Business Cycle Dependent Unemployment Insurance*

Preliminary version - comments welcome

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October 2009

Abstract

The consequences of business cycle contingencies in unemployment insurance systems are considered in a search-matching model allowing for shifts between "good" and "bad" states of nature. We show that not only is there an insurance argument for such contingencies, but also an incentive argument. If benefits are less distortionary in a recession than a boom, it follows that countercyclical benefits reduce average distortions compared to state independent benefits. We show that optimal (utilitarian) benefits are state contingent and tend to reduce the structural (average) unemployment rate, although the variability of unemployment may increase.

JEL: J6, H3

Keywords: Unemployment benefits, business cycle, insurance, incentives.

* *We are grateful to participants at a workshop in Kiel and a seminar at IFAU, Uppsala for useful comments, and to Mark Strøm Kristoffersen for efficient research assistance.*

1 Introduction

Optimal unemployment insurance systems trade off incentives and insurance. Since unemployment risk is intimately related to the business cycle situation, it is to be expected that the value of insurance is business cycle dependent. At the same time, it may be conjectured that the distortions from unemployment insurance may be larger in periods with low unemployment, and vice versa. Both of these effects go in the direction of making optimal benefit levels counter-cyclical; that is, benefit generosity is high when unemployment is high, and low when unemployment is low.

However, the key parameters of unemployment schemes are business cycle independent in most countries. Though, there are cases where elements of the unemployment insurance system is explicitly linked to the state of the labour market. Probably the most sophisticated scheme is found in Canada where benefit eligibility, levels, and duration depend on the level of unemployment according to pre-determined rules[1]. The US has a system of extended benefit duration in high unemployment periods (see Committee on Way and Means (2004)). Some countries have pursued a more discretionary approach - in some cases in a semi automatic fashion[2] - by adjusting labour market policies to the state of the labour market; i.e. extending benefits or labour market policies in general in high unemployment periods, and tightening the schemes in periods with low unemployment.

The issue of business cycle contingencies in unemployment insurance has gained further interest in perspective of the downturn induced by the financial crisis. Calls for increases in unemployment benefits or extension of benefit duration have been made by e.g. the IMF and the OECD (see Spilimbergo et al. (2008) and OECD (2009)), and if such changes are made, it is an important issue whether they should be made contingent on the business cycle to prevent that these changes become permanent.

There is a large literature on the design of unemployment insurance schemes. Since Baily (1978) it is well-known that the optimal benefit level trades-off insurance and incentives. Recent work has extended these insights in various directions (for a survey see e.g. Frederiksson and Holmlund (2006)). Surprisingly, there is neither a large theoretical literature on the effects of business cycle dependent unemployment insurance nor an empirical literature[3] exploring the state of nature dependencies in the effects of various labour market policies including the benefit level. Kiley (2003) and

[1] See http://www.hrsdc.gc.ca/eng/ei/menu/eihome.shtml.

[2] Sweden is an example of a country which has used labour market policies in this way.

[3] The few exceptions are: Moffitt (1985), Arulampalam and Stewart (1995), Jurajda and Tannery (2003), and Røed and Zhang (2005), see section 2.

Sanchez (2008) argue within a search framework that the initial benefit level should be higher and its negative duration dependence weaker in a business cycle downturn compared to an upturn. Both models are partial and rely on the assumption that benefits are more distortionary in a boom[4]. In Andersen and Svarer (2008), it is shown that the optimality of counter-cyclical benefit levels depends not only on the possibility of using the public budget as a buffer but also whether distortions move pro-cyclically. If this is the case, countercyclical unemployment benefits may also contribute to lower the structural (average) unemployment rate. However, the model is static and does not allow for changes in the business cycle situation.

This paper develops a search model in which the business cycle situation may change between "good" and "bad" states of nature[5]. Matching frictions imply a co-existence of unemployed persons and vacant jobs, but the underlying job separation rates and job finding rates are business cycle dependent. The unemployment benefit scheme is tax financed, and benefits are allowed to be business cycle dependent. Since the main issue in this paper is the trade-off between insurance and incentive, the model is cast in such a way that focus is on how unemployment benefits affect job search incentives. The paper addresses both the positive issue of how such state contingencies affect labour market performance and the normative issue of the optimal (utilitarian) state of nature contingencies to build into unemployment insurance schemes.

In addition, business cycle dependent unemployment benefits would also strengthen automatic stabilizers, which may have effects via aggregate demand effects. Such effects do not arise in the present framework which focuses on the structural consequences of business cycle dependent benefit levels.

In the search context considered in this paper the response of job search to both unemployment benefits and the business cycle situation plays a crucial role. It is shown that the distortion of search incentives caused by benefits tends to be business cycle dependent in a pro-cyclical way; i.e. a high benefit level distorts incentives more in a good than a bad business cycle situation. At the same time, insurance arguments may call for counter-cyclical bene-

[4] In a related study, Costain and Reiter (2005) analyse a business cycle model with exogenous search state allowing for contingencies in social security contributions levied on firms and unemployment benefits. In this model the public budget does not need to balance in each state due to contingent assets traded with risk neutral capitalists. It is shown that it is optimal to have pro-cyclical social security contributions, while benefits are almost state invariant.

[5] The main modelling difficulty here is to ensure stationarity of public finances under a tax financed unemployment insurance scheme. This is done by the specific assumptions concerning state transitions and the tax policy.

fit levels. This has two important implications, namely, first that optimal benefits may be counter-cyclical, and second that the structural (average) unemployment rate could be lower with business cycle contingent compared to business cycle independent benefits. However, as a consequence the actual unemployment rate may become more variable.

In addition, it is shown that the possible change in the business cycle situation has an important effect on search behaviour and therefore on unemployment and other key variables. The reason is that agents perceive the possibility of a change in the business cycle situation, and this affects the search behaviour of the unemployed. Clearly, this effect depends on both the difference between the two states of nature and the likelihood of a change in the business cycle situation. This may even imply that counter-cyclical benefits may increase search effort in both states of nature, and therefore cause a fall in unemployment in both states. This arises if the business cycle situation is not too persistent and if agents in a downturn expect a shift to an upturn with a higher job finding rate.

The paper is organized as follows: We start in section 2 by providing some indicative empirical evidence on business cycle dependent effects of unemployment benefits. A search model with business cycle fluctuations is set up and characterized in section 3. The issue of business cycle dependent incentive and insurance effects are analysed in section 4 and 5, respectively. The consequences of business cycle dependent benefits are addressed in section 6, while section 7 considers optimal benefits for a utilitarian policy maker. A few concluding remarks are given in section 8.

2 Business cycle dependent effects of UI benefits on job finding rate?

The incentive (moral hazard) effects of unemployment insurance have been extensively studied. Higher UI benefits tend to decrease the return to finding a job and hence results in reduced search effort and therefore a lower exit rate from unemployment. It is crucial to the design of unemployment insurance schemes where these incentive effects are business cycle dependent. As mentioned above only a few authors have tried to test empirically whether the effect of UI benefits are larger in booms than in recessions. This includes Moffitt (1985), Arulampalam and Stewart (1995), Jurajda and Tannery (2003), and Røed and Zhang (2005). The first three of these studies find that benefits affect incentives less in a downturn, whereas the study by Røed & Zhang (2005) does not find any differences in the effect of benefit on incentives across the business cycle. Disentangling possible business cycle dependencies in the inventice effects is very difficult, and the main empirical challenge is to find exogenous changes in UI benefits that are uncorrelated with the job finding rate not only at one point in time, but across the business cycle. Neither of the mentioned studies nor we have access to such data. As and illustration and to supplement the findings in the literature we take a closer look at the association between the level of benefits and the exit rate from unemployment across the business cycle based on a sample of Danish unemployed (reference?)[6].

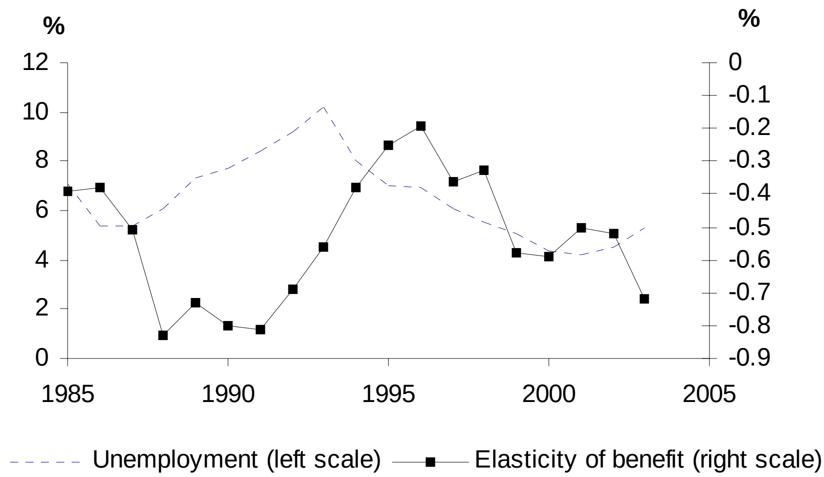
For each unemployed we use information on the level of benefit and the duration of the unemployment spell. Below we show how sensitive the weekly exit rate, e, from unemployment is to the weekly level of benefits, b. Specifically, we estimate a Cox hazard model of the following form:

$$\log e_{it} = \alpha + \beta_{1t} \log b_{it} + \beta_{2t} \log x_{it}.$$

Here α gives the baseline hazard, β_1 gives the elasticity of the hazard rate with respect to the UI benefits and β_2 gives the elasticity wrt other exogenous variables x. UI benefits are constant across the unemployment period and in Denmark the period in which unemployed can receive UI benefits is never shorter than 4 years in the observation period. Clearly, the interpretation of β_1 is contaminated by endogeneity issues and we do not pretend to make causal inference in what follows, but merely to provide an illustration of the size of β_1 across the business cycle. We estimate the Cox hazard separately for unemployment spells beginning in each of the years from 1985 to 2003 and in Figure 1 where we plot β_1 for each year along with the annual aggregate unemployment rate as a cyclical indicator.

Figure 1: The elasticity of the hazard rate with respect to UI benefits

[6] The data from Statistics Denmark covers all Danish unemployed who have received UI benefits in the period from 1985 to 2003. In order to receive unemployment insurance in Denmark, the unemployed has to be a member of a UI fund. The membership rate is around 80 %. The benefit rate amounts to 90% of the previous wage, but with a maximum level of benefits, which is reached if the wage earner is in the top 95 % of the earnings distribution. That is, most employed workers face a replacement rate below 90 %. The typical replacement rate is around 60%.



Note:

Source: Own calculations

First, notice that the unemployment rate is increasing from the mid 1980s to the early 1990s, and it has since been on a downward trend until a mild recession sat in in the beginning of the new millennium. Second, notice that the elasticity of the hazard rate with respect to benefits is on average around -0.5. That is, a 10 % increase in UI benefits is associated with a 5 % drop in the hazard rate from unemployment. This level corresponds to what e.g. Chetty (2008) finds for the U.S.. Third, the figure shows that the estimated elasticity of benefits varies pro-cyclically over the observation period. That is, the elasticity of the hazard rate wrt. benefits is higher in periods when unemployment is low and vice versa. This is interpreted as indicative evidence that the disincentive effects of benefits is lower in recessions than in booms. Clearly, Figure 1 can not claim to provide causal inference and shall merely be taken as an illustration of the raw correlation between UI benefits levels and the exit rate from unemployment. The findings here are consistent with the pattern in the abovementioned analyses that suggests that benefits are less distortionary in recessions than in booms, but the literature still awaits the - perhaps unrealistic - data set that enables a more satisfactory test of the effects of benefits on job finding across the business cycle.

3 A search matching model with business cycles

Consider a standard search matching model of the Pissarides-Mortensen type in continuous time (see e.g. Mortensen and Pissarides (1994) and Pissarides (2000)). All workers are ex-ante identical and have the same productivity. Workers search for jobs, but a matching friction implies that unemployment and vacancies coexist. Firms create vacancies, and filled jobs are destructed by some exogenous separation rate p ($p \in [0, 1]$). All probabilities are parameters of the associated time homogeneous Poisson process.

The state of nature evolves between two states, *good* (G) and *bad* (B), with the following (symmetric) transition[7] probabilities[8]

present\past state	B	G
B	π	$1-\pi$
G	$1-\pi$	π

where $0 \leq \pi \leq 1$. This formulation captures that if the economy is in a boom (recession), this state of nature may continue with probability π and terminate and turn into a recession (boom) with probability $1 - \pi$. Hence, π is also a measure of the persistence in the current business cycle situation.

The job separation rate p is in the four possible states of nature given as follows

present\past state	B	G
B	p_{BB}	p_{BG}
G	p_{GB}	$p_{GG} < p_{BB}$

i.e. the basic transition is between a regime with either a low level (p_{GG}) or high level ($p_{BB} > p_{GG}$) of job separations[9]. Upon transition there is an extraordinarily high ($p_{BG} > p_{BB}$) or low ($p_{GB} < p_{GG}$) level of job separations (see below)[10].

[7] We assume a symmetric transition matrix to simplify the analysis. Empirical evidence indicates some asymmetry with more persistence in good than in bad business cycle situations. The estimated value of π in discrete models on quarterly data is in the range 0.7 to 0.9, see Hamilton (1994). In a three state model (recession, normal and high growth) somewhat higher levels of persistence are found, see Artis et al. (2004).

[8] Note that the unconditional stationary probability of being in a given state B or G is $\Pr(G) = \Pr(B) = \frac{1}{2}$. The unconditional probabilities of the four possible states are: $\Pr(BB) = \Pr(GG) = \frac{1}{2}\pi$ and $\Pr(GB) = \Pr(BG) = \frac{1}{2}(1-\pi)$.

[9] Differences in the business cycle situation may be generated by changing other variables in the model like job creation, the costs of vacancies, matching efficiency etc., but the qualitative results would be the same, see Andersen and Svarer (2008).

[10] There has been some debate on the extent to which changes in the job separation

There is an unemployment benefit scheme providing a flow benefit b to unemployed workers, and it is financed by a proportional wage income tax (τ) and a lump sum tax (T) (see below). The inclusion of lump sum taxes facilitates the analysis involving four possible states of nature and public budget effects. The key problem is that the budget balance in general will display path dependence. To cope with this and to ensure stable debt levels, policies will in general have to be path dependent. This is captured via the lump-sum tax. The income tax rate is assumed state independent, while the benefit level may depend on whether the state is "good" or "bad". Note that there are no marginal labour supply decisions (intensive margin) in the following, so the use of lump sum taxation does not affect any results, but serves the purpose of making the analysis more simple and transparent. Search is affected by the gains from employment and thus net taxes and benefits.

3.1 Individual utility and search effort

Consider an infinite number of identical households, and normalize the population size to unity. Employed workers receive a wage w and work l hours. Both w and l are business cycle independent, and the instantaneous utility is assumed to be separable in the utility from consumption (first term) and leisure (second term), i.e.

$$g\left(w\left[1-\tau\right]-T_{ij}\right)+f(1-l)$$

where τ is the income tax rate and T_{ij} is the tax paid if the current state is i and the previous state j. Working hours l are exogenous, and the time endowment has been normalized to 1. Both $g()$ and $f()$ are concave functions. The instantaneous utility for unemployed is similarly assumed separable over consumption and leisure and given by

$$g\left(b_i-T_{ij}\right)+f(1-s_{ij})$$

where s_{ij} is time spent searching for a job if the current state is i and the previous state j.[11] Note that the separability assumption ensures that search

rate are a driver of unemployment fluctuations, especially in the US (see Shimer (2005)). Elsby et al. (2008) find that the US is an outlier compared to other OECD countries where fluctuations in both inflow and outflow rates are found to be important.

[11] The underlying utility function is assumed to be the same for employed and unemployed workers. In a more general formulation stigmatization and other factors may cause both the utility from income to depend on its source and the distulity from work to may depend on the type of time use. In an earlier version we allowed for such differences, but they did not have any qualitative implications for the results.

is not dependent on current income (see below)[12]. In addition, note that the benefit level only takes two values conditional on the current state, whereas the lump sum tax also depends on the past state. This results in four different levels of net compensation to the unemployed.

Value functions

Consider first the value functions for currently employed workers (W_{ij}^E) in a given current state (i) and past state (j).

$$\begin{aligned}
\rho W_{BB}^E &= h(w\left[1-\tau\right] - T_{BB}) + e(1-l) + \pi p_{BB}\left[W_{BB}^U - W_{BB}^E\right] \\
&\quad + (1-\pi)\left[(1-p_{GB})\left[W_{GB}^E - W_{BB}^E\right] + p_{GB}\left[W_{GB}^U - W_{BB}^E\right]\right] \\
\rho W_{BG}^E &= h(w\left[1-\tau\right] - T_{BG}) + e(1-l) + W_{BB}^E - W_{BG}^E + \pi p_{BB}\left[W_{BB}^U - W_{BB}^E\right] \\
&\quad + (1-\pi)\left[(1-p_{GB})\left[W_{GB}^E - W_{BB}^E\right] + p_{GB}\left[W_{GB}^U - W_{BB}^E\right]\right] \\
\rho W_{GG}^E &= h(w\left[1-\tau\right] - T_{GG}) + e(1-l) + \pi p_{GG}\left[W_{GG}^U - W_{GG}^E\right] \\
&\quad + (1-\pi)\left[(1-p_{BG})\left[W_{BG}^E - W_{GG}^E\right] + p_{BG}\left[W_{BG}^U - W_{GG}^E\right]\right] \\
\rho W_{GB}^E &= h(w\left[1-\tau\right] - T_{GB}) + e(1-l) + W_{GG}^E - W_{GB}^E + \pi p_{GG}\left[W_{GG}^U - W_{GG}^E\right] \\
&\quad + (1-\pi)\left[(1-p_{BG})\left[W_{BG}^E - W_{GG}^E\right] + p_{BG}\left[W_{BG}^U - W_{GG}^E\right]\right]
\end{aligned}$$

where ρ is the subjective discount rate. The value function for current unemployed workers in a given current state (i) and a past state (j) is denoted W_{ij}^U:

$$\begin{aligned}
\rho W_{BB}^U &= g(b_B - T_{BB}) + f(1-s_{BB}) + \pi\alpha_B s_{BB}\left[W_{BB}^E - W_{BB}^U\right] \\
&\quad + (1-\pi)\left[(1-\alpha_G s_{BB})\left[W_{GB}^U - W_{BB}^U\right] + \alpha_G s_{BB}\left[W_{GB}^E - W_{BB}^U\right]\right] \\
\rho W_{BG}^U &= g(b_B - T_{BG}) + f(1-s_{BG}) + W_{BB}^U - W_{BG}^U + \pi\alpha_B s_{BG}\left[W_{BB}^E - W_{BB}^U\right] \\
&\quad + (1-\pi)\left[(1-\alpha_G s_{BG})\left[W_{GB}^U - W_{BB}^U\right] + \alpha_G s_{BG}\left[W_{GB}^E - W_{BB}^U\right]\right] \\
\rho W_{GG}^U &= g(b_G - T_{GG}) + f(1-s_{GG}) + \pi\alpha_G s_{GG}\left[W_{GG}^E - W_{GG}^U\right] \\
&\quad + (1-\pi)\left[(1-\alpha_B s_{GG})\left[W_{BG}^U - W_{GG}^U\right] + \alpha_B s_{GG}\left[W_{BG}^E - W_{GG}^U\right]\right] \\
\rho W_{GB}^U &= g(b_G - T_{GB}) + f(1-s_{GB}) + W_{GG}^U - W_{GB}^U + \pi\alpha_G s_{GB}\left[W_{GG}^E - W_{GG}^U\right] \\
&\quad + (1-\pi)\left[(1-\alpha_B s_{GB})\left[W_{BG}^U - W_{GG}^U\right] + \alpha_B s_{GB}\left[W_{BG}^E - W_{GG}^U\right]\right]
\end{aligned}$$

We focus here only on risk sharing via the unemployment insurance scheme. One issue is the role private savings may play as a buffer and thus self-insurance mechanism[13]. Allowing for interaction between different forms of

[12] There is no on-the-job search since all jobs are assumed identical and have the same wage.

[13] The issue of how individual savings can be a buffer and thus a form of self-insurance

insurance will complicate the analysis, and since risk diversification offered by savings is incomplete[14], we focus only on the unemployment insurance scheme[15].

Job Search

Individuals choose search effort s_{ij} to maximize W_{ij}^U, taking all "macro" variables as given. The first order conditions to the search problem read[16]

$$f'(1-s_{BB}) = \pi\alpha_B\left[W_{BB}^E - W_{BB}^U\right] + (1-\pi)\alpha_G\left[W_{GB}^E - W_{GB}^U\right] \quad (1)$$
$$f'(1-s_{BG}) = \pi\alpha_B\left[W_{BB}^E - W_{BB}^U\right] + (1-\pi)\alpha_G\left[W_{GB}^E - W_{GB}^U\right] \quad (2)$$
$$f'(1-s_{GG}) = \pi\alpha_G\left[W_{GG}^E - W_{GG}^U\right] + (1-\pi)\alpha_B\left[W_{BG}^E - W_{BG}^U\right] \quad (3)$$
$$f'(1-s_{GB}) = \pi\alpha_G\left[W_{GG}^E - W_{GG}^U\right] + (1-\pi)\alpha_B\left[W_{BG}^E - W_{BG}^U\right] \quad (4)$$

Note that search depends, in the usual way, on the gain from shifting from unemployment into a job. However, since the business cycle situation may change, job search depends on the gain from finding a job if remaining in the current state (probability π) and the gain if there is a shift in the state of nature (probability $1-\pi$). The higher π, the more search is affected by the current state, and vice versa.

It follows immediately that search depends on the current state of nature only, and hence there are only two levels of search, i.e.

$$s_{BB} = s_{BG} = s_B$$
$$s_{GG} = s_{GB} = s_G$$

in the case of unemployment has been analysed in relation to unemployment insurance benefits in e.g. Lenz and Tranæs (2005) and the wider context of so-called welfare accounts by Bovenberg, Hansen and Sørensen (2008).

[14] The scope for self-insurance via savings is restricted both due to capital market imperfections affecting the scope for intertemporal diversification and the fact that savings and accumulation of wealth do not provide much insurance for young workers (see e.g. Bailey (1976) and Chetty (2008)). Empirical evidence shows that unemployment is associated with reductions in consumption, and that a large fraction of unemployed are liquidity constrained, see e.g. Gruber (1997) and Bloemen and Stancanelli (2005). The argument that risk diversification via savings is incomplete is here taken to the limit.

[15] However, note that in the special case where utility functions over consumption are linear ($g(w[1-\tau]-T_{ij}) = w[1-\tau]-T_{ij}$ and $g(b_i - T_{ij}) = b_i - T_{ij}$) and the discount rate ρ is interpreted as the market rate of interest, the value functions equal the expected present value of income (net of disutility from work/search). This special case can therefore be interpreted as reflecting a situation with a perfect capital market allowing individuals to smooth consumption via saving/dissaving.

[16] Concavity of the f function ensures that the second order conditions are fulfilled.

The intuition behind this implication is that the search decision is forward-looking since current search influences the future labour market status, and therefore it is independent of the past state[17].

3.2 Firms

A filled job generates an output (exogenous) y, and firms can create job vacancies at a flow cost of ky ($k > 0$). A filled job may be destroyed in the next period if there is a job separation. The value of a filled job in a given state of nature is

$$\rho J_B^E = y - w + \pi p_{BB}(J_B^V - J_B^E) + (1-\pi)\left[p_{GB}(J_G^V - J_B^E) + (1 - p_{GB})(J_G^E - J_B^E)\right] \tag{5}$$

$$\rho J_G^E = y - w + \pi p_{GG}(J_G^V - J_G^E) + (1-\pi)\left[p_{BG}(J_B^V - J_G^E) + (1 - p_{BG})(J_B^E - J_G^E)\right] \tag{6}$$

Note that the value of a filled job does not depend on the past state. A vacant job may be filled in the future if there is a job match, and hence the current value of a vacant job in a given state is

$$\rho J_B^V = -ky + \pi q_B(J_B^E - J_B^V) + (1-\pi) q_G(J_G^E - J_G^V)$$
$$\rho J_G^V = -ky + \pi q_G(J_G^E - J_G^V) + (1-\pi) q_B(J_B^E - J_B^V)$$

where q_i denotes the probability of filling a vacant job in state i (see below). Vacancies are created up to the point where the value of a vacancy is zero, i.e. $J_G^V = J_B^V = 0$.From this it follows that

$$J_B^E = \frac{q_G}{q_B} J_G^E \tag{7}$$

i.e. the relative value of having a filled job in either state (B or G) depends on the ratio of the job finding rates, and $J_G^E > J_B^E$ if $q_B > q_G$. Hence, the value of a filled job is higher in the G state than in the B state provided the job filling rate is lower $q_G < q_B$. The intuition is that the more difficult it is to fill a vacant job, the higher is the value of a filled job. The value of a filled job in the two states is therefore given as

$$J_B^E = \frac{ky}{q_B}$$
$$J_G^E = \frac{ky}{q_G}$$

[17] Note that the separability assumption is crucial for this property.

3.3 Wages

Wages are assumed to be set in a Nash-bargain after a match has been made. Employed workers are represented by unions having the objective of maximizing wages for employed workers. As has been argued in non-cooperative approaches to justify this bargaining model, the relevant outside option is what can be achieved during delay in reaching an agreement (see Binmore, Rubinstein and Wolinski (1986)). This outside option is assumed to be zero for both workers and firms, and hence the wage setting problem is given as the solution to

$$Max_w \quad [w]^{\beta} [y - w]^{1-\beta}$$

where $0 < \beta < 1$. The bargaining power of firms is thus β, and for workers $(1 - \beta)$. This wage setting model implies that the wage is given as

$$w = \beta y \tag{8}$$

The main attraction of this approach is that it gives a simple wage relation which implies that the wage is rigid across states of nature[18]. Alternative routes may be pursued in modelling wage rigidities (see e.g. Hall (2005) and Hall and Milgrom (2008) for recent work in a search matching context), and the specific formulation adopted here is to be considered as an illustrative workhorse model. The crucial property is that wages do not respond to variations in unemployment (job separations etc.)[19].

3.4 Public sector

The public sector provides the benefit level b_i to unemployed in a given state of nature i and finances this by a proportional tax rate τ and a (path dependent) lump sum tax T_{ij}. The income tax rate τ is assumed to be constant across states of nature; i.e. any business cycle dependency runs via the benefit level and the lump sum tax.

The primary budget balance in any state is

$$B_{ij} = (1 - u_{ij})\tau w + T_{ij} - b_i u_{ij}$$

[18] The issue of the cyclical properties of wages is a controversial question in macroeconomics. However, the empirical evidence on cyclical properties of wages is inconclusive (see e.g. Abraham & Haltivanger (1995) and Messina et al. (2009)).

[19] Allowing for wages to be different across states of nature may contribute to dampen unemployment variations via lower wages in downturns and higher wages in upturns, see e.g. Coles and Masters (2007).

Hence, the debt level D_{ij} in the different states is given as

$$\rho D_{BB} = b_B u_{BB} - \tau w(1 - u_{BB}) - T_{BB} + (1 - \pi)\left[D_{GB} - D_{BB}\right]$$
$$\rho D_{GB} = b_G u_{GB} - \tau w(1 - u_{GB}) - T_{GB} + \pi\left[D_{GG} - D_{GB}\right] + (1 - \pi)\left[D_{BG} - D_{GB}\right]$$
$$\rho D_{BG} = b_B u_{BG} - \tau w(1 - u_{BG}) - T_{BG} + \pi\left[D_{BB} - D_{BG}\right] + (1 - \pi)\left[D_{GB} - D_{BG}\right]$$
$$\rho D_{GG} = b_G u_{GG} - \tau w(1 - u_{GG}) - T_{GG} + (1 - \pi)\left[D_{BG} - D_{GG}\right]$$

Since the primary budget is dependent on the current state of nature nothing ensures that the debt level is stationary. A sequence of bad draws in combination with debt servicing may lead to a non-sustainable debt level. To avoid this consider the following simple policy for the lump-sum tax

$$T_{BB} = b_B u_{BB} - \tau w(1 - u_{BB})$$
$$T_{GG} = b_G u_{GG} - \tau w(1 - u_{GG})$$
$$T_{BG} = b_G u_{GB} - \tau w(1 - u_{GB})$$
$$T_{GB} = b_B u_{BG} - \tau w(1 - u_{BG})$$

This policy rule is not necessarily optimal, but it allows some diversification across states of nature while at the same time ensuring a stationary debt level in all states of nature (see Appendix A). Hence, it is useful to illustrate the basic mechanisms in a simple way. Clearly, more sophisticated schemes can deliver more insurance, and hence the present case tends to underestimate the scope for insurance.

The policy rule outlined above implies that the primary balance is given as

$$B_{BB} = 0$$
$$B_{BG} = \left[b_G u_{GB} - \tau w(1 - u_{GB})\right] - \left[b_B u_{BG} - \tau w(1 - u_{BG})\right]$$
$$B_{GB} = \left[b_B u_{BG} - \tau w(1 - u_{BG})\right] - \left[b_G u_{GB} - \tau w(1 - u_{GB})\right]$$
$$B_{GG} = 0$$

Note that $B_{BG} < 0$ and $B_{GB} > 0$ if $u_{BG} > u_{GB}$ and/or $b_B > b_G$; i.e. there is a net transfer when the state of nature shifts from low job separations (good) to high job separations (bad), and vice versa. It is thus implied that there is an across state of nature insurance mechanism when the state of nature changes, but not when it persists. Broadly speaking, this captures that transitory shocks can be diversified, while persistent shocks can not.

It is shown in Appendix A that this policy implies stationary debt levels and thus satisfies the no-Ponzi condition.

3.5 Matching

Matches are determined by a standard constant returns to scale matching function; i.e. the number of matches in state i are given as

$$m(S_{ij}, V_{ij}) \equiv AS_{ij}^{\varepsilon} V_{ij}^{1-\varepsilon} \quad ,0 < \varepsilon < 1$$

where V_i is the number of vacancies in state i and aggregate search is given as

$$S_{ij} = s_i u_{ij}$$

The job finding rate is therefore

$$\alpha_{ij} \equiv \frac{m(S_{ij}, V_{ij})}{S_{ij}} = m(1, \theta_{ij}) = A\theta_{ij}^{1-\varepsilon}$$

where $\theta_{ij} \equiv \frac{V_{ij}}{S_{ij}}$ measures market tightness, and $\alpha(\theta_{ij})$, $\alpha'_{\theta}(\theta_{ij}) > 0$.

Firms fill vacancies at the rate

$$q_{ij} \equiv \frac{m(S_{ij}, V_{ij})}{V_{ij}} = m(\theta_{ij}^{-1}, 1) = A\theta_{ij}^{-\varepsilon}$$

where $q'_{\theta}(\theta) < 0$.

3.6 Inflows and outflows

The unemployment rate is a stock variable displaying inertia due to the matching friction. Hence, in general the unemployment rate adjusts sluggishly to changes in the state of nature[20], and therefore it displays path dependence. To avoid complexities associated with this, it is assumed that job separation rates differ at state transitions so as to ensure that the unemployment rate only takes on two values, u_B and u_G. The intuition is that if there is a shift from the "good" to the "bad" state, there is an extraordinarily high job separation rate, and vice versa when shifting from a "bad" to a "good" equilibrium. Hence,

$$u_{BG} = u_{BB} = u_B$$
$$u_{GB} = u_{GG} = u_G$$

The change in unemployment is given as the difference between job separations and hires. Hence, to ensure that the economy fluctuates between two

[20] See e.g. Pissarides and Mortensen (1994) and Shimer (2005) for business cycle versions of the search model in which the unemployment rate evolves from the initial unemployment rate conditional on the realization of shocks.

levels of unemployment u_B and u_G for given exogenous job separation rates p_{BB} and p_{GG}, it is required that the following restrictions are met

$$0 = (1 - u_B)p_{BB} - \alpha_B s_B u_B \quad (9)$$

$$u_G - u_B = (1 - u_B)p_{GB} - \alpha_G s_G u_G \quad (10)$$

$$u_B - u_G = (1 - u_G)p_{BG} - \alpha_B s_B u_B \quad (11)$$

$$0 = (1 - u_G)p_{GG} - \alpha_G s_G u_G \quad (12)$$

Note that α and s only depend on the current state, and u_i is the unemployment rate in state $i(= B, G)$. It is an implication that the above conditions determine p_{GB} and p_{BG}[21]. From (9) and (11) we have

$$p_{BG} = \frac{u_B - u_G}{(1 - u_G)} + \frac{(1 - u_B)}{(1 - u_G)} p_{BB} \quad (13)$$

and from (10) and (12) that

$$p_{GB} = \frac{u_G - u_B}{(1 - u_B)} + \frac{(1 - u_G)}{(1 - u_B)} p_{GG} \quad (14)$$

It follows that $u_G - u_B < 0$ implies that a shift from the G-state to the B-state is associated with extraordinarily high job separations, i.e. $p_{BG} > p_{BB}$, and a shift from the B-state to the G-state is associated with an extraordinarily low level of job separations[22], i.e. $p_{GB} < p_{GG}$.

3.7 Equilibrium

In Appendix B it is proved that there exists anequilibrium in which market tightness is larger in a good than a bad state of nature $\theta_G > \theta_B$. This implies that i) unemployment is higher in a bad state than a good state, i.e. $u_B > u_G$, ii) the job finding rate is lower in a bad state $\alpha_B < \alpha_G$, iii) the job filling rate is higher in a bad state $q_B > q_G$, and therefore iv) the value of a filled job is higher in a good state $J_G^E > J_B^E$.

3.8 Numerical illustrations

Can we make the following somewhat more convincing/realistic?

[21] Note that this makes the job separations at "switching" states a jump variable to ensure that unemployment only varies between two levels.

[22] Conditions ensuring that $p_{GB} > 0$ are assumed fulfilled.

In the following we present some numerical results based on the following assumptions concerning the functional forms. The utility from income is

$$g(y) = \frac{(y)^{1-\alpha}}{1-\alpha}$$

and from leisure (the following figures are based on log formulation, we should change that!)

$$f(1-l) = \frac{(1-l)^{1-\beta}}{1-\beta}$$

where $\alpha = 8$ and $\beta = 1.1$. Following Frederiksson & Holmlund (2006), among others, the matching function is assumed to be Cobb-Douglas of the form $m = As^{1-\varepsilon}v^{\varepsilon}$, with $\varepsilon = 0.5$ and $A = 0.29$. Time is quarterly, and we discount utility at $\rho = 0.003$ and assume that workers spend 10% of their time at work, $l = 0.1$. The tax rate is $t = 0.01$ and $\beta = 0.9$. Finally, output is set to $y = 1$, vacancy costs are set to $k = 0.2$.

4 Business cycles and search

The key behavioural variable is job search. It turns out that the way job search depends on the business cycle situation is crucial to the effects of unemployment benefits in different business cycle situations. The standard version of the matching model with a stationary equilibrium (one state of nature) implies that a higher job separation rate and thus unemployment rate is associated with less search (see Appendix C and xxx). Making inferences from a comparison of stationary equilibria would thus lead to the conclusion that search is lower in bad than in good states of nature. This conclusion does not hold when business cycle changes are explicitly acounted for, and this underlines the need to model fluctuations explicitly[23].

To see how changes in the business cycle affect job search consider for the sake for argument search in the bad state determined by (12)

$$f'(1-s_{BB}) = \pi\alpha_B\left[W_{BB}^E - W_{BB}^U\right] + (1-\pi)\alpha_G\left[W_{GB}^E - W_{GB}^U\right]$$

Two type of factors determine the return to job search, namely, the probability of finding a job and the gain from finding a job. Both of these effects goes in the direction of strengthening job search in the bad state and weaking job search in the good state. To see this consider first the ex ante perceived job

[23] Shimer (2009) similarly argues that search intensity needs not be procyclical in a discrete time setting focussing on how the easy of finding a job affects job search.

finding probability which is given as the probability of being in a given state of nature in the future times the job finding rate in that state of nature. Suppose for the sake of argument that $W_{BB}^{E} - W_{BB}^{U} = W_{GB}^{E} - W_{GB}^{U}$ in which case it follows that the possibility of shifting to the "good" state $(0 < \pi < 1)$ will increase search in the "bad" state compared to a situation with no chance of a change in the bussiness cycle situation $(\pi = 1)$ since $\alpha_G > \alpha_B$ we have

$$\pi\alpha_B + (1-\pi)\alpha_G > \alpha_B \text{ for all } \pi < 1$$

i.e. the possibility of a shift to a state with a higher job finding rate increases, other things being equal, the search level, and the effect is stronger, the larger the difference in job finding rates between the two states. The effect is obviously the opposite for search in the good state of nature, i.e.

$$\pi\alpha_G + (1-\pi)\alpha_B < \alpha_G \text{ for all } \pi < 1$$

Moreover, shifting business cycle situation affects the gain from having a job $(W^E - W^U)$. We have from the value functions that

$$\left[W_{BB}^{E} - W_{BB}^{U}\right] = \frac{\Delta + [1-\pi]\,(1 - p_{GB} - \alpha_G s_{BB})\left[W_{GB}^{E} - W_{GB}^{U}\right]}{\rho + 1 + \pi\left[p_{BB} + \alpha_B s_{BB} - 1\right]}$$

where

$$\Delta \equiv g(w\,[1-\tau] - T_{BB}) + f(1-l) - g(b_B - T_{BB}) + f(1 - s_{BB})$$

is the instantaneous utility gain from being employed rather than unemployed. If there is no chance of a change in the business cycle situation $(\pi = 1)$ we have

$$\left[W_{BB}^{E} - W_{BB}^{U}\right]|_{\pi=1} = \frac{\Delta}{\rho + p_{BB} + \alpha_B s_{BB}}$$

Hence, using that $W_{GB}^{E} - W_{GB}^{U} > 0$

$$\left[W_{BB}^{E} - W_{BB}^{U}\right] > \left[W_{BB}^{E} - W_{BB}^{U}\right]|_{\pi=1}$$

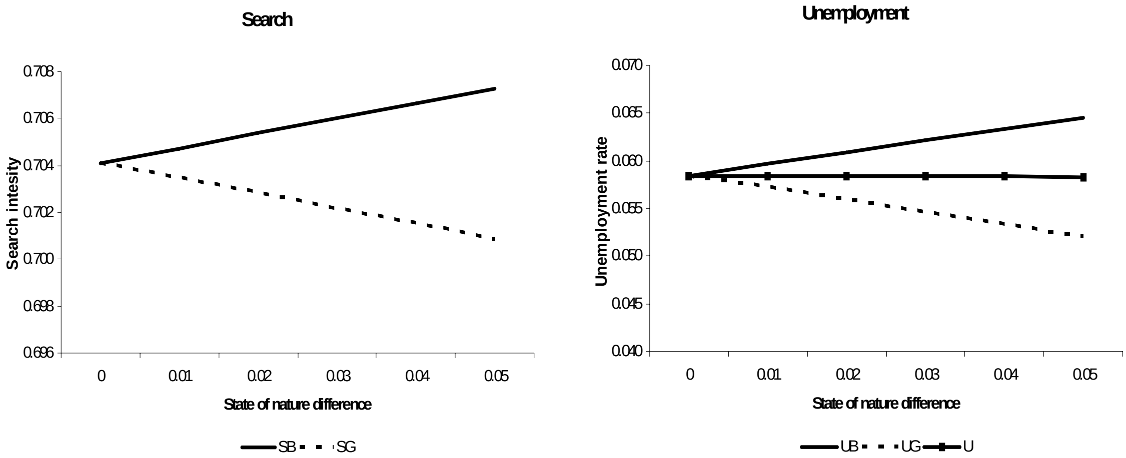
By similar reasoning it can be shown it follows

$$\left[W_{GG}^{E} - W_{GG}^{U}\right] < \left[W_{GG}^{E} - W_{GG}^{U}\right]|_{\pi=1}$$

Hence, the possibility that the business cycle situation changes tend to increase the gain from having a job in the bad state of nature, and to decrease it in the god state of nature. This goes in the direction of increasing search in the bad state and lowering it in the good state. In sum both the difference in the job finding rates and the gains from employment induced by shifts in the business cycle situation tend to induce more search in the bad state, and less search in the good state.

The role of the business cycle situation for job search is illustrated in Figure 2 showing on the x axis a widening of the difference in the job separation rate between the two states of nature (zero difference corresponds to a one state model). It is seen that job search is higher in bad states of nature. The difference widens as expected as the two states become more different. For the unemployment rate we have as expected that unemployment is higher in the bad and lower in the good state. Note that the average unemployment rate is (slightly) decreasing as the difference widens; that is, the unemployment rate is convex in the job separation rate (see also Hairault et al. (2008)), and therefore business cycle fluctuations affect the structural/average unemployment rate.

Figure 2: Widening business cycle differences: search and unemployment



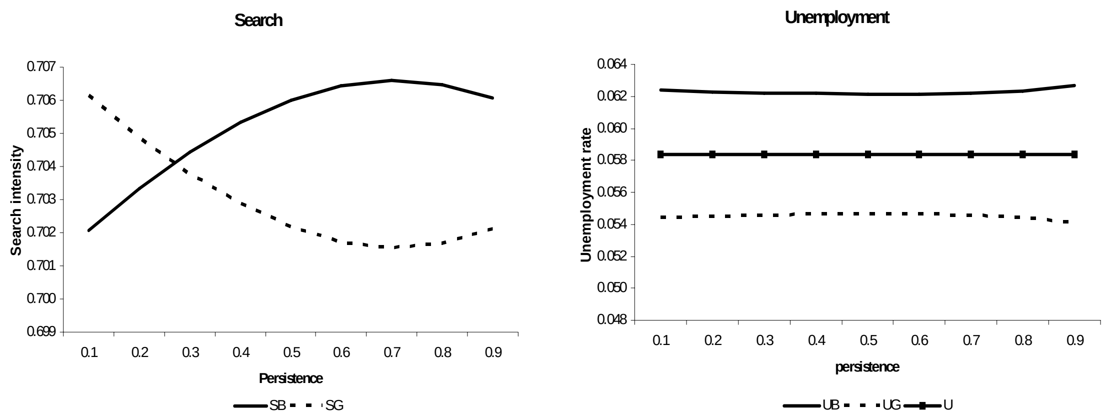
Note: For 0 the job separation rates are $p_{BB} = p_{GG} = 0.04$, and for each step 0.01 is added to p_{BB} and subtracted from p_{GG}, and the persistence is $\pi = 0.5$.

The reasoning given above points to the importance of the persistence in the business cycle situation (measured by π) via its influence on the expected gain from being employed. The larger π, the more expectations are anchored in the current state, and vice versa. Intuitively, if persistence is weak, expectations are driven by the situation in the alternate state, and oppositely if persistence is strong. This is seen by noting that

$$sign\left(\frac{\partial s_G}{\partial \pi}\right) = sign\left(\alpha_G\left[W_{GG}^E - W_{GG}^U\right] - \alpha_B\left[W_{BG}^E - W_{BG}^U\right]\right)$$
$$sign\left(\frac{\partial s_B}{\partial \pi}\right) = sign\left(\alpha_B\left[W_{BB}^E - W_{BB}^U\right] - \alpha_G\left[W_{GB}^E - W_{GB}^U\right]\right)$$

More persistence in the business cycle situation (higher π) tends to increase search effort if the expected gain from search is higher in the current state than in the new "swing" state, and vice versa. This is also seen from Figure 3 showing that there is a critical level of persistence above which search is largest in the bad state. It is a consequence that unemployment rates differ slightly more between the two states of nature if π is either low or high.

Figure 3: Persistence in business cycle situation: search and unemployment



Note: here $p_{GG} = 0.042$ and $p_{BB} = 0.038$.

5 Business cycles and distortions

The distortionary effects of the benefit level on unemployment is crucial for the optimal benefit level (see also below). Intuitively, one would expect the benefit level to be more distortionary in good states of nature with higher job finding rates than in bad states of nature. To address this issue, we can rewrite optimal search in a given state i given in (1)-(4) as

$$s_i = \phi(z_{ij}) \qquad \phi' > 0$$

where the expected gain from shifting from unemployment into employment is given as

$$z_{ij} \equiv \pi\alpha_i\left[W_{ii}^E - W_{ii}^U\right] + (1-\pi)\alpha_j\left[W_{ji}^E - W_{ji}^U\right]$$

i.e. search is increasing in the expected gain from becoming employed. It follows that

$$\frac{\partial s_i}{\partial z_{ij}}\frac{z_{ij}}{s_i} = \frac{1}{\epsilon(s_i)}\frac{1-s_i}{s_i}$$

where $\epsilon(s_i) \equiv -\frac{f''(1-s)}{f'(1-s)}(1-s) > 0$. Assuming that the latter elasticity is constant, we have that if unemployed search more in a bad than a good state $s_B > s_G$, then it follows that

$$\frac{\partial s_B}{\partial z_{BG}}\frac{z_{BG}}{s_B} < \frac{\partial s_G}{\partial z_{GB}}\frac{z_{GB}}{s_G}$$

i.e. the elasticity of search wrt. the expected gain from becoming employed is smaller in a bad than a good state.

The following tables consider this issue and report the elasticities of search and unemployment, respectively, with respect to the benefit level in the two possible states of nature. Consider first search. As expected, higher benefits lower search. There is both a direct effect in the state of nature for which the change applies and an effect in the alternate state since agents perceive the possible shift in the business cycle situation. If the business cycle situation is sufficiently persistent, the direct effect is larger than the indirect effect in the alternate state of nature. Most importantly, it is seen that in all cases the direct effect is numerically larger in the good than in the bad state; i.e. search is affected more by benefits in good than in bad states of nature.

Table 1: Effects of changing benefits: elasticity of search intensity wrt. benefit level

	$\pi = 0.3$		$\pi = 0.5$		$\pi = 0.7$	
	b_B	b_G	b_B	b_G	b_B	b_G
Elasticity of search, bad state: s_B	-0.83	-1.67	-1.17	-1.29	-1.58	-0.87
Elasticity of search, good state: s_G	-1.60	-0.92	-1.29	-1.26	-0.90	-1.69

Note: $p_{BB} = 0.042$ and $p_{GG} = 0.038$.

The effect of benefits on the unemployment rate derives from its effect on job search and we have

$$\frac{\partial u_B}{\partial b_B}\frac{b_B}{u_B} = -(1-u_B)\frac{\partial s_B}{\partial b_B}\frac{b_B}{s_B}$$

and a similar relation holds for the good state (see Appendix C). Using this we can easily characterize distortions in terms of unemployment effects, and table 2 provides numerical illustrations. As should be expected the direct effect is stronger the more persistent the business cycle situation, whereas the indirect effect on the alternate state is stronger the less persistent the business cycle situation. It is seen that the direct effect of benefit increases is larger in good than in bad states of nature; i.e. the distortions are business cycle dependent and we have that they are larger in good than in bad states. This goes in the direction of making optimal benefit levels business cycle dependent, and we explore this issue in the next section.

Table 2: Effects of changing benefits: elasticity of unemployment rate wrt. benefit level

	$\pi = 0.3$		$\pi = 0.5$		$\pi = 0.7$	
	b_B	b_G	b_B	b_G	b_B	b_G
Elasticity of unemployment, bad state: u_B	0.80	1.56	1.10	1.21	1.47	0.83
Elasticity of unemployment, good state: u_G	1.52	0.80	1.24	1.21	0.88	1.61
Elasticity of mean unemployment: u	1.12	1.27	1.16	1.21	1.20	1.18

Note: p_{BB}= 0.045 and p_{GG}= 0.035.

6 Business cycles and insurance

Turning to the insurance aspects, there are two dimensions of insurance. One is between the employed and unemployed in a given state of nature. The other dimension is across states of nature. To see this, note that disposable income for the employed (y_{ij}^E) is

$$
\begin{aligned}
y_{BB}^E &= w(1-\tau) - T_{BB} = w - (b_B + \tau w)\, u_{BB} \\
y_{GB}^E &= w(1-\tau) - T_{BG} = w - (b_G + \tau w)\, u_{GB} \\
y_{BG}^E &= w(1-\tau) - T_{GB} = w - (b_B + \tau w)\, u_{BG} \\
y_{GG}^E &= w(1-\tau) - T_{GG} = w - (b_G + \tau w)\, u_{GG}
\end{aligned}
$$

and for the unemployed

$$\begin{aligned}
y^U_{BB} &= b_B - T_{BB} = b_B + \tau w - (b_B + \tau w)u_{BB} \\
y^U_{GB} &= b_B - T_{BG} = b_B + \tau w - (b_G + \tau w)u_{GB} \\
y^U_{BG} &= b_G - T_{GB} = b_G + \tau w - (b_B + \tau w)u_{BG} \\
y^U_{GG} &= b_G - T_{GG} = b_G + \tau w - (b_G + \tau w)u_{GG}
\end{aligned}$$

It is seen that in a given state an increase in the benefit level increases the disposable income of the unemployed and decreases it for the employed. By changing the benefit level, it is thus possible to provide insurance (redistribute) between employed and unemployed[24]. Second, by running a non-balanced budget in the swing states (GB and BG), it is possible to insure across states of nature. In the present context this possibility arises when the state of nature changes, and it is seen that for $b_B > b_G$ and $u_B > u_G$ both employed and unemployed are compensated when the state shifts from G to B, and vice versa. The latter is also seen by considering how a change in the state of nature affects the overall position of employed where we have

$$\begin{aligned}
\rho \left[W^E_{BG} - W^E_{BB} \right] &= h(w\left[1-\tau\right] - T_{BG}) - h(w\left[1-\tau\right] - T_{BB}) \\
\rho \left[W^E_{GB} - W^E_{GG} \right] &= h(w\left[1-\tau\right] - T_{GB}) - h(w\left[1-\tau\right] - T_{GG})
\end{aligned}$$

Hence, if $T_{BB} > T_{BG}$ and $T_{GB} > T_{GG}$, it follows that

$$\begin{aligned}
W^E_{BG} &> W^E_{BB} \\
W^E_{GB} &< W^E_{GG}
\end{aligned}$$

i.e. employed are better off when a bad state follows a good state than when it follows a bad state, and they are worse off when a good state follows a bad state rather than a good state. To put it differently, a shift from a good to a bad state is compensated whereas a shift from a bad to good state implies a contribution.

Similarly, a change in the state of nature affects the overall position of the unemployed by

$$W^U_{BG} - W^U_{BB} = \frac{g(b_H - T_{BG}) - g(b_B - T_{BB})}{\rho + 1}$$

and

$$W^U_{GB} - W^U_{GG} = \frac{g(b_G - T_{GB}) - g(b_G - T_{GG})}{\rho + 1}$$

[24] It is easily verified that it is not possible with the state dependent policy to achieve complete insurance as defined by the Borch condition for employed and unemployed across the four different possible states of nature.

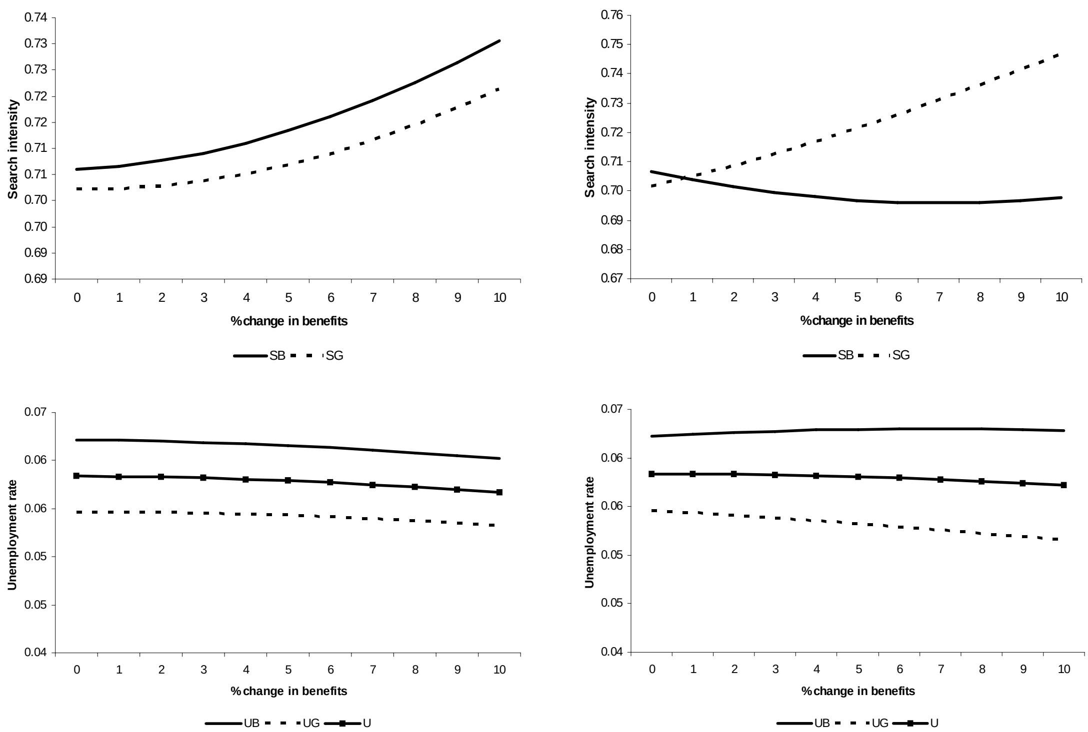
and if $T_{BB} > T_{BG}$ and $T_{GB} > T_{GG}$, it follows that

$$W^U_{BG} > W^U_{BB}$$
$$W^U_{GB} < W^U_{GG}$$

i.e. unemployed are better off when a bad state follows a good rather than a bad state, and worse off when a good state follows a bad rather than a good state.

Figure 4: Business cycle dependent benefits: search and unemployment depending on persistence

$\pi = 0.5$ $\pi = 0.7$



Note: "% change in benefits" gives the increase in benefits in the bad state and decrease in the good state relative to the initial case (0) where the benefit level is business cycle independent. Hence, the span between the two benefit levels is two times "% change in benefits".

Figure 4 shows the consequences of business cycle dependent benefits for two levels of persistence ($\pi = 0.5$ and 0.7) in the business cycle situation. The figure reports on the x-axis the % increase (decrease) in the benefit level in the bad (good) state relative to an initial situation with business cycle independent benefits. Higher benefits in the "bad" state and lower in the "good" state provide more insurance. Intuitively, it may be expected that this unambiguously would lead to less search in the "bad" state and more search in the "good" state. One striking finding is that moving from business cycle independent to business cycle dependent benefits may increase job search in both states of nature. That search increases in the G state where benefits are reduced is straightforward; it is more surprising that it also increases in the B state where benefits are increased. To see the reason for this, note that search in the B state is determined by

$$f'(1 - s_{BB}) = \pi\alpha_B \left[W_{BB}^E - W_{BB}^U\right] + (1 - \pi)\alpha_G \left[W_{GB}^E - W_{GB}^U\right]$$

The RHS gives the marginal gain from search as the probabilities of being in the various states times the job finding rate and times the gains from becoming employed. Consider for the sake of argument the case where $\pi = 1 - \pi = 1/2$ and consider a decrease in $\left[W_{BB}^E - W_{BB}^U\right]$ and an increase in $\left[W_{GB}^E - W_{GB}^U\right]$ under the constraint that $\left[W_{BB}^E - W_{BB}^U\right] + \left[W_{GB}^E - W_{GB}^U\right] = constant$. Then the RHS increases if $\alpha_G > \alpha_B$ since the job finding rate is higher in the G state. The change in the gains from finding employment in the good state matters more, ceteris paribus, than the change in the gain in the bad state even though the probability of a shift in the business cycle equals the probability of no shift ($\pi = 1/2$).

Obviously, the strength of this effect depends on the persistence in the business cycle situation. As seen from figure 4, if the business cycle situation is reasonably persistent ($\pi = 0.7$), we have that search in the good state unambiguously increases when the benefit level is lowered. In the bad state, higher benefits may first lead to lower search, but for larger increases it leads to more search. The reason is that the expected gain from shifting to the good state is lower here due to the higher persistence.

Interestingly, business cycle dependent benefits work to lower average (structural) unemployment, see Figure 4. However, the implication for unemployment fluctuations is ambiguous. If the business cycle situation is not very persistent ($\pi = 0.5$), we have that the divergence in unemployment across the two states narrows, and hence unemployment variability falls. If the business cycle situation is more persistent ($\pi = 0.7$), the divergence widens and unemployment variability goes up. It is thus in general ambiguous whether business cycle dependent benefits lead to more or less unemployment variability even if the structural (average) unemployment rate falls.

7 Optimal business cycle dependent benefits

We now turn to the issue of optimal asymmetry in benefits between the two states assessed from the utilitarian criterion. In the general case, we have that total utility can be written

$$\Psi = \sum_{i,j=B,G} \sigma_{ij} \left[(1-u_{ij}) W_{ij}^E + u_{ij} W_{ij}^U \right]$$

where σ_{ij} is the ex ante unconditional probability of being in state (i,j) and the value functions are evaluated for the tax payments implied by the budget constraints given above. Solving for the optimal benefit levels (b_B and b_G), we have the following first order condition

$$\sum_{i,j=B,G} \sigma_{ij} \left[(1-u_{ij}) \frac{\partial W_{ij}^E}{\partial b_k} + u_{ij} \frac{\partial W_{ij}^U}{\partial b_k} + \left[W_{ij}^E - W_{ij}^U \right] \frac{\partial u_{ij}}{\partial b_k} \right] = 0 \text{ for } k = B, G \tag{15}$$

7.1 One state model

Since the condition determining the optimal policies (benefit levels and taxes) is rather complex it is useful to to start by considering the one state case, i.e. there is no shift in state of nature ($\pi = 1$), or alternatively that the job separation rate is state invariant ($p_{BB} = p_{GG} = p$) (for detailes see Appendix D).

In this case there exists a stationary equilibrium (see Appendix B) with a given unemployment rate u and the budget balances. In equilibrium, unemployment is larger, the higher the job separation rate ($\frac{\partial u}{\partial p} > 0$), and the higher the benefit level ($\frac{\partial u}{\partial b} > 0$).

In the one state case there is only one policy decision since if the compensation to unemployed is determined, then the tax payment for the employed follows directly from the budget constraint. The first order condition for the optimal benefit level is

$$u \left[g'(b) - g'(w(1-\tau) \right] = \frac{\partial u}{\partial b} \left[W^E - W^U \right] \tag{16}$$

The marginal social benefits of an increase in the benefit level are given at the left hand side as the difference in marginal utilities between unemployed and employed times the unemployment rate, and the marginal social costs are given on the right hand side as the effects of the benefit level on unemployment (the distortion) times the consequences of affecting unemployment measured by the value difference between employed and unemployed. This condition shows how the trade-off between insurance and incentive (distortions) effects determine the optimal benefit level.

To interpret the condition for the optimal benefit level (16), consider first the case where there is no distortion ie. $\frac{\partial u}{\partial b}=0$ (follows if $\frac{\partial s}{\partial b}=0$, i.e. no incentive effects of unemployment benefits). In this special case optimal benefits are determined by the condition

$$g'(b)=g'(w-\frac{u}{1-u}b) \tag{17}$$

i.e. the optimal benefit level ensures that the marginal utility of income is the same for employed and unemployed[25]. This is known as the "Borch condition" for full insurance (Borch (1960)). The insurance effect is not directly related to the unemployment rate in this case but depends on the conditions prevailing as either unemployed or employed. However, there is a budget effect since the benefits are financed by taxes levied on the employed, and we have

$$\frac{db}{du}=-\frac{h''(w-\frac{u}{1-u}b)\frac{b}{(1-u)^2}}{g''(b)+h''(w-\frac{u}{1-u}b)\frac{u}{1-u}}<0$$

i.e. a higher unemployment rate is accompanied by lower benefits. The intuition is that higher unemployment raises the financing requirements to maintain a given benefit level, which in turn reduces the disposable income of employed and thus raises their marginal utility of income. To rebalance the marginal utility of consumption between the two groups, it is necessary to lower benefits. While non-distortionary benefits are a special case, this points out that a balanced budget requirement in itself implies pro-cyclical benefits.

7.2 Two state model

As shown in the previsous section the optimal policy trades-off insurance and incentives, cf (16). Importantly the value of insurance is weigthed by the unemployment rate, and the distortion of unemployment is weighted by the value of having a job rather than being unemployed. Allowing for business cycle fluctuations affect all these factors in an interesting way both directly since the unemployment rate and the value of being employed obviously differ across states of nature, but also because the possibility of business cycle changes affects these and thus search incentives, cf above. Since a bad

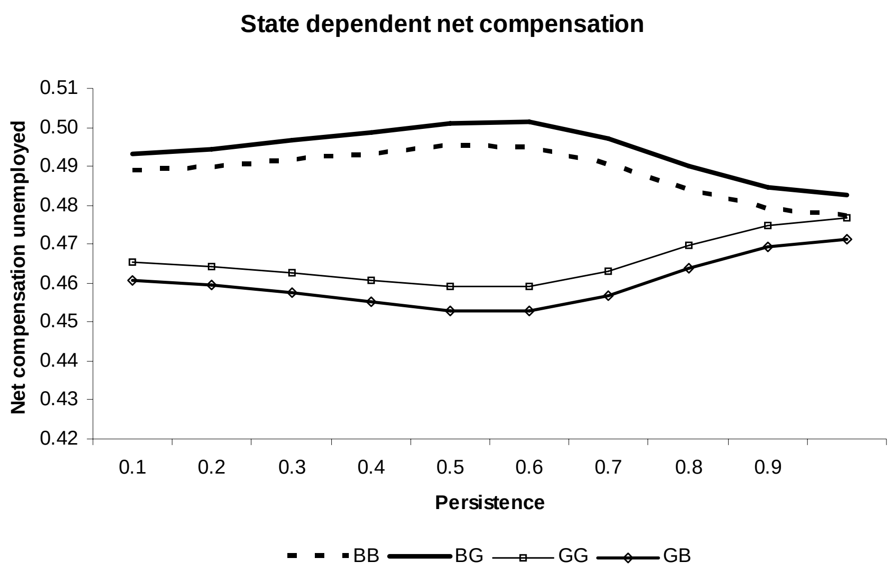
[25] Note that the participation constraint is implicitly assumed fulfilled. Otherwise there is an additional constraint in which case the benefit level is determined by the "corner" condition that

$$\left[h(w-\frac{u}{1-u}b)-e(1-l)\right]-[g(b)-f(1-s_u)]=0$$

state is characterized by a higher unemployment rate the value of insurane is larger than in a good state. At the same time distortions may be lower in a bad than a good state, and the gain from finding a job is smaller in a bad than a good state. Hence both the insurance and the incentive effects go in the direction of having benefit levels to be high in bad state of nature and low in good states of nature, that is, counter-cyclical benefit generosity.

Figure 5 below shows how the optimal net compensation (benefits less taxes paid) for the four possible states of nature depends on the underlying persistence in the business cycle situation[26]. It is seen that the net compensation is highest when a bad state follows a good state, and the intuition is that unemployed are compensated for the more bleak outlooks and lower possibilities of finding a job. Oppositely, we have the lowest net compensation when a good state follows a bad state. The net compensation offered when the bad state persists (BB) is higher than when the good state persists (GG). It is seen that the differences in net compensation are largest for intermediary levels of persistence. The intuition is that the expected gains from shifting status become lower in bad states and higher in good states of nature.

Figure 5: Business cycle dependent net compensation to unemployed and persistence



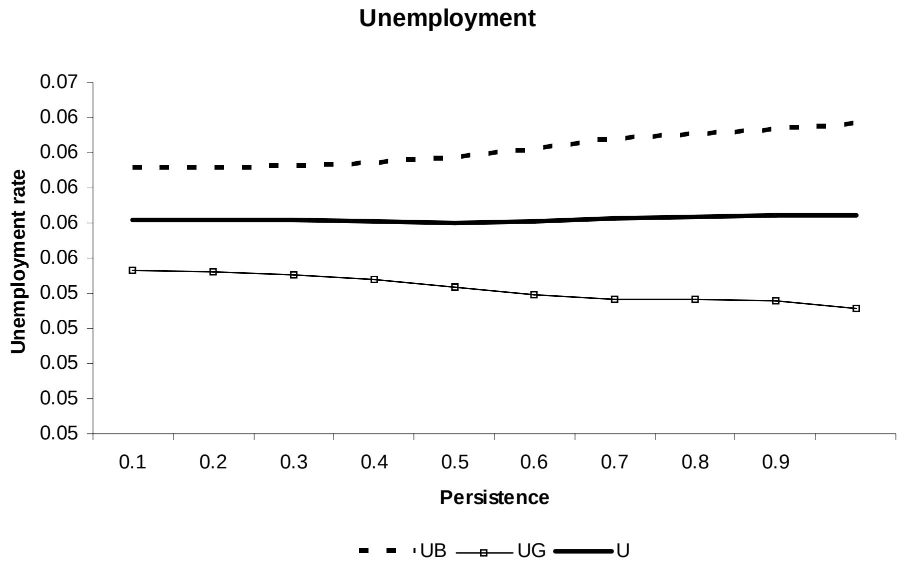
Note: The net compensation is given as $b_i - T_{ij}$. The optimal level is found

[26] We present the optimal net compensation imposing a symmetry condition; that is, increases in bad states equal decreases in good states. Considering whether optimal policies imply asymmetric adjustments, we found only small differences to the symmetric case.

in the class of symmetric business cycle dependencies in benefit levels; i.e. the increase in the bad state equals the decrease in the good state.

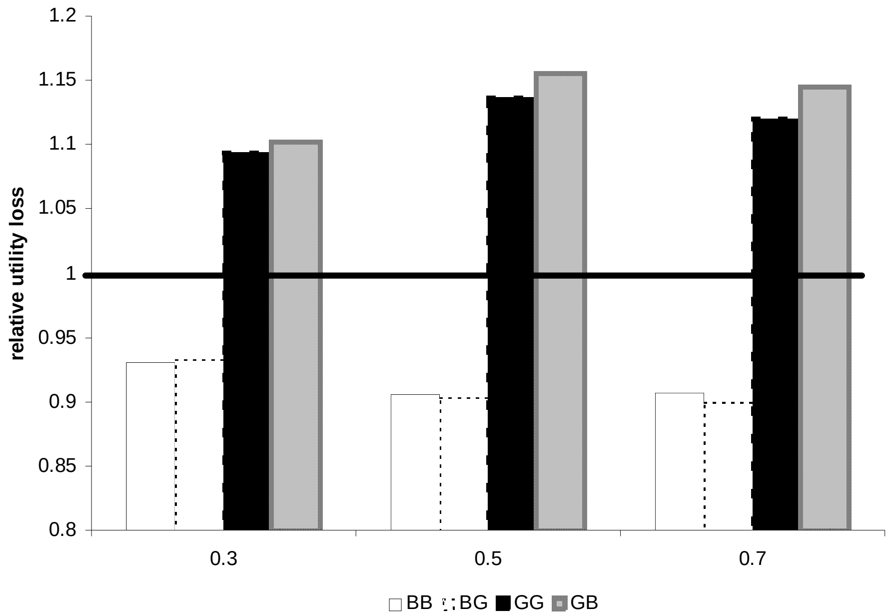
The paths for the net compensation to the unemployed are reflected in the unemployment rates in the two states of nature, and thus the average (structural) unemployment rate. Unemployment is higher in bad states and lower in good states, and the difference is widening with the persistence of the business cycle situation. The average (structural) unemployment rate is for the case considered weakly U-shaped in the persistence of the business cycle situation.

Figure 6: Business cycle dependent benefits: unemployment and persistence



One way to see the welfare consequences of business cycle dependent benefit levels is given in Figure 7. It shows that the optimal policy implies that the consequences of becoming unemployed in good states cause a larger utility loss than if benefits were business cycle independent, while in bad states the welfare loss from becoming unemployed is reduced. In this way one may say that the optimal state contingent policy effectively transfers utility from good to bad states of nature.

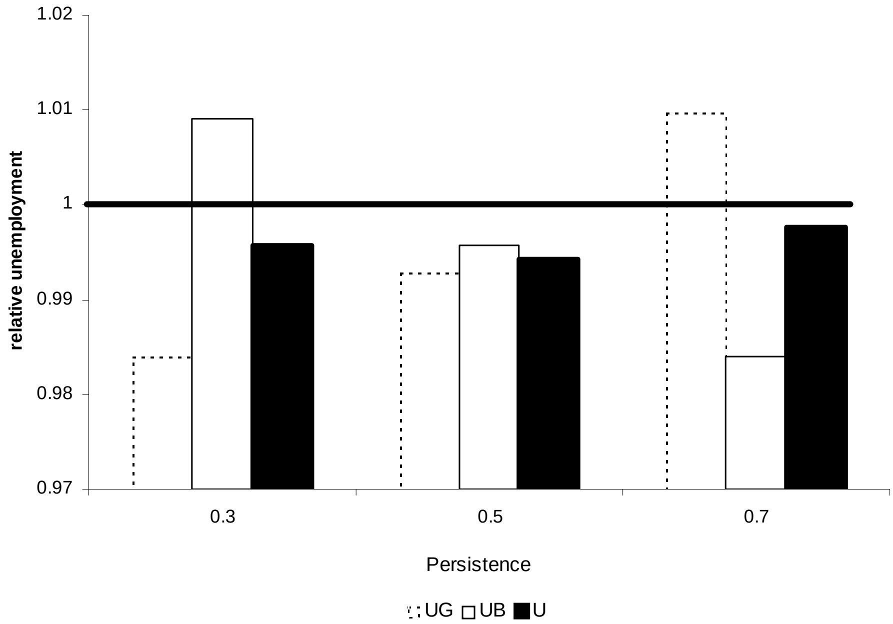
Figure 7: Relative utility loss of becoming unemployed - constant vs. business cycle dependent benefits, different levels of persistence.



Note: The figure shows the utility loss of being unemployed with business cycle dependent benefits relative to the model with business cycle independent benefits. The utility in the latter model is normalized to 1.

Finally, note that the welfare consequences differ from the consequences on the unemployment rate. Figure 8 shows that optimal business cycle dependent benefits imply more variability in unemployment rates than business cycle independent benefits. The reason is that benefits are increased in bad times with high unemployment, and decreased in good times with low employment. Hence, insurance shifts compensation from good to bad times, and search effort from bad to good times. In this way insurance and incentives are better aligned with the business cycle situation. An implication, in this illustration, is that the average unemployment rate is lower.

Figure 8: Relative unemployment: Constant vs business cycle dependent benefits



Note: The figure shows the unemployment with business cycle dependent benefits relative to the level of unemployment in a model with business cycle independent benefits. The level of unemployment in the latter model is normalized to 1.

This shows that it is possible to improve the insurance properties by making benefit levels business cycle dependent without causing an increase in the structural (average) unemployment rate. However, this gain may be achieved at the cost of more variability in unemployment.

8 Conclusion

In this paper the effects of making unemployment benefits conditional on the business cycle situation have been shown to depend not only on an insurance effect but also a budget and an incentive (distortion) effect. The budget effect tends to make benefit levels pro-cyclical since there are higher benefit expenditures in bad times with high unemployment, and vice versa. Hence, counter-cyclical benefit levels can only arise if the incentive effects of unemployment benefits are business cycle contingent. We have shown in a stylized business cycle model that if the benefit level distorts more in good than in bad times, this strengthens the argument for counter-cyclical benefit levels.

It is an important implication that such a dependency is welfare improving (utilitarian) since it shifts utility for unemployed from good to bad times. Moreover, it tends to reduce structural (average) unemployment, but it may imply that the unemployment rate becomes more sensitive to the business cycle situation. The present analysis therefore shows that a business cycle dependent unemployment insurance system can provide better insurance without resulting in higher structural unemployment.

The preceding analysis considers a very stylized unemployment insurance scheme focussing entirely on the benefit level. In practice, it may be an equally important dimension of the unemployment insurance to make the benefit duration business cycle contingent. We conjecture that the case for such a business cycle dependency is qualitatively the same as the one found in this paper for the benefit level.

The are many possible extensions of the current analysis. First, we completely ignore aggregate demand effects (automatic stabilizers) of running a business cycle dependent policy. We conjecture that incorporation of this aspect will strengthen the case for having a state dependent policy. Second, the model used in this paper relies on a very stylized description of the business cycle and a somewhat rudimentary policy rule for diversification across states of nature. It would be interesting to extent the model in these two dimensions - something which we leave for future work.

Appendix A: Stationary debt levels

To see that this policy rule ensures stationary debt levels in all states, note that the primary budget balance now can be written

$$\begin{aligned}
B_{BB} &= 0 \\
B_{BG} &= [b_G u_{GB} - \tau w(1-u_{GB})] - [b_B u_{BG} - \tau w(1-u_{BG})] \\
B_{GB} &= [b_B u_{BG} - \tau w(1-u_{BG})] - [b_G u_{GB} - \tau w(1-u_{GB})] \\
B_{GG} &= 0
\end{aligned}$$

implying

$$B_{BG} = -B_{GB}$$

i.e. if the public sector is running a budget deficit when a bad state of nature with high job separations ($B_{BG} < 0$) replaces a good state of nature with low job separations, then it will run a similar surplus when a good state of nature replaces a bad state of nature. In this way the scheme allows some risk diversification. To see that this is consistent with a stationary debt level in any state of nature, observe further that

$$\begin{aligned}
\rho D_{GB} &= b_G u_{GB} - \tau w(1-u_{GB}) - [b_B u_{BG} - \tau w(1-u_{BG})] \\
&\quad + \pi [D_{GG} - D_{GB}] + (1-\pi)[D_{BG} - D_{GB}] \\
\rho D_{BG} &= b_B u_{BG} - \tau w(1-u_{BG}) - [b_G u_{GB} - \tau w(1-u_{GB})] \\
&\quad + \pi [D_{BB} - D_{BG}] + (1-\pi)[D_{GB} - D_{BG}]
\end{aligned}$$

implying that

$$(\rho + \pi)[D_{GB} + D_{BG}] = \pi [D_{GG} + D_{BB}]$$

and since we have from the debt level equation for D_{GG} and D_{BB} that

$$(\rho + 1 - \pi)[D_{GG} + D_{BB}] = (1-\pi)[D_{GB} + D_{BG}]$$

it follows that

$$\begin{aligned}
D_{GB} + D_{BG} &= 0 \\
D_{GG} + D_{BB} &= 0.
\end{aligned}$$

The debt levels in the different states of nature can be found be using that

$$\begin{aligned}
\rho D_{BB} &= b_B u_{BB} - \tau w(1-u_{BB}) - T_{BB} + (1-\pi)[D_{GB} - D_{BB}] \\
\rho D_{GB} &= b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB} + \pi [D_{GG} - D_{GB}] + (1-\pi)[D_{BG} - D_{GB}]
\end{aligned}$$

which implies (by use of $b_B u_{BB} - \tau w(1-u_{BB}) - T_{BB} = 0$)

$$(\rho + 1 - \pi)\, D_{BB} = (1-\pi) D_{GB}$$
$$(\rho + \pi + 2(1-\pi))\, D_{GB} = b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB} - \pi D_{BB}$$

Hence

$$D_{GB} = \left[\rho + \pi + 2(1-\pi) + \pi \frac{1-\pi}{\rho + 1 - \pi}\right] \left[b_G u_{GB} - \tau w(1-u_{GB}) - T_{GB}\right]^{-1}$$

which is finite, and hence D_{BB}, D_{BG}, and D_{GG} are finite.

Appendix B: Proof of equilibrium to the two state model

Note that from (9) and (12), we have

$$\frac{m(1,\theta_B)}{(1-u_B)} = p_{BB}$$
$$\frac{m(1,\theta_G)}{(1-u_G)} = p_{GG}$$

and hence

$$\frac{(1-u_G)}{(1-u_B)} \frac{m(1,\theta_B)}{m(1,\theta_G)} = \frac{p_{BB}}{p_{GG}} \tag{18}$$

Since $\frac{p_{BB}}{p_{GG}} > 1$, it follows that a sufficient condition that $u_B > u_G$ is $\frac{m(1,\theta_B)}{m(1,\theta_G)} < 1$ or $\theta_B < \theta_G$.

From the value functions for a filled job (5) and (6), we have by use of $J_G^V = J_B^V = 0$ that

$$\rho J_B^E = y - w + \pi p_{BB}(-J_B^E) + (1-\pi)\left[p_{GB}(-J_B^E) + (1-p_{GB})(\frac{q_B}{q_G} - 1) J_B^E\right] \tag{19}$$

$$\rho J_G^E = y - w + \pi p_{GG}(-J_G^E) + (1-\pi)\left[p_{BG}(-J_G^E) + (1-p_{BG})(\frac{q_G}{q_B} - 1) J_G^E\right] \tag{20}$$

Hence

$$\left[\rho + \pi p_{BB} + (1-\pi)\left[p_{GB} + (1-p_{GB})(1 - \frac{q_B}{q_G})\right]\right] J_B^E = y - w$$
$$\left[\rho + \pi p_{GG} + (1-\pi)\left[p_{BG} + (1-p_{BG})(1 - \frac{q_G}{q_B})\right]\right] J_G^E = y - w$$

and

$$\frac{\left[\rho+\pi p_{BB}+(1-\pi)\left[1-(1-p_{GB})\frac{q_B}{q_G}\right]\right]}{\left[\rho+\pi p_{GG}+(1-\pi)\left[1-(1-p_{BG})\frac{q_G}{q_B})\right]\right]}=\frac{J_G^E}{J_B^E}=\frac{q_B}{q_G} \tag{21}$$

where the last equality follows from (7).

Using (13) and (13) we have

$$\begin{aligned}1-p_{BG} &= \frac{(1-u_B)}{(1-u_G)}(1-p_{BB})\\ 1-p_{GB} &= \frac{(1-u_G)}{(1-u_B)}(1-p_{GG})\end{aligned}$$

Implying that (21) can be written

$$\frac{\left[\rho+\pi p_{BB}+(1-\pi)\left[1-\frac{(1-u_G)}{(1-u_B)}(1-p_{GG})\frac{q_B}{q_G}\right]\right]}{\left[\rho+\pi p_{GG}+(1-\pi)\left[1-\frac{(1-u_B)}{(1-u_G)}(1-p_{BB})\frac{q_G}{q_B})\right]\right]}=\frac{q_B}{q_G}$$

and using (18) we get

$$\frac{\left[\rho+\pi p_{BB}+(1-\pi)\left[1-\frac{p_{BB}}{p_{GG}}\frac{m(1,\theta_G)}{m(1,\theta_B)}(1-p_{GG})\frac{q_B}{q_G}\right]\right]}{\left[\rho+\pi p_{GG}+(1-\pi)\left[1-\frac{p_{GG}}{p_{BB}}\frac{m(1,\theta_B)}{m(1,\theta_G)}(1-p_{BB})\frac{q_G}{q_B})\right]\right]}=\frac{q_B}{q_G} \tag{22}$$

We have that

$$\frac{q_B}{q_G}=\frac{m(\theta_B^{-1},1)}{m(\theta_G^{-1},1)}=\frac{\theta_B^{-\alpha}}{\theta_G^{-\alpha}}=\left[\frac{\theta_G}{\theta_B}\right]^{\alpha}$$

and

$$\frac{q_B}{q_G}\frac{m(1,\theta_G)}{m(1,\theta_B)}=\frac{m(\theta_B^{-1},1)}{m(\theta_G^{-1},1)}\frac{m(1,\theta_G)}{m(1,\theta_B)}=\frac{\theta_B^{-\alpha}}{\theta_G^{-\alpha}}\frac{\theta_G^{1-\alpha}}{\theta_B^{1-\alpha}}=\frac{\theta_G}{\theta_B}$$

Condition (22) can now be written

$$\frac{\left[\rho+\pi p_{BB}+(1-\pi)\left[1-\frac{p_{BB}}{p_{GG}}(1-p_{GG})\frac{\theta_G}{\theta_B}\right]\right]}{\left[\rho+\pi p_{GG}+(1-\pi)\left[1-\frac{p_{GG}}{p_{BB}}(1-p_{BB})\frac{\theta_B}{\theta_G})\right]\right]}=\left[\frac{\theta_G}{\theta_B}\right]^{\alpha} \tag{23}$$

It is seen that the LHS of (23) is decreasing in $\frac{\theta_G}{\theta_B}$, and the RHS is increasing in $\frac{\theta_G}{\theta_B}$. It follows that there is a unique solution to $\frac{\theta_G}{\theta_B}$ from which all other variables can be found. To prove that $\frac{\theta_G}{\theta_B}>1$, observe that for $\frac{\theta_G}{\theta_B}=1$ we have that the RHS of (23) equals one, whereas the LHS is larger than one.

Hence, it follows that $\frac{\theta_G}{\theta_B} > 1$. Note that this implies $\frac{q_G}{q_B} < 1$, and hence $u_G < u_B$.

Appendix C: Distortions

First, notice a recursive structure of the model. We have from (5), (6), and (8) that

$$\begin{aligned} J_B^E &= \frac{(1-\beta)\, y + (1-\pi)(1-p_{GB}) J_G^E}{[\rho + \pi p_{BB} + p_{GB}]} \\ J_G^E &= \frac{(1-\beta)\, y + (1-\pi)(1-p_{BG}) J_B^E}{\rho + \pi p_{GG} + p_{BG}} \end{aligned}$$

and using that in equilibrium that

$$J_B^E = \frac{ky}{q_B}$$
$$J_G^E = \frac{ky}{q_G}$$

we get (using that $q = q(\theta)$)

$$\begin{aligned} \frac{ky}{q(\theta_B)} &= \frac{(1-\beta)\, y + (1-\pi)(1-p_{GB}) \frac{ky}{q(\theta_G)}}{[\rho + \pi p_{BB} + p_{GB}]} \\ \frac{ky}{q(\theta_G)} &= \frac{(1-\beta)\, y + (1-\pi)(1-p_{BG}) \frac{ky}{q(\theta_B)}}{\rho + \pi p_{GG} + p_{BG}} \end{aligned}$$

From Appendix B we have

$$\frac{m(1,\theta_B)}{(1-u_B)} = p_{BB}$$
$$\frac{m(1,\theta_G)}{(1-u_G)} = p_{GG}$$

and from (13) and (14) that

$$\begin{aligned} p_{BG} &= \frac{u_B - u_G}{(1-u_G)} + \frac{(1-u_B)}{(1-u_G)} p_{BB} \\ p_{GB} &= \frac{u_G - u_B}{(1-u_B)} + \frac{(1-u_G)}{(1-u_B)} p_{GG} \end{aligned}$$

The last six equations determine the six endogenous variables:$\theta_B, \theta_G, u_B, u_G, p_{BG}$ and p_{GB} given the exogenous: p_{BB} and p_{GG}.

Using this and from (9) that

$$u_B = \frac{p_{BB}}{p_{BB} + \alpha_B s_B}$$

we have

$$\frac{\partial u_B}{\partial b_B}\frac{b_B}{u_B} = -(1-u_B)\frac{\partial s_B}{\partial b_B}\frac{b_B}{s_B}$$

and similarly for the good state of nature.

Appendix D: One state model

In the one state case ($p_{BB} = p_{GG} = p$), we have that the model is summarized by

Value function employed	$\rho W^E = h(w-T) + e(1-l) + p\left[W^U - W^E\right]$
Value function unemployed	$\rho W^U = g(b) + f(1-s) + \alpha s\left[W^E - W^U\right]$
Search	$f'(1-s) = \alpha\left[W^E - W^U\right]$
Inflow outflow	$0 = (1-u)p - \alpha(\theta)su$
Job filling rate	$[\rho + p]\frac{k}{q(\theta)} = 1-\beta$
Budget balance	$(1-u)T = ub$

Note that the job-filling rate is found from (5), which in the one state case reads

$$\rho J^E = y - w - pJ^E$$

and using (**??**) we have

$$[\rho + p]\frac{k}{q} = 1-\beta$$

This determines the job filling rate (q) and thus also the job finding rate (α). It follows straightforwardly that

$$\frac{\partial q}{\partial b} = \frac{\partial \alpha}{\partial b} = 0$$

and $\frac{\partial q}{\partial p}\frac{p}{q} = \frac{p}{\rho+p} \in [0,1]$ and $\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} < 0$.

Note for later reference that

$$\rho\left[W^E - W^U\right] = h(w-T) + e(1-l) - [g(b) + f(1-s)] + (p+\alpha s)\left[W^U - W^E\right]$$

and hence

$$\left[W^E - W^U\right] = \frac{h(w-T) + e(1-l) - [g(b) + f(1-s)]}{[\rho + p + \alpha s]} \tag{24}$$

From the inflow-outflow relation, we have

$$\frac{u}{1-u} = \frac{p}{\alpha s} \tag{25}$$

Job separation

First consider the response of the unemployment rate to the job separation rate. From (25) we have

$$\frac{\partial u}{\partial p} = [1-u]^2 \frac{1 - \left[\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} + \frac{\partial s}{\partial p}\frac{p}{s}\right]}{s\alpha}$$

where $\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} < 0$ and $\frac{\partial s}{\partial p}\frac{p}{s}$ is found from (**??**) implying

$$-f''(1-s)\frac{\partial s}{\partial p} = \left[W^E - W^U\right]\frac{\partial \alpha}{\partial p} + \alpha \frac{\partial \left[W^E - W^U\right]}{\partial p}$$

and hence

$$\frac{\partial s}{\partial p}\frac{p}{s} = \frac{1}{\epsilon(s)}\frac{1-s}{s}\left[\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} + \frac{\partial \left[W^E - W^U\right]}{\partial p}\frac{p}{\left[W^E - W^U\right]}\right]$$

where $\epsilon(s) \equiv -\frac{f''()(1-s)}{f'()} > 0$.

From (24) we have

$$\frac{\partial \left[W^E - W^U\right]}{\partial p} = \frac{f'(1-s)\frac{\partial s}{\partial p}}{[\rho + p + \alpha s]} - \frac{\partial (p + \alpha s)}{\partial p}\frac{\left[W^E - W^U\right]}{\rho + p + \alpha s}$$

Hence, using that $f'() = \alpha\left[W^E - W^U\right]$ we have

$$\begin{aligned}
\frac{\partial \left[W^E - W^U\right]}{\partial p}\frac{p}{\left[W^E - W^U\right]} &= \frac{\alpha \frac{\partial s}{\partial p} p}{[\rho + p + \alpha s]} - \left[1 + s\frac{\partial \alpha}{\partial p} + \alpha\frac{\partial s}{\partial p}\right]\frac{p}{\rho + p + \alpha s} \\
&= -\left[1 + s\frac{\partial \alpha}{\partial p}\right]\frac{p}{\rho + p + \alpha s}
\end{aligned}$$

It follows that

$$\frac{\partial s}{\partial p}\frac{p}{s} = \frac{1}{\epsilon(s)}\frac{1-s}{s}\left[\left[\frac{\rho + p}{\rho + p + \alpha s}\right]\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} - \frac{p}{\rho + p + \alpha s}\right] < 0$$

It is an implication that $s\left[1 - \left[\frac{\partial \alpha}{\partial p}\frac{p}{\alpha} + \frac{\partial s}{\partial p}\frac{p}{s}\right]\right] > 0$ and hence $\frac{\partial u}{\partial p} < 0$.

Note that

$$\begin{aligned}
\frac{\partial}{\partial p}\Big|_s \left(\frac{\alpha}{[\rho + p + \alpha s]}\right) &= \frac{\frac{\partial \alpha}{\partial p}[\rho + p + \alpha s] - \alpha\left[1 + \frac{\partial \alpha}{\partial p}s\right]}{[\rho + p + \alpha s]^2} \\
&= \frac{\frac{\partial \alpha}{\partial p}[\rho + p] - \alpha}{[\rho + p + \alpha s]^2} < 0
\end{aligned}$$

Benefits

From (25) it follows that

$$\frac{\partial u}{\partial b}\frac{b}{u} = \frac{-b}{s}\frac{\partial s}{\partial b}\left[1-u\right]$$

i.e. the elasticity of unemployment wrt. the benefit level depends on the elasticity of search wrt. the benefit level times the employment rate. To find the latter, we have from the search equation (**??**) that

$$-f''()\frac{\partial s}{\partial b} = \alpha\frac{\partial\left[W^E - W^U\right]}{\partial b}$$

and hence

$$\frac{\partial s}{\partial b}\frac{b}{s} = \frac{\alpha b}{f''()\left[\rho+p+\alpha s\right]s}\left[h'()\frac{u}{1-u}\frac{\partial\tau}{\partial b}\frac{b}{\tau}+g'(b)\right]$$

From the budget constraint we have

$$\frac{\partial T}{\partial b} = \frac{u}{1-u}+\frac{\frac{\partial u}{\partial b}}{(1-u)^2}b > 0$$

or

$$\begin{aligned}\frac{\partial T}{\partial b}\frac{b}{T} &= 1+\frac{\partial u}{\partial b}\frac{b}{u}\frac{1}{1-u}\\ &= 1-\frac{\partial s}{\partial b}\frac{b}{s}\end{aligned}$$

Hence,

$$\begin{aligned}\frac{\partial s}{\partial b}\frac{b}{s} &= \frac{\alpha b}{f''()\left[\rho+p+\alpha s\right]s}\left[h'()\frac{u}{1-u}\left[1+\frac{\partial u}{\partial b}\frac{b}{u}\frac{1}{1-u}\right]+g'(b)\right]\\ &= \frac{\alpha b}{f''()\left[\rho+p+\alpha s\right]s}\left[h'()\frac{u}{1-u}\left[1-\frac{\partial s}{\partial b}\frac{b}{s}\right]+g'(b)\right]\end{aligned}$$

and

$$\frac{\partial s}{\partial b}\frac{b}{s} = \frac{\frac{\alpha b}{f''()[\rho+p+\alpha s]s}\left[h'()\frac{u}{1-u}+g'(b)\right]}{1+\frac{\alpha b}{f''()[\rho+p+\alpha s]s}h'()\frac{u}{1-u}} < 0$$

The sign follows by noting that $1+\frac{\alpha b}{f''()[\rho+p+\alpha s]s}h'()\frac{u}{1-u} > 0$ is required for stability. To see the latter, note that search is a decreasing function of the tax rate, and that the tax rate is a decreasing function of the search level. The former gives the chosen search level for a given tax rate, and the latter is giving the required search to balance the budget for a given tax rate.

Specifically we have from (**??**)

$$\frac{\partial s}{\partial T}\mid_{\text{behaviour}}=\frac{\alpha h'()}{f''()\left[\rho+p+\alpha s\right]}<0$$

and

$$T=\frac{u}{1-u}b=\frac{p}{\alpha s}b$$

and hence

$$\frac{\partial \tau}{\partial s}\mid_{\text{budget}}=\frac{-1}{s^2}\frac{p}{\alpha}b=\frac{-1}{s}\frac{u}{1-u}b<0$$

Stability requires that the required search level exceeds the chosen search level for a tax rate below the equilibrium value, and vice versa, and this is ensured if

$$\frac{\partial s}{\partial \tau}\mid_{\text{behaviour}}<\frac{\partial s}{\partial \tau}\mid_{\text{budget}}$$

or

$$\frac{\alpha h'()w}{f''()\left[\rho+p+\alpha s\right]}<\frac{-1}{\frac{1}{s}\frac{u}{1-u}\frac{b}{w}}$$

and hence

$$\frac{\alpha h'()b}{f''()\left[\rho+p+\alpha s\right]s}\frac{u}{1-u}>-1$$

Optimal benefits

The optimal benefit level solves

$$Max_b\ \Psi\equiv(1-u)\,W^E+uW^U$$

This problem has the first order condtion

$$\digamma\equiv(1-u)\frac{\partial W^E}{\partial b}+u\frac{\partial W^U}{\partial b}+\frac{\partial u}{\partial b}\left[W^U-W^E\right]=0$$

and the second order condition

$$\digamma_b<0$$

Using the the envelope theorem we have from the value functions for employed and unemployed, respectively

$$\rho\frac{\partial W^E}{\partial b}=-h'(w-T)\frac{u}{1-u}+p\left[\frac{\partial W^U}{\partial b}-\frac{\partial W^E}{\partial b}\right]$$
$$\rho\frac{\partial W^U}{\partial b}=g'(b)+\alpha s\left[\frac{\partial W^E}{\partial b}-\frac{\partial W^U}{\partial b}\right]$$

and hence

$$\begin{aligned}
\rho(1-u)\frac{\partial W^E}{\partial b} &= -uh'(w-T) + p(1-u)\left[\frac{\partial W^U}{\partial b} - \frac{\partial W^E}{\partial b}\right] \\
\rho u \frac{\partial W^U}{\partial b} &= ug'(b) + \alpha s u\left[\frac{\partial W^E}{\partial b} - \frac{\partial W^U}{\partial b}\right]
\end{aligned}$$

It follows that

$$\rho(1-u)\frac{\partial W^E}{\partial b} + \rho u \frac{\partial W^U}{\partial b} = ug'(b) - uh'(w(1-\tau))$$

which implies

$$\mathcal{F} = u\left[g'(\cdot) - h'(\cdot)\right] + \frac{\partial u}{\partial b}\left[W^U - W^E\right] = 0$$

From (**??**) we find

$$\mathcal{F}_p = \left[g'(\cdot) - h'(\cdot)\right]\frac{\partial u}{\partial p} + uh''()\frac{\partial \tau}{\partial p} + \frac{\partial u}{\partial b}\frac{\partial\left[W^U - W^E\right]}{\partial p} + \left[W^U - W^E\right]\frac{\partial\left[\frac{\partial u}{\partial b}\right]}{\partial p}$$

or by using $u\left[g'(b) - h'(w(1-\tau))\right] = \frac{\partial u}{\partial b}\left[W^E - W^U\right]$ that

$$\mathcal{F}_p = \frac{\partial u}{\partial b}\frac{\left[W^E - W^U\right]}{p}\left[\frac{\partial u}{\partial p}\frac{p}{u} + \frac{h''()p}{g'(\cdot) - h'(\cdot)}\frac{\partial T}{\partial p} + \frac{\partial\left[W^U - W^E\right]}{\partial p}\frac{p}{\left[W^U - W^E\right]} + \frac{\partial\left[\frac{\partial u}{\partial b}\right]}{\partial p}\frac{p}{\frac{\partial u}{\partial b}}\right]$$

The key question here is how the optimal benefit level depends on the labour market situation (here the job separation rate). We have from (**??**) that

$$\frac{db}{dp} = -\frac{\mathcal{F}_p}{\mathcal{F}_b}$$

hence

$$sign\ \frac{db}{dp} = sign\ \mathcal{F}_p$$

It can be shown (see Appendix B) that

$$sign\ \mathcal{F}_p = sign\left[\underbrace{\frac{\partial u}{\partial p}\frac{p}{u}}_{\text{unempl eff }>0} + \underbrace{\frac{h''()p}{g'(\cdot) - h'(\cdot)}\frac{\partial \tau}{\partial p}}_{\text{budget eff }<0} + \underbrace{\frac{\partial\left[W^U - W^E\right]}{\partial p}\frac{p}{\left[W^U - W^E\right]}}_{\text{value effect }>0} + \underbrace{\frac{\partial\left[\frac{\partial u}{\partial b}\right]}{\partial p}\frac{p}{\frac{\partial u}{\partial b}}}_{\text{distortion}\lesseqgtr 0}\right]$$

The above suggest that benefits can be countercyclical even in the one state case. e.g. let h'' be close to zero (almost linear utility) then this will be the case unless there is a strong counterweighting effect from the distortion!

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