Search Equilibrium, Production Parameters and Social Returns to Education: Theory and Estimation[*]

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Abstract

We introduce skill groups and different production technologies with constant or increasing returns to scale into the Burdett-Mortensen model of on-the-job search. Supermodularity of the different skill groups in the production process leads to a positive intrafirm wage correlation between skill groups. Increasing returns to scale allows the theoretical earnings density to be unimodal with a long right tail even in the absence of productivity dispersion. We use the parameters of our structural estimation to evaluate the returns to education for becoming high skilled taking unemployment risk and earnings uncertainty explicitly into account. Our estimates of the production parameters demonstrate modest increasing returns to scale and imply that social returns to education exceed the individual returns.

Keywords: Search, wage correlation, social returns to education.

JEL Classification: J21, J23, J64.

1. INTRODUCTION

It is generally agreed that the shape of the wage earnings distribution is determined by the skill distribution of the work force, the production technology employed by the economy and the search and matching frictions that govern the allocation of workers to jobs. The aim of the paper is to provide a theoretical and still empirically tractable model that takes all three factors and its interactions into account. For doing so we extend the search equilibrium model of Burdett and Mortensen (1998) and derive an explicit functional form for the wage offer and earnings distributions. Our extension explicitly introduces different skill groups that are linked via a production function which permits either constant or increasing returns to scale. The extension to different skill groups allows for the analysis of firms' wage posting behavior, where firms simultaneously compete for workers of different skill groups. As we show this results in a positive correlation between the wages of workers in different skill groups within firms.

Since the endogenous wage distribution generated by the original Burdett-Mortensen model has an upward-sloping density, which is at odds with the empirical observation of a flat right tail, there has been a lot of effort to extend the original model in order to generate a more realistic-shaped wage distribution. Mortensen (1990) introduced differences in firm productivity and Bowlus et al. (1995) showed that this greatly improves the fit to the empirical wage distribution. Bontemps et al. (2000) and Burdett and Mortensen (1998) formulate a closed-form solution for a continuous atomless productivity distribution, which translates into a right-tailed wage earnings distribution, depending on the assumed productivity dispersion. Postel-Vinay and Robin (2002) extend this for both employer and worker heterogeneity.

In the present extension we demonstrate that with skill multiplicity and a production function that permits increasing returns to scale we get a unimodal right-skewed wage offer and earnings densities with a decreasing right tail. Even though we later introduce productivity dispersion our result about the shape of offer and earnings densities is true even with identical employers. While the structural estimates of models with continuous productivity dispersion as suggested by Bontemps et al. (2000) and Postel-Vinay and Robin (2002) improve the fit to the empirical wage earnings distribution and the estimates of the labor market transition rates, they tell us nothing about the production parameters governing the assumed productivity dispersion (see Manning, 2003, p.106f). Although we also introduce different production technologies in order to improve the fit of the model,

by estimating the parameters of the production functions we are able to identify the technological substitution elasticities between skill groups.

In the theoretical part of the paper we demonstrate that whenever skills are complementary in the production process we should observe a positive within-firm correlation between wages of workers with different skills. Positive intrafirm wage correlation is a well established fact, empirical evidence of which are presented in Katz and Summers (1989) and Barth and Dale-Olsen (2003) among many others. Theoretical consideration of the issue is performed by Kremer (1993). In his O-ring theory Kremer (1993) also uses a production function that exhibits complementarity of the working colleagues' abilities not to make a mistake when performing a sequence of tasks in order to complete the final good. One important consequence of the O-ring theory is a positive correlation between wages and the number of tasks and therefore the overall size of the workforce. However, recently Barth and Dale-Olsen (2002) have empirically demonstrated that the employer-size wage effect vanishes once we look at the skill-group size. In view of this result the labour market frictions approach of this paper that predicts a positive correlation between skill-group size and wages may be more favorable then the O-ring theory of Kremer (1993).

We use the our estimated parameters to analyze whether investment in education creates a positive externality of human capital, i.e. whether the increase in output pays off the individual and the government investment costs. In the model with search frictions existence of human capital externality is shown by Acemoglu (1996). Earlier, external effects of human capital accumulation were discussed by Lucas (1988). However, recent empirical investigations failed to provide undisputable support to the hypothesis that subsidizing education creates a social surplus. Within the "Mincerian" empirical framework Acemoglu and Angrist (2000) reject the existence of human capital externality, whereas Dalmazzo and de Blasio (2003) and Moretti (2004) articulate the opposite conclusion. Bils and Klenow (2000), who concentrate on output growth rather then individual earnings, use calibration to find weak support to positive schooling-growth dependence. We find that a marginal change in the skill structure of the labor force towards more high skilled workers generates an increase in output that overcompensates the society for the additional cost of education.

The paper proceeds as follows. The theory is presented in Section 2, where we extend the existing Burdett-Mortensen framework, solve for optimal strategies of workers and firms and discuss the properties of the resulting equilibrium wage offer distribution. The empirical implementation of the model is treated in Section 3. We formulate the ap-

propriate likelihood function and discuss the relevant estimation method. Whereafter we provide brief description of the data set and in detail discuss the result of the structural estimation of the model. Section 4 concludes.

2. THEORY

In this section we extend the original Burdett-Mortensen model of search equilibrium by introducing different skill groups and different technologies that link the skill groups via the production function.

2.1 *Framework*

The model has an infinite horizon, is set in continuous time and concentrates on steady states. Workers are assumed to be risk neutral and to discount at rate r. Each worker belongs to a skill group $i = 1, 2, ..., I$ whose measures are defined as q_i, satisfying $\sum q_i = m$. The measure u_i of workers is unemployed and the measure $q_i - u_i$ is employed. Before choosing a skill-group workers incur a one-off cost c_i for skill-specific education. By assuming perfect capital market workers are able to borrow the cost of education at the interest rate r.

Workers search for a job in the skill-segmented labor markets. With probability λ_i unemployed workers of skill group i encounter a firm that makes them a wage offer corresponding to their education, and with probability λ_e employed workers encounter a firm.[1] Then workers decide whether to accept or reject the job offer. Job-worker match is destroyed at an exogenous rate $\delta > 0$. Laid off workers start again as unemployed.

We assume that there exist J distinct production technologies $Y_j \left(\mathbf{l} \left(\mathbf{w} \mid \mathbf{w^r}, F \left(\mathbf{w} \right) \right) \right)$ indexed by j, where $\mathbf{l} \left(\mathbf{w} \mid \mathbf{w^r}, F \left(\mathbf{w} \right) \right)$ is the vector of skill groups $l_i \left(w \mid w_i^r, F_i \left(w \right) \right)$ employed by a firm with technology j. The size $l_i \left(w \mid w_i^r, F_i \left(w \right) \right)$ of the skill group depends on the firm's wage offer w_i, the workers' reservation wage w_i^r and the skill specific wage offer distribution $F_i(w)$. We further assume that the production function $Y_j \left(\mathbf{l} \left(\mathbf{w} \mid \mathbf{w^r}, F \left(\mathbf{w} \right) \right) \right)$ is supermodular in $\mathbf{l} \left(\mathbf{w} \mid \mathbf{w^r}, F \left(\mathbf{w} \right) \right)$, i.e. has increasing differences in $\mathbf{l} \left(\mathbf{w} \mid \mathbf{w^r}, F \left(\mathbf{w} \right) \right)$ as defined below, and is twice continuously differentiable in $l_i \left(w \mid w_i^r, F_i \left(w \right) \right)$.

Definition 1: For any $\mathbf{l} \equiv \mathbf{l} \left(\mathbf{w} \mid \mathbf{w^r}, F \left(\mathbf{w} \right) \right)$ and $\mathbf{l'} \equiv \mathbf{l'} \left(\mathbf{w} \mid \mathbf{w^r}, F \left(\mathbf{w} \right) \right)$, $Y_j \left(\mathbf{l} \right)$ is *supermodular* in $\mathbf{l}$, if

$$Y_j \left(\mathbf{l} \wedge \mathbf{l'} \right) + Y_j \left(\mathbf{l} \vee \mathbf{l'} \right) \geq Y_j \left(\mathbf{l} \right) + Y_j \left(\mathbf{l'} \right),$$

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[1] λ_e is not skill group specific, since we would otherwise not be able to derive an explicit wage offer distribution function.

where $l \vee l' \equiv (\max(l_1, l_1'), ..., \max(l_I, l_I'))$ and $l \wedge l' \equiv (\min(l_1, l_1'), ..., \min(l_I, l_I'))$.

Supermodularity in l_i implies *increasing differences* in l_i, i.e. for $\mathbf{l} \geq \mathbf{l}'$ it follows that

$$Y_j(l_i, \mathbf{l}_{-i}) + Y_j(l_i', \mathbf{l}_{-i}') \geq Y_j(l_i, \mathbf{l}_{-i}') + Y_j(l_i', \mathbf{l}_{-i}),$$

where $-i$ denotes the vector of all skill groups except i.

Firms maximize profits by offering a wage schedule $\mathbf{w} = (w_1, w_2, ..., w_I) = (w_i, \mathbf{w}_{-i})$.

2.2 *Workers' Search Strategy*

The optimal search strategy for a worker of occupation i is characterized by a reservation wage w_i^r, where an unemployed worker is indifferent between accepting or rejecting a wage offer, i.e. $U_i = V_i(w_i^r)$, where U_i is the value of being unemployed and $V_i(w_i^r)$ the value of being employed at the reservation wage w_i^r. Flow values of being unemployed and employed

$$rU_i = \lambda_i \int_{w_i^r}^{\bar{w}_i} (V_i(x_i) - U_i)\, dF_i(x_i) - rc_i \tag{1a}$$

$$rV_i(w_i) = w_i + \lambda_e \int_{w_i}^{\bar{w}_i} (V_i(x_i) - V_i(w_i))\, dF_i(x_i) + \delta(U_i - V_i(w_i)) - rc_i \tag{1b}$$

respectively, can be solved for a reservation wage[2]

$$w_i^r = (\lambda_i - \lambda_e) \int_{w_i^r}^{\bar{w}_i} \left(\frac{1 - F_i(x)}{r + \delta + \lambda_e(1 - F_i(x^-))} \right) dx. \tag{2}$$

In order to keep the analysis simple, for the remainder of the paper we assume that $r/\lambda_i \to 0$ as done in the original model by Burdett and Mortensen (1998). The wage offer distribution is given by $F_i(w) = F_i(w^-) + v_i(w)$, where $v_i(w)$ is the mass of firms offering wage w to skill group i. Since offering a wage lower than the reservation wage does not attract any worker, we assume with out loss of generality that no firm offers a wage below the reservation wage, i.e. $F_i(w) = 0$ for $w < w_i^r$.

2.3 *Steady State Flows and Skill Group Size*

Equating the flows in and out of unemployment gives the steady state measure of unemployed per skill group, i.e.

$$u_i = \frac{\delta}{\delta + \lambda_i} q_i. \tag{3}$$

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[2]The details of the derivation can be found in Mortensen and Neumann (1988).

Given the assumptions of constant Poisson arrival rates λ_i, λ_e and the constant separation rate δ Mortensen (1999) has shown that skill group size evolves according to a special Markov-chain known as stochastic birth-death process.

The birth rate of a job offered by a firm posting a wage w is given by the average rate at which a job is filled. There are u_i unemployed who leave unemployment at rate λ_i and $(q_i - u_i)$ employed workers who leave their current employer at rate $\lambda_e G_i(w^-)$ to join the firm offering a wage w, where $G_i(w) = G_i(w^-) + \vartheta_i(w)$ denotes the cumulative wage earnings distribution for skill group i. Workers quit either because they retire at rate δ or they receive a higher wage offer from another firm, which occurs at rate λ_e, and accept it, which happens with probability $\overline{F}_i(w) \equiv (1 - F_i(w))$. The death rate of a job is, therefore, given by $\delta + \lambda_e \overline{F}_i(w)$. Mortensen (1999) shows that the skill group size is Poisson distributed with mean

$$E\left[l_i\left(w \mid w_i^r, F_i\left(w\right)\right)\right] = \frac{\lambda_i u_i + \lambda_e G_i(w^-)(q_i - u_i)}{\delta + \lambda_e \overline{F}_i(w)}$$

Equating the inflow and outflow gives the steady-state measure of employed workers earning a wage less than w

$$G_i(w^-)(q_i - u_i) = \frac{\lambda_i F_i(w^-) u_i}{\delta + \lambda_e \overline{F}_i(w^-)}. \tag{4}$$

Substituting gives

$$E\left[l_i\left(w \mid w_i^r, F_i\left(w\right)\right)\right] = \frac{\delta \lambda_i \left(\delta + \lambda_e\right) / \left(\delta + \lambda_i\right)}{\left[\delta + \lambda_e \overline{F}_i(w)\right]\left[\delta + \lambda_e \overline{F}_i(w^-)\right]} q_i, \tag{5}$$

From (5) it follows that the expected skill group size $E\left[l_i\left(w \mid w_i^r, F_i\left(w\right)\right)\right]$ is increasing in w, if $w \geq w_i^r$, and upper semi-continuous. The intuition behind this result is that on-the-job search implies that the higher the wage offered by a firm the more employed workers are attracted from firms offering lower wages and the less workers quit to employers paying higher wages. This leads to a higher steady-state skill group size for firms offering higher wages. For notational simplicity from now on we use $l_i(w)$ instead of $l_i(w \mid w_i^r, F_i(w))$.

2.4 *Wage Posting*

Each firm posts a wage schedule $\mathbf{w}$ in order to maximize its profit, taking as given the workers' search strategy, i.e. the reservation wage vector $\mathbf{w^r}$, and the other firms' wage posting behavior, i.e. $F(\mathbf{w})$.

$$\pi_j\left(\mathbf{w}\right) = \max_{\mathbf{w}} E\left[Y_j\left(\mathbf{l}\left(\mathbf{w}\right)\right) - \mathbf{w}^T \mathbf{l}\left(\mathbf{w}\right)\right].$$

We assume that a firm can specify its wage policy $\mathbf{w}$ only once. This implies that we can write the maximization problem of a type j firm as

$$\pi_j(\mathbf{w}) = \max_{\mathbf{w}} \left[Y_j \left(E\left[\mathbf{l}(\mathbf{w})\right]\right) - \mathbf{w}^T E\left[\mathbf{l}(\mathbf{w})\right]\right]. \tag{6}$$

Denote by $\mathbf{W}_j$ the set of wage offers that maximize equation (6), i.e. $\mathbf{W}_j = \arg\max_{\mathbf{w}} \pi_j(\mathbf{w})$, and the corresponding I-dimensional wage offer distribution for each firm type j by $F_j(\mathbf{w}) = (F_{1j}(w), F_{2j}(w), ..., F_{Ij}(w))$, where $F_{ij}(w)$ denotes the wage offer distribution of type j firms for skill group i.

Definition 2: A *steady state wage posting equilibrium* is a wage offer distribution $F_j(\mathbf{w})$ with $\mathbf{w} \in \mathbf{W}_j$ for each firm type $j \in J$ such that

$$
\begin{aligned}
\pi_j &= Y_j\left(E\left[\mathbf{l}(\mathbf{w})\right]\right) - \mathbf{w}^T E\left[\mathbf{l}(\mathbf{w})\right] \text{ for all } \mathbf{w} \text{ on the support of } F_j(\mathbf{w}), \tag{7}\\
\pi_j &\geq Y_j\left(E\left[\mathbf{l}(\mathbf{w})\right]\right) - \mathbf{w}^T E\left[\mathbf{l}(\mathbf{w})\right] \text{ otherwise,}
\end{aligned}
$$

given the reservation wage w_i^r for each skill group $i = 1, 2, ..., I$ and a corresponding skill group wage offer distribution $F_i(w)$ such that the reservation wage w_i^r satisfies equation (2) given $F_i(w)$.

2.5 *Properties of the Wage Offer Distribution*

Following Mortensen (1990) we next describe the properties of the aggregate and the skill specific wage offer distributions.

Given the supermodularity property of the production function and the fact that the expected skill group size given in equation (5) is increasing in w and upper semi-continuous implies that profits $\pi_j(\mathbf{w})$ are supermodular in w_i. Thus, a firm paying higher wages for one skill group also pays higher wages for another skill group.

Proposition 1 *Take a firm of type $j \in [1, J]$ offering $\mathbf{w} \in \mathbf{W}_j$ and another firm of type j offering $\mathbf{w}' \in \mathbf{W}_j$ then either $\mathbf{w} \geq \mathbf{w}'$ or $\mathbf{w} \leq \mathbf{w}'$.*

Proof. We first need to define for any $\mathbf{w}$ and $\mathbf{w}'$ that $\mathbf{w} \vee \mathbf{w}' \equiv (\max(w_1, w_1'), ..., \max(w_I, w_I'))$ and $\mathbf{w} \wedge \mathbf{w}' \equiv (\min(w_1, w_1'), ..., \min(w_I, w_I'))$.

For any $\mathbf{w}$ and $\mathbf{w}' \geq \mathbf{w^r}$, $\pi_j(w_i, \mathbf{w}_{-i})$ is supermodular, i.e.

$$\pi_j\left(w_i \wedge w_i', \mathbf{w}_{-i} \wedge \mathbf{w}_{-i}'\right) + \pi_j\left(w_i \vee w_i', \mathbf{w}_{-i} \vee \mathbf{w}_{-i}'\right) \geq \pi_j\left(w_i, \mathbf{w}_{-i}\right) + \pi_j\left(w_i', \mathbf{w}_{-i}'\right),$$

because the same inequality holds for output $Y_j\left(E\left[\mathbf{l}(w_i, \mathbf{w}_{-i})\right]\right)$ and the wage cost cancel out.

Now, we prove $\mathbf{w} \geq \mathbf{w}'$ by contradiction. For any $\mathbf{w}$ and $\mathbf{w}' \in \mathbf{W}_j$ with $w_i > w_i'$, suppose $\mathbf{w}_{-i} < \mathbf{w}_{-i}'$. The following chain of inequalities results in the desired contradiction.

$$
\begin{aligned}
0 \ &< \ \pi_j\left(w_i, \mathbf{w}_{-i}\right) - \pi_j\left(w_i \vee w_i', \mathbf{w}_{-i} \vee \mathbf{w}_{-i}'\right) \\
&\leq \ \pi_j\left(w_i \wedge w_i', \mathbf{w}_{-i} \wedge \mathbf{w}_{-i}'\right) - \pi_j\left(w_i', \mathbf{w}_{-i}'\right) < 0
\end{aligned}
$$

The first and the last inequality result from optimality of $\mathbf{w}$ and $\mathbf{w}'$, the second inequality comes from the supermodularity shown above. ∎

This positive correlation between the wages of workers in different skill groups within firms is a well established fact. Katz and Summers (1989) show evidence that secretaries earn more in firms where average wages are higher. More recently, Barth and Dale-Olsen (2003) find that "[h]igh-wage establishments for workers with higher education are high-wage establishments for workers with lower education as well". The explanation provided for this empirical observation in this paper rests on two pillars. Firstly, labor market frictions lead to an upward sloping labor supply curve for each skill group which can be seen from equation (5). Secondly, we need the complementarity of the skill groups in the production process. This guarantees that increasing both labor inputs simultaneously is optimal. The empirical regularity mentioned above justifies our choice of the production function, where labor inputs are complements.

Note, that Proposition 1 does not guarantee that a firm occupies the same position in the wage offer distribution of all skill groups, because it is possible that there is a mass point in the wage offer distribution of skill group i but not in the wage offer distribution in the other $-i$ skill groups.

Given that the skill group size is increasing in the wage w_i, it would be a waist of money, if the support of the wage offer distributions was not a compact set.

Proposition 2 *The support of each skill specific wage offer distribution $F_i(w)$ is a compact set, i.e. $supp(F_i) = [w_i^r, \overline{w}_i]$.*

Proof. Suppose not, i.e. no firms offer a wage $w_i \in (w_i^*, w_i^{**}) \subset [w_i^r, \overline{w}_i]$. This cannot be profit maximizing, since the firm offering w_i^{**} can offer $\lim_{\varepsilon \to 0}(w_i^* + \varepsilon)$, have the same skill group size, i.e. $l_i\left(w_i^{**} \mid w_i^r, F_i\left(w_i^{**}\right)\right) = \lim_{\varepsilon \to 0} l_i\left(\left(w_i^* + \varepsilon\right) \mid w_i^r, F_i\left(w_i^* + \varepsilon\right)\right)$, since $\lim_{\varepsilon \to 0} F_i\left(w_i^* + \varepsilon\right) = F_i\left(w_i^{**}\right)$, and can thus make higher profit. ∎

Firms with different technologies j make potentially different profits π_j in equilibrium, compare equation (7). We index the technologies according to their profitability, i.e.

$\pi_j \geq \pi_{j-1} \forall j = 1, 2, ..., J$. The next proposition shows that for any skill group i more profitable firms pay higher wages.

Proposition 3 *Let $F_j : supp(F_j) = [\underline{\mathbf{w}}_j, \overline{\mathbf{w}}_j]$ and $F_{j-1} : supp(F_{j-1}) = [\underline{\mathbf{w}}_{j-1}, \overline{\mathbf{w}}_{j-1}]$ be the I-dimensional wage offer distributions of j and $j-1$-type firms respectively. Then, for any wage schedule $\mathbf{w}_j \in [\underline{\mathbf{w}}_n, \overline{\mathbf{w}}_n]$ and $\mathbf{w}_{j-1} \in [\underline{\mathbf{w}}_n, \overline{\mathbf{w}}_n]$ it is true that $\mathbf{w}_j \geq \mathbf{w}_{j-1}$.*

Proof. From the steady state equilibrium condition (7) it follows that:

$$\begin{aligned}
\pi_j &= Y_j(E[\mathbf{l}(\mathbf{w}_j)]) - \mathbf{w}_j^T E[\mathbf{l}(\mathbf{w}_j)] \quad \forall \mathbf{w}_j \in supp(F_j) \\
\pi_j &\geq Y_j(E[\mathbf{l}(\mathbf{w}_{j-1})]) - \mathbf{w}_{j-1}^T E[\mathbf{l}(\mathbf{w}_{j-1})] \quad \forall \mathbf{w}_{j-1} \notin supp(F_j)
\end{aligned}$$

Using the result above we can write

$$\begin{aligned}
\pi_j &= Y_j(E[\mathbf{l}(\mathbf{w}_j)]) - \mathbf{w}_j^T E[\mathbf{l}(\mathbf{w}_j)] \geq Y_j(E[\mathbf{l}(\mathbf{w}_{j-1})]) - \mathbf{w}_{j-1}^T E[\mathbf{l}(\mathbf{w}_{j-1})] \\
&\geq Y_{j-1}(E[\mathbf{l}(\mathbf{w}_{j-1})]) - \mathbf{w}_{j-1}^T E[\mathbf{l}(\mathbf{w}_{j-1})] = \pi_{j-1} \geq Y_{j-1}(E[\mathbf{l}(\mathbf{w}_j)]) - \mathbf{w}_j^T E[\mathbf{l}(\mathbf{w}_j)],
\end{aligned}$$

where the second inequality results from the fact that $\pi_j \geq \pi_{j-1}$.

The difference of the first and the last terms in this inequality is greater than or equal to the difference of its middle terms, i.e $Y_j(E[\mathbf{l}(\mathbf{w}_j)]) - Y_{j-1}(E[\mathbf{l}(\mathbf{w}_j)]) \geq Y_j(E[\mathbf{l}(\mathbf{w}_{j-1})]) - Y_{j-1}(E[\mathbf{l}(\mathbf{w}_{j-1})])$. Since $\mathbf{l}(\mathbf{w})$ is an increasing function of wages $\mathbf{w}$, the claim follows. ∎

In order to be able to identify a particular technology in the empirical estimation, we assume that technologies strictly dominate each other by profits, i.e. $\pi_j > \pi_{j-1}$. Since Proposition 2 holds true for any wage pair $\mathbf{w}_j, \mathbf{w}_{j-1}$ and thus also for $\underline{\mathbf{w}}_j = \inf \mathbf{w}_j$ and $\overline{\mathbf{w}}_{j-1} = \sup \mathbf{w}_{j-1}$, it follows that $\underline{\mathbf{w}}_j \geq \overline{\mathbf{w}}_{j-1}$. Thus, the more productive firms with technology j pay higher wages for all skill groups.

Furthermore, let γ_j denote the cumulative measure of technology j with $\gamma_j > \gamma_{j-1} > 0$ $\forall j = 1, 2, ..., J$ and $\gamma_J = 1$. Thus, Proposition 3 implies that the fraction of firms with technologies earning profit π_j or less post wages $\overline{\mathbf{w}}_j$ or below. Thus, for every skill group i the wage offer distribution at $\overline{w}_{ij}$ is given by γ_j, i.e.

$$F_i(\overline{w}_{ij}) = \gamma_j \tag{8}$$

The next proposition shows under which condition it is not optimal for a type j firm to offer the same wage w_i as a mass of other type j firms does.

Proposition 4 *The wage offer distributions $F_i(w)$ of type j firms for skill group i is continuous, if*

$$Y_j\left[E\left[l_1\left(w\mid w_1^r, F_1(w)\right)\right], E\left[\mathbf{l}(\mathbf{w}_{-1})\right]\right] - Y_j\left[E\left[l_1\left(w\mid w_1^r, F_1(w^-)\right)\right], E\left[\mathbf{l}(\mathbf{w}_{-1})\right]\right] \tag{9}$$
$$> \quad w_1\left(E\left[l_1\left(w\mid w_1^r, F_1(w)\right)\right] - E\left[l_1\left(w\mid w_1^r, F_1(w^-)\right)\right]\right).$$

Proof. Suppose a mass point exists at $w_i \in \left[\underline{w}_{ij}, \overline{w}_{ij}\right]$. Equation (6), and the fact that the cdf $F_i(w)$ is right continuous implies

$$\lim_{\varepsilon \to 0} \pi_j\left(w_i + \varepsilon, \mathbf{w}_{-i}\right)$$
$$= \quad Y_j\left[E\left[l_i\left(w\mid w_i^r, F_i(w)\right)\right], E\left[\mathbf{l}(\mathbf{w}_{-i})\right]\right] - w_i E\left[l_i\left(w\mid w_i^r, F_i(w)\right)\right] - \mathbf{w}_{-i}^T E\left[\mathbf{l}(\mathbf{w}_{-i})\right]$$
$$> \quad Y_j\left[E\left[l_i\left(w\mid w_i^r, F_i(w^-)\right)\right], E\left[\mathbf{l}(\mathbf{w}_{-i})\right]\right] - w_i E\left[l_i\left(w\mid w_i^r, F_i(w^-)\right)\right] - \mathbf{w}_{-i}^T E\left[\mathbf{l}(\mathbf{w}_{-i})\right]$$
$$= \quad \pi_j(\mathbf{w})$$

since $F_i(w) - F_i(w^-) = v_i(w) > 0$, when w_i is a mass point. ∎

The basic argument why the wage offer distributions can be continuous is given by Burdett and Mortensen (1998). If all firms offer the same wage for one skill group, then by offering a slightly higher wage a firm could attract a significantly larger expected skill group size. This wage increase can be arbitrarily small, whereas the resulting increase in the skill group size is significant, since all workers currently working for the "mass-point" wage will change to the new employer as soon as they get this higher wage offer. The deviation from a mass point is, thus, profitable if the increase in total output is higher than the increase in total wage cost induced by a slight wage increase. This is stated by the condition (9) in Proposition 4.

In order to be able to derive an explicit solution for the wage offer distribution, we continue under assumption that no mass points exist. If all wage offer distributions are continuous, then an immediate result of Proposition 1 is that a firm occupies the same position in the wage offer distribution of every skill group. To formalize this let us introduce an index k, which orders the firms of type j as they increase their wage offer for skill group 1 (i.e. firm $k = 1$ offers $\underline{w}_{1j}$), then Proposition 1 implies that for all $\mathbf{w} \in \mathbf{W}_j$

$$F_{ij}^k(w) = F_{lj}^k(w) \text{ for all } i, l = 1, 2, ..., I. \tag{10}$$

In order to have a tractable problem for each skill group i we approximate the production technology j by using a second order Taylor Expansion around the minimum wage $\underline{w}_{ij}$.

Given a technology $Y_j(\mathbf{r}_j)$ is homogeneous of degree ξ_j the Taylor Expansion is given by

$$Y_j(\mathbf{l}(\mathbf{w}_j)) = Y_j(\mathbf{r}_j) + \sum_i Y_j'(\mathbf{r}_j)[r_{ij}h_j(w) - r_{ij}] + \frac{1}{2}\sum_i \sigma_{ij}[h_j(w) - 1]^2,$$

where

$$h_j(w) = \frac{[\delta + \lambda_e(1 - \gamma_{j-1})]^2}{[\delta + \lambda_e \overline{F}_j(w)]^2}, \qquad r_{ij} = \frac{\delta(\delta + \lambda_e)\lambda_i/(\delta + \lambda_i)}{[\delta + \lambda_e(1 - \gamma_{j-1})]^2}q_i,$$

$$Y_j'(\mathbf{r}_j) = \frac{\partial Y_j(\mathbf{r}_j)}{\partial l_i} \quad \text{and} \quad \sigma_{ij} = \sum_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i \partial l_l}r_{lj}r_{ij} = (\xi_j - 1)Y_j'(\mathbf{r}_j)r_{ij}.$$

Using the results of Propositions 1-3 we invoke the equal profit condition $\pi_j = \pi_j^r$ and apply a Taylor Expansion and the first order condition to derive the skill-specific wage offer distribution. Proposition 5 provides the solution for $F_i(w)$ as a function of w.

Proposition 5 *Given that production functions $Y_j(E[\mathbf{l}(\mathbf{w})]) \; \forall j = 1, 2, ..., J$ are homogeneous of degree $\xi_j \geq 1$, a unique equilibrium wage offer distribution $F_{ij}(w)$ for each skill group $i = 1, 2, ..., I$ exists and has the following form*
(i) for $\xi_j = 1$

$$F_{ij}(w) = \frac{\delta + \lambda_e}{\lambda_e} - \frac{\delta + \lambda_e(1 - \gamma_{j-1})}{\lambda_e}\sqrt{\frac{Y_j'(\mathbf{r}_j) - w}{Y_j'(\mathbf{r}_j) - \underline{w}_{ij}}}, \tag{11}$$

(ii) for $\xi_j > 1$

$$F_{ij}(w) = \frac{\delta + \lambda_e}{\lambda_e}$$

$$- \frac{\delta + \lambda_e(1 - \gamma_{j-1})}{\lambda_e\sqrt{-\frac{(Y_j'(\mathbf{r}_j) - w)r_{ij} - \sigma_{ij}}{2(\sigma_{ij} - \mu_{ij})} + \sqrt{\left(\frac{(Y_j'(\mathbf{r}_j) - w)r_{ij} - \sigma_{ij}}{2(\sigma_{ij} - \mu_{ij})}\right)^2 + \frac{(Y_j'(\mathbf{r}_j) - \underline{w}_{ij})r_{ij} - \mu_{ij}}{(\sigma_{ij} - \mu_{ij})}}}} \tag{12}$$

for any $w \in [\underline{w}_{ij}, \overline{w}_{ij}]$, where

$$\mu_{ij} = \frac{r_{ij}}{\sum_i r_{ij}}\frac{1}{2}\sum_i \sigma_{ij},$$

A necessary condition for an upward sloping wage offer density $\partial F_i(w)/\partial w$ is

$$(2 - \xi_j)\frac{\partial Y_j(\mathbf{r}_j)}{\partial r_{ij}} - w > 0. \tag{13}$$

Proof. See Appendix. ∎

The aggregate wage offer distribution is given by

$$F(w) = \sum_{i=1}^{I} \frac{q_i}{m} F_i(w) = \sum_{i=1}^{I} \frac{q_i}{m} \sum_{j=1}^{J} (\gamma_j - \gamma_{j-1}) F_{ij}(w).$$

A special case for $F_{ij}(w)$ when $\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right) r_{ij} = \mu_{ij}$ is shown in the proof of Proposition 5. Since it implies artificial restrictions on ξ_j considering this case here is neither interesting nor useful.

For a production function with homogeneity of degree one the explicit wage offer distribution resembles the distribution derived in Burdett and Mortensen (1998) and has its typical increasing density. Since an upward-sloping earnings density is at odds with the empirical observation of a flat right tail, Mortensen (1990) introduces differences in firm productivity by allowing for different productivity levels in order to improve the fit to the empirical wage earnings distribution. Bowlus et al. (1995) demonstrate that this greatly improves the fit to the empirical earnings distribution. Bontemps et al. (2000) and Burdett and Mortensen (1998) formulate a closed-form solution for a continuous atomless productivity distribution, which translates into a right-tailed wage earnings distribution, depending on the assumed productivity dispersion.[3]

Our wage offer distribution allows technologies having constant or increasing returns to scale. The novelty is that the wage offer distribution given in Proposition 5 can have an increasing and a decreasing density for a given production technology. Although we allow for the possibility that heterogeneous production technologies are used, we do not need any technology dispersion to get a hump-shaped density. As stated in condition (13) only technologies with homogeneity of degree $2 > \xi_j \geq 1$ can have an increasing density. Notice further that as the wage w increases condition (13) is more likely to be violated implying that the wage offer density can have an upward sloping part for small wages and an downward sloping part for large wages. A production technology with decreasing returns to scale would result in a negative wage offer density for at least one skill group.

Mortensen (2000) makes implicitly a similar restriction to production functions with increasing returns to scale when deriving endogenously the employer heterogeneity based on match specific capital. He assumes that the production technology has constant returns

---

[3]However, tail behavior of the productivity density, hence offer and earnings densities, in this case is subject to additional restrictions (see Bontemps et al., 2000; Proposition 8).

with respect to labor but on whole increasing returns, i.e. $Y(l(w)) = k^{\alpha}l(w)$. By simulation he shows that for positive α the wage offer distribution has a flat right tail.

The comparative statics of the original Burdett-Mortensen model are still valid for the general wage offer distribution function. If the arrival rate of on-the-job offers, i.e. λ_e, goes to zero, then the wage offer distribution $F_i(w)$ collapses to a mass point at the reservation wage w_i^r, which equals the Diamond (1971) monopsony solution. If moving from one job to another becomes very easy, i.e. λ_e goes to infinity, the competition among firms drives wages up and the wage earnings distribution $G_i(w)$ converges to a mass point at the marginal product of the skill group.

3. ECONOMETRIC MODEL

Here we consider in detail the structural econometric model based on the theory presented above. We assume a Cobb-Douglas production technology which allows for constant and increasing returns to scale, i.e.

$$Y_j(l(\mathbf{w}_j)) = p_j \prod_{i=1}^{I} l_i(w_j)^{\alpha_{ij}} \tag{14}$$

with $\sum_i \alpha_{ij} = \xi_j \geq 1$, $\alpha_{ij} > 0$.

In general, we build upon the model developed by Bowlus et al. (1995), (2001). In the discussion to follow we put special emphasis on such new features as parameter identification and related modification of the estimation procedure.

3.1 *The Likelihood Function*

Let us start from the formulation of the likelihood function. For Poisson process with rate θ the joint distribution of the elapsed (t_e) and residual (t_r) duration of time spent by an individual in a certain state of the labour market is $f(t_e, t_r) = \theta^2 e^{-\theta(t_e+t_r)}$. For an individual that belongs to i-th skill group the appropriate Poisson rates are λ_i if the person is unemployed and $\delta + \lambda_e [1 - F_i(w)]$ if the person is employed at wage w. Furthermore:

- *For Unemployed:* Equilibrium probability of sampling an unemployed agent who belongs to i-th skill group is $m^{-1}q_i\delta/(\delta + \lambda_i)$. In case the subsequent job transition is observed we know the offered wage and can record the value of the wage offer density $f_i(w)$.

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- *For Employed:* Equilibrium probability of sampling an agent who belongs to i-th skill group and earns wage w is $m^{-1}q_i g_i(w)\lambda_i/(\delta + \lambda_i)$. In case the transition to the next state is observed we record the destination state. The probabilities of exit to unemployment and to next job are $\rho_{j\to u} = \delta/\left(\delta + \lambda_e \overline{F}_i(w)\right)$ and $\rho_{j\to j} = \lambda_e \overline{F}_i(w)/\left(\delta + \lambda_e \overline{F}_i(w)\right)$ respectively.

For convenience of estimation, define $\kappa_i = \lambda_i/\delta$, $\kappa_e = \lambda_e/\delta$. Then the likelihood contributions of unemployed $(\mathcal{L}_{(i)\,u})$ and employed $(\mathcal{L}_{(i)\,e})$ individuals affiliated with i-th skill group is:

$$\mathcal{L}_{(i)\,u} = \frac{q_i}{m\left(1 + \kappa_i\right)} \left[\delta\kappa_i\right]^{2-d_r-d_l} e^{-\delta\kappa_i[t_e+t_r]} \left[f_i(w)\right]^{1-d_r},\qquad(15)$$

$$\mathcal{L}_{(i)\,e} = g_i(w)\frac{q_i}{m}\frac{\kappa_i}{1 + \kappa_i} \left[\delta\left(1 + \kappa_e\overline{F}_i(w)\right)\right]^{1-d_l} e^{-\delta\left(1+\kappa_e\overline{F}_i(w)\right)[t_e+t_r]}$$
$$\times \left[\left[\delta\kappa_e\overline{F}_i(w)\right]^{d_t} \delta^{1-d_t}\right]^{1-d_r}\qquad(16)$$

In (15) and (16) $d_l = 1$, if a spell is left-censored, 0 otherwise; $d_r = 1$, if a spell is right-censored, 0 otherwise; $d_t = 1$ if there is a job-to-job transition, 0 otherwise. Substitution of the appropriate $g_i(w)$, $f_i(w)$ and $F_i(w)$ into (15) and (16) completes the formulation of the likelihood function, where $g_i(w)$ is obtained from $F_i(w)$ using (4).

Notice that except of probability terms $m^{-1}q_i/(1 + \kappa_i)$ and $m^{-1}q_i\kappa_i/(1 + \kappa_i)$ (15) and (16) are the same as in Kiefer and Neumann (1993) or Bowlus et al. (1995). The main differences are rather driven by the functional forms of the offer and earnings distributions.

3.2 *Homogeneous Firms*

We find it instructive to start with the model with no productivity dispersion, since the theory allows obtaining an earnings density with a decreasing right tail even with homogeneous employers. This density will have $I-1$ jumps at infimum wages and $I-1$ spike at supremum wages of each skill group.

Under employer homogeneity the assumed production function modifies to $Y(l(\mathbf{w})) = p\prod_{l=1}^{I} l_l(w)^{\alpha_l}$. Functional form of the wage offer distribution with homogeneous employers is $F(w) = \sum_{i=1}^{I}\frac{q_i}{m}F_i(w)$, where $F_i(w)$ is given in Proposition 5 with $J = 1$. Rewritten in terms of $\kappa_{i,e}$ the skill-specific offer distribution becomes

$$F_i(w) = \frac{1 + \kappa_e}{\kappa_e} - \frac{1 + \kappa_e}{\kappa_e\sqrt{-\frac{\left(Y_i'(\mathbf{r})-w\right)r_i-\sigma_i}{2(\sigma_i-\mu_i)} + \sqrt{\left(\frac{\left(Y_i'(\mathbf{r})-w\right)r_i-\sigma_i}{2(\sigma_i-\mu_i)}\right)^2 + \frac{\left(Y_i'(\mathbf{r})-\underline{w}_i\right)r_i-\mu_i}{(\sigma_i-\mu_i)}}}},\qquad(17)$$

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where

$$
r_i = \frac{\kappa_i}{(1 + \kappa_e)(1 + \kappa_i)} q_i, \qquad Y_i'(\mathbf{r}) = \frac{\alpha_i}{r_i} p \prod_{i=1}^{I} r_i^{\alpha_i},
$$

$$
\sigma_i = \alpha_i (\xi - 1) Y(\mathbf{r}), \quad \text{and} \quad \mu_i = \frac{r_i}{\sum_i r_i} \frac{1}{2} \sum_i \sigma_i.
$$

Recognizing that $F_i(\overline{w}_i) = 1$ we use $Y(l(\mathbf{w}))$ to get the following solution for the common productivity parameter

$$
p = \frac{r_i}{\prod_{i=1}^{I} q_i^{\alpha_i}} \left(\frac{(1 + \kappa_e)^2 \overline{w}_i - \underline{w}_i}{(1 + \kappa_e)^2 - 1} \right)
$$
$$
\times \left[\alpha_i - (\xi - 1) \left(\frac{\xi \left((1 + \kappa_e)^2 + 1 \right) r_i}{2 \sum_i r_i} - (\kappa_e + 1)^2 \alpha_i \right) \right]^{-1} \quad (18)
$$

Consider the unknowns of the econometric model. The skill measures $\{q_i\}_{i=1}^{I}$ are known from the data and they are nothing else but sample sizes of each skill group. Furthermore Kiefer and Neumann (1993) suggest using $\underline{w}_i = \min(w_i)$ and $\overline{w}_i = \max(w_i)$ as the consistent estimates for the bounds of the skill-specific wage offer distributions. Finally, from the fact that (18) holds for any i one can show that any α_i can be represented as a function of ξ. Thus the parameter space eventually reduces to $\left\{ \{\kappa_i\}_{i=1}^{I}, \kappa_e, \delta, \xi \right\}$.

In the present paper we estimate the model with two different skill levels. For $I = 2$ we get

$$
\alpha_1 = \xi \frac{(\xi - 1) \left(C \left[(\kappa_e + 1)^2 - D_2 \right] + D_1 \right) + C}{\left[(\xi - 1)(\kappa_e + 1)^2 + 1 \right] (C + 1)}
$$

where

$$
C = \frac{(1 + \kappa_2) \kappa_1 q_1}{(1 + \kappa_1) \kappa_2 q_2} \frac{(\kappa_e + 1)^2 \overline{w}_1 - \underline{w}_1}{(\kappa_e + 1)^2 \overline{w}_2 - \underline{w}_2} \quad \text{and} \quad D_i = \left((\kappa_e + 1)^2 + 1 \right) \frac{\kappa_i q_i / (1 + \kappa_i)}{2 \sum_{l=1}^{2} \kappa_l q_l / (1 + \kappa_l)}
$$

For $I > 2$ the production parameters α_{-I} are a solution to a non-linear system determined by $\left\{ \{\kappa_i\}_{i=1}^{I}, \kappa_e, \delta, \xi \right\}$.

To establish the (local) identifiability of the model with identical employers it is sufficient to show that the matrix of the expected outer product of the log-likelihood has full rank (see Rothenberg, 1971, Theorem 1). However, even with $I = 2$ the derivations become too cumbersome. Numerically we find that this condition is met in the sufficiently wide neighborhood of the maximizer.

Given $\min(w_i)$ and $\max(w_i)$ the model has a regular likelihood function which can be maximized using standard gradient-based methods.

3.3 *Heterogeneous Firms*

For heterogeneous employers the production functions are given in (14). The relevant occupation-specific wage offer distribution $F_i(w)$ is given in Proposition 5. Rewritten in $\kappa_{i,e}$ terms it becomes

$$F_i(w) = \frac{1 + \kappa_e}{\kappa_e}$$
$$- \frac{1 + \kappa_e\left(1 - \gamma_{j-1}\right)}{\kappa_e\sqrt{-\frac{\left(Y_j'(\mathbf{r}_j) - w_{ij}\right)r_{ij} - \sigma_{ij}}{2(\sigma_{ij} - \mu_{ij})} + \sqrt{\left(\frac{\left(Y_j'(\mathbf{r}_j) - w_{ij}\right)r_{ij} - \sigma_{ij}}{2(\sigma_{ij} - \mu_{ij})}\right)^2 + \frac{\left(Y_j'(\mathbf{r}_j) - \underline{w}_{ij}\right)r_{ij} - \mu_{ij}}{(\sigma_{ij} - \mu_{ij})}}}} \quad (19)$$

where

$$r_{ij} = \frac{\kappa_i/\left(1 + \kappa_i\right)\left(1 + \kappa_e\right)}{\left[1 + \kappa_e\left(1 - \gamma_{j-1}\right)\right]^2}q_i, \qquad Y_j'(\mathbf{r}_j) = \frac{\alpha_{ij}}{r_{ij}}p_j\prod_{i=1}^{I}r_{ij}^{\alpha_{ij}},$$

$$\sigma_{ij} = \alpha_{ij}\left(\xi_j - 1\right)Y_j(\mathbf{r}_j), \quad \text{and} \quad \mu_{ij} = \frac{r_{ij}}{\sum_i r_{ij}}\frac{1}{2}\sum_i\sigma_{ij}$$

for all $w \in [\underline{w}_{ij}, \overline{w}_{ij}]$, $i = 1, ..., I$ and $j = 1, ..., J$.

We also assume that for any i and j none of α_{ij} need to equal each other. Remembering that $\gamma_j = F_i(\overline{w}_{ij})$ we use this production function and the expression for the skill-specific wage offer distribution in (19) to derive the productivity level of the firm

$$p_j = \frac{r_{ij}}{\prod_{i=1}^{I}r_{ij}^{\alpha_{ij}}}\left[\alpha_{ij} - \frac{\xi_j - 1}{\eta_j}\left(\frac{\xi_j\left(1 + \eta_j\right)r_{ij}}{2\sum_i r_{ij}} - \alpha_{ij}\right)\right]^{-1}\left(\frac{\overline{w}_{ij} - \eta_j\underline{w}_{ij}}{1 - \eta_j}\right) \quad (20)$$

where $\eta_j = [(1 + \kappa_e[1 - \gamma_j])/(1 + \kappa_e[1 - \gamma_{j-1}])]^2$.

Consider the unknowns of the econometric model with heterogeneous firms. As before, skill group size and group-specific bounds for the offer distributions are available from the data. At the same time there appears an additional set of unknown cutoff wages $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$ for the firm-specific wage offer. Existence of unknown cutoff wages does not allow using (20) to write down α_{ij} as a function of exclusively ξ_j. Though, the fact that $\overline{w}_{ij} = \underline{w}_{ij-1}$ provides us with additional cross-restrictions on p_{j-1} and p_j. Along with (20) being the same for all i we use these cross-restrictions to express cutoff wages as a function of production parameters $\{\alpha_{ij}\}_{i,j=1}^{I-1,J}$ and ξ. Conversely, one can express production parameters

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as a function of $\{\overline{w}_{ij}\}_{i,j=1}^{I,J-1}$ and ξ, but it turns out that for the estimation of the model with $J \leq I$ the first option is more convenient. Thus, the parameter space of the model eventually reduces to $\left\{\theta, \{\gamma_j\}_{j=1}^{J-1}\right\}$, where $\theta = \left\{\{\kappa_i\}_{i=1}^{I}, \delta, \kappa_e, \{\alpha_{ij}\}_{i,j=1}^{I-1,J}, \{\xi_j\}_{j=1}^{J}\right\}$. To demonstrate identifiability of the model we would need a theory similar to that of Rothenberg (1971), but generalized for nonregular models.

Since in the present application we estimate the symmetric case of $I = J = 2$ the cutoff wages become

$$
\begin{bmatrix} \overline{w}_{11} \\ \overline{w}_{21} \end{bmatrix} = \begin{pmatrix} \frac{\kappa_1 q_1/(1+\kappa_1)}{\alpha_{11}(\eta_1+\xi_1-1)-C_{11}} & -\frac{\kappa_2 q_2/(1+\kappa_2)}{(\xi_1-\alpha_{11})(\eta_1+\xi_1-1)-C_{21}} \\ -\frac{\eta_2\kappa_1 q_1/(1+\kappa_1)}{\alpha_{12}(\eta_2+\xi_2-1)-C_{12}} & \frac{\eta_2\kappa_2 q_2/(1+\kappa_2)}{(\xi_2-\alpha_{12})(\eta_2+\xi_2-1)-C_{22}} \end{pmatrix}^{-1} \times
$$
$$
\begin{bmatrix} \frac{\eta_1\kappa_1 q_1/(1+\kappa_1)}{\alpha_{11}(\eta_1+\xi_1-1)-C_{11}}\underline{w}_{11} - \frac{\eta_1\kappa_2 q_2/(1+\kappa_2)}{(\xi_1-\alpha_{11})(\eta_1+\xi_1-1)-C_{21}}\underline{w}_{21} \\ \frac{\kappa_2 q_2/(1+\kappa_2)}{(\xi_2-\alpha_{12})(\eta_2+\xi_2-1)-C_{22}}\overline{w}_{22} - \frac{\kappa_1 q_1/(1+\kappa_1)}{\alpha_{12}(\eta_2+\xi_2-1)-C_{12}}\overline{w}_{12} \end{bmatrix}
$$

where

$$
C_{ij} = \frac{\kappa_i q_i}{1+\kappa_i} \frac{\xi_j\left(\xi_j-1\right)\left(1+\eta_j\right)}{2\sum_{l=1}^{2}\kappa_l q_l/(1+\kappa_l)}
$$

The fact that the theoretical structure of our extension allows expressing cutoff wages as a function of other parameters also brings about the changes into the estimation strategy of the model. To estimate an equilibrium search model with discrete productivity dispersion Bowlus et al. (1995) suggest a stepwise procedure. At the first step given the starting values for the structural parameters cutoff wages are estimated by simulated annealing. At the second step structural parameters are estimated using the previously obtained cutoff wages (and the second step is a "smooth" optimization). Afterwards the results of both steps are used to calculate new point mass values for the productivity distribution, which is done through the empirical distribution of earnings, and the cycle repeats again. Estimation of our extension is no longer stepwise, because the cutoff wages (conversely, production parameters) are known functions of the rest of the parameters of the model. This means that all the parameters are estimated in one shot. Having obtained the maximizer θ we use it along with (4) and (8) to calculate the new point mass values γ_j by finding a zero of a function

$$
\gamma_j = 1 - \sum_{i=1}^{I} \frac{1-\hat{G}_i(\overline{w}_{ij}|\theta,\gamma_j)}{1+\kappa_e\hat{G}_i(\overline{w}_{ij}|\theta,\gamma_j)} \tag{21}
$$

where $\hat{G}_i$ is a nonparametric estimate of the skill-specific earnings distribution evaluated at the cutoff wages implied by θ. Given the new values of $\{\gamma_j\}_{j=1}^{J-1}$ the cycle repeats again.

To find θ we apply a Nelder-Mead direct search method. To avoid a possible error due to a too early termination we repeatedly restart the algorithm at the claimed maximum until the function value does not improve any longer (see Press et al., 1995).

Finally we use bootstrap to find the confidence intervals for the estimated parameters (see Chernozhukov and Hong (2004) for the theoretical validation of bootstrap methods in nonregular models).

3.4 Specification Check

We have derived the wage offer distribution (12) under the assumption that all skill specific wage offer distributions $F_i(w)$ are continuous. As argued in Proposition 4 a mass point can only exist, if increasing the wage further would imply that the additional wage cost outweighs the additional output produced with the additionally recruited workers. Consider an arbitrary skill group h. Proposition 4 implies that the distribution function $F_h(w)$ is continuous, if for a type j firm $\lim_{\varepsilon \to 0} \pi_j(w_h + \varepsilon, \mathbf{w}_{-h}) > \pi_j(\mathbf{w})$, i.e.

$$p_j \left(\frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_h(w)\right]^2} q_h \right)^{\alpha_{hj}} \prod_{\substack{i=1 \\ i \neq h}}^{I} l_i(w)^{\alpha_{ij}} - \frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_1(w)\right]^2} w_h q_h >$$

$$> \left(\frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_h(w)\right]\left[1+\kappa_e \overline{F}_h(w^-)\right]} q_h \right)^{\alpha_{hj}} p_j \prod_{\substack{i=1 \\ i \neq h}}^{I} l_i(w)^{\alpha_{ij}} - \frac{\kappa_h(1+\kappa_e)/(1+\kappa_h)}{\left[1+\kappa_e \overline{F}_h(w)\right]\left[1+\kappa_e \overline{F}_h(w^-)\right]} w_h q_h.$$

First, note that this condition is satisfied for $\alpha_{hj} \geq 1$. For $\alpha_{hj} < 1$ the concavity of the production function implies that if a mass point exists at $w_h \in [\underline{w}_{hj}, \overline{w}_{hj}]$, then increasing the wage by ε still implies that the additional wage cost outweighs the additional output produced. Thus, if a mass point exists, then it exists at the upper bound of the support of $F_{hj} : supp(F_{hj}) = [\underline{w}_{hj}, \overline{w}_{hj}]$. Together with the fact that $F_h(\overline{w}_{hj}) = \gamma_j$ this implies that $F_h(\overline{w}_{hj}^-) = \gamma_j - \upsilon_h(\overline{w}_{hj})$. Substituting $\overline{w}_{hj}$ for w_h in the equation above and rearranging gives the following inequality:

$$\frac{1 - \left(\frac{1+\kappa_e(1-\gamma_j)}{1+\kappa_e\left(1-\gamma_j+\upsilon_1(\overline{w}_{hj})\right)} \right)^{\alpha_{hj}}}{1 - \frac{1+\kappa_e(1-\gamma_j)}{1+\kappa_e\left(1-\gamma_j+\upsilon_1(\overline{w}_{hj})\right)}} > \frac{\overline{w}_{hj} l_h(\overline{w}_{hj})^{1-\alpha_{hj}}}{p_j \prod_{\substack{i=1 \\ i \neq h}}^{I} l_i(w)^{\alpha_{ij}}}. \tag{22}$$

From (22) a necessary condition for continuity follows whenever $\lim_{\upsilon_h(\overline{w}_{hj}) \to 0} (lhs) > (rhs)$. Taking limit of the lhs and applying (10) to the rhs we get

$$a_{hj} > \frac{\overline{w}_{hj} l_h(\overline{w}_{hj})}{p_j \prod_{i=1}^{I} l_i(\overline{w}_{ij})^{\alpha_{ij}}} \tag{23}$$

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The estimated parameters are consistent only when the model is properly specified, i.e. when (23) holds.

In the model with productivity dispersion another way to see whether (10) holds is to consider $\hat{G}_i \left(\overline{w}_{ij} \middle| \underset{\{\theta, \gamma_j\}}{\arg \max} \left(\mathcal{L} \right) \right)$. Both (10) and (4) imply that $\hat{G}_i = \hat{G}_l \ \forall i, l \in [2, I]$. At the same time (21) does not restrict $\hat{G}_i$ to be equal to each other. Thus, if $\{\theta, \gamma_j\}$ $\forall j \in [2, J-1]$ is a consistent estimate of the true parameters the values of the empirical earnings distribution at the skill-specific cutoff wages must not be significantly different from each other.

4. EMPIRICAL APPLICATION

4.1 *The Data*

We use data from the German Socio-Economic Panel – a longitudinal survey of German households, which was started at 1984 and conducted on the annual basis ever since. Our sample contains information from the waves of 1984 to 2001. The analysis is restricted to working age population of native West Germans and major groups of foreigners living in West Germany.

According to the theoretical model we have only two states, namely "full time employment" and "unemployment". Since utility maximizing behavior of the representatives of the other groups, such as part-time employed, self-employed or non-participants can be different from behavior of the individuals considered by the model we exclude all the agents who are neither full time employed nor unemployed from the sample (see Koning et al., 1995, van den Berg and Ridder, 1998).

To estimate the model we need have information on both duration and wages. We get duration information by choosing a reference year and sampling all employed and unemployed individuals at this year. After doing so for each observation we track the individual history backwards and forwards to restore the elapsed and residual duration of his/her staying in the current state of the market. Both elapsed and residual spells can be incomplete due to overshooting the starting and terminal dates of the observation period while the spell is still in progress. To minimize the number of incomplete spells and at the same time provide the most recent information about the length of total unemployment or job duration we choose 1995 as a reference year. Whenever residual spell is complete we

also record information about the exit state (one should keep in mind that in the setting of the model, job-to-job changes are also considered as "change of state").

Unemployment duration is calculated from the retrospective labour force status calendarium of the GSOEP, in which respondents have to provide their labour force status for every month of the previous calendar year.

Retrieving job duration requires a bit more elaboration. First of all every currently employed individual provides information about the calendar month and year of the job start. Though for those who have undergone a job change we need to check additionally the date and the type of this job change. Apart from job changes to a new employers or within firm job changes with wage promotion, which classifies as change of state, this can also be company takeover, return to work etc. Thus only simultaneous application of both sources of information allows us to find the correct starting date. Similarly we find the endpoint of the job spell. The calendar end of job spells is set to the first reported job end in subsequent waves or to the first reported job start with new employer or within the same firm.

We also need to comment on incomplete spells. Those incomplete from the left can be seldom observed in the data. In our data set, the main reason for a spell being incomplete from the left is that it is not always possible to determine its exact calendar month (sometimes even year), because the respondent was simply not interviewed prior to the start of the spell. There are much more spells incomplete from the right. This happens because of the two reasons. First of all, the spell can still be in progress by the end of the available observation period. Secondly, spells that terminate by exit to non-participation are treated as right-censored.

The final bit of information necessary for the estimation of the model is earnings. We use the data on net wages provided by the GSOEP. Individuals who are employed at their interview provide the monthly net wage in the month prior to the interview. For the sample of job spells we use wage information provided by respondents at the year for which the sample is drawn. For the sample of unemployment spells we use the first reported wage after the end of unemployment, given that the transition to the job is observed. All wages are deflated by the West German consumer price index at prices of 1998.

In our application we estimate the model with two different productivity levels and two different skill groups. Skill stratification of the sample is performed on the basis of the

Table 1: "Descriptive Statistics of Event History and Earnings Data *"

	Full Sample	Low-Skilled	High-Skilled
Number of individuals:	3977	2903	1074
Employed:	3638	2602	1036
Unemployed:	339	301	38
Employed Agents:			
Uncensored observations with:			
job → job transition:	420	240	180
job → unemployment transition:	275	234	41
mean time spell between two states [job duration]:	107.086	115.464	89.118
(std. deviation):	(101.28)	(107.04)	(85.16)
Censored observations			
a) left-censored durations only			
with job → job transition:	21	15	6
with job → unemployment transition:	16	14	2
b) right-censored durations only:	2822	2033	789
c) both left- and right-censored durations:	84	66	18
Mean time spell [both uncensored and censored]:	155.518	156.177	153.864
(std. deviation):	(119.28)	(118.84)	(120.43)
Unemployed Agents:			
Uncensored observations (u → j transition):	104	90	14
mean time spell between two states [job duration]:	20.971	22.500	11.143
(std. deviation):	(23.00)	(23.95)	(11.96)
Censored observations			
a) left-censored durations (u → j transition) only:	3	3	-
b) right-censored durations only:	219	195	24
c) both left- and right-censored durations:	13	13	-
Mean time spell [both uncensored and censored]:	35.490	36.924	24.132
(std. deviation):	(33.67)	(34.57)	(22.77)
Earnings Information:			
Sample Minimum:	603.	603.	952.
Mean Wage:	3095 (1356)	2757 (1034)	3962 (1663)
Sample Maximum:	11535.	9524.	11535.

* Duration data in Months. Earnings in DM.

"International Standard Classification of Education (ISCED)" code. We identify as "low-skilled" all individuals who have inadequate, general elementary or middle vocational training. As "high-skilled" we qualify all the rest, i.e. those with higher vocational training, university education etc.

Summary of duration and wage data is presented in Table 1. Apart from the information about the full sample we also present summary statistics for the two skill groups. The data on skills reflect such basic facts about less skilled in comparison to high-skilled as higher level of unemployment, higher rate of job loss and longer unemployment duration. Additionally wage data are summarized by the kernel density plot (see Figure A.1 in the Appendix). As expected, earnings density of low-skilled is more skewed to the right than that of the high-skilled. Also mean net wage of high-skilled workers amounts to DM 3962 which exceeds those of less skilled by more then 40%.

4.2 *Estimation Results: Overall Fit of the Model*

Firstly, we estimate the model with identical employers (see Table A.1 in the Appendix). When doing so we also fit the original Burdett-Mortensen model with no productivity dispersion to compare it with the results provided by our extension. From Table A.1 one can see that the structural parameters estimated with both original and constant returns specifications do not significantly differ from each other (bootstrap confidence intervals in square brackets). Predicted theoretical offer and earnings densities (Figures A.2-A.3 respectively) have a jump at the reservation wage of the high-skilled and a spike at the maximum wage of the low-skilled, which generates a "falling" right tail of the aggregate density despite that skill-specific ones are strictly increasing. However, even with large I the model with constant returns has limited ability to fit the data. The results crucially change when we introduce increasing returns (the third column in Table A.1). The estimate of κ_1 almost precisely matches the observed 10.37% unemployment rate of low-skilled predicting 10,19%. The model underestimates κ_2 but at the same time provides much more realistic results for κ_e and δ. Addressing Figures A.2-A.3 we see that the model with constant returns implies offer and earnings densities with strictly decreasing right tail even in absence of productivity dispersion. At the same time the predicted earnings density is too flat implying that the formulation with identical firms also has limited capacity of fitting the data. Furthermore, the initial unrestricted estimates of the model with increasing returns to scale did not meet the no-mass-point condition of Proposition 4, so in Table A.1 we present the estimates which are obtained by maximizing

the likelihood function subject to (23). Failure to meet the requirements of Proposition 4 implies that even despite a great improvement in comparison to the specifications with constant returns technology the assumption that firms are identical is too restrictive.

Next we estimate the model with two skill groups and two distinct productivity levels. As before, we also fit the original Burdett-Mortensen model with $J = 2$. The first remarkable result is that unconstrained estimates of both constant returns and increasing returns specifications meet the the no-mass-point condition. Again, the results of the original Burdett-Mortensen model and our extension with constant returns almost do not differ from each other. Even though two jumps at the left tail and two spikes at the right one improve the fit of the aggregate earnings density predicted by the constant returns specification (see Figure A.4) locally increasing right tail of individual-specific densities still makes this fit being far from satisfactory. Relaxing the assumption of constant returns again changes the picture completely (see Table A.2, column 3). First, the estimates of $\kappa_{1,2}$ imply the unemployment rates of 10.07% for low-skilled and 3,67% for high-skilled, which is not significantly different from the sample measures of 10.37% and 3,54% respectively.[4] Secondly, the model provides quite satisfactory results for on-the-job search frictions κ_e and the separation rate δ.[5] Finally the predicted aggregate earnings density has not only the decreasing right tail but also the sufficiently high right skew (see Figure A.4). Skill-specific earnings mean and standard deviation computed with the predicted theoretical densities make DM 2987.4 (1229.7) for the low-skilled and DM 4216.9 (2045.0) for the high-skilled workers thereby showing somewhat overestimated second moment (see Table 1 for the corresponding sample measures). The shape of the predicted wage offer density (Figure A.4) is also consistent with the density of observed wages.

Along with frictional parameters we also estimate the size of the returns to scale. We find that the degree of homogeneity in our model is 1.06 for the "low-productive" firms and 2.01 for the "high-productive" (see Table A.2) which implies the economy-wide returns to scale at the level of 1.07. Increasing returns to scale of the production technology is a well-established fact. Evidence of which can be found in Färe at al. (1985), Kim (1992)

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[4]Note that in this specification $\kappa_{1,2}$ directly influence the shape of $F(w)$ through the cutoff wages, which, unlike in the formulation of Bowlus et al. (2001), makes them sensible indicator of the fit of the model.

[5]Previous experience of the authors with the same data set and the model of Bowlus et al. (2001) with $J \geq 10$, which makes the predicted earnings distribution almost indiscernible from the observed one, shows that the estimates of κ_e and δ lie around 4.5 and 0.004 respectively.

or Zellner and Ryu (1998) among others. For all these authors estimates of the returns to scale range from about 1.1 to about 1.4. Biorn et al. (2002) find either "constant or moderately increasing" returns for different industries. Their typical point estimate of the degree of returns is of about our size, but in some cases the confidence intervals are big enough to encompass unity.

We also experiment with the model were one of the two technologies is assumed to have constant returns using both (11) and (12) to write down the skill-specific offer distribution. The hypothesis of one constant and one increasing returns technology is always rejected in favor of the two technologies with increasing returns.

4.3 *Estimation Results: Returns to Education*

The individual rate of return to schooling is one of the most studied topics in empirical economics. With few exceptions the estimates are based on the Mincer earnings equation, where the years of schooling, working experience and other individual characteristics are regressed on wages.[6] The coefficient on years of schooling is then interpreted as the individual return to schooling. The coefficient on working experience is usually interpreted as the return to learning on-the-job. In an on-the-job search model wages increase with working experience because workers move from lower paid jobs to better paid jobs. The interpretation of the coefficient on working experience in an on-the-job search model would be one of returns to on-the-job search.[7] The return to educating a worker from low-skill one to high-skill one measured in wages should, therefore, be the sum of the Mincer estimates of returns to schooling and to experience.

The fact that estimates are based on wages rather when life-time income neglects the effect that the difference in unemployment risk between low and high skilled workers has on their expected life-time earnings. We try to explicitly account for unemployment risk by using the frictional parameters estimated for the different skill groups. Let $i = 1$ stand for the low-skilled and $i = 2$ for the high-skilled. By assuming that workers start their working career as unemployed we can measure the return to becoming a high skilled worker by

$$R\left(U_i\right) = \frac{U_2 - U_1}{U_1},$$

---

[6]Heckman et al. (2003) present and estimate a more general model and show under which conditions the Mincer coefficients estimate the individual returns to schooling correctly.

[7]Since we classify a promotion at one employer as a job-to-job move, any learning on-the-job that takes place is captured by the returns to on-the-job search.

where the value of unemployment U_i is given by

$$U_i = \frac{\kappa_i}{r} \int_{w_i^r}^{\bar{w}_i} \frac{1 - F_i(w)}{1 + \kappa_e \left(1 - F_i(w)\right)} dw.$$

The total return to becoming a high skilled worker taking into account the lower unemployment risk for high skilled workers is according to our estimated parameters $37,0\%$.

In addition our framework enables us to analyze whether there is a pecuniary human capital externality namely that social returns to education are higher than the individual returns to education. Increasing education in our model is reflected by a shift of the skill structure towards a larger fraction of skilled labour. Positive pecuniary externality to the subsidy will arise if such shift would imply that the increase in total output is big enough to overcompensate both the increase of the total labour cost due to the change of the skill structure and the education cost, without reducing the equilibrium profit of the firm. To learn if this is the case with our data consider the comparative statics induced by an education subsidy.

Suppose the cost $\Delta c = c_2 - c_1$ of becoming high skilled is distributed according to some continuous function among individuals. Then, in the steady state all workers with cost below the difference in the value of being unemployed $U_2 - U_1$ should become high skilled workers, the rest remains low skilled. The marginal worker is exactly indifferent between the two skill groups, i.e.

$$
\begin{aligned}
\Delta c &= U_2 - U_1 \\
&= \frac{\kappa_2}{r} \int_{w_2^r}^{\bar{w}_2} \frac{1 - F_2(w)}{1 + \kappa_e \left(1 - F_2(w)\right)} dw - \frac{\kappa_1}{r} \int_{w_1^r}^{\bar{w}_1} \frac{1 - F_1(w)}{1 + \kappa_e \left(1 - F_1(w)\right)} dw.
\end{aligned}
\tag{24}
$$

Subsidizing education by one cent induces the marginal low skilled worker to educate. The cost the marginal worker has to incur to become high skilled equals the difference in the value of being unemployed between the two skill groups given in equation (24). The cost to the marginal worker using our estimated parameters is DM 749. In addition to the individual cost of education comes the cost to the government, which already subsidies higher education. According to OECD statistics the expenditures for educating one low skilled worker to a high skilled worker by 1998 were DM 164 per month.[8]

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[8]Following the International Standard Classification of Education (ISCED) of the OECD 1998 there where about 2.5 million (full-time equivalent) individuals enrolled in post secondary or tertiary education. The total expenditure for these education levels amounted to around 25 million DM per year. Given an

In order to have social returns to education the change in the skill structure of the labor force has to increase profits by more than the joint cost to the individual and the government. So consider the change in the expected profit

$$\frac{\partial}{\partial q_2} \left[\int \pi(w) dF(w) \right] \quad \text{s.t. } q_1 + q_2 = m.$$

Notice that $\mathbf{q}$ also takes part in the determination of the cutoff wages, and, since the wage offer densities are discontinuous at the cutoff wages, we have evaluate the derivative holding cutoff wages constant.

Evaluating the effect on profits, it turns out that not only the marginal increase in the fraction of the high-skilled workers increases the expected total output, but also the resulting marginal change in output is big enough to pay off all the costs and generate a positive excess value of about DM 88700. The resulting pecuniary human capital externality creates a scope for increasing the expected earnings of the whole population without making the firms worse off. The second derivative of the expected profit with respect to q_2 is negative (amounting to DM 253), which implies decreasing returns of a welfare improving education subsidy.

Predictions of our model related to the optimal skill structure are broadly consistent with the evidence of an overall shift of the demand towards skilled labour (see, for instance, Kugler et al., 1989 and Falk and Koebel, 1999). Moreover, the results above appear to be in line with those of Falk and Koebel (1999) who show that output is a positive and increasing function of skills and output effect dominates in explaining the shift away from unskilled labour in Germany.

Considering the literature on the increasing social returns to education one may also notice that the present paper introduces an additional dimension into estimation of human capital externalities. Up to now the major part of empirical work in this area has been performed using a Mincer regression approach. Acemoglu and Angrist (2000) estimate a reduced-form "Mincerian" equation that is consistent with the model of Acemoglu (1996). Applying an instrumental variable technique they find almost no support for social increasing returns. Moretti (2004) provides additional elaboration on the method of Acemoglu and Angrist (2000) and develops own version of the appropriate econometric

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average enrolment duration of 4.9 years to complete a tertiary education we estimate that the government spends around 49,000 DM for the education of an individual enrolled in tertiary education. Assuming the annual interest rate of 4% the flow value of the government expenditure on a monthly basis is given by DM 164 per month.

model. Considering in detail the issues of omitted variable bias and endogeneity Moretti (2004) backs the existence of positive externality using the same data as Acemoglu and Angrist (2000). Finally, within the same framework Dalmazzo and de Blasio (2003) consider locally disaggregated markets rather then one aggregate labour market and find significant positive effect of human capital externalities.

The "Mincerian" approach has, however two main drawbacks that can possibly lead to the ambiguity found in the empirical studies mentioned above. First of all, its' performance depends of the availability of appropriate instrumental variables and on the identifying assumptions concerning the time-varying nature of endogenous dependence between pecuniary human capital externality and unobserved individual abilities (see Moretti, 2004). Secondly, Ciccone and Peri (2002) theoretically demonstrate that even with the constant returns to scale technology the "Mincerian" estimates are biased whenever workers of different skill groups are imperfect substitutes. Empirical investigation of Ciccone and Peri (2002) backs their finding. Furthermore, the latter authors develop a "constant-composition" approach which assumes that human capital accumulation does not change the skill structure of the workforce so that positive externality can appear only due to the increasing average level of human capital in the society. Implementing this approach empirically Ciccone and Peri (2002) do not find support to this hypothesis. Contrary to them we address the shift in skill composition and find that accumulation of higher skills can indeed create a positive pecuniary externality.

5. CONCLUSION

This paper extends the on-the-job search equilibrium model by Burdett and Mortensen (1998) by introducing different skill groups and linking them via a production function which permits constant and increasing returns to scale.

The main theoretical contribution of this paper is that allowing production function to exhibit increasing returns to scale we generate a decreasing wage offer density. Subsequent introduction of employer heterogeneity leads to further improvement of the shape of wage offer and earnings distributions predicted by the model. Another important result of the extended model is that local monopsony power of firms and complementarity of skills in the production function imply that firms occupy the same position in the wage offer distribution for each skill group. This fact makes our theory consistent with the empirical

findings that wages of workers of different skill groups employed at the same firm are positively correlated.

Theoretical solution of our extension suggests a structural econometric model that allows estimating not only search frictions inherent to the labor market but also the parameters of the production function. Richness of the theoretical model enables us to estimate all parameters of interest using wage and duration data only, which requires no additional information on the output.

We apply our model to learn whether on the aggregate level there exists an externality from investing into human capital. Our results suggest that education subsidy aimed at shifting the skill composition of the workforce towards the higher share of skilled workers is able to generate a positive human capital externality and social increasing returns. We also demonstrate that the second order effect of the marginal change in skill composition on output is negative. In view of the existing literature that shows no support to the externality that arises from the overall increase of human capital given constant skill composition our results imply that financing education should target the accumulation of high rather low skills.

REFERENCES

ACEMOGLU, D., "A Microfoundation for Social Increasing Returns to Human Capital Accumulation", *Quarterly Journal of Economics* 111 (1996).

ACEMOGLU, D., AND J., ANGRIST "How Large are Human Capital Externalities? Evidence from Compulsory Schooling Laws", NBER Macroannual (2000), 9-59 .

BARTH, E., AND H., DALE OLSEN, "Skill-Group Size and Wages", *New Zealand Economic Papers* 36 (2002), 83-84.

BARTH, E., AND H., DALE OLSEN, "Assortative Matching in the Labor Market", mimeo, Institute for Social Research, Norway (2003).

BILS, M., AND P., KLENOW, "Does Schooling Cause Growth?", *American Economic Review* 90 (2000), 1160-1183.

BIØRN, E., LINDQUIST, K.-G., AND T., SKJERPEN, "Heterogeneity in Returns to Scale: A Random Coefficient Analysis with Unbalanced Panel Data", *Journal of Productivity Analysis* 18 (2002), 39-57.

BONTEMPS, C., J.-M., ROBIN, AND G.J., VAN DEN BERG, "Equilibrium Search with Productivity Dispersion: Theory and Estimation", *International Economic Review* 41 (2000), 305-358.

BOWLUS, A., KIEFER, N., AND G., NEUMANN, "Estimation of Equilibrium Wage Distributions with Heterogeneity", *Journal of Applied Econometrics* 10 (1995), S119-S131.

BOWLUS, A., KIEFER, N., AND G., NEUMANN, "Equilibrium Search Models and the Transition from School to Work", *International Economic Review* 42 (2001), 317-343.

BURDETT, K., AND D.T., MORTENSEN, "Wage Differentials, Employer Size and Unemployment", *International Economic Review* 39 (1998), 257-273.

CHERNOZHUKOV, V., AND H., HONG, "Likelihood Estimation and Inference in a Class of Nonregular Econometric Models", *Econometrica* 72 (2004), 1445-1480.

CICCONE, A., AND G., PERI, "Identifying Human Capital Externalities: Theory with Application to US Cities", IZA Discussion Paper No. 488 (2002).

DIAMOND, P., "A Model of Price Adjustment", Journal of Economic Theory 3 (1971), 156-168.

DALMAZZO, A., AND G., DE BLASIO, "Social Returns to Education: Evidence from Italian Local Labor Market Areas", IMF Working Paper WP/03/165 (2003).

FALK, M., AND B., KOEBEL, "Curvature Conditions and Substitution Pattern among Capital, Energy, Materials and Heterogeneous Labour", ZEW Discussion Paper No. 99-06 (1999).

FÄRE, R., JANSSON, L., AND K., LOVELL "Modelling Scale Economies with Ray-Homothetic Production Functions", *Review of Economics and Statistics* 67 (1985), 624-629.

HECKMAN, J.J., LOCHNER L.J., AND P.E., TODD, "Fifty Years of Mincer Earnings Regressions", IZA Discussion Paper No. 775 (2003).

KATZ, L., AND L., SUMMERS, "Industry Rents: Evidence and Implications," *Brookings Papers on Economic Activity: Microeconomics* (1989), 209-275.

KIEFER, N.M., AND G.R., NEUMANN, "Wage Dispersion with Homogeneity: The Empirical Equilibrium Search Model", in H. Bunzel et al., eds., *Panel Data and labor market analysis*, (Amsterdam: North Holland, 1993).

KIM, Y., "Translog Production Functions and Variable Returns to Scale", *Review of Economics and Statistics* 74 (1992), 546-552.

KONING, P., G., RIDDER, AND G.J., VAN DEN BERG, "Structural and Frictional Unemployment in an Equilibrium Search Model with Heterogeneous Agents", *Journal of Applied Econometrics* 10 (1995), 133-151.

KREMER, M., "The O-ring Theory of Economic Development", *Quarterly Journal of Economics* 108 (August 1993), 551-575.

KUGLER, P., MULLER, U., AND G., SHELDON "Non-Neutral Technical Change, Capital, White-Collar and Blue-Collar Labour. Some Empirical Results", *Economics Letters* 31 (1989), 91-94.

LUCAS, R., "On the Mechanics of Economic Development", *Journal of Monetary Economics* 22 (1988), 3-42.

MANNING, A., "Monopsony in Motion", (Princeton University Press, 2003).

MORETTI, E., "Estimating the Social Return to Higher Education: Evidence from Longitudinal and Repeated Cross-Sectional Data", *Journal of Econometrics* 121 (2004), 175-212.

MORTENSEN, D.T., "Equilibrium Wage Distribution: a Synthesis", in J. Hartog et al., eds. *Panel Data and Labor Market Studies* , (Amsterdam: North Holland 1990).

MORTENSEN, D.T., "Modelling Matched Job-Worker Flows", mimeo (1999).

MORTENSEN, D.T., "Equilibrium Unemployment with Wage Posting: Burdett-Mortensen Meet Pissarides", in H. Bunzel, B.J. Christensen, P. Jensen, N.M. Kiefer, and D.T. Mortensen, eds., *Panel Data and Structural Labour Market Models* (Amsterdam: Elsevier, 2000).

MORTENSEN, D.T., AND G.R., NEUMAN, "Estimating Structural Models of Unemployment and Job Duration", in W.A., Barnett et al., eds., *Dynamic Econometric Modelling, Proceedings of the Third International Symposium in Economic Theory and Econometrics* (Cambridge: Cambridge University Press, 1988).

POSTEL-VINAY, F., AND J.-M., ROBIN, "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity", *Econometrica* 70 (2002), 2295-2350.

PRESS, W., TEUKOLSKY, S., VETTERLING, S., AND B., FLANNERY, "Numerical Recipes in C: The Art of Scientific Computing", (Cambridge University Press, 1995).

ROTHENBERG, T., "Identification in Parametric Models", *Econometrica* 39 (1971), 577-591.

VAN DEN BERG, G.J., AND G., RIDDER, "An Empirical Equilibrium Search Model of the Labor Market", *Econometrica* 66 (1998), 1183-1221.

ZELLNER, A., AND H., RYU, "Alternative Functional Forms for Production, Cost and Returns to Scale Functions", *Journal of Applied Econometrics* 13 (1998), 101-127.

APPENDIX

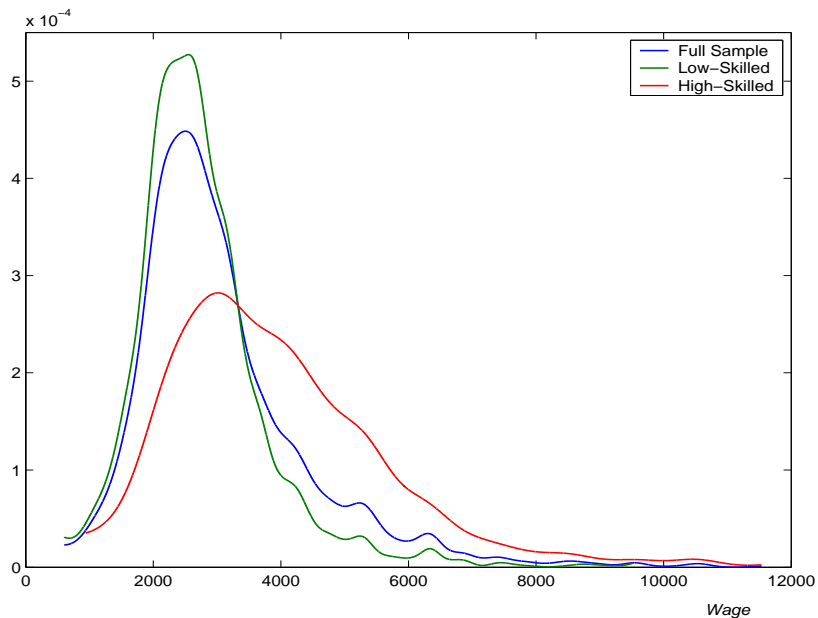Figure A.1: "Kernel Estimates of Earnings Densities"

Table A.1: "Estimation Results: Homogeneous Firms"

	Standard BM		Extension		
			Constant Returns		Increasing Returns
κ_u	7.2490 [6.6976, 7.8623]	κ_u	7.2658 [6.7077, 7.8635]	κ_{u1}	8.8110 [8.0227, 9.6474]
				κ_{u2}	19.2835 [15.5806, 24.3741]
κ_e	0.1370 [0.1280, 0.1534]	κ_e	0.1484 [0.1370, 0.1651]	κ_e	2.9167 [2.7861, 2.9422]
δ	0.0066 [0.0064, 0.0068]	δ	0.0065 [0.0064, 0.0067]	δ	0.0038 [0.0037, 0.0039]
				ξ	1.6213 [1.5969, 1.6240]
		α_1	0.7260	α_1	1.1314
log(L):	−64412.17		−66132.87		−64277.48

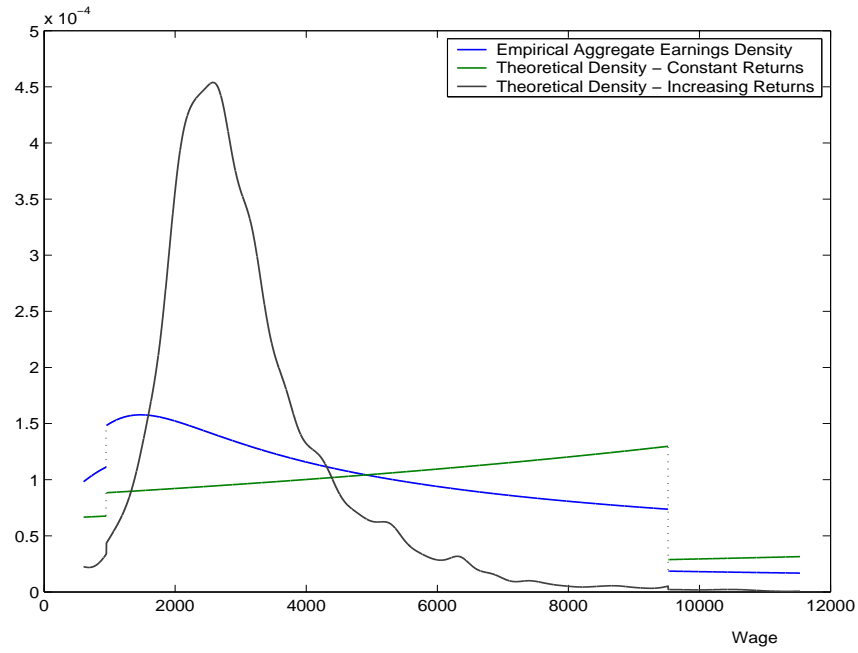Figure A.2: "Theoretical Earnings Densities Predicted by the Model: Homogeneous Firms"

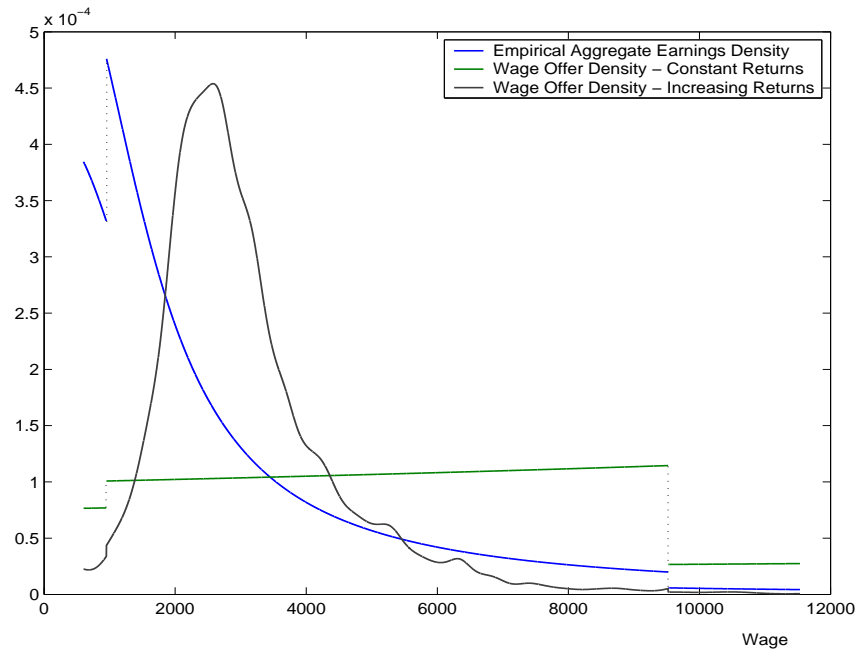Figure A.3: "Aggregate Wage Offer Densities: Homogeneous Firms"

Table A.2: "Estimation Results: 2-Point Employer Heterogeneity"

Standard BM			Extension				
			Constant Returns		Increasing Returns*		
κ_u	8.1544 [7.4125, 8.8117]		κ_u	7.9816 [7.3937, 8.8740]	κ_{u1}	8.9304 [,]	
					κ_{u2}	26.2769 [,]	
κ_e	0.9956 [0.7773, 1.0608]		κ_e	0.7863 [0.7037, 1.0274]	κ_e	5.6861 [,]	
δ	0.0054 [0.0052, 0.0055]		δ	0.0056 [0.0053, 0.0058]	δ	0.0037 [,]	
					ξ_1	1.0597 [,]	
			α_{11}	0.6324 [0.5717, 0.6502]	α_{11}	0.6984 [,]	
					ξ_2	2.0135 [,]	
			α_{12}	0.7260 [0.7099, 0.7781]	α_{12}	1.4366 [,]	
$\overline{w}_1$	3704.		$\overline{w}_{11}$	3704.	$\overline{w}_{11}$	4233.	
			$\overline{w}_{21}$	5821.	$\overline{w}_{21}$	6351.	
			$\langle G_1(\overline{w}_{11}):\ 0.8709\rangle$		$\langle G_1(\overline{w}_{11}):0.92684\rangle$		
			$\langle G_2(\overline{w}_{21}):\ 0.8756\rangle$		$\langle G_2(\overline{w}_{21}):0.91630\rangle$		
γ_1	0.8685		γ_1	0.9242	γ_1	0.9878	
$\log(\mathrm{L})$:	-61539.42			-63279.61		-62564.01	

———

* Bootstrapping the model with increasing returns is still in progress

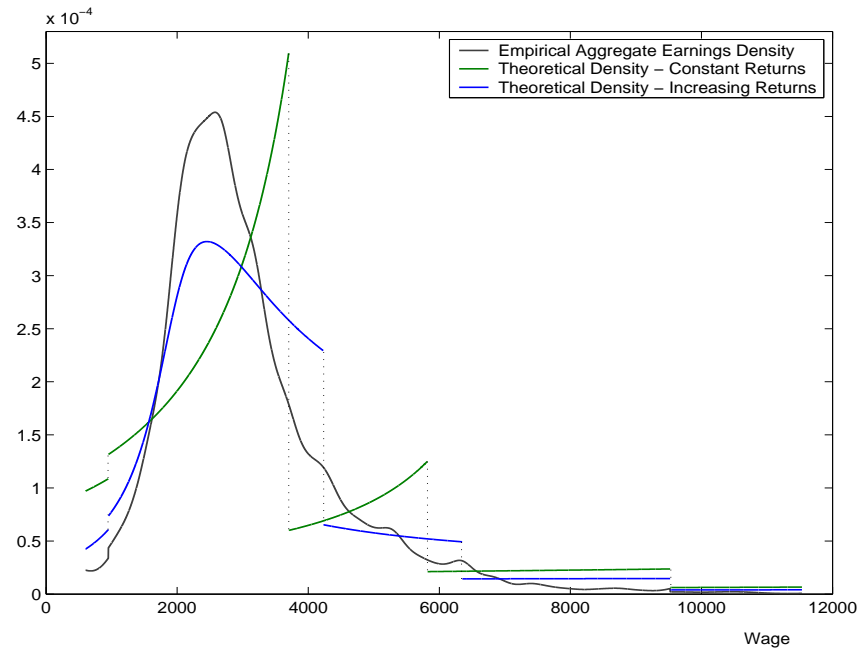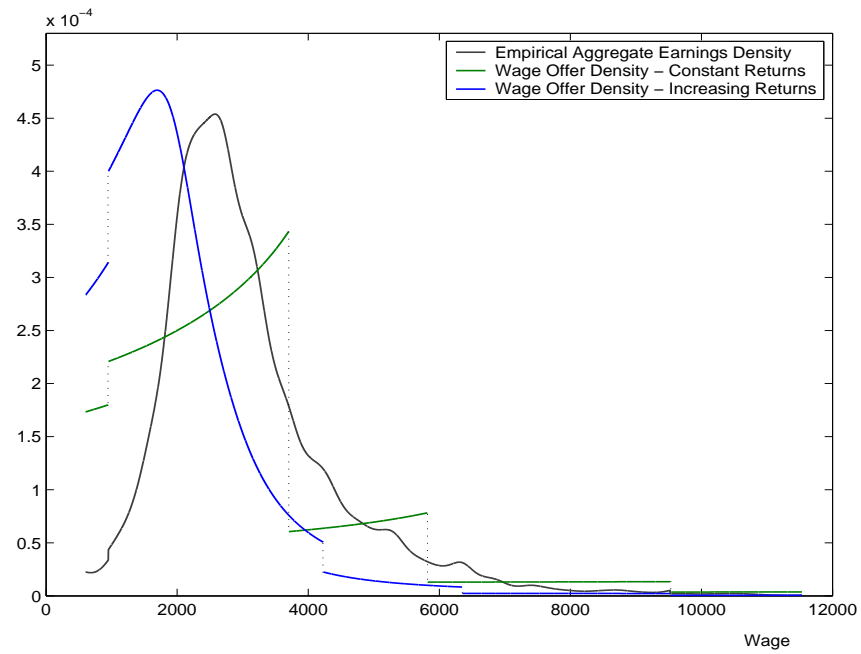Figure A.4: "Theoretical Earnings Densities Predicted by the Model: 2-Point Employer Heterogeneity"

Figure A.5: "Aggregate Wage Offer Densities: 2-Point Employer Heterogeneity"

Proof of Proposition 5.

Define

$$h_j(w) = \frac{[\delta + \lambda_e(1 - \gamma_{j-1})]^2}{[\delta + \lambda_e \overline{F}_j(w)]^2}, \; r_{ij} = \frac{\delta \lambda_i(\delta + \lambda_e)}{(\delta + \lambda_i)[\delta + \lambda_e(1 - \gamma_{j-1})]^2} q_i$$

$$Y_j'(\mathbf{r}_j) = \frac{\partial Y_j(\mathbf{r}_j)}{\partial l_i}, \text{ and } \sigma_{ij} = \sum_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i \partial l_l} r_{lj} r_{ij}.$$

The second order Taylor-Expansion of the production function around r_j is given by

$$Y_j(\mathbf{l}(\mathbf{w}_j)) = Y_j(\mathbf{r}_j) + \sum_i Y_j'(\mathbf{r}_j)[r_{ij}h_j(w) - r_{ij}] + \frac{1}{2}\sum_i \sigma_{ij}[h_j(w) - 1]^2.$$

Note, that $h_j(w)$ is independent of the skill group i, because of equation (10). Using the equal profit condition for the equilibrium, i.e. $\pi_j(\mathbf{w}_j) = \pi_j(\underline{\mathbf{w}}_j)$, and substituting gives

$$D = \sum_i (Y_j'(\mathbf{r}_j) - w_{ij}) r_{ij} h_j(w) + \frac{1}{2}\sum_i \sigma_{ij}(h_j(w) - 1)^2 \qquad (A.1)$$

$$- \sum_i (Y_j'(\mathbf{r}_j) - \underline{w}_{ij}) r_{ij} = 0$$

The first order condition for wage w_{ij} satisfies,

$$\left(\frac{\partial Y_j(\mathbf{l}(\mathbf{w}))}{\partial l_i(w_{ij})} - w_{ij}\right) l_i(w_{ij}) = l_i(w_{ij})^2 \left[\frac{dl_i(w_{ij})}{dw_{ij}}\right]^{-1}, \qquad (A.2)$$

where rhs can be written as

$$l_i(w_{ij})^2 \left[\frac{dl_i(w_{ij})}{dw_{ij}}\right]^{-1} = [r_{ij}h_j(w)]^2 \left[r_{ij}\frac{dh_j(w)}{dw_{ij}}\right]^{-1}$$

According to the result that all firms occupy the same position in all wage offer distribution, changing the wage for one skill group implies a change of all other wages in the same direction, i.e. according to equation (A.1)

$$[r_{ij}h_j(w)]^2 \left[r_{ij}\frac{dh_j(w)}{dw_{ij}}\right]^{-1} = r_{ij}h_j(w)^2 \left(\frac{-\partial D/\partial h_j(w)}{-\sum_i \partial D/\partial w_{ij}}\right)$$

$$= \frac{r_{ij}}{\sum_i r_{ij}}\left(\sum_i (Y_j'(\mathbf{r}_j) - w_{ij}) r_{ij}h_j(w) + \sum_i \sigma_{ij}(h_j(w)^2 - h_j(w))\right).$$

Using a Taylor-Expansion for the first derivative of the production function and substituting $l_l(w_{lj})$ out gives

$$Y_j'(\mathbf{l}(\mathbf{w})) = Y_j'(\mathbf{r}_j) + \sum_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i \partial l_l}(r_{lj}h_j(w) - r_{lj}).$$

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The first order condition can therefore be written as

$$
\left(Y'_j\left(\mathbf{r}_j\right) - w_{ij}\right) r_{ij} h_j\left(w\right) + \sigma_{ij}\left(h_j\left(w\right)^2 - h_j\left(w\right)\right)
$$
$$
= \frac{r_{ij}}{\sum_i r_{ij}} \left(\sum_i \left(Y'_j\left(\mathbf{r}_j\right) - w_{ij}\right) r_{ij} h_j\left(w\right) + \sum_i \sigma_{ij}\left(h_j\left(w\right)^2 - h_j\left(w\right)\right) \right).
$$

Substituting $\sum_i \left(Y'_j\left(\mathbf{r}_j\right) - w_{ij}\right) r_{ij} h_j\left(w\right)$ from equation (A.1) gives

$$
\left(Y'_j\left(\mathbf{r}_j\right) - w_{ij}\right) r_{ij} h_j\left(w\right) + \sigma_{ij}\left(h_j\left(w\right)^2 - h_j\left(w\right)\right)
$$
$$
= \frac{r_{ij}}{\sum_i r_{ij}} \sum_i \left(Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}\right) r_{ij} + \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} \sum_i \sigma_{ij}\left[h_j\left(w\right)^2 - 1\right].
$$

Evaluating this equation at $\underline{w}_{ij}$ and substituting $\frac{r_{ij}}{\sum_i r_{ij}} \sum_i \left(Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}\right) r_{ij}$ gives

$$
\left(Y'_j\left(\mathbf{r}_j\right) - w_{ij}\right) r_{ij} h_j\left(w\right) + \sigma_{ij}\left(h_j\left(w\right)^2 - h_j\left(w\right)\right)
$$
$$
= \left(Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}\right) r_{ij} + \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} \sum_i \sigma_{ij}\left[h_j\left(w\right)^2 - 1\right].
$$

Rearranging gives

$$
\left(\sigma_{ij} - \mu_{ij}\right) h_j\left(w\right)^2 + \left(\left(Y'_j\left(\mathbf{r}_j\right) - w\right) r_{ij} - \sigma_{ij}\right) h_j\left(w\right) = \left(Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}\right) r_{ij} - \mu_{ij}, \quad \text{(A.3)}
$$

where $\mu_{ij} = \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} \sum_i \sigma_{ij}$.

For a production function with homogeneity of degree one $\sigma_{ij} = 0$ for all i we get

$$
F_{ij}(w) = \frac{\delta + \lambda_e}{\lambda_e} - \frac{\delta + \lambda_e(1 - \gamma_{j-1})}{\lambda_e} \sqrt{\frac{Y'_j\left(\mathbf{r}_j\right) - w}{Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}}}.
$$

Apart from this a special cases appear if $\left(Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}\right) r_{ij} - \mu_{ij} = 0$. In this case we get

$$
F_{ij}(w) = \frac{\delta + \lambda_e}{\lambda_e} - \frac{\delta + \lambda_e(1 - \gamma_{j-1})}{\lambda_e} \sqrt{\frac{\left(Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}\right) r_{ij} - \sigma_{ij}}{\left(Y'_j\left(\mathbf{r}_j\right) - w\right) r_{ij} - \sigma_{ij}}}.
$$

Otherwise, we get the following solution for the quadratic function, i.e.

$$
h_j\left(w\right) = -\frac{\left(Y'_j\left(\mathbf{r}_j\right) - w_{ij}\right) r_{ij} - \sigma_{ij}}{2\left(\sigma_{ij} - \mu_{ij}\right)}
$$
$$
\pm \frac{\sqrt{\left(\left(Y'_j\left(\mathbf{r}_j\right) - w_{ij}\right) r_{ij} - \sigma_{ij}\right)^2 + 4\left(\sigma_{ij} - \mu_{ij}\right)\left(\left(Y'_j\left(\mathbf{r}_j\right) - \underline{w}_{ij}\right) r_{ij} - \mu_{ij}\right)}}{2\left(\sigma_{ij} - \mu_{ij}\right)}. \quad \text{(A.4)}
$$

The wage offer density implied by the quadratic function (A.3) has to be positive, i.e.

$$\frac{dF_{ij}(w)}{dw} = -\frac{-r_{ij}h_j(w)}{\left(2(\sigma_{ij}-\mu_{ij})h_j(w) + \left((Y_j'(\mathbf{r}_j)-w)r_{ij}-\sigma_{ij}\right)\right)\frac{\partial h_j(w)}{\partial F_{ij}(w)}} > 0$$

Since $\frac{\partial h_j(w)}{\partial F_{ij}(w)} > 0$, it follows that $2(\sigma_{ij}-\mu_{ij})h_j(w) + \left((Y_j'(\mathbf{r}_j)-w)r_{ij}-\sigma_{ij}\right)$ has to be greater than zero. Rewriting equation (A.4) implies that only the positive solution is valid, i.e.

$$+\sqrt{\left((Y_j'(\mathbf{r}_j)-w_{ij})r_{ij}-\sigma_{ij}\right)^2 + 4(\sigma_{ij}-\mu_{ij})\left((Y_j'(\mathbf{r}_j)-\underline{w}_{ij})r_{ij}-\mu_{ij}\right)}$$
$$= 2(\sigma_{ij}-\mu_{ij})h_j(w) + \left(Y_j'(\mathbf{r}_j)-w_{ij}\right)r_{ij}-\sigma_{ij} > 0. \tag{A.5}$$

Notice firstly that $h_j(w) \geq 1$ implies

$$\sqrt{\left((Y_j'(\mathbf{r}_j)-w_{ij})r_{ij}-\sigma_{ij}\right)^2 + 4(\sigma_{ij}-\mu_{ij})\left((Y_j'(\mathbf{r}_j)-\underline{w}_{ij})r_{ij}-\mu_{ij}\right)}$$
$$\geq 2(\sigma_{ij}-\mu_{ij}) + \left(Y_j'(\mathbf{r}_j)-w_{ij}\right)r_{ij}-\sigma_{ij} > 0, \tag{A.6}$$

and secondly that the following inequality

$$\left((Y_j'(\mathbf{r}_j)-w_{ij})r_{ij}-\sigma_{ij}\right)^2 + 4(\sigma_{ij}-\mu_{ij})\left((Y_j'(\mathbf{r}_j)-\underline{w}_{ij})r_{ij}-\mu_{ij}\right)$$
$$= \left((Y_j'(\mathbf{r}_j)-w_{ij})r_{ij}-\sigma_{ij}\right)^2 + 4(\sigma_{ij}-\mu_{ij})\left((Y_j'(\mathbf{r}_j)-\underline{w}_{ij})r_{ij}-\sigma_{ij}\right) + 4(\sigma_{ij}-\mu_{ij})^2$$
$$> \left((Y_j'(\mathbf{r}_j)-w_{ij})r_{ij}-\sigma_{ij}\right)^2 + 4(\sigma_{ij}-\mu_{ij})\left((Y_j'(\mathbf{r}_j)-w_{ij})r_{ij}-\sigma_{ij}\right) + 4(\sigma_{ij}-\mu_{ij})^2$$
$$= \left((Y_j'(\mathbf{r}_j)-w_{ij})r_{ij}-\sigma_{ij}+2(\sigma_{ij}-\mu_{ij})\right)^2$$

with an equality at $w_{ij} = \underline{w}_{ij}$ holds only for $(\sigma_{ij}-\mu_{ij}) > 0$. Hence the cumulative wage offer distribution is given by

$$F_{ij}(w) = \frac{\delta+\lambda_e}{\lambda_e} - \frac{\delta+\lambda_e(1-\gamma_{j-1})}{\lambda_e\sqrt{-\frac{(Y_j'(\mathbf{r}_j)-w)r_{ij}-\sigma_{ij}}{2(\sigma_{ij}-\mu_{ij})} + \sqrt{\left(\frac{(Y_j'(\mathbf{r}_j)-w)r_{ij}-\sigma_{ij}}{2(\sigma_{ij}-\mu_{ij})}\right)^2 + \frac{(Y_j'(\mathbf{r}_j)-\underline{w}_{ij})r_{ij}-\mu_{ij}}{(\sigma_{ij}-\mu_{ij})}}}}.$$

The restriction $(\sigma_{ij}-\mu_{ij}) > 0$ provides a lower bound on the degree of homogeneity of the production function for which the wage offer distribution of skill group i is defined. Substituting gives

$$\sum_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i \partial l_l}r_{lj}r_{ij} > \frac{r_{ij}}{\sum_i r_{ij}}\frac{1}{2}\sum_i\sum_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i \partial l_l}r_{lj}r_{ij}.$$

If $Y_j(\mathbf{r}_j)$ is homogenous of degree ξ_j, then Euler's Theorem gives

$$\sum_l \frac{\partial^2 Y_j(\mathbf{r}_j)}{\partial l_i \partial l_l} r_{ij} r_{lj} = (\xi_j - 1) \frac{\partial Y_j(\mathbf{r}_j)}{\partial l_i} r_{ij}.$$

This implies

$$(\xi_j - 1) \frac{\partial Y_j(\mathbf{r}_j)}{\partial l_i} r_{ij} > \frac{r_{ij}}{\sum_i r_{ij}} \frac{1}{2} (\xi_j - 1) \sum_i \frac{\partial Y_j(\mathbf{r}_j)}{\partial l_i} r_{ij}.$$

Summing over i gives a restriction on the degree ξ_j of homogeneity of the production function, i.e.

$$(\xi_j - 1) \xi_j > \frac{1}{2} (\xi_j - 1) \xi_j$$

which is only valid for $\xi_j > 1$.

In order to see that the wage offer density can be increasing and decreasing consider the explicit solution to the wage offer density

$$f_{ij}(w) = \frac{(\delta + \lambda_e(1 - \gamma_{j-1})) r_{ij}}{2\lambda_e \sqrt{\left((Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}\right)^2 + 4(\sigma_{ij} - \mu_{ij})\left((Y_j'(\mathbf{r}_j) - \underline{w}_{ij}) r_{ij} - \mu_{ij}\right)}}$$

$$\times \frac{1}{\sqrt{-\frac{(Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}}{2(\sigma_{ij} - \mu_{ij})} + \frac{\sqrt{\left((Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}\right)^2 + 4(\sigma_{ij} - \mu_{ij})\left((Y_j'(\mathbf{r}_j) - \underline{w}_{ij}) r_{ij} - \mu_{ij}\right)}}{2(\sigma_{ij} - \mu_{ij})}}}.$$

The slope of the wage offer density is given by

$$\frac{\partial f_{ij}(w)}{\partial w} = -\frac{\left((Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}\right)^2 + 4(\sigma_{ij} - \mu_{ij})\left((Y_j'(\mathbf{r}_j) - \underline{w}_{ij}) r_{ij} - \mu_{ij}\right) - 2r_{ij}\left((Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}\right)}{\left((Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}\right)^2 + 4(\sigma_{ij} - \mu_{ij})\left((Y_j'(\mathbf{r}_j) - \underline{w}_{ij}) r_{ij} - \mu_{ij}\right)}$$

$$\times \frac{\frac{(\delta + \lambda_e(1 - \gamma_{j-1})) r_{ij}^2}{4\lambda_e \sqrt{\left((Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}\right)^2 + 4(\sigma_{ij} - \mu_{ij})\left((Y_j'(\mathbf{r}_j) - \underline{w}_{ij}) r_{ij} - \mu_{ij}\right)}}}{\sqrt{-\frac{(Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}}{2(\sigma_{ij} - \mu_{ij})} + \frac{\sqrt{\left((Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij}\right)^2 + 4(\sigma_{ij} - \mu_{ij})\left((Y_j'(\mathbf{r}_j) - \underline{w}_{ij}) r_{ij} - \mu_{ij}\right)}}{2(\sigma_{ij} - \mu_{ij})}}}$$

Thus, a necessary condition for the wage offer density to be upward sloping is that $(Y_j'(\mathbf{r}_j) - w) r_{ij} - \sigma_{ij} > 0$. Substituting σ_{ij}, and using the Euler Theorem gives the stated condition. ∎