Market Power and Efficiency in a Search Model*

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Abstract

We build a theoretical model to study the welfare effects and resulting policy implications of firms' market power in a frictional labor market. Our environment has two main characteristics: wages play a role in allocating labor across firms and there is a finite number of agents. We find that the decentralized equilibrium is inefficient and that the firms' market power results in the misallocation of workers from the high- to the low-productivity firms. A minimum wage forces the low-productivity firms to increase their wage, leading them to hire even more often thereby exacerbating the inefficiencies. Moderate unemployment benefits can increase welfare because they limit firms' market power by improving the workers' outside option.

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1 Introduction

In this paper we study the welfare effects and resulting policy implications that arise when firms have market power in the labor market. To that end, we examine a directed search model with a finite number of agents. We find that the firms' market power results in inefficiencies by causing the misallocation of workers across firms. The policy implications are that a minimum wage is always detrimental to welfare while unemployment benefits may improve welfare.

In our model workers are homogeneous, firms differ in their productivity and each firm has one vacancy. The split of the surplus is endogenously determined during the hiring process: first every firm posts a wage and then each worker observes all postings and applies for one job.[1] Frictions are introduced by assuming that workers cannot coordinate their application decisions with each other. To study a setting where firms have market power, we focus our attention on a version of this model with a finite number of agents. We show that the decentralized allocation is inefficient: the more (less) productive firms hire less (more) often than is optimal; furthermore, unemployment is *too low* because workers apply to safe low-wage/low-productivity jobs too frequently.

The culprit for the inefficiency is the market power that firms enjoy in a a finite market, in the sense that a single firm's action affects the equilibrium outcomes of all agents. Indeed, constrained efficiency obtains in large markets where firms do not have market power (Moen (1997), Shi (2001), Shimer (2005)). Market power reduces the elasticity of the hiring probability with respect to the wage and, as a result, all firms post wages that are lower than at the efficient benchmark. This redistributes surplus from workers to firms which is a feature that is shared with frictionless models of monopsony (Bhaskar, Manning and To (2002)). A novel feature of our model is that the incentive to reduce wages is stronger for high productivity (and in equilibrium high-wage) firms which leads them to reduce wages disproportionally. This results in the reallocation of workers from the high- to the low-productivity firms which turns out to be inefficient. Furthermore, since high-wage jobs are harder to get, a side-effect of this misallocation is that the unemployment rate is lower in the decentralized equilibrium than in the efficient benchmark. Therefore, market power increases employment but reduces output (and hence welfare) by decreasing the average productivity of employed workers. This source of inefficiencies is new to the directed search literature.

We examine the welfare effects of two labor market interventions: minimum wages and

[1]Most of the directed search literature restricts attention to a single application as a way of capturing the time-consuming aspect of the job-finding process. Multiple applications were recently introduced in continuum directed search models by Albrecht, Gautier and Vroman (2006), Galenianos and Kircher (2009a) and Kircher (2008).

unemployment benefits. A minimum wage constraints the low productivity firms to offer higher wages than they otherwise would and hence hire even more often than in the original equilibrium, thereby exacerbating the inefficiencies. Introducing unemployment benefits results in the reallocations of workers towards high productivity firms because it improves the outside option of workers effectively reducing the firms' market power. In one sense, unemployment benefits act as a subsidy to search and reduce firms' market power. This is on top of any additional benefits that may be derived from better match formation (e.g. Marimon and Zilibotti (1999)). In terms of employment effects, introducing a minimum wage (unemployment benefits) reallocates workers to the low (high) productivity firms; since low productivity firms offer a higher probability of employment, the minimum wage *reduces* the unemployment rate while benefits increase it.[2] Therefore, our model shows that evaluating the welfare implications of labor market policy based on their employment effects alone can lead to misleading results.

We emphasize that even though both policies redistribute surplus towards the workers, they have diametrically opposing effects on aggregate welfare. This is interesting because it contrasts with the predictions of many recent frictional models where both policy instruments yield the same qualitative welfare implications (Acemoğlu and Shimer (1999), Acemoğlu (2001), Manning (2004)).[3] However, wages have no allocative role in these models, either because agents are assumed to be identical or because wages are set through bargaining *after* a match has formed. When we introduce an allocative role for wages by giving firms a non-trivial wage-setting decision in an environment with productivity heterogeneity the qualitative similarity across policy instruments disappears.[4]

The next section describes the model. Most of the insights of our model can be conveyed in the simple setting with two workers and two firms which is examined in Section 3. Section 4 generalizes our results. We discuss the relevance of our results and conclude in Section 5.

2 The Environment

We begin with a brief description of the environment.

[2]These employment effects do not hinge on the finite nature of the market, as we discuss in the conclusions. Of course, the policies' normative effects do depend on the size of the market.

[3]These papers span the three most popular classes of labor search models: directed search, random search with bargained wages and random search with posting, respectively. Acemoğlu and Shimer (1999) do not explicitly consider a minimum wage but it is easy to show that it has the same effect as their prescribed unemployment benefits, as also remarked in Acemoğlu (2001).

[4]A different modeling approach is taken by Kaas and Madden (2008). They consider a two-firm Hotelling model and show that a minimum wage reduces the firms' market power and leads to a welfare improvement. In that model the wage does play an allocative role.

The economy is populated with a finite number of risk-neutral workers and firms, denoted by $N = \{1, ..., n\}$ and $M = \{1, ..., m\}$ respectively, where $n \geq 2$ and $m \geq 2$. Each firm j has one vacancy and is characterized by its productivity level x_j, where $x_j > 0$ for all j. We assume without loss of generality that $x_m \leq x_{m-1} \leq ... \leq x_1 \equiv \overline{x}$. The productivity of all firms is common knowledge.[5] The profits of firm j are equal to $x_j - w_j$ if it employs a worker at wage w_j and zero otherwise. All workers are identical and the utility of a worker is equal to his wage if employed and zero otherwise.

The hiring process has three stages:

1. Each firm j posts a wage $w_j \in [0, \overline{x}]$.
2. Workers observe the wage announcement $\mathbf{w} = \{w_1, w_2, ..., w_m\} \in [0, \overline{x}]^m$ and each worker simultaneously applies to one firm.
3. A firm that receives one or more applicants hires one of these workers at random. A firm without applicants remains idle.

We focus our attention on equilibria with pure wage-posting strategies by the firms and we solve for the subgame perfect equilibria of the game. As is standard in the directed search literature, we restrict attention to equilibria where workers follow symmetric strategies. Symmetric strategies mean that, following any wage announcement, every worker applies to firm j with the same probability for all $j \in M$. This assumption rules out coordination among workers and it is a natural way of introducing trading frictions.[6] It is straightforward to extend this environment to introduce our two policy variables: a minimum wage puts a lower bound on the wages that firms can post; unemployment benefits increase the value of remaining unemployed. This is the standard directed search environment, for instance as in Burdett, Shi and Wright (2001, henceforth BSW).[7]

[5] The case where productivity levels are private information is examined in Galenianos and Kircher (2007).

[6] Lack of coordination may seem incompatible with a finite (or, small) labor market. What we have in mind is that the labor market for some occupation may have a small number of participants while the total number of agents in the geographical vicinity is large enough to preclude coordination among them.

[7] We take the trading mechanism and the associated coordination failures as given. The coordination problem would be less severe if the contracts were posted by the workers rather than the firms, as in Coles and Eeckhout (2003a, 2003b). However, in that environment the firms do not obtain any surplus and they would therefore prefer to offer the contracts themselves rather than apply for workers' services if given the choice. Analyzing the effects of competing markets is beyond the scope of our paper (see Halko, Kultti and Virrankoski (2008) for such a model). Virag (2008) considers a model of competing mechanisms with finite markets where the firms take their market power into account, but that paper assumes that firms are homogeneous.

3 The Case of Two Workers and Two Firms

We begin our analysis by examining the case where $n = m = 2$ and $x_1 > x_2$.[8] We find it fruitful to start with this case as it allows for a simple characterization of the subgame while preserving the strategic interaction among the agents. The general case is considered in section 4.

This section's results are the following.

Proposition 3.1 *When $n = m = 2$ and $x_1 > x_2$:*

1. *Equilibrium Characterization: A unique directed search equilibrium exists. The more productive firm posts a higher wage ($w_1 > w_2$).*

2. *Efficiency Properties: Constrained efficiency does not obtain in equilibrium. The low productivity firm hires too often and unemployment is too low from an efficiency viewpoint. The firms' market power is the source of the inefficiency.*

3. *Policy Implications: Introducing a binding minimum wage reduces welfare. There exists a (strictly positive) level of unemployment benefits that leads to the constrained efficient allocation.*

3.1 Equilibrium Characterization

We show that there exists a unique equilibrium and that the high productivity firm offers a higher wage. Even though this result is not new (or surprising), we think the proof is useful for the efficiency analysis of the following section.

The model is solved by backwards induction. The first step is to derive the equilibrium response of the two workers for an arbitrary wage announcement $\{w_1, w_2\}$. To facilitate exposition the two workers are named A and B. Suppose that worker B's strategy is to visit firm j with probability p_j^B. We proceed to derive the probability that worker A is hired conditional on applying to firm j, which we denote by $G(p_j^B)$. Worker B applies to firm k $(\neq j)$ with probability $1 - p_j^B$, in which case A is hired for sure; with probability p_j^B, B applies to firm j and A is hired with probability $1/2$. Therefore:

$$G(p_j^B) = (1 - p_j^B) + \frac{p_j^B}{2} = \frac{2 - p_j^B}{2}. \tag{1}$$

[8]The case where $x_1 = x_2$ and $n = m = 2$ is exhaustively analyzed by BSW.

The expected utility that worker A receives from applying to firm j is equal to $G(p_j^B)w_j = [(2 - p_j^B)\ w_j]/2$ and similarly for B. Finally, in a symmetric subgame we have $p_j^A = p_j^B = p_j$.

We define *market utility* to be the utility that workers expect to receive in the equilibrium of the subgame and denote it by $U(w_1, w_2) \equiv \max_j G(p_j)\ w_j$. When $w_j \geq 2w_k$, applying to firm j is a dominant strategy for the workers which leads to

$$p_j = 1,\ p_k = 0 \ \text{ and } \ U(w_1, w_2) = \frac{w_j}{2}. \tag{2}$$

When $w_j/w_k \in [1/2, 2]$, workers mix ($p_l > 0$ for $l = 1, 2$). Their strategies and market utility are given by:

$$\frac{(2 - p_j)\ w_j}{2} = \frac{(2 - p_k)\ w_k}{2} = U(w_1, w_2). \tag{3}$$

Equations (2) and (3) define the optimal response of workers $\{p_1(w_1, w_2), p_2(w_1, w_2)\}$ for arbitrary wages $\{w_1, w_2\}$. We shall show that only equation (3) is relevant for equilibrium.

We now turn to the firms' problem in the first stage of the hiring process. Let $H(p_j)$ denote the probability that firm j fills its vacancy when the workers' strategy is to apply to j with probability p_j. Firm j hires a worker unless it receives no applicants, which occurs with probability $(1 - p_j)^2$. Therefore, $H(p_j) = 1 - (1 - p_j)^2$.

Firm j takes as given the wage of firm k and the response of workers and maximizes

$$\Pi_j(w_j, w_k) \equiv (x_j - w_j)H(p_j(w_j, w_k)) \tag{4}$$

over $w_j \in [0, x_j]$. Note that firm j has no incentive to post a wage above $2w_k$ since that wage attracts both workers with probability 1. Therefore, in equilibrium $p_j(w_j, w_k)$ is determined by equation (3) alone.

Differentiating equation (4) with respect to w_j yields:

$$\frac{d\Pi_j(w_j, w_k)}{dw_j} = (x_j - w_j)H'(p_j(w_j, w_k))\frac{dp_j(w_j, w_k)}{dw_j} - H(p_j(w_j, w_k)) \tag{5}$$

The first term captures the marginal benefit of raising the wage, i.e. the increase in the hiring probability times productivity. The second term represents the cost of having to pay a higher wage to workers. We will use this expression extensively for the efficiency analysis.

It will prove convenient to optimize over p_j rather than w_j. Using equation (3) one can express w_j as a function of p_j and w_k which, recalling that $p_k = 1 - p_j$, leads to $w_j(p_j, w_k) =$

$(1 + p_j)\ w_k/(2 - p_j)$. With a bit of algebra, equation (4) can be rewritten as

$$\Pi_j[w_j(p_j, w_k), w_k] = x_j(1 - (1 - p_j)^2) - w_k p_j(1 + p_j), \tag{6}$$

and it is maximized over $p_j \in [0, 1]$. The first derivative is:

$$\frac{d\Pi_j[w_j(p_j, w_k), w_k]}{dp_j} = 2\ (1 - p_j)\ x_j - (1 + 2\ p_j)\ w_k. \tag{7}$$

We equate (7) to zero for both firms, solve for x_j and combine terms, using equation (3) to substitute out wages, to get:

$$\frac{x_1}{x_2} = \frac{1 - p_2}{1 - p_1} \frac{1 + 2\ p_1}{1 + 2\ p_2} \frac{2 - p_1}{2 - p_2} \tag{8}$$

Equation (8) implicitly characterizes the equilibrium. It is straightforward to show that there are unique $\{p_1^*(x_1, x_2), p_2^*(x_1, x_2)\}$ satisfying (8). Let $R(p_1)$ denote the right hand side of equation (8), where $p_2 = 1 - p_1$. Simple calculations show that $R(0) = 0$, $R'(p_1) > 0$ and $\lim_{p_1 \to 1}\ R(p_1) = \infty$ proving that there is a unique p_1^* (and p_2^*) satisfying (8) and hence the equilibrium exists and it is unique.

The equilibrium wages are given by using equation (7):

$$w_k^*(x_1, x_2) = \frac{2(1 - p_j^*(x_1, x_2))\ x_j}{1 + 2\ p_j^*(x_1, x_2)}. \tag{9}$$

The symmetry of (8) and $x_1 > x_2$ imply $p_1^* > 1/2 > p_2^*$ and hence $w_1^* > w_2^*$. Note that p_1^* is strictly interior regardless of x_1 and x_2. In other words, the high productivity firm finds it suboptimal to price its competitor out of the market.[9] Finally, recalling equation (1), the expected unemployment rate of the decentralized equilibrium is:

$$u^* = p_1^*\ (1 - p_1^* + \frac{p_1^*}{2}) + p_2^*\ (1 - p_2^* + \frac{p_2^*}{2}) = \frac{1}{2}\ [1 + 2\ p_1^* - 2\ (p_1^*)^2] \tag{10}$$

3.2 Efficiency Properties of Equilibrium

We now examine the efficiency properties of the equilibrium. We have two main results: first, efficiency does not obtain; second, the pattern of inefficiency is that the low productivity firm hires too often and unemployment is too low. We identify the market power enjoyed by firms in a finite market as the culprit for the inefficiency.

[9]This result is particular to the two firm case. In the general m-firm model additional assumptions on productivity are needed to guarantee that all firms attract applications. See Galenianos and Kircher (2009b).

Our benchmark for efficiency is the solution to the following problem: a social planner chooses the strategies of the agents to maximize output subject to the constraint that workers' strategies are symmetric.[10] The constraint means that the planner is subject to the same frictions as the agents, and we call the solution to his problem the constrained efficient benchmark. This is the standard notion of constrained efficiency in a decentralized matching process as in Shi (2001) or Shimer (2005).

The firms' strategies (wage-posting) are irrelevant for efficiency since they only affect the distribution of the surplus. Therefore, the planner chooses the workers' strategies to solve

$$\begin{aligned} \max_{p_1, p_2} \quad & x_1 \ H(p_1) + x_2 \ H(p_2) \\ \text{s.t.} \ \ p_1 + p_2 = 1 \quad & \text{and} \ \ p_j \geq 0, \ j = 1, 2. \end{aligned} \tag{11}$$

Let $\{p_1^P, p_2^P\}$ denote the solution of this problem.

The trade-off faced by the social planner is the following: by increasing p_1, he raises the average productivity of an employed worker because the high productivity firm is left idle less often; however, he also reduces the number of employed workers since they crowd each other out more often at the high productivity firm ($p_1 > 1/2$ is clearly a necessary condition for efficiency). We proceed to show that the decentralized equilibrium does not strike the welfare-maximizing balance between these two forces.

Setting the first derivative of (11) to zero yields

$$\frac{x_1}{x_2} = \frac{1 - p_2^P}{1 - p_1^P}, \tag{12}$$

Comparing equation (12) with equilibrium condition (8) it is clear that constrained efficiency does not obtain in equilibrium, except in the special case $x_1 = x_2$.

Furthermore, simple calculations show that the product of the second and third ratios on the right hand side of (8) is larger than 1 which implies that $(1-p_2^*)/(1-p_1^*) < (1-p_2^P)/(1-p_1^P)$, and therefore

$$p_2^P < p_2^* < \frac{1}{2} < p_1^* < p_1^P. \tag{13}$$

That is, in equilibrium workers apply to the less productive firm too often. As a result, the unemployment rate is *too low* from an efficiency viewpoint: equation (10) is minimized at $p_1 = 1/2$ which, together with equation (13), leads to $u^* < u^P$.

[10] In other words, the planner has a utilitarian welfare function.

Market power is the source of the inefficiency. Market power refers to the fact that an individual firm's action alters the workers' market utility. To clarify this point, use equation (3) to rewrite the probability that a worker applies to firm j as

$$p_j = p_j[U, w_j] = 2(1 - U/w_j), \tag{14}$$

where, of course, $U = U(w_1, w_2)$. A change in w_j affects $p_j[w_j, U]$ through two distinct channels. The directed search channel is that as a firm raises its wage, workers increase their probability of applying there. The market power channel is that a single firm's wage affects the workers' market utility and hence their strategies. Mathematically:

$$\frac{dp_j}{dw_j} = \frac{\partial p_j}{\partial w_j} + \frac{\partial p_j}{\partial U}\frac{\partial U}{\partial w_j} \tag{15}$$

It is straightforward to show that the constrained efficient allocation obtains if the market power channel is shut off (i.e. if $\partial U/\partial w_j = 0$). For instance, this is the case in Montgomery (1991) who considers a similar model but assumes that firms behave competitively in that they take market utility as fixed when deciding what wage to post. Furthermore, Peters (2000) shows that market power diminishes as the number of agents grows which is consistent with the findings of Moen (1997), Shi (2001) and Shimer (2005) that constrained efficiency obtains in large markets.

To see why market power leads low productivity firms to "overhire" we examine equation (15) in some more detail. First, note that $\partial p_j/\partial U < 0$: a better outside option makes workers more picky. Second, observe that in a finite market we have $\partial U/\partial w_j > 0$: as w_j increases, workers apply more often to firm j and, therefore, less often to firm k; this makes it easier to be hired at firm k, which leads to an increase in $G(p_k)w_k$ and hence in market utility. Note that when the number of agents becomes large this argument ceases to hold because the queues at other firms are affected infinitesimally as workers increase their probability of applying to j. Therefore, in a large market the expected utility of applying to some other firm k remains unchanged and p_j increases sufficiently to bring the payoffs of applying to j down to its previous level.

These observations imply that market power decreases the elasticity of hiring with respect to the wage by reducing the right-hand side of equation (15). As a result, firms face less competition and they post lower wages than they would if they did not take their market power into consideration, as in Montgomery (1991). The reason why the high productivity firm is affected by this feature to a larger amount has to do with the strict concavity of the hiring function $H(p_j)$. Recalling equation (5), it is easy to see that a unit decrease in dp_j/dw_j

has a smaller effect on the hiring probability of the high wage (and hence high productivity) firm. Therefore, the high productivity firm will respond to market power by decreasing its wage by a larger amount, which leads to the misallocation of workers.

The firms' market power therefore leads to a redistribution of surplus from workers to firms, which does not enter our welfare criterion, but also to a reduction in expected output due to the misallocation of workers across heterogeneous firms, which does. The source of the inefficiencies is different from the underutilization of labor suggested by more common frictionless models of monopsony (e.g. Bhaskar, Manning and To (2002)). The result that in equilibrium workers under-apply to the more productive firms and face lower unemployment than is optimal is similar in flavor to Acemoğlu and Shimer (1999). The driving force in that paper, however, is the workers' risk aversion, while in this paper it is the firms' market power. More importantly, the focus of the two papers is quite different: we focus on the interaction between policy and firms' pricing decisions while Acemoğlu and Shimer (1999) concentrate on how to counter the effects of workers' risk aversion.

3.3 Policy implications

The next step is to examine whether policy can improve on the decentralized allocation. We consider two policy interventions: a minimum wage and unemployment benefits. Our findings are that the introduction of a minimum wage exacerbates the misallocation of workers while an unemployment benefits scheme can achieve constrained efficiency. We provide a discussion of these results at the end of the section.

We first consider the minimum wage and show that a binding minimum wage results in the reductions of more productive firm's hiring probability. Fix the original economy $\{x_1, x_2\}$, label the equilibrium before the introduction of a (binding) minimum wage as *unconstrained* and denote the equilibrium wages and application probabilities by $\{w_1^*(x_1, x_2), w_2^*(x_1, x_2)\}$ and $\{p_1^*, p_2^*\}$. Introduce a minimum wage in the interval $\underline{w} \in (w_2^*(x_1, x_2), x_2)$,[11] and label the resulting equilibrium as *constrained* with associated wages and probabilities $\{w_1^C(x_1, x_2, \underline{w}), \underline{w}\}$ and $\{p_1^C, p_2^C\}$.[12]

The constrained equilibrium of economy $\{x_1, x_2\}$ features the same wages and probabilities as the unconstrained equilibrium of an alternative economy $\{x_1, \tilde{x}_2\}$ where $\underline{w}$ is the low productivity firm's profit maximizing wage. In other words, the alternative economy is such

[11] A minimum wage below $w_2^*(x_1, x_2)$ has no effect because it does not bind, while if $\underline{w} \geq x_2$ the low productivity firm is priced out of the market. It is clear from equation (12) that it is never efficient to leave one of the firms without applications. Depending on parameter values, closing firm 2 down could improve efficiency but this is not a policy that we will consider.

[12] Set $w_2 = \underline{w}$ in equation (7) and solve for w_1.

that $w_2^*(x_1, \tilde{x}_2) = \underline{w}$ and $w_1^*(x_1, \tilde{x}_2) = w_1^C(x_1, x_2, \underline{w})$. It is straightforward to see that $\tilde{x}_2 > x_2$ which implies that the low productivity firm of the alternative economy hires more often than its counterpart in the original economy. Therefore, introducing a minimum wage leads to low productivity firm to hire more often, pushing the economy further away from efficiency: $p_1^P > p_1^* > p_1^C$. Note, however, that the expected unemployment rate decreases as a result of the minimum wage: $u^C < u^* < u^P$.

We now consider an unemployment benefits scheme that gives b $(< x_2)$ to every worker who was unable to find a job and is financed by lump-sum taxation.[13] This scheme is simply a redistribution of resources and therefore it does not affect the efficient allocation. For the equilibrium analysis, we normalize all values by the unemployment benefits: let $\hat{x}_j = x_j - b$ and $\hat{w}_j = w_j - b$ be the productivity and wage, respectively, in excess of the workers' unemployment benefits (or, outside option). Treating $\hat{x}_j$ as the firms productivity and $\hat{w}$ as the wage, the equilibrium can be characterized in the same way as in section 3.1. Equation (8) becomes

$$\frac{x_1 - b}{x_2 - b} = \frac{1 - p_2(b)}{1 - p_1(b)} \, \frac{1 + 2\, p_1(b)}{1 + 2\, p_2(b)} \, \frac{2 - p_1(b)}{2 - p_2(b)} \tag{16}$$

Equation (16) defines the equilibrium worker strategies for given b, $p_1(b)$. The ratio $(x_1 - b)/(x_2 - b)$ is strictly increasing in b and $p_1(b)$ can achieve any value in $[p_1^*, 1)$ by varying b within $[0, x_2)$. $p_1(x_2) = 1$ is too high because it is inefficient to price the low productivity firm out of the market and $p_1(0)$ is too low, as was shown in section 3.2. Therefore, there is a unique $b^* \in (0, x_2)$ such that $p_1(b^*) = p_1^P$ and efficiency is restored.

The main lesson of section 3.2 is that the market power of firms leads to inefficiencies. This, of course, is not a new result. What is novel in our model is how the inefficiencies manifest themselves and the resulting implications with respect to two policies that can reduce the firms' market power. In contrast to frictionless models of monopsony where the inefficiencies are due to the underutilization of labor and where a (carefully chosen) minimum wage helps move towards efficiency, our model shows that the allocative inefficiencies are important and they are actually made worse by a minimum wage. A minimum wage constraints the low productivity firms to offer higher wages than they otherwise would and hence hire even more often than in the original, already inefficient, equilibrium. In some sense, the minimum wage

[13] Of course, in reality taxation is distortionary. Our goal is to examine whether unemployment benefits can improve the allocation in the most favorable environment possible: if the answer is negative, then a more realistic assessment of taxation is unnecessary; if, as already anticipated, the answer is positive, then one can consider different taxation schemes –which we leave for future research.

mostly affects the low productivity firms which are not principally responsible for the inefficiency. Therefore, even though the minimum wage results in a redistribution of surplus from firms to workers, it also reduces aggregate welfare. Introducing an appropriately measured unemployment benefits scheme can help overcome the inefficiencies. The reason is that unemployment benefits introduce a positive fallback option for the workers in case they do not get the job and this option is exercised with higher probability when a worker applies to the high productivity firm. Therefore, the workers are willing to take more "risk" which induces high productivity firms to offer higher wages.

It is worth reiterating that in other frictional models that exhibit inefficiencies, but where prices do not have an allocative role, the welfare effects of introducing a minimum wage are qualitatively similar to those of introducing unemployment benefits, unlike the results of our model. For instance, see Acemoğlu and Shimer (1999), Acemoğlu (2001) or Manning (2004).

4 The General Case

We now extend our results to the general case of arbitrary but finite numbers of workers and firms. We replicate the analysis of section 3. The existence and characterization of equilibrium is analyzed in Galenianos and Kircher (2009b) so section 4.1 simply describes the model. The subsequent sections generalize our inefficiency result and examine the effects of policy.

4.1 Equilibrium Characterization

In this section we describe the agents' maximization problem for the general case of n workers and m firms.

The strategy of worker i specifies the probability with which he applies to each firm after observing a particular announcement $\mathbf{w} = (w_1, w_2, ..., w_m)$. Let $p_j^i(\mathbf{w})$ denote the probability that worker i applies to firm j after observing $\mathbf{w}$. Since workers follow symmetric strategies, we have $p_j^i(\mathbf{w}) = p_j^l(\mathbf{w}) = p_j(\mathbf{w})$, for all $i,\ l \in N$. We denote the strategy of all workers with the vector $\mathbf{p}(\mathbf{w}) = (p_1(\mathbf{w}), ..., p_m(\mathbf{w}))$.

Consider a worker who applies to firm j. The probability that he is hired depends on the number of other workers that have applied for the same job. When there are exactly n_j other workers at firm j, our worker gets the job with probability $1/(n_j + 1)$. The number of *other* workers that visit firm j follows a binomial distribution with parameters $(p_j, n-1)$ when their strategy is to apply to firm j with probability p_j. It is straightforward to sum over the binomial coefficients and derive that the probability of being hired by firm j is given by

$$G(p_j) = \frac{1-(1-p_j)^n}{n\ p_j},$$

where $G(0) \equiv \lim_{p_j \to 0} G(p_j) = 1$.[14] Also, let $g(p_j) \equiv G'(p_j) = -[G(p_j) - (1-p_j)^{n-1}]/p_j$ for $p_j > 0$ and $g(0) \equiv \lim_{p_j \to 0} g(p_j) = -(n-1)/2$.

The worker's expected utility from applying to firm j is $G(p_j)\ v_j$. Utility maximization implies that the expected utility received by a worker is the same at all the firms where he applies, i.e. $G(p_j)w_j = G(p_k)w_k = U(\mathbf{w})$, whenever $p_j, p_k > 0$. Define $u_j(\mathbf{w}) \equiv g(p_j(\mathbf{w}))\ w_j$.

Firm j hires a worker unless it receives no applicants which occurs with probability $(1-p_j)^n$. Therefore, firm j hires with probability

$$H(p_j) = 1 - (1-p_j)^n.$$

Define $h(p_j) \equiv H'(p_j) = n\ (1-p_j)^{n-1}$.

Firm j takes as given the announcements of the other firms, $\mathbf{w}_{-j}$, and the response of workers in the subgame, $\mathbf{p}(\mathbf{w})$. Firm j maximizes its expected profits:

$$\Pi_j(w_j, \mathbf{w}_{-j}) = (x_j - w_j)H(p_j(\mathbf{w})). \tag{17}$$

Profits are uniquely determined given $\mathbf{w}$ since each announcement leads to a unique set of application probabilities in the subgame (Peters (1984)).

Galenianos and Kircher (2009b) prove the existence of an equilibrium in pure strategies by firms. Furthermore, under an additional condition it is shown that the equilibrium is characterized by the firms' first order conditions and that more productive firms post higher wages and firms with the same productivity post the same wage. The condition (Assumption 3 in that paper) guarantees that all firms attract applicants with positive probability ($p_j > 0$ for all j) and it is given by:

C1: For all $j \in M$ we have $p_j(\overline{\mathbf{w}}) > 0$ where $\overline{\mathbf{w}} = (x_1, ..., , x_m)$.

It is easy to show that Condition C1 holds as long as the maximum wages that firms are willing to offer are not too far apart, i.e. there exists parameter $\gamma < 1$ such that C1 holds if $\min_j x_j > \gamma \max_j x_j$. Note that Condition C1 is sufficient but not necessary for our results.

[14] See Burdett, Shi and Wright (2001) for a detailed derivation.

Consider the firms' problem. Under (C1), all firms attract applications and the solutions to the firms' problem are characterized by their first order conditions. The first order conditions of the firm's problem are

$$\frac{d\Pi_j}{dw_j} = -H(p_j) + h(p_j)\ [x_j - w_j]\ \frac{dp_j}{dw_j}. \tag{18}$$

Equating (18) to zero for all j and rearranging leads to:

$$\frac{x_j}{x_k} = \frac{h(p_k)}{h(p_j)}\ \frac{h(p_j)\ w_j + \frac{H(p_j)}{dp_j/dw_j}}{h(p_k)\ w_k + \frac{H(p_k)}{dp_k/dw_k}}, \tag{19}$$

which characterizes the equilibrium. Let $\{p_1^*, ..., p_m^*\}$ denote the equilibrium allocation.

4.2 Efficiency Properties of Equilibrium

We now generalize the results of section 3.2. We show that constrained efficiency does not obtain except for the special case of homogeneous firms and that in equilibrium the more productive firms hire less frequently than is efficient.

The planner's optimization problem is given by

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{j=1}^{m} H(p_j)\ x_j \\ \text{s.t.} \quad & \sum_{j=1}^{m} p_j = 1 \text{ and } p_j \geq 0\ \forall j \in M \end{aligned} \tag{20}$$

Let $\{p_1^P, ..., p_m^P\}$ denote the planner's constrained efficient allocation.

Equation (20) yields the following first order conditions:

$$\begin{aligned} h(p_j)\ x_j \ &\leq\ \lambda \\ &=\ \lambda \text{ if } p_j > 0, \quad \forall j \in M, \end{aligned}$$

where λ is the Lagrange multiplier. This condition requires that the shadow value in terms of expected output is equal across all firms that attract applications. Therefore, for any two firms j and k with $p_j, p_k > 0$ the following has to hold:

$$\frac{x_j}{x_k} = \frac{h(p_k^P)}{h(p_j^P)} = (\frac{1 - p_k^P}{1 - p_j^P})^{n-1}. \tag{21}$$

Comparing the efficiency requirement (21) with the equilibrium condition (19) reveals that efficiency is only achieved when the second ratio of (19) is equal to one for all pairs of firms. This can be easily verified to hold for the case when firms are homogeneous ($x_1 = ... = x_m$) and workers apply to each firm with identical probability (Galenianos and Kircher (2009b) establish that homogeneous firms post the same wage and therefore efficiency obtains in that case). However, we will show that when firms are heterogeneous, this ratio is different from one.

The proposition states our results regarding the efficiency properties of equilibrium. The proof is in the appendix.

Proposition 4.1 *Assume (C1) holds.*
(1) If $x_j \neq x_k$ for some $j, k \in M$, then constrained efficiency does not obtain in equilibrium. Furthermore, there exists an $r \in \{1, 2, ..., m\}$ such that $p_j^* < p_j^P$ for $j \in \{1, ..., r\}$ and $p_j^* > p_j^P$ for $j \in \{r+1, ..., m\}$.
(2) If $x_j = x_k$ for all $j, k \in M$, then constrained efficiency obtains in equilibrium.

Proof. See the appendix. ■

4.3 Policy Implications

We now generalize the policy implications of section 3.3. We first show analytically that the results of section 3.3 hold when firms have two productivity levels. We then provide some computational evidence that they extend to more general productivity distributions, although we have not been able to provide a proof.

Let $x_1 > x_2$ and suppose that m_1 and m_2 is the number of high and low productivity firms, respectively. Assume that condition (C1) holds. The characterization results in Galenianos and Kircher (2009b) guarantee that in the unconstrained equilibrium all high productivity firms post w_1^* and all low productivity firms post w_2^*.

We first consider the welfare effect of imposing a binding minimum wage:

Proposition 4.2 *Aggregate welfare is strictly higher in any unconstrained equilibrium $\{w_1^*, w_2^*\}$ than at an equilibrium with a minimum wage $\underline{w} \in (w_2^*, x_2)$.*

Proof. See the appendix. ■

In contrast, unemployment benefits can implement the efficient outcome.

Proposition 4.3 *There exist unemployment benefits $b^* > 0$, such that an equilibrium with these unemployment benefits is constrained efficient.*

Proof. See the appendix. ■

The following figures provide numerical evidence that the logic behind the above results holds in the general environment with a larger number of different productivity levels. In both figures there are five equidistant productivity levels ($x_1 = 3, x_2 = 2.5, ..., x_5 = 1$) with s firms at each level ($m_j = s$ for all j). In each case the number of worker is equal to the number of firms ($n = 5s$). The vertical axis denotes the percentage change in welfare due to a policy change as a proportion of the original inefficiency.

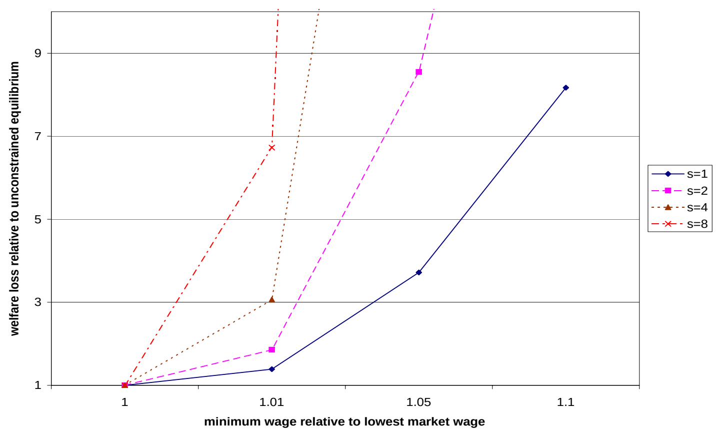

Figure 1: Illustration of the efficiency loss relative to the unconstrained equilibrium when a minimum wage is introduced.

In figure 1, the policy in question is the minimum wage which is denoted as a ratio over the lowest unconstrained equilibrium wage on the horizontal axis. Consistent with our previous results, it is clear that the efficiency loss increases when the minimum wage is introduced. Different specifications of productivity and s yield qualitatively similar graphs which leads us to believe that this is a more general result.

In figure 2, the level of the unemployment benefit is on the horizontal axis. The productivity levels and number of agents are the same as above. This figure shows two things: first, moderate unemployment benefits improve welfare; second, the optimal level of unemployment benefits decreases in the market size (s) which reflects the fact that the decentralized allocation approaches efficiency as the market becomes larger. Note that it is not always possible to fully achieve efficiency due to the interaction between various productivity levels that cannot be completely fine-tuned with a single policy instrument. However when unemployment

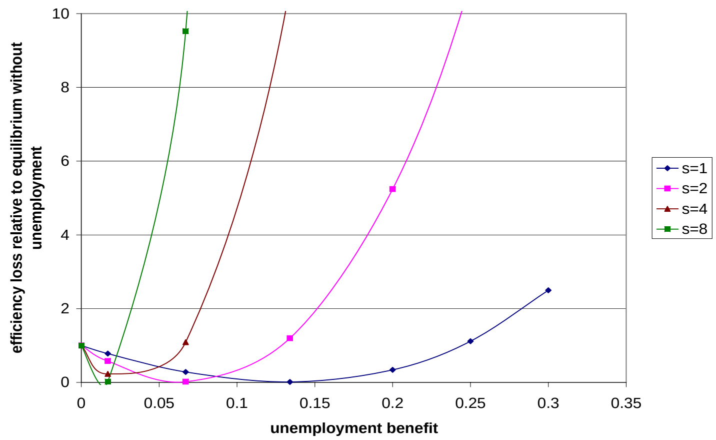

Figure 2: Illustration of the efficiency loss (efficiency gain if the numbers are smaller than 1) from the introduction of unemployment benefits.

benefits are chosen optimally the efficiency losses in our example are substantially reduced even under multiple productivity levels. Again, the features we present are representative of the numerical examples with different number of firms and productivity levels.

5 Conclusions

We develop a frictional model of the labor market with two main features: firms enjoy market power which leads to inefficiencies in the decentralized allocation; and wages play an important role in allocating labor. The nature of the inefficiency is that low-productivity firms hire too often and unemployment is too low from a welfare point of view. We show that unemployment benefits can increase welfare because they increase workers' willingness to risk unemployment and look for better employment options, thus, in effect, limiting firms' market power. In contrast, a minimum wage forces the low-productivity firms to increase their wages thereby exacerbating the inefficiencies. The employment results put the recent debate on minimum wages into perspective (e.g. Card and Krueger (1994)): while most papers have focused on the employment effects of a minimum wage, we show that the welfare implications of such policy can be more complicated. Even when it is desirable to transfer surplus towards workers (in our case because of the firms' market power), a minimum wage may have additional undesirable distortionary effects.

We highlight a particular channel through which inefficiencies may arise, namely the market power that firms enjoy in the context of a small market. Since many labor markets are fragmented across occupational and geographical lines, we think that this is a case worth studying. For instance, following Shimer (2007) and restricting a labor market to an occupation and geographical area combination leads to as few as 10 unemployed workers per labor market on average with a correspondingly small number of hiring firms.[15] While workers arguably search across some geographical borders and across some occupational boundaries, this calculation suggests that a strategic view of the hiring process might be relevant for the labor market experience of a significant proportion of workers.

The positive implications of the two policies that we consider do not depend on the finite nature of the market. We conjecture that models where there is some surplus to the employment relationship (e.g. due to frictions) and where wages play some role in allocating labor share our model's positive predictions: a (moderate) binding minimum wage constraints the low productivity firms, forcing them to offer better wages and hence leads to a reallocation of labor towards such firms. Unemployment benefits, on the other hand, may reallocate workers toward high-productivity firms, depending on the particulars of the model. The welfare effects of these policies depend on whether the original equilibrium is efficient or not. In the case of a large market with risk neutral agents (e.g. Shi (2001) or Shimer (2005)), constrained efficiency obtains in the decentralized equilibrium and policy can only reduce welfare. If one thinks that workers' risk aversion plays a significant role (as in Acemoğlu and Shimer (1999)) then we conjecture that our main normative insights carry over: in an extension of that model with heterogeneous firms, low productivity firms hire too often from an efficiency viewpoint, a minimum wage worsens the inefficiency and unemployment benefits can help. Of course, it may turn out that firm entry, from which we have abstracted, is important in the this context.

[15]Shimer (2007) proposes the combination of occupation and geographical unit as a labor market. With a total of 362 metropolitan and 560 micropolitan statistical areas (regions with at least one urbanized area of more than 50,000 inhabitants and 10,000 to 50,000 urban inhabitants, respectively) and about 800 occupations listed in the Occupational Employment Statistics (OES) and he obtains a total of about 740,000 combinations of occupations and geographic areas. For an unemployment level of 7.6 million in the Current Employment Statistics (CES) of December 2007 this yields on average 10.4 unemployed people per combination of occupation and geographical area.

6 Appendix

Proposition 4.1:

Proof. For simplicity, assume that no two firms have the same productivity level, though the complimentary case can be handled with minor modifications. Note that the characterization result from Galenianos and Kircher (2009b) implies that $p_1 > p_2 > ... > p_m > 0$.

We proceed to compare the probabilities implied by the solution to the planner's problem to the ones from the decentralized firms' problem. First, note that $p_j^P = 0$ is a possibility for some $j \in M$. In that case, it is straightforward to show that there exists some t such that $p_1^P > p_2^P > ..., p_t^P > 0$ and $p_{t+1}^P = ... = p_m^P = 0$, which means that the low productivity firms (below the tth) hire too often in the decentralized allocation, as the statement of the proposition suggests.

In what follows, we restrict attention to those firms that is efficient to attract applications in the constrained efficient allocation. The efficient probabilities are given by equation (21) which we compare to the probabilities from the decentralized allocation (19). We want to show that for all $j < k$

$$\frac{1 - p_k^P}{1 - p_j^P} > \frac{1 - p_k}{1 - p_j}, \tag{22}$$

which implies the claim of the proposition. We establish equation (22) for $j = 1$ and $k = 2$ but the proof is identical for other values of j and k.

Recalling that $U(\mathbf{w}) = G(p_i(\mathbf{w}))\ w_i$, for all $p_i > 0$, the problem of firm 1 is

$$\begin{aligned} &\max_{w_1}\ H(p_1(\mathbf{w}))\ (x_1 - w_1) \\ \text{s.t.}\ \ & G(p_1(\mathbf{w}))\ w_1 = G(p_2(\mathbf{w}))\ w_2 \end{aligned}$$

which is equivalent to: $\max_{w_1}\ [(1 - (1 - p_1)^n)\ x_1 - n\ p_1\ G(p_2)\ w_2]$ where the argument $\mathbf{w}$ has been omitted for brevity. Setting the first order condition of this problem to zero yields

$$(1 - p_1)^{n-1} x_1 \frac{\partial p_1}{\partial w_1} = w_2[g(p_2)\frac{\partial p_2}{\partial w_1} p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}].$$

Performing the same calculation for firm 2 and combining the results yields

$$\frac{w_1}{w_2} = (\frac{1 - p_2}{1 - p_1})^{n-1} \frac{x_2}{x_1} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2)\frac{\partial p_2}{\partial w_1} p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1)\frac{\partial p_1}{\partial w_2} p_2 + G(p_1)\frac{\partial p_2}{\partial w_2}]}.$$

Using the indifference condition of the buyers, $G(p_1)\ w_1 = G(p_2)\ w_2$, leads to

$$\frac{x_1}{x_2} = (\frac{1-p_2}{1-p_1})^{n-1}\frac{G(p_1)}{G(p_2)}\frac{\frac{\partial p_2}{\partial w_2}[g(p_2)\frac{\partial p_2}{\partial w_1}p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1)\frac{\partial p_1}{\partial w_2}p_2 + G(p_1)\frac{\partial p_2}{\partial w_2}]}.$$

If

$$\frac{G(p_1)}{G(p_2)}\frac{\frac{\partial p_2}{\partial w_2}[g(p_2)\frac{\partial p_2}{\partial w_1}p_1 + G(p_2)\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1)\frac{\partial p_1}{\partial w_2}p_2 + G(p_1)\frac{\partial p_2}{\partial w_2}]} > 1 \tag{23}$$

then equation (22) holds and we have our result. The rest of the proof establishes (23).

Equation (23) holds if and only if

$$\frac{p_1\ g(p_2)\ \frac{\partial p_2}{\partial w_1}/\frac{\partial p_1}{\partial w_1}}{p_2\ g(p_1)\ \frac{\partial p_1}{\partial w_2}/\frac{\partial p_2}{\partial w_2}} > \frac{G(p_2)}{G(p_1)}. \tag{24}$$

We want to characterize $\partial p_i/\partial w_l$. Note that $p_1+...+p_m = 1 \Rightarrow \partial p_1/\partial w_i+...+\partial p_m/\partial w_i = 0$. Let $\rho_i \equiv g(p_i)/G(p_i)$. We can differentiate the equality $G(p_1)\ w_1 - G(p_i)\ w_i = 0$ for $i > 2$ with respect to w_2 to get (where the equality was used again to substitute out the wages)

$$\frac{\partial p_i}{\partial w_2} = \frac{\partial p_1}{\partial w_2}\ \frac{\rho_1}{\rho_i}$$

Therefore,

$$\begin{aligned}\frac{\partial p_2}{\partial w_2} = -[\frac{\partial p_1}{\partial w_2} + \frac{\partial p_2}{\partial w_2} + ... + \frac{\partial p_m}{\partial w_2}] &= -\frac{\partial p_1}{\partial w_2}\ \rho_1\ [\frac{1}{\rho_1} + \frac{1}{\rho_3} + ... + \frac{1}{\rho_m}] \\ \Rightarrow \frac{\partial p_1}{\partial w_2}/\frac{\partial p_2}{\partial w_2} &= -\frac{\frac{1}{\rho_1}}{\frac{1}{\rho_1} + \frac{1}{\rho_3} + \frac{1}{\rho_4} + ... + \frac{1}{\rho_m}}.\end{aligned}$$

We can characterize $(\partial p_2/\partial w_1)/(\partial p_1/\partial w_1)$ in a similar way. Using these results, we can rewrite (24) as

$$\begin{aligned}&\frac{p_1\rho_1(1+2(1-p_2)+...+(n-1)(1-p_2)^{n-2})}{p_2\rho_2(1+2(1-p_1)+...+(n-1)(1-p_1)^{n-2})}\frac{\frac{1}{\rho_1}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}}{\frac{1}{\rho_2}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}} \\ &\qquad > \frac{1+(1-p_2)+...+(1-p_2)^{n-1}}{1+(1-p_1)+...+(1-p_1)^{n-1}}.\end{aligned} \tag{25}$$

The definition of ρ implies that

$$\frac{\rho_1}{\rho_2} = \frac{(1+2(1-p_1)+...+(n-1)(1-p_1)^{n-2})}{(1+2(1-p_2)+...+(n-1)(1-p_2)^{n-2})}\frac{1+(1-p_2)+...+(1-p_2)^{n-1}}{1+(1-p_1)+...+(1-p_1)^{n-1}}. \tag{26}$$

Case 1: $0 > \rho_1 > \rho_2$

In this case

$$\frac{\frac{1}{\rho_1}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}}{\frac{1}{\rho_2}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}} > 1$$

and thus (25) follows from

$$\frac{p_1\rho_1(1+2(1-p_2)+...+(n-1)(1-p_2)^{n-2})}{p_2\rho_2(1+2(1-p_1)+...+(n-1)(1-p_1)^{n-2})} > \frac{1+...+(1-p_2)^{n-1}}{1+...+(1-p_1)^{n-1}}.$$

However, using (26) this last inequality is equivalent to $p_1 > p_2$ which holds since $x_1 > x_2$.

Case 2: $0 > \rho_2 \geq \rho_1$

In this case it holds that

$$\frac{\rho_1}{\rho_2}\frac{\frac{1}{\rho_1}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}}{\frac{1}{\rho_2}+\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m}} = \frac{1+\rho_1(\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m})}{1+\rho_2(\frac{1}{\rho_3}+\frac{1}{\rho_4}+...+\frac{1}{\rho_m})} \geq 1,$$

because $\rho_i < 0$ for all i. Then (25) follows from

$$\frac{p_1(1+2(1-p_2)+...+(n-1)(1-p_2)^{n-2})}{p_2(1+2(1-p_1)+...+(n-1)(1-p_1)^{n-2})} > \frac{1+(1-p_2)+...+(1-p_2)^{n-1}}{1+(1-p_1)+...+(1-p_1)^{n-1}},$$

which can be rewritten as

$$\frac{1-(1-p_1)^n}{1+...+(n-1)(1-p_1)^{n-2}} > \frac{1-(1-p_2)^n}{1+...+(n-1)(1-p_2)^{n-2}}.$$

This inequality holds holds because $p_1 > p_2$. ■

Proposition 4.2:

Proof. Let p_i^* be the equilibrium application probability of workers to firm i when no minimum wage is introduced, and let $p_i(\underline{w})$ be the application probability when the minimum wage is introduced. Let $(w_1(\underline{w}), w_2(\underline{w}))$ refer to an equilibrium wage offer profile if a minimum wage requirement is introduced.

First, it is easy to show that

$$w_1(\underline{w}) \geq w_2(\underline{w}).$$

Moreover, $w_1(\underline{w}) = w_2(\underline{w})$ implies that $w_1(\underline{w}) = w_2(\underline{w}) = \underline{w}$, i.e. the minimum wage is so high that it is binding even for the high productivity firm. In this case obviously $p_1(\underline{w}) = p_2(\underline{w})$ holds and thus the equilibrium application levels are further from the constrained efficient allocation than without a minimum wage, since without the minimum wage at least $p_1 > p_2$ could be ensured.

Consider now the case where $w_1 > w_2 \geq \underline{w}$. In this case a high productivity firm chooses his wage offer such that his marginal profit is zero, since such a firm does not face a binding minimum wage. Let $w_1(p_1, w_2, \widehat{w}_1)$ denote the (unique) wage level that a high productivity firm needs to offer to obtain an application probability of p_1, if low productivity firms offer a wage of w_2 and the other high productivity firms offer a wage of $\widehat{w}_1$. It is easy to show that

$$w_1(p_1, \alpha w_2, \alpha \widehat{w}_1) = \alpha w_1(p_1, w_2, \widehat{w}_1).$$

Now, we show that if w_2 increases, then the high productivity firms obtain lower application probabilities in equilibrium, which would prove the result, since this means that increasing the minimum wage (and thus w_2) moves the allocation even further from the constrained efficient allocation. To prove this claim take any given p_1 and let $\widehat{w}_1$ be such that if all high productivity firms offer this wage, then each of them is visited with probability p_1. Let us denote this value as $w_1^*(p_1, w_2)$ and note that

$$w_1(p_1, w_2, w_1^*(p_1, w_2)) = w_1^*(p_1, w_2).$$

Suppose that a high productivity firm, firm i considers a deviation in his wage to change the application probability he receives. If he achieves an application probability of $\widetilde{p}_1$, then his profit can be written as

$$\Pi_1(\widetilde{p}_1, w_2, p_1) = (x_1 - w_1(\widetilde{p}_1, w_2, w_1^*(p_1, w_2)))H_1(\widetilde{p}_1).$$

Linearity of w_1^* in w_2 implies that

$$w_1^*(p_1, \alpha w_2) = \alpha w_1^*(p_1, w_2),$$

and thus linearity of function w_1 in w_2 and $\widehat{w}_1$ implies that

$$w_1(\widetilde{p}_1, \alpha w_2, w_1^*(p_1, \alpha w_2))) = \alpha w_1(\widetilde{p}_1, w_2, w_1^*(p_1, w_2))).$$

Now, let us study the marginal profit of firm i from increasing $\widetilde{p}_1$ when $\widetilde{p}_1 = p_1$ and the low productivity firms offer w_2. This marginal profit can be written as

$$\beta(w_2, p_1) = \frac{\partial(x_1 - w_1(\widetilde{p}_1, w_2, w_1^*(p_1, w_2)))H_1(\widetilde{p}_1)}{\partial \widetilde{p}_1} |_{\widetilde{p}_1 = p_1} =$$

$$= \frac{\partial(x_1 - w_2 w_1(\widetilde{p}_1, 1, w_1^*(p_1, 1)))H_1(\widetilde{p}_1)}{\partial \widetilde{p}_1} |_{\widetilde{p}_1 = p_1} .$$

It is immediate that

$$\frac{\partial \beta(w_2, p_1)}{\partial w_2} < 0$$

holds. Therefore, if $w_2 > w_2^U$ then $\beta(w_2, p_1^U) < 0$. Moreover, it holds that $\beta(w_2, 0) > 0$, since attracting an extra customer when no one is planning to visit is always profitable. By continuity of function β it follows that there exists a value $\overline{p}_1 \in (0, p_1^U)$, such that $\beta(w_2, \overline{p}_1) = 0$. By construction, if all other high productivity firms offer a wage of $w_1^*(\overline{p}_1, w_2)$ and low productivity firms offer w_2, then any given high productivity firm cannot gain by changing his wage offer by deviating from $w_1^*(\overline{p}_1, w_2)$ slightly. However, Galenianos and Kircher (2009b) show that under assumption C1 the profit function is concave in the own wage variable and thus offering $w_1^*(\overline{p}_1, w_2)$ is a best reply for all high productivity firms.

Therefore, for every wage level of the low productivity firms, such that $w_2 > w_2^U$, there is an equilibrium in the game between only the high productivity firms (i.e. taking w_2 as given) such that the workers' application probability to high productivity firms goes down if low productivity firms all offered wage w_2, because $\overline{p}_1 < p_1^U$.

The proof is completed by showing that for every w_2 the best response in the game of high productivity firms is unique. The tedious algebra for this result can be obtained from the authors upon request (or see the technical appendix on http://galenian.googlepages.com/research). ■

Proposition 4.3:

Proof. Let $p_j(b)$ denote the probability with which a worker applies to some firm of productivity x_j when the level of unemployment benefits is given by b. Assume without loss of generality that $x_1 > x_2$. Under (C1) both firms attract applications when $b = 0$. If $p_2^* = 0$, then setting $b = x_2$ implements the constrained efficient allocation. The rest of the proof considers the case where $p_2^P > 0$.

The strategy of the proof is to show that there is a b^* such that the firms' first order conditions coincide with the ones of the planner. Constrained efficiency is given by

$$\frac{x_1}{x_2} = (\frac{1 - p_2^P}{1 - p_1^P})^{n-1} > (\frac{1 - p_2^*}{1 - p_1^*})^{n-1},$$

where the inequality follows from the argument used in the proof of Proposition 4.1 (noting that in our notation $p_j^* = p_j(0)$).

As shown in Section 3, an unemployment benefit is mathematically equivalent to lowering the productivity of every firm by b leading to the following equilibrium condition for the decentralized economy:

$$\frac{x_1 - b}{x_2 - b} = (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} \frac{G(p_1(b))}{G(p_2(b))} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2(b))\frac{\partial p_2}{\partial w_1} p_1(b) + G(p_2(b))\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1(b))\frac{\partial p_1}{\partial w_2} p_2(b) + G(p_1(b))\frac{\partial p_2}{\partial w_2}]}$$

We use the intermediate value theorem to conclude the proof. Note that for b close enough to x_2

$$\frac{x_1 - b}{x_2 - b} > (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} \frac{G(p_1(b))}{G(p_2(b))} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2(b))\frac{\partial p_2}{\partial w_1} p_1(b) + G(p_2(b))\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1(b))\frac{\partial p_1}{\partial w_2} p_2(b) + G(p_1(b))\frac{\partial p_2}{\partial w_2}]}.$$

If $b = 0$, then

$$\frac{x_1 - b}{x_2 - b} = (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} < (\frac{1 - p_2(b)}{1 - p_1(b)})^{n-1} \frac{G(p_1(b))}{G(p_2(b))} \frac{\frac{\partial p_2}{\partial w_2}[g(p_2(b))\frac{\partial p_2}{\partial w_1} p_1(b) + G(p_2(b))\frac{\partial p_1}{\partial w_1}]}{\frac{\partial p_1}{\partial w_1}[g(p_1(b))\frac{\partial p_1}{\partial w_2} p_2(b) + G(p_1(b))\frac{\partial p_2}{\partial w_2}]}.$$

Therefore, an appropriate value of $b \in [0, x_2)$ works to replicate the constrained efficient allocation. ■

References

[1] Acemoğlu, Daron (2001): "Good Jobs versus Bad Jobs," *Journal of Labor Economics*, 19, pp. 1-22.

[2] Acemoğlu, Daron and Robert Shimer (1999): "Efficient Unemployment Insurance," *Journal of Political Economy*, 107, 893-928.

[3] Albrecht, James, Pieter A. Gautier, and Susan Vroman (2006): "Equilibrium Directed Search with Multiple Applications." *Review of Economic Studies*, 73, 869-891.

[4] Bhaskar, V., Alan Manning and Ted To (2002): "Oligopsony and Monopsonistic Competition in Labor Markets." *Journal of Economic Perspectives*, 16 (2), 155-174.

[5] Burdett, Kenneth, Shouyong Shi and Randall Wright (2001): "Pricing and Matching with Frictions," *Journal of Political Economy*, 109, 1060-1085.

[6] Card, David and Alan B. Krueger (1994): "Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania," *American Economic Review*, 84 (4), 772-793.

[7] Coles, Melvyn and Jan Eeckhout (2003a): "Indeterminacy and Directed Search," *Journal of Economic Theory*, 111, 265-276.

[8] Coles, Melvyn and Jan Eeckhout (2003b): "Heterogeneity as a Coordination Device,"mimeo.

[9] Galenianos, Manolis and Philipp Kircher (2007). "Heterogeneous Firms in a Finite Directed Search Economy." PIER working paper.

[10] Galenianos, Manolis and Philipp Kircher (2009a): "Directed Search with Multiple Job Applications." *Journal of Economic Theory*, 114(2), 445-471.

[11] Galenianos, Manolis and Philipp Kircher (2009b). "Game-Theoretic Foundations of Competitive Search Equilibrium," mimeo.

[12] Halko, Marja-Liisa, Klaus Kultti and Juha Virrankoski (2008): "Search Direction and Wage Dispersion," *International Economic Review*, 49, 111-134.

[13] Kaas, Leo and Paul Madden (2008): "Minimum wage and welfare in a Hotelling duopsony," *Economic Theory*, forthcoming.

[14] Kircher, Philipp (2008): "Efficiency of Simultaneous Search." mimeo.

[15] Manning, Alan (2004): "Monopsony and the Efficiency of Labour Market Interventions," *Labour Economics*, 11, 145-163.

[16] Marimon, Ramon and Fabrizio Zilibotti (1999): "Unemployment vs. Mismatch of Talents: Reconsidering Unemployment Benefits," *Economic Journal*, 109, 266-291.

[17] Moen, Espen R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105, 385-411.

[18] Montgomery, James D. (1991): "Equilibrium Wage Dispersion and Interindustry Wage Differentials," *Quarterly Journal of Economics*, 106, 163-179.

[19] Peters, Michael (1984): "Bertrand Competition with Capacity Constraints and Mobility Constraints," *Econometrica*, 52, 1117-1128.

[20] Peters, Michael (1991): "Ex Ante Price Offers in Matching Games: Non-Steady States." *Econometrica*, 59 1425-1454.

[21] Peters, Michael (2000): "Limits of Exact Equilibria for Capacity Constrained Sellers with Costly Search," *Journal of Economic Theory*, 95, 139-168.

[22] Shi, Shouyong (2001): "Frictional Assignment I: Efficiency," *Journal of Economic Theory*, 98, 232-260.

[23] Shimer, Robert (2005): "The Assignment of Workers to jobs in an Economy with Coordination Frictions," *Journal of Political Economy*, 113/5, 996-1025.

[24] Shimer, Robert (2007): "Mismatch," *American Economic Review*, 97/4, 1074-1101.

[25] Virag, Gabor (2008): "Collusive equilibria in directed search models," mimeo.