates. In principle, an individual-specific growth term in addition to a permanent component and a transitory component is identifiable in our model.

shock to the value of the match that is determined by transmission of transitory firm-level shocks ξ^t and transitory idiosyncratic shocks $\psi^t \sim N\left(0, \sigma_{\psi^t}^2\right)$. We will estimate the distribution of firm level shocks ξ^p and ξ^t directly from the data and treat them as exogenous in the model estimation.⁵

The existence of a match-specific effect has been motivated theoretically within the search and matching framework and empirically by papers such as Topel and Ward (1992) and Abowd, Kramarz and Margolis (1999). Most studies on earnings dynamics, however, have not explicitly modeled the firm side. Low, Meghir and Pistaferri (2010) include a match-specific component in the wage process, but in their paper the match is not allowed to change within the firmworker relationship and is not subject to shocks that could be related to firmlevel productivity. These additions to the match-specific component are one of the contributions of our work compared to earlier studies.

Participation and Mobility One of the key issues is controlling for selection into work and for job mobility, both of which may truncate the distributions of shocks. We model participation decisions E and mobility choices J similar to Low, Meghir and Pistaferri (2010) and Altonji, Smith and Vidangos (2013) as

$$E_{i,a,t} = \mathbb{1}\left\{z_{i,a,t}'\delta + \phi_1\left(P_{i,a,t} + \varepsilon_{i,a,t} + v_{i,j(a_0),a,t}\right) + u_{i,a,t}^E > 0\right\}$$
(8)

and

$$J_{i,a,t} = \mathbb{1}\left\{z_{i,a,t}'\theta + b\left(v_{i,j(a),a,t}^{init} - v_{i,j(a_0),a,t}\right) + u_{i,a,t}^J > 0\right\}$$
(9)

where $u^E \sim N(0,1)$ and $u^J \sim N(0,1)$. These equations are "reduced form" in the sense that we do not fully specify the structure of the problem in terms of intertemporal utility taking dynamics into account.

 $^{{}^{5}}$ In a previous benchmark case, we alternatively model the match effect as an AR(1) process without distinguishing different types of firm shocks.

Note that there is a vector of observable characteristics z that affects both of these choices, where $x_{i,a,t}$ is a subset of $z_{i,a,t}$. Participation is affected by the entire wage residual as determined by the various individual-level and matchlevel shocks. The coefficient ϕ_1 is the incentive effect of working, it reflects the importance of unobserved heterogeneity in participation choices. Job mobility on the other hand only depends on the difference in match values because permanent and transitory productivity shocks do not depend on a particular firm match but are portable characteristics of a worker across jobs. The importance of wage differences as opposed to worker observables in determining mobility is captured by b.

We assume that u^E and u^J are normally distributed with unit variances (a normalization) and that all shocks $\{\epsilon, \zeta, \psi^t, \psi^p, \psi^{init}, u^E, u^J\}$ are uncorrelated with each other.

Labor Market Frictions and Job Offers Finally transitions occur in a frictional labor market where workers receive job offers with arrival rate λ_U during unemployment and λ_E while on the job. Furthermore, existing matches are exogenously separated at quarterly rate λ_S . If a worker receives a new offer, we also model the origin of the offer. We classify firms in bins based on their sector and firm size and we assume that the offer can originate from the same bin, from a firm of similar size but different sector, from a firm in the same sector of different size or from a firm in a different sector and size group entirely,

$$Pr(sector, size) = \omega_0 + \omega_1 \mathbb{1}\left\{sect_{j(t)} = sect_{j(t_0)}\right\} + \omega_2 \mathbb{1}\left\{size_{j(t)} = size_{j(t_0)}\right\}.$$
(10)

3 Data

3.1 The Data Set

We have put together a matched employer-employee data set that combines information from four different data sources, compiled by Statistics Sweden. The first is the Longitudinal Database on Education, Income and Employment (LOUISE) that contains information on demographic and socioeconomic variables for the entire working age population in Sweden from 1990 onwards. We use information about age, gender, municipality of residence, number and ages of children, marital status and education level as well as the collection of public transfers such as disability, public pension, sickness, unemployment and parental leave benefits. All variables in LOUISE are registered on a yearly basis.

The second data set is the Register-Based Labour Market Statistics (RAMS) that contains information about the universe of employment spells in Sweden from 1985 onwards. On the worker side, RAMS registers the gross yearly earnings and the first and last remunerated month for each employment, as well as firm and plant identifiers. On the firm side, RAMS registers information about institutional sector, industry and the type of legal entity for all firms with employees. The third data set is the Structural Business Statistics (SBS) that contains accounting data and balance sheet information for all non-financial corporations in Sweden from 1997 onwards, and a subset of corporations during 1990–1996. The fourth data set is the Unemployment Register that contains all spells of unemployment registered at the Public Employment Service.

Since the Structural Business Statistics covers all non-financial corporations in Sweden only from 1997 onwards, we focus the analysis on the period 1997– 2008. We include all firms with the legal entity being limited partnership, limited company other than banking and insurance companies or economic association, but exclude personal companies because data for these firms does not cover the entire sample period. The final sample represents 83 percent of value added and 83 percent of employment in the Swedish private sector over 1997–2008.

Table 1 presents descriptive statistics of the firms in our data set. The data contains 98,630 unique firms and 678,792 firm-year observations from 1997 to 2008. The four sectors construction, manufacturing, retail and services account for 17%, 19%, 24% and 40% of firms in the sample respectively. Within sectors, larger firms on average have higher revenue per worker. The growth rate for revenue per worker does not follow the same pattern across sectors. For construction, larger firms grow more slowly on average, whereas growth rates are higher for larger firms in the other sectors.

We include all individuals who work at firms in our sample at some point during 1997–2008. We use the data from RAMS together with registrations of unemployment at the Public Employment Service to define employment on a quarterly basis. We keep the main employment per quarter, that is, the employment accounting for the largest share of quarterly earnings, and define a worker as employed if working at least 2 months for any employer during the quarter. In each quarter, we record if an individual is a job mover, a job stayer or an entrant from non-employment. Monthly earnings are recorded based on the yearly earnings and the number of remunerated months as registered in the RAMS data.

We exclude individuals until the last year they receive public study grants at the beginning of their working life. We also exclude individuals from the first year they receive disability benefits, occupational pension or public pension at the end of their working life. We further exclude individuals when they move to a workplace that is not in the firm sample, mainly to the public sector, to self-employment or to a financial corporation. Importantly, however, we keep all the records of non-employment that are in connection with employment spells

		Firm size: num	ber of employee	s
	5 - 20	20 - 50	50 - 100	100 +
A. Construction				
Number of unique firms	$14,\!532$	1,556	360	343
Number of firm-year obs	89,824	8,711	1,805	1,413
Revenue per worker	$1,\!384,\!364$	$1,\!535,\!071$	1,771,355	$2,\!355,\!350$
	$(1,\!854,\!185)$	$(3,\!196,\!727)$	$(3,\!238,\!043)$	(3, 542, 072)
Growth, log revenue/worker	0.0367	0.0432	0.0204	0.0199
	(0.3932)	(0.4609)	(0.4169)	(0.4864)
B. Manufacturing				
Number of unique firms	$13,\!156$	2,788	$1,\!130$	1,213
Number of firm-year obs	$97,\!404$	$22,\!638$	9,604	10,286
Revenue per worker	$1,\!662,\!272$	$1,\!823,\!414$	$2,\!070,\!341$	$3,\!219,\!025$
	$(5,\!793,\!189)$	$(2,\!203,\!141)$	$(2,\!736,\!472)$	(41, 400, 000)
Growth, log revenue/worker	0.0261	0.0326	0.0321	0.0385
	(0.3827)	(0.3550)	(0.2831)	(0.2846)
C. Retail Trade				
Number of unique firms	20,393	2,322	636	488
Number of firm-year obs	149,027	$17,\!590$	4,652	3,583
Revenue per worker	$3,\!313,\!787$	$4,\!322,\!443$	$4,\!949,\!602$	5,503,985
	$(7,\!885,\!840)$	$(8,\!932,\!304)$	$(9,\!464,\!565)$	(8, 890, 912)
Growth, log revenue/worker	0.0126	0.0172	0.0303	0.0413
	(0.4025)	(0.4144)	(0.4698)	(0.4291)
D. Services				
Number of unique firms	$33,\!197$	4,330	1,188	998
Number of firm-year obs	$217,\!202$	29,777	8,099	7,177
Revenue per worker	1,468,543	$1,\!815,\!600$	$2,\!518,\!315$	2,344,788
	$(4,\!915,\!948)$	$(7,\!053,\!932)$	$(12,\!000,\!000)$	(10, 300, 000)
Growth, log revenue/worker	0.0289	0.0313	0.0420	0.0348
	(0.4938)	(0.5556)	(0.5740)	(0.5261)

Table 1: Summary statistics, firms

Note: Revenue per worker is reported in real SEK for base year 2007.

at the firms in our sample.

We perform separate estimations for men and women in two education groups: workers with at most high school education and workers with at least some college education. Table 2 presents summary statistics for each group of workers. Individuals with less than high school or high school education are included from age 21 and individuals with some college are included from age 26. Table 2 shows that workers with lower education are on average slightly older, which reflects changes in years of schooling across cohorts. Workers with lower education are also less likely to have children living at home. The employment rate increases with education, but the fraction of the employed who remain at their current job each quarter is fairly constant across groups. More educated workers are more likely to move from job to job, and less likely to enter a new job from non-employment. The data indicate that job to job mobility and transitions between employment and non-employment are fairly common. Each quarter, 2–3 percent of employed workers change jobs and 2–3 percent enter employment after a period of non-employment.

The comparison between men and women shows that both female education groups are less likely to participate in the labor market compared to their male counterparts. Female workers are also slightly younger and more likely to have children living in the household. Women are employed in services and retail trade much more frequently, whereas men are more likely to work in manufacturing or construction.

Table 2 shows that the average level of earnings differs quite a bit across education groups, but also across gender for a given level of education. We can take a more detailed look at life-cycle earnings profiles in Figure 1, using observations across different cohorts in the data. In particular, within each gender-education group we construct five-year cohort groups and separately plot

Table 2: Summary statistics, workers				
	Me	n	Wome	en
	\leq High school	College	\leq High school	College
Number of unique workers	$1,\!405,\!298$	476,007	$736,\!982$	$329,\!946$
Number of quarter-year obs	$38,\!876,\!150$	$11,\!663,\!760$	$17,\!890,\!145$	$6,\!465,\!541$
Monthly earnings, 2007 SEK	$25,\!165$	$37,\!623$	19,033	$26,\!994$
	$(10,\!756)$	(26, 764)	(7, 986)	(15,039)
Age	41.3	40.8	41.1	39.6
	(11.8)	(9.64)	(11.6)	(9.6)
Married	0.5463	0.6240	0.5948	0.6092
Having children living at home	0.3651	0.4613	0.4497	0.4953
Employed, of which	0.8722	0.9066	0.8105	0.8603
Job stayer	0.9543	0.9529	0.9488	0.9462
Job mover	0.0222	0.0294	0.0202	0.0262
Re-entrant	0.0235	0.0176	0.0310	0.0276
Industry				
Construction	0.1591	0.0660	0.0337	0.0271
Manufacturing	0.4290	0.3800	0.3066	0.2402
Retail Trade	0.1698	0.1260	0.2729	0.1748
Services	0.2420	0.4280	0.3867	0.5579
Firm size				
5-20	0.2906	0.2386	0.2695	0.2658
20-50	0.1392	0.1228	0.1401	0.1227
50 - 100	0.1064	0.0903	0.1030	0.0837
100+	0.4638	0.5482	0.4874	0.5277

Table 2: Summary statistics, workers

their average log earnings over the age period in which we observe this particular cohort group. The vertical distance between different cohort groups' earnings for a given age can then be interpreted as cohort-group effects. Overall, we observe the familiar life-cycle earnings profile increasing quite quickly early in the career and then flattening or slightly decreasing towards the end of the lifecycle. Level-differences show the absolute gain from achieving a higher level of education. The comparison between men and women reveals a level advantage for men in each education group that is consistent with different occupational choices, different hours choices or higher labor market experience for men with more women choosing part-time work or taking maternity leave. For women, earnings appear to be flat or slightly decreasing in their main childbearing years, which occurs in their late 20s for women with at most high school degrees and in their mid-30s for women with at least some college education. We will come back to this pattern when we estimate selection patterns into the labor market as the first step of our analysis.

Figure 2 presents the development of the variance of residual log real earnings, when year and age effects have been removed. For men, the variance of earnings increases over the life-cycle for each successive cohort, but much more strongly for higher educated workers. For women in both education groups, we observe a strong increase in wage variation early in their careers, followed by a decline in wage dispersion and another increase towards the end of their life-cycle. The largest increase in the variance of earnings by age takes place for men with college education. This might indicate a larger degree of risk taking for this group, which also seems to pay off in terms of average earnings, as indicated in Figure 1.

Figure 3 presents the employment rate by age for each education group and gender. As a result of our sample selection to capture working-age individuals



Figure 1: The development of log real monthly earnings by five-year cohort groups and average age, 1997-2008

Figure 2: The development of the residual variance of log real monthly earning by five-year cohort groups and average age, 1997-2008





Figure 3: Quarterly employment rates by gender, age and education group

Figure 4: Quarterly re-entry rates from non-employment by gender, age and education group



with some attachment to the labor market, participation rates are above 70% for all age groups. The lower the achieved level of education, the lower participation at young ages. Interestingly, there is an increase in participation from the beginning of individuals' careers until their mid-50s for male high-school graduates, whereas participation for male workers with some college education quickly levels off at 90%. For women, we see the familiar pattern of temporary absence from the labor market in their mid-20s for high school or less and in their mid-30s for some college education. The figure also shows a substantial drop in employment above age 60 in all education groups. Figure 4 shows that young workers across all gender and education groups have high quarterly reentry rates when out-of-work. The entry rate from non-employment is particularly large for younger and low educated workers. As Figures 3 and 4 illustrate, transitions in and out of employment seem to be an important feature of the



Figure 5: Quarterly job-to-job transition rates by gender, age and education group

labor market. Figure 5 presents the quarterly job-to-job transition rates by age for each gender-education group. The importance of job to job transitions is particularly high at younger ages and more so for higher educated workers over most of the life-cycle.

Finally, Figure 6 presents the average number of employers for each cohort and age that we observe over the sample period from 1997 to 2008.⁶ The figure confirms that the change of employer is an important feature of the labor market. Individuals of age 20–25 in 1997 had on average more than 2 employers until 2008.

4 Estimation Strategy

The estimation of the model is quite complex because of the dynamics and because of transitions between work and different jobs. Hence we will proceed in several steps. In particular, we will estimate the dynamics of productivity shocks ignoring wages and use these estimation results in the subsequent estimation of the model. Furthermore, we will decompose the model estimation into a twostage procedure.

⁶A previous version of this graph used the entire longitudinal data on firms and workers (instead of starting in 1997) and showed a steady increase in the average number of employers as cohorts were in the labor market for a longer period of time.



Figure 6: Number of jobs by age, education and gender in five-year cohort groups

4.1 First Stage: Selection Model

In the first stage, we estimate a selection model of earnings and labor market participation in order to get estimates for γ and δ in equations (1) and (8) respectively. These coefficients are identified from the cross-section of workers without taking job transitions into account. Estimation follows a Heckman twostep procedure that has to take into account that there is a discrepancy between the frequency of employment decisions and earnings in the model and the frequency at which we observe participation and wages in the data. In theory, we assume that all decisions of individuals and firms happen at a quarterly basis. Yet, in the data we only observe wages as an annual monthly average over all quarters. As a result, our observed outcome variable in levels is the average quarterly wage,

$$w_t = \frac{\sum_{qt=1}^4 w_{qt}}{\sum_{qt=1}^4 E_{qt}}$$

To make the model consistent with the data, we need to aggregate quarterly selection correction terms in the annual wage model. More formally, the conditional expectation of observed average quarterly wages assuming a lognormal distribution of the error term yields the empirical specification

$$\log E[w_t | x_t, z_t, E = 1] = x_t'\beta + \log\left[\sum e^{\rho\lambda(z_{qt}'\delta)} / \sum E_{qt}\right] + \frac{1}{2}\sigma_v^2.$$
(11)

The last term in this equation explicitly shows the bias from aggregating individual wage information at annual frequency, even though wages are determined at a higher frequency. This aggregation bias term is reminiscent of the bias due to individual heterogeneity that Blundell, Reed and Stoker (2003) find when aggregating individual wages to aggregate wage measures.⁷

Moreover, the second term is a nonlinear function of quarterly Mills ratios $\lambda \left(z'_{qt}\delta\right)$. This term implies that seasonality of participation decisions can introduce a second bias when running a simple linear specification of log wages on individual characteristics, even when controlling for selection. If some of the decision criteria for participation z_{qt} change at quarterly frequency, a nonlinear specification is needed that takes seasonal changes in participation into account when aggregating employment choices to the annual level. The estimation approach based on equation (11) then controls for these two sources of aggregation bias that occur because of data availability and can be used to get consistent estimates of γ and δ .

To estimate the selection effects we use region-time fixed effects in the quarterly participation equation as excluded instruments. These instruments are motivated by the fact that income taxes in Sweden are determined at a community level and inflation, in particular housing or rental prices, differ widely

⁷Note that under the assumption of a lognormal distribution of the error term, the additional heteroskedastic term $\frac{1}{2}\sigma_v^2$ will be absorbed by the constant in the regression.

across regions. As a consequence, individuals' opportunity cost of working differs across regions and time. However, we assume that the labor market is integrated and that other than fixed regional effects and time effects the interactions can be excluded (see for example Blundell, Duncan and Meghir, 1998)

Based on this first stage, we can treat the wage residual $P_{i,a,t} + \varepsilon_{i,a,t} + v_{i,j(a_0),a,t}$ as given. We use these residuals as input into the second stage where we estimate the remaining parameters of the model,

$$\boldsymbol{\beta} = \left\{ \boldsymbol{\delta}, \boldsymbol{\theta}, \kappa^p \kappa^t, \boldsymbol{b}, \boldsymbol{\rho}, \sigma_{\zeta}^2, \sigma_{\varepsilon}^2, \sigma_{\psi^p}^2, \sigma_{\psi^t}^2, \sigma_{\psi^{init}}^2, \sigma_P^2, \lambda_U, \lambda_E, \lambda_S, \omega \right\}$$

4.2 Second Stage: Indirect Inference

We estimate the remaining model parameters in the second stage by indirect inference (Smith (1993), Gourieroux, Monfort and Renault (1993)). We use MCMC techniques following Chernozhukov and Hong (2003) to fit the model to auxiliary moments that capture the main aspects of labor market dynamics.

4.2.1 Simulation Algorithm: Initial Match Effect

One of the key computational advantages of the proposed theoretical framework is that we do not need to simulate the entire life-cycle of workers. Workers who start a new job after an unemployment spell have lost their search capital and the quality of the new match does not depend on previous experience in the labor market. Furthermore, we can use the selection model estimated in the first stage to measure the annual wage residual and treat it as observed for the second-stage estimation. We would like to condition on the match effect at the end of the first year of a given worker in the labor market and simulate forward. Yet the annual wage residual is based on the sum of quarterly wages and is thus a nonlinear combination of quarterly shocks over the year,

$$u_{it} = \sum_{q=1}^{4} e^{(P_{qit} + \epsilon_{qit} + v_{qit})}.$$

This structure poses a challenge to this type of forward simulation approach for the following reason. Take someone who works in the firm for the whole year. The problem is that not all combinations of draws over the year are consistent with what we observe because we know the worker did not go into unemployment and did not change jobs. In order to simulate the match effect at the end of the year, we need to condition on the total residual we observe, and on the entire employment history during that year. All residuals in every quarter must be consistent with the person working in the four quarters in that firm. Hence this procedure would require a complex algorithm that draws a sequence of residuals that add up to the total required *and* are consistent with the observed behavior.

Given these insights, we restrict the estimation sample to those workers who were previously unemployed and start a new job in the fourth quarter of a given year. As a result, we directly observe the wage residual of these workers that corresponds to their fourth-quarter salary. We elegantly avoid issues of consistency within the year and back out the match effect at the current job given the observed quarterly wage residual. Based on a guess for the remaining parameters β , we can simulate the permanent and transitory wage shocks for each worker in the first period of their new job. The variance of the permanent shock is only a function of age. Conditional on simulated individual and firmlevel shocks and observing the wage residual, we know the value of v_{ijt}^{init} in the first period of the new job. This value combined with future shock draws can be used to simulate a person's behavior forward.

4.2.2 Auxiliary Model

The key feature of indirect inference is the use of an auxiliary model to form the objective function. As a simulation-based estimation procedure, we compute auxiliary moments based on the simulated population and in the real data. Indirect inference chooses the structural model parameters that make the simulated population as similar as possible to the real data through the lens of the auxiliary model. As a result, the key task and art of this estimation procedure is to choose a set of auxiliary moments that are able to capture the important data features that the economic model tries to explain. Ideally changes in auxiliary moments can be directly related to changes in structural parameters in an intuitive way since formal identification proofs are mostly absent in the literature using these simulation techniques (see Altonji, Smith and Vidangos (2013) or Bagger, Fontaine, Postel-Vinay and Robin (2011) for recent examples).

This section describes the choice of the auxiliary model and explains in detail how the moments are computed in the real data and in the simulation. It is important to make these two counterparts consistent in order to get consistent estimates. One important issue is the frequency of the data. Employment and job mobility decisions are observed at a quarterly frequency. Yet wages and firm shocks are only observed yearly. We want to exploit the additional variation in choices and transition probabilities in order to estimate quarterly variances of permanent and transitory shocks to the worker. At the same time we need to acknowledge the coarser structure of the data with respect to wages in order to estimate the wage equation for example. In practice, we will assume quarterly processes for all shocks in the simulation, but in order to compute moments corresponding to the outcome in the real data, we need to aggregate firm shocks and wages within each year. For wages we need to be careful to maintain the properties of the shock process by aggregating in levels and taking

Table 5. Quarterly I articipation model							
$1 \left\{ F = 1 \right\}$	Μ	ale	Female				
$\mathbb{I}\left\{ L=1\right\}$	low educ	high educ	low educ	high educ			
constant	0.5591	0.8381	0.4518	0.7817			
age	0.0646	0.0085	0.1140	0.0237			
age^2	-0.0111	-0.0025	-0.0207	-0.0042			
$V(U_t^E E_t = E_{t-1} = 1)$	0.1087	0.0836	0.1449	0.1170			

Table 3: Quarterly Participation Model

Table 4. Quarterly Job Mobility Model						
$1 \int I = 1$	М	lale	Female			
1 (J = 1)	low educ	high educ	low educ	high educ		
constant	0.0605	0.0744	0.0618	0.0636		
age	-0.0128	-0.0142	-0.0117	-0.0096		
age^2	0.0013	0.0012	0.0010	0.0000		
	0.0001	0.0000	0.0000	0.00.01		
$V(U_t^{o} E_t = E_{t-1} = 1)$	0.0221	0.0289	0.0203	0.0261		

Table 4: Quarterly Job Mobility Model

logs afterwards.

The auxiliary model first contains linear probability models for quarterly participation and job mobility which relate directly to the structural parameters δ and θ respectively. Table 3 shows that participation is a concave function of age, whereas Table 4 shows that mobility for both men and women is negatively related to age. Of course these are no structural estimates but auxiliary patterns in the data that we wish to replicate in the simulation. For these quarterly regressions, we assume that demographics are fixed within a year. This is in line with what we observe in the data because we only have information about individual characteristics at one specific date each year.

Furthermore, we compute quarterly transition rates into and out of unemployment and across jobs that relate to job offer and separation rates $\lambda_U, \lambda_E, \lambda_S$ as well as to transitory and permanent shock variances and the variance of new offers. Participation rates by age groups are an important indicator of how important wage residuals are (ϕ_1) relative to observables $z'\delta$ and shock variances.

Quarterly job transition rates across sectors and firm size groups help identify job offer probabilities ω . Finally, in order to identify firm-draw probabilities, we need to add moments that describe the distribution of workers across bins in our sample and that describe transition probabilities conditional on destination. We assume that offer arrival probabilities are fixed across the lifecycle, so differences in transition rates conditional on destination by age should only be driven by the variance of the initial match effect. If workers have been in the labor market longer, they have moved to better matches over time, so now they are only willing to switch sectors if they receive a very good offer. This is more likely with thicker tails of the normal distribution of match effects, i.e. higher variance in a given firm bin. These auxiliary moments by gender-education groups can be found in Tables 4-6. Note that all transition and participation probabilities are computed for different age groups [21-25, 26-30, 31-40, 41-50, 51-60, 61-64]to improve the fit of the estimation. The data show a clear pattern of decreasing unemployment, reentry and job-to-job mobility as workers get older. Switching industries and firm types is also more common for younger workers across all gender-education groups.

We complement these quarterly moments by annual moments related to earnings. Since the model assumes quarterly processes for all shocks, all simulation outcomes are quarterly as well. As a result, we need to aggregate simulated outcomes such as firm shocks and wages within each year to make the simulation comparable to the observed moments. Only then can we simulate annual moments such as a regression of wage residuals on observed firm-level shocks⁸

$$\tilde{e}_t = a + b \cdot \Delta q_t + v_t,$$

⁸These results have to be updated depending on the assumptions we make about the types of shocks! At the moment we only estimate the effect of firm-level shocks on stayers, not on new entrants in the firm.

		Μ	lale	Female	
	age	low educ	high educ	low educ	high educ
$1 \{E_t = 0\}$	21-25	0.2023		0.2843	
Unemployment	26 - 30	0.1576	0.1123	0.2856	0.1642
by age groups	31 - 40	0.1246	0.0908	0.2092	0.1590
	41 - 50	0.1067	0.0843	0.1371	0.1098
	51 - 60	0.0920	0.0862	0.1160	0.0982
	61-64	0.1356	0.1207	0.1765	0.1236
$1 \{ E_t = 0 E_{t-1} = 1 \}$	21 - 25	0.0427		0.0590	
Quarterly Job	26 - 30	0.0262	0.0184	0.0477	0.0305
separation by age	31 - 40	0.0191	0.0138	0.0300	0.0262
	41 - 50	0.0158	0.0121	0.0185	0.0158
	51 - 60	0.0150	0.0121	0.0167	0.0134
	61 - 64	0.0260	0.0184	0.0289	0.0183
$\mathbb{1}\left\{E_t = 1 E_{t-1} = 0\right\}$	21 - 25	0.2258		0.1835	
Quarterly	26 - 30	0.1750	0.2413	0.1399	0.2122
Job creation	31 - 40	0.1571	0.1816	0.1348	0.1727
	41 - 50	0.1416	0.1462	0.1242	0.1488
	51 - 60	0.1280	0.1134	0.0961	0.1096
	61 - 64	0.0834	0.0734	0.0507	0.0690

Table 5: Job Creation and Separation Moments

		М	ale	\mathbf{Female}	
	age	low educ	high educ	low educ	high educ
	21 - 25	0.6794		0.6911	
Enter new sector	26 - 30	0.6137	0.7468	0.5769	0.7095
by age group	31 - 40	0.5721	0.6245	0.5600	0.5566
	41 - 50	0.5382	0.5854	0.6163	0.6388
	51 - 60	0.4649	0.5351	0.5445	0.5955
	61 - 64	0.3172	0.4479	0.3899	0.4582
	21 - 25	0.7251		0.7465	
Enter new firm size	26 - 30	0.6682	0.7860	0.6320	0.7579
by age group	31 - 40	0.6242	0.6770	0.6092	0.6033
	41 - 50	0.5931	0.6393	0.6675	0.6957
	51 - 60	0.5136	0.5872	0.5923	0.6602
	61 - 64	0.3558	0.4979	0.4324	0.5246
	21 - 25	0.6185		0.6454	
Enter new bin	26 - 30	0.5450	0.7060	0.5308	0.6750
by age group	31 - 40	0.5070	0.5701	0.5123	0.5139
	41 - 50	0.4789	0.5279	0.5664	0.5972
	51 - 60	0.4087	0.4799	0.4918	0.5574
	61-64	0.2684	0.4074	0.3474	0.4347

		М	ale	Female	
	age	low educ	high educ	low educ	high educ
$P(J_t = 1 E_{t-1} = 1, E_t = 1)$	21 - 25	0.0383		0.0393	
Job mobility	26 - 30	0.0310	0.0441	0.0290	0.0411
by age group	31 - 40	0.0244	0.0336	0.0218	0.0284
	41 - 50	0.0187	0.0241	0.0168	0.0203
	51 - 60	0.0143	0.0170	0.0123	0.0128
	61 - 64	0.0109	0.0123	0.0090	0.0086
	21 - 25	0.4252		0.3810	
Sectoral Switch	26 - 30	0.4006	0.4115	0.3692	0.3887
by age group	31 - 40	0.3894	0.4083	0.3796	0.3877
	41 - 50	0.3729	0.3990	0.3697	0.3541
	51 - 60	0.3558	0.3747	0.3589	0.3100
	61 - 64	0.3429	0.3533	0.3485	0.2370
	21 - 25	0.6056		0.6194	
Firm Size Switch	26 - 30	0.5828	0.5719	0.5989	0.5889
by age group	31 - 40	0.5706	0.5469	0.5757	0.5622
	41 - 50	0.5454	0.5138	0.5505	0.5456
	51 - 60	0.5040	0.4841	0.5174	0.5383
	61 - 64	0.4882	0.4422	0.4968	0.5530
	21 - 25	0.2709		0.2449	
$\operatorname{Bin}\operatorname{Switch}$	26 - 30	0.2465	0.2407	0.2294	0.2309
by age group	31 - 40	0.2329	0.2271	0.2251	0.2153
	41 - 50	0.2137	0.2084	0.2110	0.1910
	51 - 60	0.1867	0.1836	0.1904	0.1670
	61-64	0.1705	0.1641	0.1699	0.1242

Table 7: Job Mobility Moments

Table 8: Firm and Sector Shocks						
	Μ	lale	Fer	nale		
	low educ	high educ	low educ	high educ		
firm shock for stayers	0.0022	0.0069	0.0102	0.0145		
Var(shock residual)	0.0899	0.1757	0.1182	0.1822		
ACV1 (shock res stay)	0.0655	0.1398	0.0838	0.1300		
ACV2 (shock res stay)	0.0594	0.1285	0.0758	0.1204		
intraclass correlation	0.1113	0.1263	0.0733	0.0628		

autocovariances of the residual from this exercise, $\tilde{e}_t - \tilde{a} - \tilde{b} \cdot \Delta q_t$, or the intraclass correlation of wage growth,

$$\rho = \frac{\sum_{\text{firms } j} \sum_{\text{worker } k \in j} \sum_{l \in j, k \neq l} (\Delta \tilde{e}_{kt} - \Delta \bar{e}) (\Delta \tilde{e}_{lt} - \Delta \bar{e})}{Var(\Delta \tilde{e}_{it}) \sum_{j} n_j (n_j - 1)}$$

that are related to the structural parameter κ . The intraclass correlation coefficient describes the share of variation in wage growth that is due to variation across firms, i.e. the share of wage growth explained by a common factor, firm affiliation.

In order to understand the time series properties of match effects, we compute variances and autocovariances of wage growth for stayers and movers. Wage information in transition years is not very reliable because we do not know the exact timing of the switch. Instead we only know the quarter in which mobility occurred. We therefore choose to not use wage information for these years and instead use mover information by looking at residual wage growth across periods before and after the switch occurred, $\tilde{e}_{+1} - \tilde{e}_{-1}$. In doing so, it is important to control for the number of periods we are skipping ys and the number of job moves that occurred JJ, as more periods of job mobility and more job switches will tend to rise wages further,

$$\tilde{e}_{+1} - \tilde{e}_{-1} = const + c_1 \cdot JJ + c_2 \cdot ys + \epsilon_{jj}.$$

0	J			
	Μ	lale	Fei	nale
	low educ	high educ	low educ	high educ
$V(\tilde{e}_t E_t = 1, a = 0)$	0.1655	0.2401	0.2127	0.2225
$V(\Delta \tilde{e}_t E_{t-1} = 1, E_t = 1, J_t = 0)$	0.0412	0.0540	0.0606	0.0826
$C(\Delta \tilde{e}_t, \Delta \tilde{e}_{t-1} J_t = 0)$	-0.0382	-0.0309	-0.0513	-0.0445
$C(\Delta \tilde{e}_t, \Delta \tilde{e}_{t-2} J_t = 0)$	-0.0020	-0.0013	-0.0039	-0.0023
$C(\Delta \tilde{e}_t, \Delta \tilde{e}_{t-3} J_t = 0)$	-0.0006	-0.0007	-0.0003	0.0003
$C(\Delta \tilde{e}_t, \Delta \tilde{e}_{t-4} J_t = 0)$	-0.0001	0.0001	-0.0001	0.0004
$\operatorname{constant}$	0.0300	0.0369	0.0414	0.0642
mover eq number of moves	0.0302	0.0269	0.0385	0.0279
years skipped	-0.0214	-0.0178	-0.0263	0.0001
Var(movers' residual wage growth)	0.0853	0.0958	0.1198	0.1309
$Cov(U_t^E, U_t^w E_t = E_{t-1} = 1, J_t = 0)$	0.0023	0.0077	0.0222	0.0232
$Cov(U_t^E, U_t^w E_t = E_{t-1} = 1, J_t = 1)$	0.0102	0.0227	0.0013	0.0125
$Cov(U_t^J, U_t^w E_t = E_{t-1} = 1, J_t = 0)$	-0.0008	-0.0032	-0.0031	-0.0051
$Cov(U_t^J, U_t^w E_t = E_{t-1} = 1, J_t = 1)$	0.0099	0.0304	0.0244	0.0399

Table 9: Wage Dynamics Moments

The residual from this regression, $\tilde{\epsilon}_{jj}$ can then be used to compute the variance of between-jobs wage growth, which in turn will be informative about the variance of match-specific effects for example.⁹ Finally, the covariance between wage residuals and mobility residuals helps us identify the variance of match-specific effects whereas the covariance between wage residuals and participation residuals is affected by transitory and permanent shock variances.

5 Results

5.1 First Stage: Selection Model

5.1.1 Specification

We first estimate the wage equation (11) controlling for endogenous selection into work as modeled by equation (8). This first stage will yield consistent

⁹In principle, this residual could also be used to compute correlations with previous periods' wage growth in a similar spirit as autocorrelations. This will help us to sort out whether a worker just moved because of a preference shock and his wage growth reflects productivity changes that would have lead to wage increases in the previous job as well or whether the worker received a good match-effect and switched because of that.

estimates for the model parameters γ and δ that can then be treated as known in the subsequent second-stage estimation.

The basic observable controls in the wage regression will be a constant, a fourth-order polynomial in age, education dummies, children dummies by age groups,¹⁰ marital status, region-fixed effects and time-fixed effects. The latter are included because our model is entirely stationary and time effects are our best way of controlling for aggregate shocks that affect participation and wages.¹¹

The same set of control variables will also be included in the participation choice equation,¹² but we add region-time interactions by year that are excluded from the wage regression.¹³ Wages are allowed to vary across regions and time directly, for example between rural and urban areas in Sweden but we assume that differential changes across regions do not affect wages but only participation decisions. In particular, our argument is based on differential outof-job income across different regions because of differential changes in rental and housing prices over time. Hence workers' outside options and therefore their participation decisions are affected by a particular region-time trend. Figure 7 illustrates the large differential trends in housing prices over time for the six most-populated counties in Sweden that together account for more than 60 percent of the population.

¹⁰Note that we do not distinguish between different numbers of children within the same age group. The simplifying assumption here is that the effect of children on labor force participation of a parent is the same for any nonzero number of children in a particular age group in a particular period in time.

¹¹Preliminary descriptive analysis as in Figure 4 revealed the importance of cohort effects but each cohort is described by a unique combination of time effect and age. Hence, as is well known, including a separate cohort fixed-effect would lead to perfect multicollinearity.

 $^{^{12}}$ Because of data limitations, all of these control variables are fixed for all quarters of a given year. As a result, the nonlinearity in the selection term of equation (11) disappears and we can estimate a linear model.

 $^{^{13}}$ Yet note that since we impose parametric assumptions on structural errors following Heckman (1979), these exclusion restrictions are not necessary to guarantee identification of the model. As is well understood from the literature, the model is identified based on functional form assumptions. Nevertheless we want to exploit additional identifying power based on exclusion restrictions.



Figure 7: Housing Price Changes

Finally, we need to acknowledge the role of measurement error in employment. For example, it is quite common for individuals in Sweden to receive some payments from their employers while on parental leave. If these payments are considered the regular wage, then those individuals will be falsely considered to be employed and will appear as particularly bad working types in the data even though they should be considered out of work during that period. These cases would lead to overestimating the amount of low-productivity types in the labor market and will bias the estimation results.¹⁴ In order to address this type of measurement error, we directly include controls for parental leave and sickness benefits into our wage and participation equations.

5.1.2 Results

The results for the first-stage estimation are presented in Tables 10 and 11. For readability, we suppress region effects, time effects and region-time interactions in the participation equation and time and region effects in the wage regression. Instead we only report the coefficients for personal characteristics.

First, consider the results for participation choices in Table 10. The table

 $^{^{14}}$ Note that the familiar result of consistent estimates despite measurement error in the dependent variable does not apply for the participation equation because we estimate a nonlinear model. See Hausman (2001) for details.

	N	lale	Fe	male
	High School	Some College	High School	Some College
constant	0.1940***	0.9833***	0.3825***	1.0144***
	(0.008)	(0.019)	(0.013)	(0.026)
age	0.9059 * * *	0.4661 ***	0.7258***	0.3402^{***}
	(0.005)	(0.010)	(0.007)	(0.012)
age^2	-0.7315^{***}	-0.5345^{***}	-0.4293 * * *	-0.3274***
	(0.004)	(0.011)	(0.006)	(0.014)
age^3	0.2609^{***}	0.2313^{***}	0.1456^{***}	0.1445^{***}
	(0.001)	(0.004)	(0.002)	(0.006)
age^4	-0.0314***	-0.0326^{***}	-0.0187^{***}	-0.0210***
	(0.000)	(0.001)	(0.000)	(0.001)
child 0-3 yrs	-0.0576***	-0.0062***	-0.2948***	-0.1838***
	(0.001)	(0.002)	(0.001)	(0.002)
child 4-6 yrs	0.0264^{***}	0.0525 * * *	-0.0995***	0.0022
	(0.001)	(0.002)	(0.001)	(0.002)
child 7-10 yrs	0.0262***	0.0495^{***}	-0.0815***	-0.0017
	(0.001)	(0.002)	(0.001)	(0.002)
child 11-17 yrs	0.0341***	0.0983^{***}	-0.1007***	0.0108^{***}
	(0.001)	(0.002)	(0.001)	(0.002)
married	0.3060 * * *	0.2279^{***}	0.1476^{***}	0.1532^{***}
	(0.001)	(0.001)	(0.001)	(0.002)
parental leave	0.0205***	0.0318***	-0.0853***	-0.0583***
	(0.000)	(0.001)	(0.000)	(0.000)
sickness benefits	-0.0900***	-0.0987***	-0.0979***	-0.0872***
	(0.000)	(0.000)	(0.000)	(0.000)
Observations	$38,\!824,\!193$	$11,\!653,\!681$	$17,\!881,\!403$	6,463,989
Wald test $[df=220]$	19425.29	4102.50	5881.99	1623.23
Wald test [p-value]	0.0000	0.0000	0.0000	0.0000
Pseudo R-squared	0.0744	0.0406	0.1132	0.0688

Table 10: First-Stage Results: Participation Equation

*** p<0.01, ** p<0.05, * p<0.1

Note: All specifications include region, year and region-year fixed effects. Standard errors in parentheses. Wald tests report test statistics and p-values for the exclusion restriction of region-time interactions in each specification.

reports probit estimates, so the magnitude of the coefficients cannot be interpreted directly. Yet the sign patterns of the results are as expected. For men, having children up to three years of age significantly decreases the probability of participating in the labor market, but older children increase participation. Women with at most high school education are less likely to work if they have children and the effect is stronger the younger the children are. Highly educated women often seem to combine having children and a career. They are likely to temporarily leave the workforce when the child is very young but they reenter soon afterwards and are more likely to participate than women without children. This behavior is consistent with high-productivity types achieving higher education and being more likely to work. Temporary absence is facilitated by the Swedish system of parental leave benefits that offers 80% of previous earnings for up to 13 months with a very generous cap. The full benefit period only applies if the father also stays with the child for some time, which is consistent with the lower participation probability for men with young children. Interestingly, married men are more likely to work, but the same is true to a lesser extent for women as well.

The coefficients on parental leave and sickness benefits confirm the measurement problems in employment status described above. For example, parental leave payments increase the probability of being employed for men. The reason is that men usually only take out parental leave benefits for a few months. Yet employers are likely to add some bonus payments during this time, which makes these fathers appear working at low wages. For women the relationship is negative, but relatively small. Hence these results suggest that as for men, some women will be considered employed at low wages during their parental leave. The coefficient for sickness benefits is negative and significant for all gender-education groups, but a similar caveat applies: Short time sickness benefits will make individuals appear to be working nevertheless, but at a lower average wage.

Table 11: Wage equation							
	N	fale	Fei	male			
	High School	Some College	High School	Some College			
constant	9.6343***	9.9937***	9.1751***	9.9701***			
	(0.0075)	(0.0097)	(0.0103)	(0.0075)			
age	0.6124^{***}	0.7015 * * *	0.7811***	0.4164^{***}			
	(0.003)	(0.008)	(0.005)	(0.009)			
age^2	-0.3627***	-0.3701***	-0.4204***	-0.2532***			
	(0.002)	(0.007)	(0.004)	(0.009)			
age^3	0.1000^{***}	0.1095^{***}	0.1066^{***}	0.0730^{***}			
	(0.001)	(0.003)	(0.001)	(0.003)			
age^4	-0.0102***	-0.0133 * * *	-0.0108***	-0.0086***			
	(0.000)	(0.000)	(0.000)	(0.000)			
child $0-3 \text{ yrs}$	-0.0373***	-0.0169***	-0.1694***	-0.1351***			
	(0.000)	(0.001)	(0.001)	(0.002)			
child 4-6 yrs	-0.0059***	0.0234***	-0.0888***	-0.0514***			
	(0.000)	(0.001)	(0.001)	(0.001)			
child 7-10 yrs	-0.0069***	0.0167^{***}	-0.0785***	-0.0569***			
	(0.000)	(0.001)	(0.001)	(0.001)			
child 11-17 yrs	0.0006*	0.0230***	-0.0644***	-0.0525***			
	(0.000)	(0.001)	(0.001)	(0.001)			
$\operatorname{married}$	0.0838 * * *	0.1425***	-0.0213***	0.0388^{***}			
	(0.001)	(0.001)	(0.001)	(0.001)			
parental leave	-0.0421***	-0.0402***	-0.0916***	-0.0830***			
	(0.000)	(0.000)	(0.000)	(0.001)			
sickness benefits	-0.0698***	-0.1006***	-0.0855***	-0.1005***			
	(0.000)	(0.001)	(0.000)	(0.001)			
Mills ratio	0.2161^{***}	0.9770 * * *	0.4693^{***}	0.4431^{***}			
	(0.006)	(0.021)	(0.008)	(0.023)			
Mills ratio * age	-0.1113***	-0.5110***	-0.0534***	0.3696***			
	(0.004)	(0.017)	(0.005)	(0.015)			
Mills ratio * age ²	0.0148^{***}	0.0896***	0.0116^{***}	-0.0654***			
	(0.001)	(0.004)	(0.001)	(0.004)			
Observations	9,010.548	2,796.200	3,921.223	1.514.611			
R-squared	0.199	0.168	0.275	0.251			

*** p < 0.01, ** p < 0.05, * p < 0.1Note: All specifications include region and year fixed effects. Standard errors in parentheses. Wald tests report test statistics and p-values for the exclusion restriction of region-time interactions in each specification.

Next, consider the results for wages in Table 11. The results highlight the measurement issues related to parental leave and sickness benefits mentioned above and they also confirm the familar concave life-cycle profile of wages. These profiles are illustrated graphically in Figure 8. As we can see from the comparison with simple OLS earnings profiles in Figure 8, the model predicts that selection has an effect on the slope of the earnings profile. Positive selection into the labor market is stronger at early ages, which means that without selection correction, wage growth at the beginning of the life-cycle will be underestimated by looking at cross-sectional worker data as lower ability individuals enter the labor force later. This is an important finding that needs to be taken into account for analyses of wage inequality for example. Furthermore, we find increasing positive selection at the end of workers' careers again. One explanation could be early retirement based on disability, which is very common in Sweden and is more likely to be chosen by low-ability types. As a result, the wage decrease in the life-cycle of earnings is underestimated. Finally, the selection patterns for women are consistent with higher-educated women having children later in their lives, thereby leading to peak positive selection in their early to mid-thirties as illustrated in Figure 10 below.

Since the main focus of this analysis is on selection patterns across the lifecycle, we allow for a fairly flexible specification of the Mills ratio in the wage regression. In particular, we interact the Mills ratio with a second-order polynomial in age to illustrate life-cycle patterns.¹⁵ The overall selection coefficients by age corresponding to the regression results in Table 11 can be found in Figure 9. For male workers, selection is highest early in the life-cycle and decreases over time as lower-productivity types enter the labor market. Finally selection increases again as workers get closer to retirement age. The same qualitative

¹⁵However, it turns out that alternative models using (i) a simple Mills ratio or (ii) an even more flexible interaction with age dummies provide qualitatively very similar selection and earnings patterns. The results of these alternative specifications are available upon request.



Figure 8: Predicted Wages controlling for parental leave and sickness benefits (f0 = male, e0 = low-educated)

pattern holds for women with at most high-school education even though selection increases again quite strongly after the age of 45. Highly educated females are the exception here, they display increasing selection in their 30s and 40s as lower productivity types are more likely to decide to stay out of work to bring up children for example. These patterns directly mirror the results for earnings profiles taking selection into account in Figure 8.

Overall, the wage regression implies a positive and significant selection effect for all samples. As Figure 10 suggests, wage differences because of selection are in the range of 0-20 log points, where these effects are higher for groups with higher education. Interestingly, selection among women tends to be larger than for their male counterparts conditional on education group.



Figure 9: Selection Coefficient by Age (f0 = male, e0 = low-educated)

Figure 10: Average Selection Effects by Age (f0 = male, e0 = low-educated)



5.2 Firm Shocks

Before proceeding to the second-stage estimation of the remaining model parameters, we need to estimate the process of firm-level shocks that will be related to wages in the model. As a first approach, we use log output per worker as a firm-level measure of productivity. As in Guiso, Pistaferri and Schivardi (2005), we assume that the quarterly stochastic process of firm productivity can be decomposed into permanent and transitory components,

$$q_{jt} = q_{jt}^p + u_{jt}$$

where

$$q_{jt}^{p} = q_{jt-1}^{p} + \xi_{jt}$$
$$\xi_{jt} \sim N\left(0, \sigma_{\xi}^{2}\right)$$
$$u_{jt} \sim N\left(0, \sigma_{u}^{2}\right).$$

Note that in the data, we can only observe the annual aggregate shock

$$Q_t = q_t + q_{t-1} + q_{t-2} + q_{t-3}$$

where we drop notation for firm j for convenience.

We apply simulation-based estimation to estimate the quarterly firm-shock process. Given the parametric assumptions of the quarterly shock process, we guess the parameter vector $\{\sigma_{\xi}^2, \sigma_u^2\}$ and simulate firm productivity q_t for a set of hypothetical firms. Then we aggregate these simulated shocks to replicate the structure of the real data. Finally, we define a set of auxiliary moments that can easily be computed in the data as well as from the simulation and we minimize the distance between model and data in terms of these moments. In particular,

Table 12: Autocovariance of Output per wo	orker
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	Data Moment
$\operatorname{Var}\left(\Delta Q_{t}\right)$	0.1926
	(0.002)
$\operatorname{Cov}\left(\Delta Q_t, \Delta Q_{t-4}\right)$	-0.0481
	(0.0011)
$\operatorname{Cov}\left(\Delta Q_t, \Delta Q_{t-8}\right)$	-0.0039
	(0.0007)
$\operatorname{Cov}\left(\Delta Q_t, \Delta Q_{t-12}\right)$	-0.0030
	(0.0006)

Table 13 <u>:</u>	Results:	Firm-Shock	Process
	σ_{ξ}	σ_u	
_	0.2656	0.4075	
	(0.0008)	(0.0012)	

we identify the underlying parameters of the shock process from the variance and first-order autocovariance for the annual change in firm productivity. Table 12 shows that this is not a bad approximation because the autocovariances for productivity growth in the data are close to zero for the second and third order moments already. ¹⁶

Table 13 shows the estimation results. The implied process for quarterly output per worker shows considerable mean reversion in productivity innovations that matches the first-order autocovariance reported in Table 12 very closely. These results will be used to simulate firm shocks in the second-stage estimation procedure below.

16 Note that under the random walk assumption, we can rewrite
$\Delta Q_t = q_t + q_{t-1} + q_{t-2} + q_{t-3} - (q_{t-4} + q_{t-5} + q_{t-6} + q_{t-7})$
$= e_t + 2e_{t-1} + 3e_{t-2} + 4e_{t-3} + 3e_{t-4} + 2e_{t-5} + e_{t-6}$
$\Delta Q_{t-4} = e_{t-4} + 2e_{t-5} + 3e_{t-6} + 4e_{t-7} + 3e_{t-8} + 2e_{t-9} + e_{t-10}$

where $e_s = \xi_s + u_s$. Hence $Var(\Delta Q_t)$ and $Cov(\Delta Q_t, \Delta Q_{t-4})$ identify the underlying permanent and transitory shock variances.

5.3 Second Stage

5.3.1 Estimation Method

Finally, we use the results from the first stage and from the firm shock process to conduct the main estimation. We treat wage residuals as observed and start the simulation when a worker enters a new job out of unemployment. Given worker observables and a parameter guess for

$$\beta = \left\{ \delta, \theta, \kappa^p \kappa^t, b, \phi_1, \sigma_\zeta^2, \sigma_\varepsilon^2, \sigma_{\psi^p}^2, \sigma_{\psi^t}^2, \sigma_{\psi^{init}}^2, \sigma_P^2, \lambda_U, \lambda_E, \lambda_S, \omega \right\},\$$

we draw unobserved transitory and permanent productivity for each worker in the first period of the new job and back out the implied initial match value. In the second period, we draw updates to productivity and match effect and simulate the worker's response in terms of job mobility and participation given the parameter guess. Firm shocks are drawn from the productivity process estimated in Table 13. We simulate workers' behavior for the number of periods that we observe them in the data and compute the auxiliary moments of participation, mobility and wages for the simulated economy.

We maximize the GMM objective function

$$L_{n}(\theta) = -\frac{n}{2} (g_{n}(\theta))' W_{n}(\theta) (g_{n}(\theta))$$

where $g_n(\theta) = \frac{1}{n} \sum_{i=1}^n m_i(\theta)$ and $m_i(\theta)$ is a vector of differences between simulated moments $\Gamma^S(\theta)$ and data moments Γ^D such that

$$E[m_i(\theta_0)] = E\left[\Gamma^D - \Gamma^S(\theta_0)\right] = 0.$$

We use equally weighted minimum distance, W = I, for the reasons discussed in Altonji and Segal (1996). The objective function will not necessarily be a smooth function of the underlying model parameters and there are likely to be multiple local optima. As a result, we use a Laplace type estimator (LTE) as proposed by Chernozhukov and Hong (2003) to estimate the remaining model parameters.

The main computational advantage of the LTE approach is that it uses functions of the criterion function that can be computed by Markov Chain Monte Carlo methods (MCMC). In particular, we use the Metropolis-Hastings algorithm with uniform priors. We transform the objective function $L_n(\theta)$ into a quasi-posterior

$$p_{n}\left(\theta\right) = \frac{e^{L_{n}\left(\theta\right)}}{\int_{\Theta} e^{L_{n}\left(\theta\right)} d\theta}$$

and evaluate this function at the current parameter guess $\theta^{(j)}$ and an alternative draw ξ from a multivariate normal distribution. The parameter guess is then updated according to

$$\theta^{(j+1)} = \begin{cases} \xi & \text{with probability} \quad \rho\left(\theta^{(j)},\xi\right) \\ \theta^{(j)} & \text{with probability} \quad 1 - \rho\left(\theta^{(j)},\xi\right) \end{cases}$$

where

$$\rho(x,y) = \min\left(\frac{p_n(y)}{p_n(x)}, 1\right) = \min\left(e^{L_n(y) - L_n(x)}, 1\right)$$

Our estimator follows as the quasi-posterior mean

$$\hat{\theta} = \int_{\Theta} \theta p_n\left(\theta\right) d\theta,$$

which in practice can be computed as the average over all N_S elements of the converged Markov chain

$$\hat{\theta}_{MCMC} = \frac{1}{N_S} \sum_{j=1}^{N_S} \theta^{(j)}.$$

This estimation strategy is a good fit for our problem because MCMC only requires many function evaluations $L_n(\theta)$ at different parameter guesses but the method is derivative-free and can deal with multiple local minima quite well.¹⁷

5.3.2 Results [Please take the following results with caution. The estimation is in progress and highly preliminary.]

We report preliminary point estimates for high-educated male workers. We estimate a chain of 100,000 elements and report average coefficients across the entire chain, dropping the first 20,000 elements.¹⁸

The results for participation in Table 14 show the expected concave pattern in age as well as a positive selection effect in the labor market. Mobility is decreasing in age and the positive coefficient for b shows that mobility choices are highly influenced by the wage difference between incumbent and poaching firm. The large value of ϕ_1 is by itself not worrying because we normalize the variance of the error term in the participation equation to 1. Hence, instead of the absolute magnitude of the coefficients, we have to interpret the relative importance of ϕ_1 times the variances of productivity shocks on the one hand and worker observables like age on the other hand.

The results in Table 15 for standard deviations of productivity shocks are consistent with previous findings in the literature, for example Low, Meghir and Pistaferri (2010). These authors also report standard deviations for permanent and transitory shocks in the range of 15-20%. The transitory productivity shock is relatively high in our estimation, which can be related to measurement error in wages especially in transition years or imprecise estimation of ϕ_1 for example.

¹⁷See the discussion in Chernozhukov and Hong (2003) for more details.

¹⁸The first 20,000 elements of the chain are computed based on a preset error variance. For the subsequent chain, we use an adaptive procedure to take the covariance structure of the previous chain into account and to target an acceptance rate of 23.4%.

Table 14: I	Participation and Job Mobility
	Men with College Education
const_p	-2.0959
age_p	3.0952
${ m age2_p}$	-0.1089
$const_m$	-0.4826
age_m	0.3610
$age2_m$	-0.1217
ϕ_1	15.7786
b	0.8683

Table	15:	Worl	ker-le	vel l	Prod	uctivity	y Shocks
		Men	with	Col	lege	Educat	ion

The main contribution of our estimation is the law of motion for the match effect in Table 16. We allow for additional idiosyncratic transitory and permanent components that can be compared to the effect of firm-level shocks. The standard deviations of these shocks are an order of magnitude smaller than the productivity shocks in Table 15, which means a large share of permanent shocks to worker productivity is transferred to other jobs and is not specific to the current match. Firm-level shocks as estimated in Table 13 will affect wages at rates κ^t and κ^p . Transmission of transitory shocks is estimated to be close to zero, whereas permanent firm-level shocks positively affect workers' wages. The current estimates are still very preliminary, but the qualitative pattern is consistent with the findings in Guiso, Pistaferri and Schivardi (2005). Finally, the estimate for the standard deviation of initial match effects is smaller than the estimate reported in Low, Meghir and Pistaferri (2010) for example. Yet in our specification, the variance of the match effect grows with tenure, thus leading to a higher cross-sectional variance of match effects. Finally, we estimate quarterly offer arrival rates while unemployed and on-the-job as well as

Table	16:	Matc	h Eff	ect	and	Firm	Shocks
		Mon	mith	Cal	llogo	Edua	otion

Men with College Education
0.0004
0.0239
0.0059
0.0080
0.0272

 Table 17: Offer Arrival Rates and Job Separation

 Men with College Education

 λ^u

 0.1587

λ^u	0.1587	
λ^e	0.1532	
λ^s	0.0010	

an exogenous separation rate. Interestingly, offers arrive at very similar rates both for unemployed and employed workers. Exogenous separations are negligible because the model generates a sufficient amount of endogenous separations through participation choices.

6 Conclusion

[to be completed]

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