Wage Rigidity and Labor Market Dynamics with Sorting*

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Abstract

This paper adds two-sided ex-ante heterogeneity and a production technology inducing sorting to the canonical Diamond-Mortensen-Pissarides (DMP) search and matching model. Ex-ante heterogeneity and sorting have important implications for the dynamic properties of the model. The modifications solve the problem that standard DMP models do not generate enough volatility in response to shocks, also known as the "Shimer Puzzle" (Shimer, 2005). Amplification to overcome the volatility puzzle stems from an endogenously generated wage rigidity, which is of reasonable magnitude given empirical evidence from the U.S. labor market. Additionally, endogenous matching sets fluctuate in response to shocks and amplify job-creation. Using a standard Nash sharing rule, I show that the surplus function of the model, which depends on both workers' and firms' outside options, exhibits an asymmetry in equilibrium that stems from unequal bargaining powers. Using the standard calibration of the model, the firms' matching sets are wider in equilibrium than the workers' matching sets and fluctuate more in response to shocks.

Keywords: Sorting, Mismatch, Wage Rigidity, Heterogeneity, Unemployment, Search and Matching, Aggregate Fluctuations

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1 Introduction

A large literature in labor economics shows how frictions and information imperfections prevent labor markets from clearing. This explains the existence of equilibrium unemployment. Job seekers and employers have to spend time and money to locate a suitable counterpart and negotiate before eventually forming a match and starting production. This coordination friction can be understood as an outcome of heterogeneity. Workers and firms differ, for instance, with respect to their location or the supply and demand of specific skills, which makes finding a suitable partner and forming a match costly. This is the essence of the Diamond-Mortensen-Pissarides (DMP) search and matching framework. It incorporates the coordination friction elegantly without making heterogeneity across workers and firms explicit.[1] In this class of models, the wage is determined by simple Nash bargaining. Shimer (2005) points out that it is due to this sharing rule that a simple dynamic DMP model largely fails to generate sufficient volatility in response to aggregate shocks. He argues that this shortcoming, also known as the "Shimer Puzzle", has a distinct connection to the wage formation mechanism. Cyclical fluctuations in the value of unemployment, which is the worker's outside option, are absorbed entirely by the fully-flexible wage. This reduces the incentive to create new jobs on the firm side, thereby limiting the overall responsiveness of the model to shocks. Many papers show, in turn, that this volatility puzzle can be solved by making wages less responsive to shocks, either by simply assuming rigid wages (Hall (2005)), by modifying the calibration to increase the worker's outside option (Hagedorn and Manovskii (2008)), or by replacing the Nash sharing rule with a more credible bargaining game (Hall and Milgrom (2008)).[2]

The model I propose in this paper solves this problem in a new way. It generates rigid wages endogenously while sticking to the simple Nash sharing rule at the same time. This property arises as a result of explicit heterogeneity across workers and firms. I assume complementarity of types in production and, hence, positive assortative matching. This modification enables the model to overcome the Shimer Puzzle via adjustments of both the wage-formation mechanism and the firms' job-creation condition. In the light of recent advances and new insights into empirically identifying the sign and strength

[1] In the baseline DMP model, the arrival of suitable match opportunities is governed by a Poisson process which depends on the aggregate state of the labor market. In the dynamic setting, representative workers and firms base their optimal, forward-looking decisions solely on the expected law of motion of this aggregate state, which is the tightness of the labor market, defined as job-openings per unemployed worker.

[2] The volatility puzzle itself and approaches to solve it are still highly debated. Hornstein et al. (2005) and other papers discuss the modifications necessary to align the standard search and matching model with the data from the perspective of empirical plausibility.

of sorting in labor market data (Gautier and Teulings (2006), Andrews et al. (2008), Lopes de Melo (2009), Eeckhout and Kircher (2011), Hagedorn et al. (2012), Bartolucci and Devicienti (2013), Bagger and Lentz (2014)), the model provides a both micro-founded and empirically supported complement to existing approaches to better align search and matching models with the data.

My equilibrium framework is inspired by Hagedorn et al. (2012). Sorting makes the allocation of workers to jobs meaningful, both in terms of match-specific output and overall welfare. It implies that an optimal counterpart exists for every market participant. In the absence of labor market frictions, a firm (worker) would instantaneously match with the optimal employee (employer). This would lead to the Walrasian first-best allocation, just like in Gary Becker's neoclassical marriage market model (Becker (1973)). Due to frictions and costly search, however, this optimal, welfare-maximizing allocation of workers to jobs can never be realized. As shown by Shimer and Smith (2000), an equilibrium property in this class of models is that workers and firms are picky, that is, they are willing to match only with a subset of types in the vicinity of their optimal counterpart. The cardinality of these so-called matching sets depends on the degree of complementarity in production. The matching sets are endogenously determined and have important implications for the dynamics of the model. They directly influence both the values of being unemployed and maintaining an unfilled vacancy which are, in turn, the outside options in the Nash bargaining solution.

This paper is, to the best of my knowledge, one of the first two that consider business cycle fluctuations in a search and matching model with two-sided ex-ante heterogeneity and sorting. The other, Lise and Robin (2014), builds on Postel-Vinay and Robin (2002) and Robin (2011) to show that a model with on-the-job search and aggregate uncertainty matches the data well, also with respect to unemployment fluctuations. The key difference to my model is that the main theoretical result of Lise and Robin (2014) does not hold with a Nash sharing rule. In my model, the surplus function depends not only on the current state of the aggregate shock (as in Lise and Robin (2014)) but additionally on the agents' outside options which are functions on the matching sets. This leads to richer dynamics via two channels: The matching sets fluctuate in response to shocks and constitute an additional margin of adjustment in the dynamic job-creation condition. Additionally, the surplus function exhibits the following interesting property: unequal bargaining powers lead to an asymmetry between the outside options of workers and firms in equilibrium. Since workers typically get a larger share of the surplus in the standard calibration of search and matching models, firms' matching sets are wider in equilibrium and more volatile in response to shocks. In the wage equation, this appears as a relative, i.e. match-specific labor market tightness term which is on average lower than aggregate labor market tightness. This property shields match-specific wages from fluctuations of the outside option and, thus, generates a real rigidity.

To solely focus on the consequences of sorting for the dynamics of the model and analyze them transparently, I stick to most of the standard model's assumptions. Thus, I also abstract from search on the job. While this is without doubt a strong assumption, I argue that search on the job would not alter the conclusions I draw and, hence, can be left aside for the sake of clarity. On-the-job search takes effect in this class of models simply by relaxing the agents' pickiness. This is due to the possibility of continued search after matching with a non-optimal partner. The degree of pickiness, however, can be controlled by the degree of complementarity in production. I can thus analyze the influence of different degrees of pickiness in my model without the additional layer of complexity that search on the job would add. This allows me to analyze the mechanism behind the amplification I find as transparent as possible. For the same reason, I calibrate the model along the lines of Shimer (2005) for the U.S. labor market to conduct numerical simulations under aggregate uncertainty. My results are therefore directly comparable to both previous work on the dynamics of DMP models and the wider field of macroeconomic models with frictional labor markets. In fact, the link is so close that the model can be reduced to the well-known textbook version of the basic DMP model simply by collapsing the type space into a single point.[3]

The wage rigidity, my main finding, arises solely out of equilibrium properties of the search and matching model with sorting: I consider positive sorting based on both absolute and comparative advantage and show that the augmented model creates an elasticity of wages with respect to labor productivity of less than unity in both cases. This implies that wages do not fully adjust and do not follow labor productivity one-to-one as in the standard model. I find a moderate and empirically plausible elasticity of 0.75 (absolute advantage) and 0.87 (comparative advantage). These elasticities are close to the benchmark estimates for the U.S. labor market reported by Haefke et al. (2013), who find an elasticity of 0.8 with a standard error of 0.4. The moderate rigidity alone, however, does not generate sufficient amplification to overcome Shimer's volatility puzzle. I show that the second adjustment of the model with sorting, the modified job-creation condition, is the main source of additional volatility in response to shocks.

[3] I use a series or robustness checks to show that the simplifying assumptions do not induce any loss of generality with respect to the key mechanism at the core of the augmented model. The abstraction from search on-the-job is also further discussed in Section 4.

Firms' expected surplus changes along two margins: First, in response to a positive shock the surplus function shifts upwards. Second, additional workers with positive surplus enter the equation due to wider matching sets. Therefore, a higher number of additional vacancy postings is the optimal response to a positive shock. I conclude that a simple dynamic DMP model with heterogeneity and sorting does not need any additional modifications to generate rigid wages and amplification. The DMP model's volatility problem vanishes with sorting. In the presence of aggregate uncertainty, two channels contribute to the amplification effect: the endogenously generated wage rigidity and the effect of sorting on the forward-looking job creation decision.

The remainder of this paper is structured as follows: Section 2 develops the model and derives the wage-formation mechanism in the presence of heterogeneous agents and sorting as a bargaining outcome in the recursive framework. The equilibrium is characterized and the computational strategy is briefly outlined. Section 3 presents numerical simulations under aggregate uncertainty and compares the results of the augmented model to the baseline model in the light of empirical moments from the U.S. labor market data. The interplay between sorting, wage formation, and the model's dynamics is analyzed in more detail and robustness checks are presented. Section 4 discusses the results in the light of related theoretical and empirical literature. Section 5 concludes.

2 The Model

2.1 Setup

I construct a fully dynamic two-sided equilibrium search and matching model with discounting and transferable utility. Workers and firms are heterogeneous. Uncertainty with respect to worker and firm types is not considered, that is, agents know their own type and the types of all possible matching partners. Time is discrete. Workers and firms are risk neutral, infinitely lived, and seek to maximize their expected discounted future income streams. The parameter β represents the common discount factor. As in the canonical search and matching model, only unemployed individuals search randomly in this model; employed workers do not search. In case of unemployment, workers receive constant unemployment benefits b every period, which can also be interpreted as the value of non-market activity. Vacant firms have to pay per-period vacancy posting costs κ, representing expenses for posting vacancies, screening applications, and so forth. Productive activity commences when a firm and a worker who meet in the labor market are able to jointly produce a non-negative surplus given their types. If this is the case, the surplus is shared according to the standard Nash bargaining solution with workers having bargaining power α. Every period, matches between firms and workers may terminate for two possible reasons: One the one hand, they are subject to idiosyncratic separation shocks, which lead to immediate dissolution of the employment relationship. These shocks hit a match with an exogenous per-period probability δ. On the other hand, in the presence of aggregate uncertainty and business cycle fluctuations, the model allows for endogenous separations, which may occur at the margins of the agents' matching sets. When a negative productivity shock hits the economy, the surplus of previously marginally profitable employment relationships may become negative. Since a negative surplus is always less than both parties' outside option, they prefer to separate. This mechanism is in line with Mortensen and Pissarides (1994) only that in my model, aggregate rather than idiosyncratic productivity shocks trigger separations.

2.2 The Type Space

Following Shimer and Smith (2000), I assume atomless distributions of both worker and firm types. The population of infinitely-lived agents is constant, that is, the model is stationary. Workers are endowed with heterogeneous skills, indexed by s, and firms differ from each other in terms of job complexity, indexed by c. Both skills and job complexity can be viewed as a one-dimensional representation of a larger, multi-dimensional set of worker and firm characteristics. I assume that both firms and workers can be unambiguously ranked based on skills and job complexity.[4] Skills, s, and job complexity, c, are distributed uniformly on the open interval. The overall probability density function for workers and firms are time constant, normalized to 1, and denoted by, $g^w(s)$ and $g^f(c)$. Table 1 shows how these densities relate to the distribution of unemployed workers of type s, $g^u(s)$, and the distribution of vacant firms of type c, $g^v(c)$, which are both equilibrium objects that vary over time in the presence of aggregate uncertainty. They entail the probability of meeting a specific worker or firm type. The aggregate number of vacancies, V, and unemployed workers, U, are computed by taking the integral over the distribution of vacant firms or unemployed workers, respectively. $g^m(s, c)$ is the two-dimensional distribution of active matches. The integral over either dimension yields the probability distributions of employed workers, $g^e(s)$, and producing firms, $g^p(c)$. The

[4] In an empirical setting, constructing such a ranking given the available matched employer-employee data is indeed the most difficult part of identifying the sign and the strength of sorting, see also the discussions in Eeckhout and Kircher (2011) and Hagedorn et al. (2012).

Table 1: Labor market states, density and distribution functions, and aggregation

Description	Density/Distribution Function	Aggregate Value
Active matches	$g^m(s,c)$	$M = \iint g^m(s,c)\ \mathrm{d}s\mathrm{d}c$
Employed workers	$g^e(s) = \int g^m(s,c)\ \mathrm{d}c$	$E = \int g^e(s)\ \mathrm{d}s$
Unemployed workers	$g^u(s) = g^w(s) - g^e(s)$	$U = \int g^u(s)\ \mathrm{d}s$
Producing firms	$g^p(c) = \int g^m(s,c)\ \mathrm{d}s$	$P = \int g^p(c)\ \mathrm{d}c$
Vacant firms	$g^v(c) = g^f(c) - g^p(c)$	$V = \int g^v(c)\ \mathrm{d}c$

sum of both distributions of matched agents ($g^e(s)$ and $g^p(c)$) and unmatched agents ($g^u(s)$ and $g^v(c)$) has to equal the overall density functions for workers and firms, $g^w(s)$ and $g^f(c)$.

2.3 Production

The model economy is coined by complementarities between worker and firm types in production. Hence, output is match-specific. It is commonly assumed in this class of models that every firm has exactly one job to fill and I stick to this assumption. The worker's labor is the only input to production, that is, I ignore capital. Based on the complementarity in production, this paper's central assumption is positive assortative matching (PAM). PAM requires a supermodular production function.[5] Let $F(s,c)$ be this supermodular function which is increasing in both its arguments, the worker skill's s and the firm's job complexity c. It is non-negative, twice continuously differentiable, and strictly concave. Intuitively, this structure implies that the output, which a firm and a worker can jointly produce, is the higher, the more the complementarity in production can be exploited by a given firm-worker couple. Specific modeling strategies for this feature that have been suggested in the literature are either based on absolute advantage of high types (e.g. in Shimer and Smith (2000)) or on comparative advantage in the sense that each worker type has its own optimal firm (e.g. in Teulings and Gautier (2004)).

[5] Supermodularity necessitates positive cross-derivatives of the production function, $F_{sc} > 0$. That is, the higher the first argument of the function, the higher is the marginal value of an increase in the second argument. In the presence of costly search, Shimer and Smith (2000) show that for any search frictions and type distribution, log-supermodularity is a necessary condition for existence of a search equilibrium. This condition also implies convexity of the matching sets. Topkis (1998) provides more details on the application of closely related mathematical concepts from lattice theory.

This is further discussed in the empirical part of the paper in Section 3. For the derivation of the model, I can remain agnostic about the specific form of the production function.

To apply this kind of model to a study of business cycle dynamics, an additional aggregate component of production is necessary. It influences the output of every match in equal measure. This kind of aggregate uncertainty, driving the business cycle in this class of models, is commonly modeled as a stochastic process that resembles aggregate labor productivity. Let Y_t denote this process, which is normalized to one and calibrated to match an empirical measure of labor productivity. Being agnostic about the specific functional form of this process, it is sufficient for the derivation of the model to use subscript t to denote in the following that a model object depends on the state of aggregate labor productivity Y_t in a given period of time. In the recursive formulation, next period's state is denoted t'. I assume a simple multiplicative relation to link the match-specific component of output to the aggregate state:

$$F_t(s,c) = F(s,c) \times Y_t. \tag{1}$$

The output of a match between worker s and firm c in state t, denoted $F_t(s,c)$, is thus simply the product of the time-constant and match-specific part, $F(s,c)$, and the time-varying and type-independent aggregate labor productivity process, Y_t.

The recursive setup of the model developed in the following is largely inspired by Hagedorn et al. (2012). However, I simplify the timing structure to be in line with the baseline setup as in Pissarides (2000).[6] A period begins when the state of aggregate labor productivity is revealed. Workers and firms form their optimal decision rules based on this state. Both endogenous and exogenous separations take place. Then, new matches are formed but workers and firms separated in the same period start their search not until the next period. Finally, production commences.

2.4 The Dynamic Model

The model economy's labor market is characterized by random search of unemployed agents only. Labor market frictions prevent meeting the first-best counterpart. Thus, workers and firms rationally form matches as long as the surplus to share is weakly

6 I assume that matches cannot break up in the period they are formed. Hagedorn et al. (2012) assume a more complex timing structure with two subperiods: Matches are formed in the first subperiod. In the second subperiod, before production takes place, separation can occur for all matches, including newly formed ones. This assumption is inconsequential for the properties of the model.

positive. In the event of an encounter that would yield a negative surplus, no match is formed and both parties continue to search. The search technology is linear, that is, meetings are governed by a standard Cobb-Douglas type matching function with constant returns to scale, $M_t(U_t, V_t) = \vartheta U_t^{\xi} V_t^{1-\xi}$, where ξ $(1-\xi)$ represents the elasticity of new matches with respect to unemployment (vacancies). ϑ is a scale parameter. Since the matching function is homogenous of degree one, standard Poisson arrival rates arise as functions of labor market tightness $\theta_t = {}^{V_t}/_{U_t}$. In the frictional labor market, $q^v(\theta_t)$ is the rate at which vacant firms meet unemployed workers and, correspondingly, $q^v(\theta_t)$ represents the probability that unemployed workers meet vacant firms. $q^v(\theta_t)$ is decreasing and $q^u(\theta_t)$ is increasing in labor market tightness θ_t. To reduce notational clutter, let the two meeting rates simply be q_t^v and q_t^u. In the search and matching model with heterogeneity and sorting, not all meetings result in a productive employment relationship. If a firm and a worker meet and find that they are too different in terms of their types, that is, the surplus of the match would be negative, both parties prefer to continue their search. Thus, for every firm (worker) only a subset of the type space of workers (firms) qualifies for a match. Formally, two matching sets, $M_t^s(c)$ and $M_t^c(s)$, delimit the types of suitable counterparts for a firm c or worker s given the state t of the aggregate productivity process. Thus, the set $M_t^s(c)$ ($M_t^c(s)$) contains all worker (firm) types with which a given firm (worker) of type c (s) is willing to match. The surplus, $S_t(s, c)$, is weakly positive for those matches.[7]

$$S_t(s, c) \geq 0 \leftrightarrow s \in M_t^s(c) \leftrightarrow c \in M_t^c(s). \tag{2}$$

A joint, two-dimensional matching set $\mathcal{M}_t$ contains all matches (s, c) that form given the state in period t. Matching sets defined this way are mutually consistent, that is, no worker or firm is willing to match with a counterpart that would not agree to the match as well.

$$\mathcal{M}_t \equiv \{(s, c) : s \in M_t^s(c) \wedge c \in M_t^c(s)\}. \tag{3}$$

The possibility of meeting an unsuitable counterpart, i.e., $S_t(s, c) < 0$, has to be incorporated in the set of recursive equations that pin down the option values for both workers and firms in both states (matched and unmatched). Following Hagedorn et al. (2012), albeit with small modifications to notation, these four value functions are written as follows:

[7] I assume that a match is formed in case of indifference ($S_t(s, c) = 0$).

$$V_t^e(s,c) = W_t(s,c) + \beta \mathbb{E}_t \left[\underbrace{\delta V_{t'}^u(s)}_{\text{separation}} + \underbrace{(1-\delta)\max\{V_{t'}^e(s,c), V_{t'}^u(s)\}}_{\text{continued employment}} \right] \tag{4}$$

$V_t^e(s,c)$ is the value of an employed worker of type s at firm c in state t. It is determined by the match-specific wage, $W_t(s,c)$, plus the future expected value of two contingencies, discounted with β. First, the value of unemployment in case of an exogenous separation shock (probability δ) and, second the value of non-separation and continued employment at firm c (probability $(1-\delta)$). Note that the max operator entails the possibility of an endogenous separation: the match will also terminate once the value of continued employment falls short of the value of unemployment. The asset value of an unemployed worker, $V_t^u(s)$, has to incorporate three contingencies:

$$V_t^u(s) = b + \beta \mathbb{E}_t \left[\underbrace{(1-q_{t'}^u)V_{t'}^u(s)}_{\text{no meeting}} + \underbrace{q_{t'}^u \int\limits_{M_{t'}^c(s)} \frac{g_{t'}^v(c)}{V_{t'}} V_{t'}^e(s,c)\mathrm{d}c}_{\text{successful match}} + \underbrace{q_{t'}^u V_{t'}^u(s) \int\limits_{\overline{M_{t'}^c(s)}} \frac{g_{t'}^v(c)}{V_{t'}} \mathrm{d}c}_{\text{meet unacceptable firm}} \right] \tag{5}$$

In case of unemployment all workers receive the same flat payment b. In the next period, with probability $(1-q_{t'}^u)$ the worker will not meet any firm, stay unemployed, and continue to receive the value of unemployment. With probability $q_{t'}^u$, a meeting with a firm will occur. In case the firm's type is an element of the worker's matching set and vice versa, a match is formed and production starts. Note that $g_t^v(c)/V_t$ represents the probability of meeting a single firm type in the labor market in a given period. The type-specific value of this fraction works like a weight on the surplus. The firm could also be an unsuitable match, i.e., be of a type that is not an element of the worker's matching set, $M_t^c(s)$, but of the complementary set, $\overline{M_t^c(s)}$. The complementary set contains all firms the worker is not willing to match with. The value of this eventuality is captured by the third term. The firms' asset value equations are constructed in a similar way:

$$V_t^p(s,c) = F_t(s,c) - W_t(s,c) + \beta \mathbb{E}_t \left[\underbrace{\delta V_{t'}^v(c)}_{\text{separation}} + \underbrace{(1-\delta)\max\{V_{t'}^p(s,c), V_{t'}^v(c)\}}_{\text{continued employment}} \right] \tag{6}$$

A productive employment relationship generates match-specific output $F_t(s,c)$ for the firm. The firm has to pay the bargained match-specific wage to the employee. In the following period, the match breaks up with probability δ, leading to the option value of a vacancy, or is sustained with probability $(1-\delta)$, yielding the same value also in the next period, but discounted with β. Again, the max operator allows for an endogenous termination of the employment relationship. Finally, the asset value of a vacant firm is as follows:

$$V_t^v(c) = -\kappa + \beta \mathbb{E}_t \left[\underbrace{(1-q_{t'}^v) V_{t'}^v(c)}_{\text{no meeting}} + \underbrace{q_{t'}^v \int_{M_{t'}^s(c)} \frac{g_{t'}^u(s)}{U_{t'}} V_{t'}^p(s,c) \mathrm{d}s}_{\text{successful match}} + \underbrace{q_{t'}^v V_{t'}^v(c) \int_{\overline{M_{t'}^s(c)}} \frac{g_{t'}^u(s)}{U_{t'}} \mathrm{d}s}_{\text{meet unacceptable worker}} \right] \tag{7}$$

The constant cost of maintaining an open vacancy, κ, which must be paid every period, enters the value function with a negative sign. In the next period, there is the possibility of not meeting a worker, the possibility of meeting a suitable worker and filling the vacant job, and the possibility of meeting an unsuitable worker and continuing search, constructed similarly to Equation (5). To determine how the surplus is divided in case of a suitable match, I apply the standard Nash bargaining solution of the baseline search and matching model. For both the worker and the firm, the respective share of surplus equals the additional value of being matched compared to the value of continued search, which serves as threat point in the bargaining game. The split of the surplus from a match between worker s and firm c is governed by the parameter $\alpha \in [0,1]$, which is defined as the workers' bargaining power:

$$S_t(s,c) = V_t^p(s,c) - V_t^v(c) + V_t^e(s,c) - V_t^u(s) \tag{8}$$

$$\alpha S_t(s,c) = V_t^e(s,c) - V_t^u(s) \tag{9}$$

$$(1-\alpha) S_t(s,c) = V_t^p(s,c) - V_t^v(c) \tag{10}$$

To simplify Equations (4) to (7) I plug in the surplus sharing rules of Equations (9) and (10). This leads to a simplified system of four value functions, now with the surplus function under the integral sign.

$$V_t^e(s,c) = W_t(s,c) + \beta \mathbb{E}_t \left[V_{t'}^u(s) + \alpha(1-\delta) \max\{S_{t'}(s,c), 0\} \right], \tag{11}$$

$$V_t^u(s) = b + \beta \mathbb{E}_t \left[V_{t'}^u(s) + \alpha q_{t'}^u \int\limits_{M_{t'}^c(s)} \frac{g_{t'}^v(c)}{V_{t'}} S_{t'}(s,c) \mathrm{d}c \right], \tag{12}$$

$$V_t^p(s,c) = F_t(s,c) - W_t(s,c) + \beta \mathbb{E}\left[V_{t'}^u(c) + (1-\alpha)(1-\delta)\max\{S_{t'}(s,c), 0\} \right], \tag{13}$$

$$V_t^v(c) = -\kappa + \beta \mathbb{E}_t \left[V_{t'}^v(c) + (1-\alpha) q_{t'}^v \int\limits_{M_{t'}^s(c)} \frac{g_{t'}^u(s)}{U_{t'}} S_{t'}(s,c) \mathrm{d}s \right]. \tag{14}$$

On the basis of these value functions, I derive the firms' job-creation condition and an expression for match-specific wages. The adjustments to these key equations establish the mechanisms leading to improved dynamics of the search and matching model with sorting.

2.5 Job creation

As is commonly assumed in search and matching models, firms will post vacancies as long as there is an opportunity to realize additional profits. Thus, the value of a vacant job, as represented by Equation (14), has to equal zero in equilibrium. Hence, the following free-entry condition holds (expectation operator dropped):

$$\frac{\kappa}{q_{t'}^v} = \beta(1-\alpha) \int\limits_{M_{t'}^s(c)} \frac{g_{t'}^u(s)}{U_{t'}} S_{t'}(s,c) \ \mathrm{d}s. \tag{15}$$

The job-creation condition is simply a modification of Equation (14) with $V_t^v(c) = V_{t'}^v(c) \stackrel{!}{=} 0$. As in the baseline model, the expected cost of hiring a worker, $\kappa/q_{t'}^v$ has to equal the future discounted expected profits of a job. In the model, this value is a function of the integral over the firm-specific matching set, taking into account the surplus with every possible worker type the firm would be willing to match with because when posting a job, the firm does not know the type of worker it will match with. This has to be appropriately weighted by next period's probability of meeting every specific worker type, $g_{t'}^u(s)/U_{t'}$. Due to the endogenous matching set of the firm and the distribution function, the job-creation condition in the model with heterogeneity is richer than in standard search and matching models. The standard model also relates the expected surplus of an additional job to the expected cost of hiring: $\kappa/q_{t'}^v = \beta(1-\alpha)S_{t'}$. Note that the additional objects on the RHS of equation (15) influence the response to a shock in the dynamic setting with aggregate uncertainty: all three endogenous variables of the augmented equation adjust, not only the surplus function. For a given firm, the surplus function shifts upwards after a positive shock as in the standard model. Additionally, the cardinality of the matching sets must increase as well since more potential matches yield a positive surplus. Hence, there is an additional surplus to be realized both with workers that have been in the firm's matching set before and with new workers on the margins of the matching set. Since the job-creation condition determines how many vacancies a firm creates, these additional margin of adjustment plays an important role for the model's dynamics as we will see in Section 3.[8]

2.6 Wage Formation

To derive the equation for match-specific wages, I again use the Nash bargaining solution. Using Equations (9) and (10) and equating them via the surplus the following must hold true:

$$V_t^e(s,c) - V_t^u(s) = \frac{\alpha}{1-\alpha}(V_t^p(s,c) - V_t^v(c)). \tag{16}$$

$V_t^v(c)$ drops out in equilibrium due to the imposed free-entry condition. Plugging in the value functions (11), (12), and (13), maximizing the Nash product, and performing some algebra yields an expression that determines the match-specific wage, $W_t(s,c)$:

$$W_t(s,c) = \alpha F_t(s,c) + (1-\alpha)\beta\alpha\mathbb{E}_t\left[q_{t'}^u(\theta)\int\limits_{M_{t'}^c(s)} \frac{g_{t'}^v(c)}{V_{t'}} S_{t'}(s,c)\ \mathrm{d}c\right] + (1-\alpha)b. \tag{17}$$

The wage of a given worker is thus a convex combination of the match-specific output, $F_t(s,c)$, produced jointly with the present firm and the worker's outside option, that is, the value of being unemployed. The same logic as from the firms' perspective in job-creation applies: the outside option depends on the expected value of the surplus with all other potential employers in his matching set, hence the integral term. Depending on the distribution of vacant firms, $g_{t'}^v(c)$, the surplus is weighted with the probability of actually meeting a specific firm type. The wider the matching set and the higher the probability of meeting every specific firm type in the set, the higher is the bargained wage

[8] Note that this is an important difference to Lise and Robin (2014). As they state, job creation is essentially a static problem in their model because the surplus function is independent of the workers' and the firms' outside options (equations (12) and (14) in my model). In this case, no integral term appears in the job-creation condition. This also implies that vacancies jump in response to shocks and return to their steady state value immediately, whereas my model generates a persistent response of vacancies.

because the worker has more valuable outside options that he needs to be compensated for. Note that for all workers in the matching set, the surplus is weakly positive by definition. After factoring out α, equation (15) can be plugged into this wage equation to arrive at the following expression:

$$W_t(s,c) = \alpha\left(F_t(s,c) + \kappa\mathbb{E}_t\left[\theta_{t'}\frac{\int_{M^c_{t'}(s)}\frac{g^v_{t'}(c)}{V_{t'}}S_{t'}(s,c)\ \mathrm{d}c}{\int_{M^s_{t'}(c)}\frac{g^u_{t'}(s)}{U_{t'}}S_{t'}(s,c)\ \mathrm{d}s}\right]\right) + (1-\alpha)b. \tag{18}$$

Now, the same logic regarding the integral over the matching set applies also from the firm's perspective. The integral over all workers in the matching set $M^s_{t'}(c)$ of a firm type c influences the negotiated wage negatively through the denominator. The wider the firm's matching set the better is the firm's outside option of continued search and thus the lower the match-specific wage of worker s employed at firm c. Note that the aggregate labor market tightness $\theta_{t'}$ in front of the quotient cancels out with $1/V_{t'}$ in the numerator and $1/U_{t'}$ in the denominator:

$$W_t(s,c) = \alpha\left(F_t(s,c) + \kappa\mathbb{E}_t\left[\frac{\int_{M^c_{t'}(s)} g^v_{t'}(c)S_{t'}(s,c)\ \mathrm{d}c}{\int_{M^s_{t'}(c)} g^u_{t'}(s)S_{t'}(s,c)\ \mathrm{d}s}\right]\right) + (1-\alpha)b. \tag{19}$$

Therefore, the match-specific wage $W_t(s,c)$ does not depend on aggregate labor market tightness as in the standard model. Instead, the quotient, call it *relative* labor market tightness, depends on the expectations about the distributions of vacancies and unemployed workers and surpluses for the types within the respective matching sets. Let relative labor market tightness be denoted by $\Theta_t(s,c)$:

$$\Theta_t(s,c) = \frac{\int_{M^c_t(s)} g^v_t(c)S_t(s,c)\ \mathrm{d}c}{\int_{M^s_t(c)} g^u_t(s)S_t(s,c)\ \mathrm{d}s}.$$

The close relationship to the baseline DMP model—and the key difference—now becomes apparent. Compare Equation (19) with its textbook counterpart for a DMP model with constant labor productivity and homogenous firms and workers:[9]

$$W_t = \alpha(F_t + \kappa\theta_t) + (1-\alpha)b. \tag{20}$$

[9] Note that this equation corresponds to Shimer's Equation (7) (Shimer (2005), p. 41), which is a slightly generalized version of Equation (1.20) in Pissarides (2000), p. 17.

In the baseline version of the model, the wage positively responds to changes in aggregate labor market tightness. The higher the labor market tightness ($\theta = {}^{V}/{U}$), the more difficult it is for firms to hire a worker. If there is fierce competition between many firms for relatively few unemployed workers, wages are higher. Equation (19) generalizes this notion for a framework with heterogeneous agents.[10] The key difference between wage formation in the baseline DMP model and in the augmented model with sorting is that aggregate labor market tightness is replaced by relative labor market tightness, the quotient in equation (19). In particular, the integrals represent both workers' and firms' option value of search, that is, the value of the surplus function over the respective matching sets, properly weighted by the distribution function of suitable types in the relevant subspaces of the type space.[11] The intuition behind the modified wage formation mechanism in the presence of sorting is straightforward: aggregate tightness may be high, but if there are many unemployed workers with types lying within a given firm's matching set, this firm has no incentive to pay a high wage and the worker extracts less, even if unemployment outside the firm's matching set is low. Thus, the worker's bargaining position does not depend on scarcity or abundance of other unemployed workers outside the matching set of his potential employer and this mechanism unties match-specific wages from the influence of aggregate labor market conditions. This shields the wage bargain from the influence of fluctuations in the outside option to some extent: In the dynamic setting, fluctuations of aggregate tightness have a smaller impact on the wage bargain if and only if relative labor market tightness is lower than aggregate labor market tightness:

$$\Theta_t(s, c) < \theta_t.$$

If this holds true—Section 2.7 will elaborate on this—the standard model's link between aggregate fluctuations and the bargained wage is being muted and the wage does no longer respond to shocks in a fully flexible manner. It is well known that mechanisms to disconnect wages and aggregate fluctuations are key to enable search and matching models to generate empirically credible dynamics. The finding that it can endogenously arise in a setting with heterogeneity, sorting, and Nash bargaining is, however, new to

[10] Note that Equation (19) collapses to the textbook Equation (20) for the case of constant labor productivity and homogenous workers and firms. This close connection between this model with sorting and the baseline model is key to understanding the augmented model's dynamics. The simulation results presented in Section 3 include a baseline case which confirms that the model without heterogeneity and sorting exactly reproduces the results presented in Shimer (2005).

[11] The same integrals show up in the value functions of unmatched workers and firms, see (12) and (14).

the literature and the central contribution of this paper.[12]

To further analyze the consequences of the augmented wage formation mechanism and the adaption of the firm's job creation decision for the dynamics of the model, I need to compute the stationary equilibrium. The equilibrium is the pivotal point for the numerical simulations and empirical analysis in Section 3.

2.7 Equilibrium

Due the integrals in the values of unemployment (12) and a vacant firm (14), in which both the integrand and the domain of integration are endogenous, this model cannot be solved analytically. It has been shown in the literature, however, that iterative fixed point procedures can be applied to numerically approximate the stationary equilibrium in this class of models. The solution of Shimer and Smith's model (2000) is computed iteratively on a discrete grid.[13] The iterative procedures used to pin down equilibrium in Lise et al. (2013) and Hagedorn et al. (2012) are similar to their approach. I use of the same computational strategy, that is, value function iteration on a discrete grid, to find the stationary equilibrium of the model. Using this technique, the equilibrium can be computed reliably with high precision in a relatively short amount of time even for a quite finely spaced grid. For the computation of equilibrium, the stochastic process for aggregate labor productivity, Y_t, is in steady state, that is, unity. Due to the simple multiplicative link with match-specific output introduced in Equation (1) it can thus be ignored for the computation of the equilibrium and the state index t as well as the expectation operator are dropped. The equilibrium of the search and matching model with ex-ante heterogeneity and sorting consists of a quadruplet $(S(s,c), \mathcal{M}, g^u(s), g^v(c))$. Equilibrium surpluses, $S(s,c)$, determine the optimal matching sets $\mathcal{M}$ via the simple non-negativity condition, $S(s,c) \geq 0$. The following functional equation for match-specific surpluses in equilibrium is the result of simply summing up Equations (11) to (14):

[12] The intuition behind this key property is comparable to the mechanism described in Hall and Milgrom (2008), who propose to replace the standard Nash bargaining solution by a repeated game in the spirit of Rubinstein (1982). They generate empirically reasonable dynamics by limiting the influence of fluctuations in the value of the outside option on the wage bargain via the additional possibility of making a counteroffer instead of walking away from the match. In my model, the impact of the outside option on the wage is limited because only a subset of the type space is relevant for the wage bargain.

[13] I thank Robert Shimer for sharing the code used to produce the numerical results in Shimer and Smith (2000).

$$
\begin{aligned}
S(s,c) = F(s,c) + \beta(1-\delta)S(s,c) - &\left(b + \beta\alpha q^u(\theta) \int\limits_{M^c(s)} \frac{g^v(c)}{V} S(s,c)\ \mathrm{d}c \right) \\
- &\left(-\kappa + \beta(1-\alpha) q^v(\theta) \int\limits_{M^s(c)} \frac{g^u(s)}{U} S(s,c)\ \mathrm{d}s \right).
\end{aligned}
\tag{21}
$$

This is the equilibrium surplus flow equation.[14] The joint distribution of active matches in equilibrium, which pins down the type distributions of unemployed workers and vacant firms, $g^u(s)$ and $g^v(c)$, is computed using the steady-state flow condition of a stationary search and matching model. Computing the integrals of the latter two probability distribution functions yields the aggregate flows of unemployed workers and vacant firms, which in turn determine aggregate labor market tightness, arrival rates, and the flow of matches.

$$
\delta g^m(s,c) = g^u(s)\ q^u(\theta)\ \frac{g^v(c)}{V}\ \mathbb{1}\{S(s,c) \geq 0\}. \tag{22}
$$

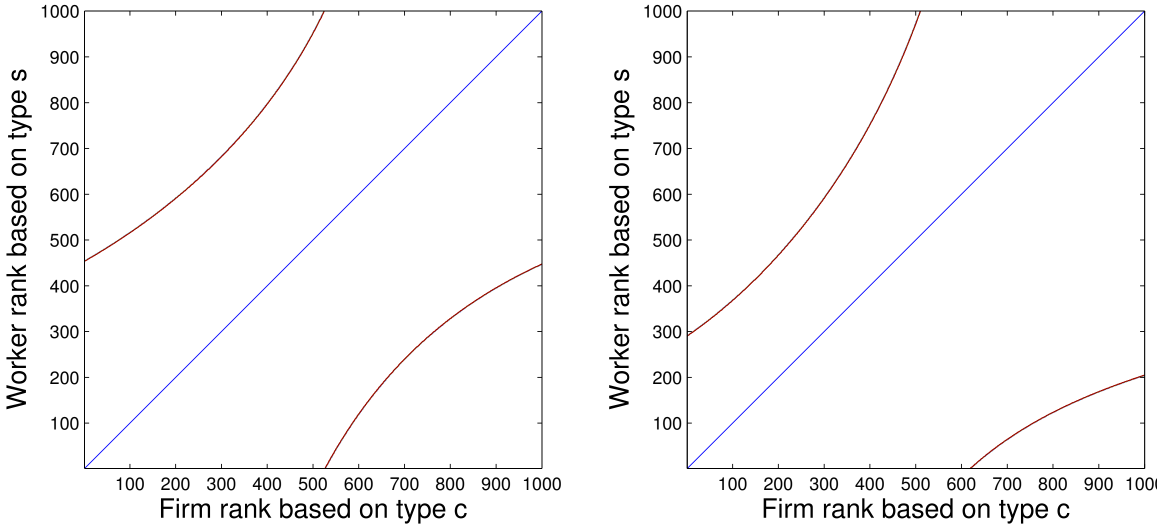
The LHS represents the number of dissolved matches[15], which has to equal new match formation for all pairs (s,c) that form in equilibrium, i.e., $\forall (s,c) \in \mathcal{M}$. The indicator function ensures that new matches are only created in the subspace of the type-space that features weakly positive surpluses in equilibrium. If this condition is satisfied, new match formation arises for every possible pair as the product of the density of an unemployed worker of type s, the aggregate job-finding rate $q^u(\theta)$, and the probability of meeting any specific firm type c in the labor market, which is nothing but the density of this type of vacant firm divided by the overall number of vacancies. To approximate the equilibrium, I alternate between computing the fixed point of the surplus flow Equation (21) and updating the distribution of active matches using the steady-state flows in Equation (22). This procedure is repeated until the decision rule converges as described in Hagedorn et al. (2012).[16]

[14] Note that the terms in brackets, which represent the outside options of both workers and firms are absent in the surplus function of Lise and Robin (2014). Their main theoretical result is that using the sequential auction framework developed in Postel-Vinay and Robin (2002), the surplus function depends on the aggregate state of the productivity shock only. This result does not hold for the case of Nash bargaining considered in this paper.

[15] In the stationary equilibrium, all match dissolutions happen exogenously with probability δ. Endogenous separations, which play a role in the dynamic setting, can thus be ignored here.

[16] See Hagedorn et al. (2012), Appendix II. Convergence is achieved once the absolute difference of the surplus between two iterative steps is less than 10^{-12}.

Figure 1: Equilibrium matching sets $\mathcal{M}$



(a) Equilibrium Matching Sets for $\alpha = 0.72$

(b) Equilibrium Matching Sets for $\alpha = 0.95$

The key element, driving the results of this paper, are the endogenously chosen matchings sets $\mathcal{M}$ which are depicted in Figure 1. To illustrate $\mathcal{M}$ graphically, I assume a very simple production function that is in line with the existence conditions formulated in Shimer and Smith (2000), $\log F(s,c) = s \times c$. This functional form implies a hierarchy, that is, it features absolute advantage of high-skilled workers and highly productive firms. Thus, a high-skilled worker produces more output at any firm.[17] I solve the model on a grid entailing 1000 worker and firm types which are ranked based on their types s and c. All the parameter values necessary for the solution of the model are taken from the calibration of the search and matching model presented in Shimer (2005), see also Table 2 in the next section.

First of all, it becomes apparent that workers and firms are picky in equilibrium as they will not match with every possible partner. Figure 1 shows a top down view on the surface of the equilibrium surplus function, what we see is the cutoff for $S(s,c) = 0$, that is, the margins of the matching sets for all types. The surplus is negative for a number of pairings, outside those margins, both in the upper-left and lower-right corners of the type space, as expected for matches between high-type firms and a low-type workers and vice versa. To make an important point, I show the solution for two different values of the workers' bargaining power: $\alpha = 0.72$, as in the standard calibration[18], and a

[17] For now, this is for illustrative purposes only. I discuss other choices for the production function as well as advantages and disadvantages in Section 3.

[18] Following the Hosios (1990) condition for socially efficient vacancy posting, the bargaining power

more extreme value of $\alpha = 0.95$. The graphical illustration in Figure 1, particularly for $\alpha = 0.95$, shows that the plane of equilibrium surpluses is skewed and asymmetric. The diagonal line, which represents the Walrasian first-best allocation in a frictionless market, is meant to increase visibility. The explanation for this asymmetry in equilibrium lies in the surplus flow Equation (21) and in the way the bargaining power influences it. α $(1-\alpha)$ acts like a weight on the workers' (firms') outside option. In case of $\alpha = 0.5$, the shape of the matching sets in Figure 1 would be perfectly symmetric, that is, the diagonal line would cut the surface within the margins of the matching sets exactly in half. This does not hold with unequal bargaining powers: If the workers' bargaining power is higher, i.e. $\alpha > 0.5$, the lower-right cutoff of the matching set (where $S(s,c)$ becomes negative) is visibly shifted outwards. The upper-left cutoff is stretched towards the axis. The area below the diagonal becomes larger than the area above. Using the standard calibration of search and matching models with $\alpha = 0.72$, this asymmetry is a feature of labor market equilibrium. Intuitively, a higher bargaining power on the workers' side implies that workers can afford to be more picky in equilibrium because they extract a higher fraction of the surplus once they are in a match. As compared to the firms, workers have a higher option value of search and, thus, optimally choose narrower matching sets. Firms, in turn, optimally choose wider matching sets because they can command only a small share of the match-specific surplus. Thus, with a small bargaining power for the firm, a wider matching set is necessary to secure the same option value as in case of $\alpha = 0.5$.

Since it has now been established that the cardinality of the workers' matching sets is smaller in equilibrium than the cardinality of the firms' matching sets, recall the augmented wage formation mechanism in Equation (19) and the condition under which bargained wages are lower in the search and matching model with sorting as compared to the standard model:

$$\Theta(s,c) = \frac{\int_{M^c(s)} g^v(c)S(s,c)\ \mathrm{d}c}{\int_{M^s(c)} g^u(s)S(s,c)\ \mathrm{d}s} < \theta.$$

The standard calibration normalizes aggregate labor market tightness, θ, to a value of 1. Considering that the matching set of the firm, which enters the denominator of $\Theta(s,c)$ via the integral is in equilibrium wider than the worker's matching set in the numerator,

of the worker is set equal to the matching function's elasticity parameter ξ. Based on empirical evidence, this parameter value is usually set to 0.72, implying a higher bargaining power and, thus, a larger share of surplus for the worker.

the quotient, known as relative labor market tightness from the previous section, must be smaller than 1. This implies that using the standard calibration of the model, bargained wages will be lower in the equilibrium of the search and matching value with sorting as compared to the standard model and the property derived in Section 2.6, $\Theta(s, c) < \theta$, holds true, so wages also do not fully adjust to shocks. Interestingly, this is because (and not instead) of the higher bargaining power on the worker side. The following numerical simulations under aggregate uncertainty show how this feature translates into an endogenous wage rigidity which influences the model's dynamics in conjunction with the augmented job-creation condition. The magnitude of these effects and their impact on the model's ability to generate empirically credible dynamics, evaluated in the next section, are the main results of this paper.

3 Numerical Simulations and Empirical Analysis

The Shimer Puzzle revolves around the search and matching model's ability (or lack thereof) to explain stylized facts of labor market data. Methodologically, the empirical performance of this class of structural models is tested using numerical simulations in a stochastic environment where aggregate uncertainty is driving the business cycle. The stochastic process typically represents aggregate labor productivity as introduced in Section 2.3. Shimer (2005) shows that the standard model fails to generate a sufficient degree of volatility in response to shocks to labor productivity.[19] Shimer (2005) emphasizes that his approach "is not an attack on the search approach to labor markets, but rather a critique of the commonly-used Nash bargaining assumption for wage determination."[20] My results show, however, that it is indeed possible to solve the volatility puzzle without discarding Nash bargaining by generalizing the model to account for explicit heterogeneity and complementarity. I find that second moments of simulated data from the augmented model are considerably closer to the data than are those generated using the baseline model. This is due to both the augmented wage formation mechanism and the additional margins in the job-creation condition. Both channels critically depend on the endogenous matching sets which, importantly, also fluctuate in response to shocks. The search and matching model with heterogeneity and sorting is thus able to solve the Shimer puzzle. The degree of model-generated wage rigidity, as detailed below, turns out

[19] Shimer (2005) also considers separation rate shocks, and comes to essentially the same conclusions. Stochastic labor productivity is far more common in the literature, which is why concentrate on this concept.

[20] Shimer (2005), p. 45.

to be supported by empirical evidence for the U.S. labor market as reported by Haefke et al. (2013).

To proceed further with the analysis, a functional form assumption for the match-specific production function is necessary. As mentioned before, sorting can be based either on comparative or absolute advantage. I consider both cases: A circular model as proposed by Marimon and Zilibotti (1999) and Gautier et al. (2010) with comparative advantage and a hierarchical model like in Shimer and Smith (2000) featuring absolute advantage. In this setting, the most productive firm's output is the highest no matter which worker is hired. For this case, I stick to the very simple log-supermodular production function already used to produce Figure 1:

$$\log F(s,c) = s \times c. \tag{23}$$

In the second, circular model every firm has one optimal employee type and, due to comparative advantage, a most productive firm and most highly skilled worker do not exist. As in Gautier et al. (2010), I calculate match output as a function of the "distance" between workers and firms, defined along the circumference of a circle. This production function features comparative advantage, that is, output is determined solely by a relative metric defined for every (s, c) pair on the type-space:

$$F(s,c) = F(x) = 1 - \frac{1}{2}\gamma x^2, \;\; x \equiv |s - c|. \tag{24}$$

This metric, the distance x, can be seen as a measure of mismatch between workers and firms. Thus, the function is maximized for $x = 0$, i.e., for the case of an optimal match. The parameter γ governs the degree of complementarity of worker types and, thus, the cost of mismatch. Conceptually, γ is related to the "complexity dispersion parameter" discussed in Teulings and Gautier (2004) and Teulings (2005). The lower γ, the more substitutable are different types in production. A small γ, i.e., a small degree of complementarity is sufficient to achieve the desired amplification effect. A minimum constraint on γ ensures that workers do not accept every job in equilibrium and, hence, the matching sets do not cover the whole type-space. The higher γ the more narrow are the matching sets, that is, the smaller is the number of counterparts that a given type is willing to match with. For γ going to 0, in turn, the output of a match becomes independent of worker and firm types, just like in the baseline case homogenous workers and firms. I provide a robustness check for different values of γ in Section 3.5. The interior maximum of the circular production function delivers a nice intuition for the non-monotonicity of wages in firm type in this class of structural models (as emphasized in Eeckhout and Kircher (2011)) because it represents the optimal match that maximizes output and wage. The critical property of this model—the equilibrium asymmetry of the surplus function due to unequal bargaining powers—does not hinge on the choice of the production function. It arises both with the circular model and with the simple hierarchical function. In the following, results from numerical simulations are presented.[21] I compare the dynamics of a baseline model, which reproduces Shimer's results, and the dynamic search and matching model with sorting for both types of production function. Due to log-linearization around the steady state, numerical results must be understood as elasticities, i.e., percentage deviations from the steady state. As in Shimer (2005), one period of time is set to be one quarter.

3.1 Calibration Based on Business Cycle Properties

Table 2 shows the calibration of the model based on quarterly U.S. labor market data used for the simulation exercise. To ensure direct comparability of the dynamics of the augmented model with the results in Shimer (2005), identical parameter values are chosen. The value of non-market activity b is computed using U.S. replacement rates in relation to mean labor income. It is assumed that this value is entirely determined by unemployment benefits since there is no leisure in the model. Vacancy posting costs κ are set to resemble average U.S. labor market tightness. A value of 0.1 for the separation rate represents an average employment spell of about 2.5 years in the United States in the relevant period (1951—2003). In the period relevant for the model, annual interest rates were roughly 5% in the United States. Thus, the quarterly discount rate is set to 0.012, which translates to a discount factor (as it appears in the model equations) of $\beta = {}^{1}/_{1.012} \approx 0.99$. The Hosios (1990) condition for socially efficient vacancy posting in the decentralized equilibrium leads to the equalization of workers' bargaining power and the elasticity parameter of the matching function. The matching function elasticity, in turn, is set by Shimer (2005) at 0.72, which is within the empirically-supported range reported by Petrongolo and Pissarides (2001) and resembles the average job-finding rate in U.S. labor market data. The only parameter that is not part of the baseline search and matching model and its calibration proposed in Shimer (2005) is the degree of

[21] I apply standard solution methods and simulate the model in log-linearized form using Dynare (Adjemian et al. (2014)). The steady state is the equilibrium computed in Section 2.7. To account for fluctuations of the additional endogenous objects in the model with sorting (surplus, matching sets, probability distributions of types), I use a number of functions external to Dynare to compute the numerical differentials for every state of the labor productivity process.

Table 2: Parameter values for the quarterly calibration of the search and matching model for the U.S. labor market (1951—2003)

Parameter	Symbol	Value	Source
Substitutability	γ	1	Gautier and Teulings (2014)
Discount factor	β	0.99	Shimer (2005) (discount rate 0.012)
Separation rate	δ	0.1	Shimer (2005)
Workers' bargaining power	α	0.72	Hosios (1990) condition: $\alpha = \xi$,
Matching function elasticity	ξ	0.72	Shimer (2005)
Value of non-market activity	b	0.4	Shimer (2005)
Vacancy posting costs	κ	0.213	Shimer (2005)
First order autocorrelation	ρ	0.765	Hagedorn and Manovskii (2008)
Standard deviation	σ_ϵ	0.013	Hagedorn and Manovskii (2008)

complementarity γ of the production function. I use a value of 1, which lies within the range considered by Gautier and Teulings (2014), as baseline and check the robustness of my results with respect to the choice of γ in Section 3.5. Given the production function (24), this degree of complementarity suffices to achieve a large amplification effect. The value of γ has to be high enough, though, to induce the agents to be picky in equilibrium.[22]

As introduced in Section 2.3, the labor productivity process Y_t can be imagined as an underlying technology that enables labor to be used productively. It is a stochastic process which varies over time. It is type-independent and affects all matches in equal measure. As in Shimer (2005), it is normalized to 1 in steady state and calibrated to resemble empirical labor productivity in the U.S. over the relevant period of time. Regarding the functional form, I follow Hagedorn and Manovskii (2008) and set up stochastic labor productivity as a first-order autoregressive process:[23]

$$Y_t = Y_{t-1}^{\rho}\, e^{\epsilon_t} \leftrightarrow y_t = \rho\, y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2). \tag{25}$$

[22] A very low degree of complementarity leads to an equilibrium with matching sets that comprise the whole type space and hence, agents would not be picky.

[23] Many closely related papers use more general Markov chains to add a stochastic dimension to the model. An AR(1) process is a homogenous Markov process iff the error terms are i.i.d.. I prefer to use the AR(1) process under this assumption for computational reasons.

Table 3: Actual and simulated standard deviations of labor market variables

	Standard deviations	U_t	V_t	θ_t	q_t^u
1.	U.S. data	0.190	0.202	0.382	0.118
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010
3.	This paper (no sorting, no heterogeneity)	0.009	0.026	0.035	0.010
4.	This paper (sorting, hierarchical model)	0.102	0.277	0.380	0.168
5.	This paper (sorting, circular model, $\gamma = 1$)	0.201	0.544	0.745	0.329

Note: Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28/39. Calculated based on quarterly U.S. data, 1951—2003. Rows 3 to 5: Standard deviations of simulated data from the author's model with and without sorting. All moments come from HP-filtered data with $\lambda = 10^5$.

Lower-case letters represent labor productivity in logs. $\rho \in (0, 1)$ captures the degree of first-order autocorrelation of this AR(1) process. Innovations are drawn from a Gaussian distribution with mean 0 and standard deviation σ_ϵ. Both parameters are set to match quarterly U.S. labor productivity.[24] All values in Table 2 are based on quarterly data. Shimer's (2005) simulation results as well as my own results are reported as deviations from a HP trend, which is conventional in the literature.

3.2 The Amplification Effect of Sorting

I find large amplification of shocks using the presented search and matching model with sorting. Second moments of simulated time series data are of the same order of magnitude as the volatility we see in U.S. labor market data for the relevant period of time. In particular, the simulated standard deviations of unemployment, vacancies, labor market tightness, and the job-finding rate are much closer to empirical second moments than the results Shimer (2005) reports using the standard search and matching model. Table 3 compares the main results of this paper to those of Shimer (2005) and the true data moments. The first two rows of Table 3 show the well-known result reported in Shimer (2005). The original Shimer Puzzle is immediately apparent. The standard deviations of unemployment, U_t, vacancies, V_t, labor market tightness, θ_t, and the job-finding rate, q_t^u, in simulated time series data miss the standard deviations in the data

[24] Shimer (2005), Hornstein et al. (2005) and Hagedorn and Manovskii (2008) report the parameter values necessary to represent U.S. labor productivity "as seasonally adjusted quarterly real average output per person in the non-farm business sector constructed by the BLS" (Hagedorn and Manovskii (2008), p. 1694).

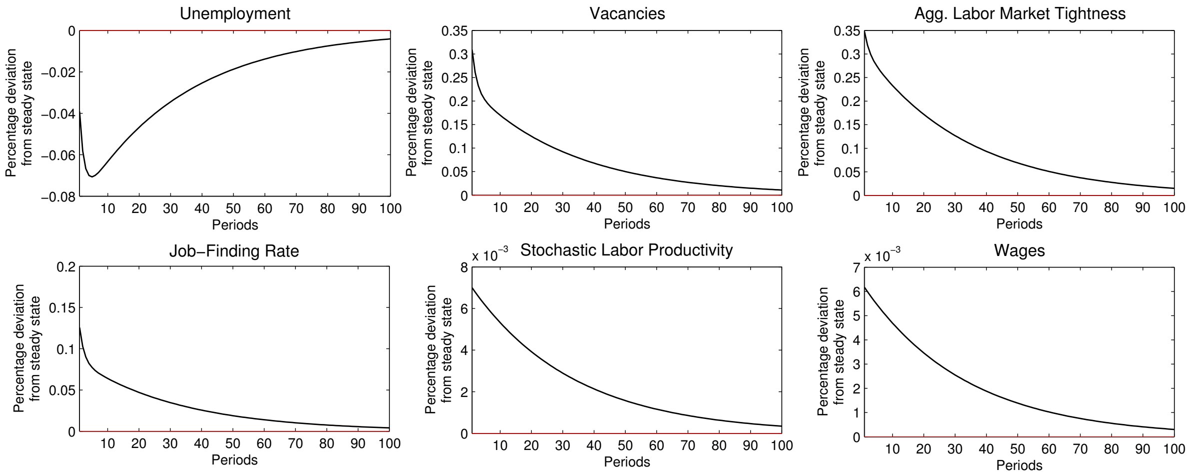
by a factor of about 10 to 20. As a simple test, the third row shows that my model reproduces Shimer's results for the case of no sorting and no heterogeneity. It is thus ensured that the results are directly comparable to Shimer (2005).[25]

The main results are reported in the fourth and fifth row of Table 3. The second moments of time series data for the main labor market aggregates from the augmented model with sorting are much closer to the data than the corresponding numbers generated using the model without sorting. Using the hierarchical model with absolute advantage, the standard deviation of the HP-filtered time series of labor market tightness (0.380) is very close to the data (0.382). Thus, the search and matching model with sorting is able to generate realistic dynamics of labor market tightness via the two channels that have been shown to distinguish the model with sorting from the standard model: job-creation and wage formation. The simulated standard deviations of vacancies (V_t) and the job-finding rate (q_t^u)) are also much higher than in the baseline model. They even overshoot their empirical counterparts to some extent. The standard deviation of unemployment (U_t) is amplified as well but remains somewhat lower than the empirical value. The results using the circular model with comparative advantage overcome this shortcoming to some extent, see the fifth row of Table 3. Unemployment dynamics are now closely matched: 0.201 generated by the model and 0.190 in the data. The circular model leads, however, to standard deviations of vacancies, labor market tightness and the job-finding rate which are too high; they overshoot their empirical counterparts by a factor of 2 to 3. Recall that both the tightness and the job-finding rate are simple functions of V_t. Thus, the excess volatility generated both using the hierarchical and, too a larger extent, using the circular model must stem from the firm's vacancy posting decisions which are obviously amplified too much by combining the simple structure of the standard model, the standard parametrization from the literature, and sorting. While it is interesting to think about ways that avoid the relative discrepancy between the volatilities of unemployment and vacancies, which show roughly the same volatility in the data, it is important to note that the aim of the exercise presented in this paper is not to match the empirical moments as closely as possible; rather, the main message is that a very simple search and matching model generates amplification sufficient to match empirical second moments with just one additional element: Sorting.[26]

[25] Note that due to the complexity of model with heterogeneity, results reported here come from a single stochastic simulation on the entire type space, which is approximated by a discrete grid of dimensions 50×50. The Shimer (2005) results are bootstrapped standard errors from 10,000 simulations of the baseline model.

[26] One approach to resolve this discrepancy would be to think carefully about vacancy posting costs in the setting with sorting. To dampen excessive vacancy posting in response to shocks, an additional

Figure 2: Impulse Response Functions of key variables in the search and matching model with sorting

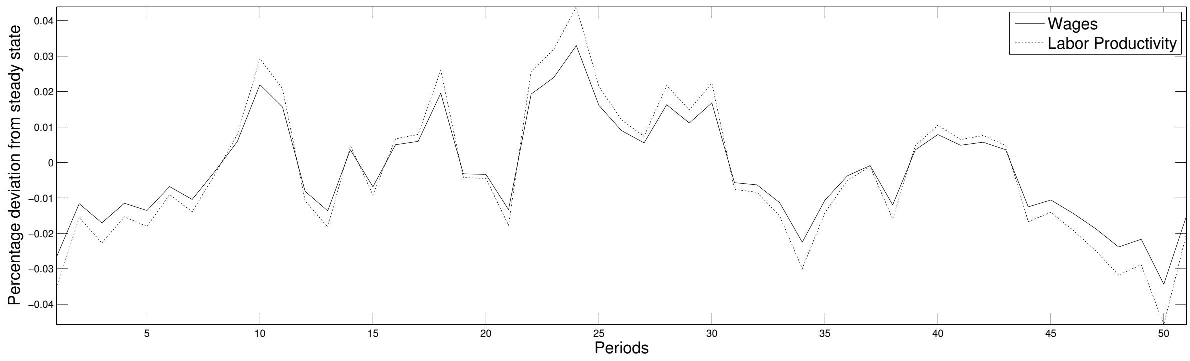


To illustrate the dynamics of the search and matching model with sorting, Figure 2 shows impulse response functions of key variables: unemployment, vacancies, aggregate labor market tightness, the job-finding rate as well as the autoregressive labor productivity process and wages. In response to a positive shock to labor productivity, which fades out slowly due to the autoregressive nature of the process, unemployment falls and shows a countercyclical hump-shaped response. Strong procyclical reactions in vacancy posting, job-finding, and aggregate tightness of the labor market materialize directly in the aftermath of the shock. All impulse responses show a high degree of persistence and both correlations and cyclical properties are realistic. For instance, unemployment and vacancies move in opposite directions in response to the shock and are thus highly negatively correlated (Beveridge curve). Note also the underproportional adjustment of wages to the shock in labor productivity. The endogenously generated wage rigidity becomes apparent: The initial adjustment of wages is roughly only 85% of the jump in labor productivity in the depicted example.

I now take a closer look at the simulation results presented. As shown in Section 2.5 and 2.6, the augmented search and matching model with sorting entails two additional channels that lead to amplification of the dynamic model: The additional margin in job-creation and the endogenous wage rigidity. How do both channels contribute quantitatively to the improved empirical performance and how large is the implied degree of wage rigidity?

"fixed" cost component, which should be non-proportional to the expected duration of maintaining the vacancy, as suggested by Pissarides (2009), would be one promising way to go in future work.

Figure 3: Simulated time series of wages and labor productivity



Note: Fluctuations around trend of simulated data for labor productivity and wages, HP-filtered with $\lambda = 10^5$. Hierarchical model. The 50-period timeframe has been randomly chosen.

3.3 The Degree of Wage Rigidity

The point that rigid wages can amplify the search and matching model's response to shocks has been made repeatedly in the literature (Hall (2005), Hagedorn and Manovskii (2008), Hall and Milgrom (2008)). The model with sorting developed in this paper is complementary. It has been shown in Section 2.7 that rigid wages originate from an an equilibrium property of the model with sorting, namely the asymmetric values of workers' and firms' outside options that leads to a skewed surplus function and a difference in the cardinalities of worker and firm matching sets. This generates a wage rigidity of empirical reasonable magnitude without disregarding Nash bargaining and without changing Shimer's (2005) calibration strategy. A closer look at the interrelation between my and other approaches to solving the volatility puzzle will be provided in Section 4.

Shimer (2005) does not analyze wage data for the U.S. labor market and does not use the wage-formation mechanism to characterize the equilibrium of his model. Therefore, I analyze the degree of model-generated wage rigidity separately. Perfectly flexible wages, as in the baseline model, respond directly to stochastic changes of labor productivity in the form of a one-to-one co-movement. The elasticity of wages with respect to labor productivity is thus simply 1. Totally rigid wages, on the other hand, would not react at all to changes in labor productivity, implying an elasticity of 0 (as in Hall (2005)). Figure 3 shows how simulated wages (solid) and labor productivity (dotted) deviate from trend in the model with sorting. The data stem from a simulation of the hierarchical model and the 50-period timeframe is randomly chosen. In the standard search and matching model, these two time series would simply be congruent. Wages are fully flexible and instantaneously adjust whenever stochastic labor productivity changes. In the depicted model with sorting, however, wages turn out to be less volatile and do not fully adjust, as can be seen in Figure 3. The amplitude of labor productivity deviations (dotted) is considerably larger. Hence the elasticity of wages with respect to labor productivity must be somewhat smaller than 1. This is exactly the result one would expect. In the baseline model, wages are too responsive. After a favorable shock, workers soak up most of the extra productivity via wages since they immediately adjust what leads to insufficient responsiveness of other model variables, particularly vacancies, due to a lack of incentives for the firm. In the model with sorting, however, the one-to-one link between wages and labor productivity is dampened. The arising wage rigidity, which is visible in Figure 3, limits the extent to which wages adjust in response to the shock. This increases firm's incentives to create new jobs and leads to amplification.

To check whether the model-generated rigidity is of a reasonable magnitude, I refer to the empirical literature on this issue. I rely on Haefke et al. (2013), who focus primarily on the different degrees of wage rigidity for newly hired workers as compared to established employment relationships. I can compare the reported wage-elasticity with respect to labor productivity for new hires to my results. Haefke et al. (2013) note that this value "is an appropriate and informative calibration target for search and matching models."[27] The authors find an elasticity of wages with respect to labor productivity of 0.8 with a relatively large standard error of 0.4. Using data from simulations of the model with sorting, this elasticity can simply be computed as the coefficient η_1 of a simple linear regression of wages on labor productivity in logs and first differences:

$$\Delta \log W_t(s,c) = \eta_0 + \eta_1 \Delta \log Y_t + \varepsilon_t \tag{26}$$

Running this simple regression yields a wage elasticity of $\eta_1 = 0.751$ for the hierarchical model and $\eta_1 = 0.871$ in the circular case. These elasticities lie well within the empirically supported range. Hagedorn and Manovskii (2008) also compute an elasticity from U.S. wage and productivity data and report a coefficient of 0.449, which also lies

[27] Haefke et al. (2013), p. 898. Since the baseline search and matching model with Nash bargaining is essentially a model of new hires, the elasticity of wages with respect to labor productivity for this group is a reasonable metric to assess the plausibility of the theoretical model and the simulation results. Note that wages do not play any allocational role in a random search model. The Nash bargaining solution simply determines how the surplus is shared in every time-period given the state of the model in the same period. Thus, the length of an employment spell does not influence wages and there is no meaningful distinction between new hires and existing employment relationships.

within the supported range, albeit at the lower end. The elasticity/derivative implied by the alternating-offer bargaining game proposed by Hall and Milgrom (2008) is about 0.7, again in line with both the empirical evidence and the endogenous wage rigidity generated by the model with sorting. It is reassuring that the rigidity lies close to these benchmarks.

Recall that the endogenous wage rigidity is, as described above, not the only modification of the model with sorting. Given the strong amplification effect, the degree of wage rigidity I find appears to be small at first sight. However, the value is well in line with empirical evidence on wage rigidities, a more extreme kind of rigidity would not be justified. It is thus instructive to decompose the effect of sorting in the dynamic model according to the two sources of amplification: The endogenous wage rigidity and the additional margin of adjustment in job-creation.

3.4 Rigid Wages vs. Job-Creation

Besides amplification due to rigid wages, sorting unfolds a second effect that leads to amplification via forward-looking vacancy posting decisions. Recall the firms' job creation condition, equation (15), with ${}^{M_{t'}}\!/\!_{V_{t'}}$ plugged in for $q^v_{t'}$:

$$\frac{\kappa V_{t'}}{M_{t'}} = \beta(1-\alpha) \int\limits_{M^s_{t'}(c)} \frac{g^u_{t'}(s)}{U_{t'}} S_{t'}(s,c)\ \mathrm{d}s. \tag{27}$$

After a positive productivity shock hits the economy, the inversely U-shaped surplus function shifts upwards. Firms choose wider matching sets because the surplus from an increased number of potential matches is larger than zero. This leads to a higher expected value of future surpluses on the side of the firm and, consequentially, amplification. The difference to the standard model is precisely this effect of the fluctuating matching sets. Not only the surplus increases in response to the shock. The margins of the firms' matching sets get wider too. This implies that for a productivity shock of equal magnitude, the RHS of (27) in the model with sorting experiences a larger increase than the corresponding equation in the standard model would lead to. This additional margin of adjustment via the matching set has to be offset by a larger number of vacancy postings, V, which is the firm's control variable. Thus, in this model wages do not need to be extremely rigid (as in Hall (2005)) to generate sufficient volatility. A higher number of *additional* vacancy postings is the optimal response to a positive shock as compared to the standard model. This generates more amplification than the wage-rigidity alone.

Table 4: Amplification effect of an imposed wage rigidity

	Standard deviations	U_t	V_t	θ_t	q_t^u
1.	U.S. data	0.190	0.202	0.382	0.118
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010
3.	This paper (sorting, circular, $\gamma = 1$)	0.201	0.544	0.745	0.329
4.	This paper (rigidity only, $\eta_1 = 0.751$)	0.031	0.084	0.114	0.051
5.	This paper (rigidity only, $\eta_1 = 0.871$)	0.016	0.043	0.059	0.026

Note: Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28/39. Calculated based on quarterly U.S. data, 1951—2003. Rows 3 to 5: Standard deviations of simulated data from the author's model. All moments come from HP-filtered data with $\lambda = 10^5$.

To find out to what degree both the wage rigidity and the larger number of vacancy postings contribute to the documented amplification, I take the calculated elasticities of wages with respect to labor productivity, both for the hierarchical and the circular model, and impose them on a standard model without sorting and heterogeneity. The gap in volatility, which cannot be accounted for by the effect of rigid wages alone, must then be the effect of sorting on job-creation. The last two rows of Table 4 show the results of this exercise.

The volatility of labor market variables with an imposed wage rigidity alone remains too small as compared to the data. As expected, the rigidity amplifies the model, but even for the stronger wage rigidity in case of the hierarchical model ($\eta_1 = 0.751$) a factor of about 2-6 is missing (depending on the variable) between simulated standard deviations and data. Thus the relatively small model-generated wage rigidity alone does not suffice to generate sufficient amplification. This is not surprising. I conclude that the large amplification effect of sorting must be primarily driven by the effect of additional job-creation, which is quantitatively dominant. Since there are not additional modifications in the model, the gap in volatility between the third (sorting, circular model) and the fourth and fifth rows (no sorting, rigidity imposed) in Table 4 shows how large the effect of additional job-creation is in comparison.

3.5 The Choice of γ

As a final robustness check, I examine to what extent the results I document for the circular model depend on the assumed degree of complementarity in the production function,

Table 5: The influence of γ on amplification

	Standard deviations	U_t	V_t	θ_t	q_t^u
1.	U.S. data	0.190	0.202	0.382	0.118
2.	Results of Shimer (2005)	0.009	0.027	0.035	0.010
3.	This paper (sorting, circular, $\gamma = 1$)	0.201	0.544	0.745	0.329
4.	This paper (sorting, circular, $\gamma = 1.5$)	0.179	0.485	0.664	0.294
5.	This paper (sorting, circular, $\gamma = 2$)	0.166	0.451	0.617	0.273

Note: Rows 1 & 2: Based on Tables 1 and 3 in Shimer (2005), pp. 28/39. Calculated based on quarterly U.S. data, 1951—2003. Rows 3 to 5: Standard deviations of simulated data from the author's model. All moments come from HP-filtered data with $\lambda = 10^5$.

Equation (24). As mentioned before, γ is the only parameter of the model with sorting that is not easily adaptable from the literature, because reliable empirical evidence on the extent of sorting and complementarities in the labor market is still rare, particularly for the U.S. Some recent innovative empirical studies are discussed in Section 4. Note, however, that the simple hierarchical model does not add any additional parameter to the model. This could be seen as an advantage in the light of limited empirical evidence. In Section 1, I claimed that the impact of search on the job in this class of models can be controlled for by allowing different degrees of complementarity. This is due to the fact that intuitively, on-the-job search reduces the pickiness of agents. If a worker had the possibility of continuing to search after being matched, he would optimally choose wider matching sets. Thus, results with a low value of γ intuitively resemble a high degree of search on the job. Of course this is just a very rough approximation. Nevertheless, it allows us to understand the impact that on-the-job search would have in a model that makes it explicit. For the robustness check, I compare three values of γ in the range proposed by Gautier and Teulings (2014), who make extensive use of the circular model. These values are $\gamma = 1$ (low degree of complementarity, used in Table 3), $\gamma = 1.5$ (medium degree of complementarity), and $\gamma = 2$ (high degree of complementarity). Recall that the higher the value of γ, the stronger is the degree of complementarity in production and the more picky the agents will be in equilibrium. This leads to narrower matching sets due to high costs from not being matched optimally in the form of foregone output. As Table 5 shows, the lowest degree of complementarity considered ($\gamma = 1$) yields the highest amplification. Search on the job would, therefore, add amplification to the model. Increasing γ lowers the amplification effect of sorting and reduces the excess volatility of vacancies. This is intuitive: Very picky agents in equilibrium and a corresponding high degree of complementarity imply that for the same magnitude of the productivity shock the matching sets will adjust less. In other words, a higher additional value of surplus is necessary to increase the cardinality of the matching sets by the same amount. This pattern emerges simply as an equilibrium property of the surplus function. The surface of positive surpluses is much steeper in case of high complementarity due to the large gains from being optimally matched. Hence, matching sets fluctuate less in response to shocks and the additional margin in the job-creation condition is dampened, what leads to less amplification with large complementarity. The discrepancy between the volatility of unemployment and vacancies remains, however. It is not possible to perfectly adjust the search and matching model with sorting to the data by calibrating γ alone. Instead, empirical evidence for the importance of complementarity and sorting in the U.S. labor market is necessary to make an informed choice.

4 Discussion in Relation to the Literature

This paper constructs an equilibrium search and matching model with two-sided heterogeneity and sorting. Gautier et al. (2006) provide an insightful overview of the development of this class of labor search models. The relationship to the previous literature is best understood from two reference points. The first reference point is the assignment game among heterogeneous agents in a frictionless market following Becker (1973). Becker characterizes the equilibrium of a marriage market, that is, a market with non-transferable utility. In this frictionless, neoclassical model, all types are only willing to match with their one optimal counterparts to form matches which jointly maximize payoffs. Thus, matching sets consist of only a single element. Shimer and Smith (2000) take this model out of its Walrasian equilibrium by adding frictions and costly search. Additionally, they consider transferable utility, thereby building a bridge to labor market applications of the theory. In their paper, technical conditions for positive assortative matching in equilibrium are derived and existence is proven: in a nutshell, types need to be complements in production. Thus, models featuring positive sorting rely on a supermodular production function which is increasing in its two arguments, the worker's skill and the firm's productivity, and twice continuously differentiable.The second reference point is the baseline search and matching model of the labor market as outlined in the textbook by Pissarides (2000) and used in—among many other papers—Shimer (2005). The DMP model has been very successful due to its ability to explain equilibrium unemployment and a number of important stylized facts of labor market data (e.g. the Beveridge curve). Due to the fact that it abstracts from heterogeneity and thus considers only one representative type of workers and firms, the matching sets of those types embrace the whole type space.[28] In other words, every agent is willing to match with every other agent, but frictions limit the number of encounters in the labor market and, therefore, unemployment exists. Wages are totally flexible in this model. In a dynamic framework with aggregate uncertainty, they adjust instantaneously and follow labor productivity one-to-one. This point is central to the discussion started by Shimer (2005) and Hall (2005). Shimer (2005) points out that even though the standard search and matching framework appealingly predicts the long-run equilibrium of the labor market, the volatility of data from stochastic simulations of the baseline model is too small by a factor of up to 20 compared to empirical second moments of U.S. labor market data. This obstacle has become known as the Shimer Puzzle. Departure from totally flexible wages, that is, incorporating rigid wages into the baseline model, has been used to solve this problem since rigid wages increase firms' incentives to create jobs in a cyclical upswing, that is, in response to a positive productivity shock.[29] Due to the rigidity, wages no longer follow labor productivity one-to-one. Thus, wages become less volatile and the dynamics of the model are amplified since more vacancies will be posted until the firms' free entry condition is satisfied. However, the assumptions used in the literature to justify rigid wages are highly debated. Hall (2005) shows that the volatility puzzle vanishes once wages are made fully inflexible. This, however, implies a counterfactual wage volatility of zero. Hagedorn and Manovskii (2008) show that the dynamics can be amplified by increasing the value of the workers' outside option of non-market activity. A higher calibrated value of the respective model parameter leads to lower wage outcomes in the Nash bargaining game, however, with rather unrealistic consequences.[30]

I construct a model in between the two reference points: in a labor market with search frictions and assortativeness through complementarity, suboptimal matches between heterogeneous jobs and workers arise in equilibrium and persist through time. This setup generates sufficient volatility in a search and matching model without the additional

28 Which is simply a point.

29 The baseline model is calibrated to ensure that vacancy posting is socially efficient according to the Hosios (1990) condition. This leads to a too-high share of the additional surplus for the worker and diminishes firms' incentives to post additional jobs given free entry, which leads to the well-known problem that labor market variables react insufficiently to a shock.

30 Hornstein et al. (2005) and Costain and Reiter (2008) conclude that the calibration strategy proposed by Hagedorn and Manovskii (2008) is implausible because a 15% increase in the value of non-market activity implies that the unemployment rate doubles.

assumptions that earlier approaches made.[31]

Frictions rationalize the formation of matches beyond the optimal allocation of workers to jobs because heterogeneity and match-specific output give rise to the possibility that the option value of continued search for the optimal partner is lower than the option value of starting production immediately. The key object of this optimal stopping problem is the production function and the degree of complementarity between its inputs, worker skills and firm productivity. It governs the acceptable degree of suboptimality, that is, the acceptable divergence from the first-best allocation. The higher the degree of complementarity between the inputs of the production function is, the higher is the penalty from a suboptimal match in form of foregone output leading to narrower matching sets and a higher aggregate welfare-loss relative to the optimal allocation. The production function's degree of complementarity is thus a parameter of high empirical interest, also because the degree of sorting contains important implications for policies which are concerned with the optimal allocation of scarce resources in the labor market. A large literature attempts to identify the sign and strength of sorting using matched employer-employee data and reduced form empirical models with worker and firm fixed effects representing unobservable worker and firm characteristics. (see Abowd et al. (1999) and follow-up papers). While adding fixed effects considerably improves those models' explanatory power regarding wage dispersion, many authors inferred that sorting does not systematically affect wage formation because the correlation between the two fixed effects is typically statistically indistinguishable from zero. If positive sorting played a significant role in explaining wage formation, one would expect to find a strong positive correlation between the two fixed effects.

An empirical and intuitive explanation for this small correlation of worker and firm fixed effects is the so-called limited mobility bias (Andrews et al. (2008), Andrews et al. (2012)). It implies that estimated correlations are biased downwards. A conclusion regarding the importance of sorting for wage determination would thus be misleading. The fewer observations of movers between firms in matched employer-employee data there are, the larger the bias and the smaller is the estimated correlation.[32]

[31] The impact of worker heterogeneity on the dynamics of a search and matching model is also in the focus of an interesting article by Pries (2008). He analyzes how worker heterogeneity can impact the dynamics of a search and matching model, however, without analyzing wage formation in particular. He introduces heterogeneity, two types of workers and homogenous firms, in a much simpler way than this paper. Pries (2008) finds some amplification, which he ascribes to a changing composition of the pool of unemployed workers over the business cycle. However, this modification does not to sufficiently amplify the model.

[32] Using German data, Andrews et al. (2012) find positive correlations between worker and firm fixed effects for establishments with a large number of movers and thus conclude that the true correlation

Eeckhout and Kircher (2011) present a theoretical argument why one cannot conclude that sorting is irrelevant for match formation and wage determination in the labor market based on inference from reduced form empirical models. The key argument is that, in theory, assortative matching leads to type-specific wages which, for a given worker, do not monotonically increase in the type of the firm. The fixed effects methodology used in studies that build on Abowd et al. (1999) cannot account for such non-linear relations between the unobserved time-invariant determinants of wages as it necessitates additive separability. A theoretical model with sorting, however, implies an inverted U-shape pattern of a given worker's wages in the firm type. This is also a feature of the model presented in this paper. Beyond the extremum of this non-monotonous function, which represents the first best allocation of this worker and his highest achievable wage, remuneration is lower because the worker has to compensate the firm for hiring him and not waiting to match with a more suitable candidate. Due to search frictions, the probability of matching with the optimal type on a continuous type space is zero for both workers and firms. They have to decide about the acceptable degree of mismatch by optimally choosing the width of their matching sets. The condition for a type to be an element of the matching set is a high enough surplus to compensate both parties for the foregone option value of continued search. While it is impossible to identify the sign of sorting in the simple two-period model used by Eeckhout and Kircher (2011) with wage data alone, Hagedorn et al. (2012) show that the production function—and thus the sign and strength of sorting—is non-parametrically identifiable using standard matched employer-employee data and a fully-dynamic search model with discounting. In a different paper that focuses on workers' search intensity as driving force of sorting, Bagger and Lentz (2014) also solve this identification problem and document significant positive sorting for the Danish labor market using state-of-the-art structural econometric techniques. Due to a renewed and broad interest in the empirical identification of labor market sorting, more evidence for different countries can be expected in the near future. From this papers perspective, these new insights are critically important. I show that sorting has the potential to increase our understanding of labor market dynamics. Since positive sorting is the central assumption of the model, the key parameter of the production function, which governs sorting via the degree of complementarity, has to be calibrated in an informed way using sound empirical evidence, see Section 3.5.

should be positive for Germany. Mendes et al. (2010) document comparable evidence in support of positive sorting. Using very detailed Portuguese data, the authors construct a measure of firm-specific productivity and thus do not have to rely on firm fixed effects. They find strong evidence for PAM in Portugal.

Rigidities of prices and wages play a key role in modern macroeconomics. Blanchard and Galí (2010) show how nominal rigidities influence aggregate fluctuations in a DSGE model with a search and matching labor market component. The key insight is that the central bank faces a trade-off between inflation and unemployment fluctuations if and only if inefficient unemployment fluctuations arise in the presence of productivity shocks. This can only be the case, however, if the labor market block of the DSGE model generates a sufficient degree of amplification in response to shocks. Otherwise unemployment would simply not be responsive on a meaningful scale, contrary to the data. Thus, the Shimer puzzle plays a key role also in enabling larger macroeconomic models to generate dynamics which are consistent with the data and, thus, allow for model-based policy advice in the presence of macroeconomic fluctuations. Blanchard and Galí (2010) also survey a number of earlier papers that explored the implications of real and nominal rigidities in different variants of real business cycle (RBC) or New-Keynesian macro (NKM) models and also link them to the Shimer puzzle literature. The models developed by Merz (1995), Andolfatto (1996), Christoffel and Linzert (2005), Krause and Lubik (2007), Faia (2008), and Gertler and Trigari (2009), among many others, are all examples of complex DSGE models, studied through calibration and simulation, that integrate variants of price and wage staggering to create persistence and sufficient amplification for the analysis of different kinds of shocks. Gertler and Trigari (2009), for example, adapt the well-known Calvo (1983) pricing structure for wage formation in the labor market. Rigid wages are generated by only allowing a calibrated fraction of matches to renegotiate wages in every period. This complex and empirically reasonable modification brings the model's dynamics considerably closer to the data. The key contribution of this paper, in the light of the literature surveyed by Blanchard and Galí (2010), is that the search and matching model with two-sided heterogeneity and sorting is able to generate rigid wages as well as large and inefficient unemployment fluctuations, in line with the data, without additional model extensions. Intuitively, the channels of transmission are similar to those explored in the literature. This shows that sufficient amplification in the presence of shocks and the simple Nash bargaining solution are not necessarily mutually exclusive.

The model developed in this paper is closely related to the standard DMP models that have been extensively analyzed in the literature mentioned above. It can even be shown that it nests the textbook model. Once the continuous type-space is collapsed into a single point, that is, in case of one representative worker and firm, the equations simplify to their textbook versions. Therefore, the mechanism of transmission in the presence of productivity shocks are transparent and the results are directly applicable to larger macroeconomic models. Thus, incorporating a frictional labor market with sorting into a large macroeconomic model with heterogeneous agents is a fascinating topic for future research. One should, however, keep in mind the well-known limitations of the standard search and matching model. First, the model uses a linear search technology in the form of an aggregate matching function with constant returns to scale. While this approach has not been rejected by the data in numerous empirical studies (for a summary, see Petrongolo and Pissarides (2001)), it is by no means undisputed.[33] Second, the assumption that only unemployed individuals search might be less convincing in a model with heterogeneity. The literature contains some empirical evidence on job-to-job flows (for the United States, see, e.g., Fallick and Fleischman (2004) and Nagypál (2005)). I emphasized, however, that search on-the-job would not alter the main results of this paper qualitatively since it would simply imply wider matching sets in equilibrium, similar to a smaller value of γ in my model. Lise et al. (2013) and Lise and Robin (2014) indeed show that apply a sequential bargaining framework in the spirit of Postel-Vinay and Robin (2002) to allow for on-the-job search and poaching also lead to empirically sound dynamics. However, these richer models cannot be applied directly to the literature on wage rigidity and the Shimer Puzzle, which is the focus and central contribution of this paper.

5 Conclusions

Explaining the market imperfections that one can observe in the data in form of sluggish and imperfect price adjustments is a key challenge of modern macroeconomics. Only taking these rigidities into account enables macroeconomic models to generate dynamics which are consistent with empirical facts. Thus, many papers rely on complex model extensions to generate rigid prices and wages to match the data. Often, the labor market is in the center of attention because rigid wages can be the source of inefficient unemployment fluctuations over the business cycle, what is by many considered to be a key labor market policy concern. Search and matching models are the workhorse

[33] Kohlbrecher et al. (2014) show in a recent working paper how the characteristic co-movement of matches and unemployment/vacancies can also arise in a model without a matching function due to idiosyncratic labor productivity and free entry. Thus, matching function estimations might be seriously biased. Additionally, Borowczyk-Martins et al. (2013) emphasize that empirical models that estimate the elasticities of a standard matching function can be severely biased due to endogeneity. They propose an empirical strategy that eliminates this bias by using lagged values of the key variables as instruments.

environment in this kind of application. However, the most widely applied version features fully flexible wages and cannot create rigidities and sufficient amplification by itself.

This paper shows how two-sided ex-ante heterogeneity and a production function with complementarity, i.e., positive assortative matching, can help to overcome this shortcoming and improve the canonical search and matching model's ability to match empirical moments of labor market data. The model endogenously generates a wage rigidity and does not rely on any additional assumptions besides heterogeneity and sorting. Additionally, the recent advances in capturing the extent of assortative matching in labor markets empirically (Andrews et al. (2008), Lopes de Melo (2009), Eeckhout and Kircher (2011), Hagedorn et al. (2012), Bartolucci and Devicienti (2013), Bagger and Lentz (2014)) will increasingly enable the researcher to plausibly measure and calibrate the degree of sorting and complementarity. Some evidence in favor of positive sorting is already presented in some of those studies. The approach to generate rigid wages and sufficient amplification in a search and matching model that is presented in this paper is thus a both microfounded and empirically supported complement to existing approaches to better align search and matching models with the data.

The endogenous wage rigidity I find is of empirically reasonable magnitude. Together with the effect of sorting on firms' vacancy posting decisions, it enables the model to overcome Shimer's volatility puzzle. Both effects provide additional incentives for firms to create jobs in response to a favorable productivity shock. This mechanism significantly amplifies the simulated standard deviations of unemployment, vacancies, labor market tightness, and the job-finding rate compared to the baseline model of Shimer (2005). Aside from the added heterogeneity, the model's elements are standard in the literature and it collapses to the Pissarides (2000) textbook search and matching model for the case of homogenous workers and firms. Thus, I can rely on the empirical results of Shimer (2005) to calibrate the model and comparability of my results is ensured. The improved dynamic properties are solely a result of explicitly accounting for assortative matching.

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