

# Household Time Allocation and Modes of Behavior: A Theory of Sorts<sup>1</sup>

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# 1 Introduction

Most analyses of household behavior conducted at the microeconomic level posit cooperative behavior by spouses (for an exception, see Chen and Woolley (2003)). In fact, Chiappori and his coauthors (e.g., Chiappori (1992), Chiappori and Browning (1998)) have argued that all such models should posit that household allocations are associated with welfare values on the Pareto frontier as an identifying assumption. Unfortunately, further sticky identification issues arise since the dependent variable, household allocations, is not uniquely determined. An additional function, the sharing rule, is added to the analysis by these authors in order to close the model.

McElroy (1990) and others have argued for the use of Nash bargaining instead of the sharing rule concept. The Nash bargaining formulation of the model requires the specification of an explicit value for each spouse's outside option, and still generates nonunique household allocations if we allow for the possibility of asymmetries in bargaining power. It shares this problem with the sharing rule approach.

From an econometric perspective, noncooperative equilibria are attractive since it is often straightforward to demonstrate existence and uniqueness given common specifications of spousal objectives, household production technologies, and constraint sets. Macroeconomists investigating intergenerational patterns of behavior in a household context (e.g., Aiyagari et al. (2000)) often invoke noncooperative behavior in solving their models.<sup>1</sup> Del Boca and Flinn (1995) assume noncooperative behavior between divorced parents, while Flinn (2000) allows them to choose whether to cooperate with the assistance of institutional agents. Del Boca and Flinn (2004) investigate the labor supply behavior of married couples in a model that makes the decision to cooperate endogenous, based on a comparison of the costs of coordinating and policing the cooperative equilibrium versus the efficiency gain from moving to the Pareto frontier.

In this paper we explore the issue of household behavior, and for simplicity focus on only two alternatives, Nash equilibrium (NE) and symmetric Nash bargaining (NB). We first show that after allowing for general forms of population heterogeneity in preferences, household productive ability, mar-

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<sup>1</sup>Greenwood et al (2003) utilize much of the Aiyarari et al (2000) framework but allow for Nash bargaining within the household. They make the important point that the impact of policy interventions on welfare will often crucially depend on household decision-making mechanisms. The point of the present paper is that it is empirically difficult to determine the mechanism that households are using given the nature of the data to which we typically have access.

ket productivity, and time endowments, it is not possible to distinguish between NE and NB on the basis of household time allocation decisions. To do so requires imposing homogeneity restrictions that may not be justifiable and are rarely tested.

Nevertheless, we show that the patterns of marital sorting observed in the data do contain information on the way in which household members interact. Assuming that spouses interact in some manner  $R$ , we use the observed time allocations, wages, and nonlabor incomes to “back out” the parameters characterizing all husbands and wives in the sample. We then apply the Gale and Shapley (1962) - henceforth GS - bilateral matching algorithm to determine the predicted equilibrium matches under  $R$ . We compare the correspondence between the predicted matches and the observed ones for  $R = NE$  and  $R = NB$ , and offer some (weak) evidence in favor of the hypothesis that husbands and wives interact in a noncooperative manner.

Household formation has been recognized to be important in explaining household allocation. Following the view of Becker (1991) that marriage is a partnership for joint production and consumption, several authors have analyzed aspects of the marriage market to explore marital behavior and the gains to marriage (e.g., Choo and Siow (2003), Dagsvik et al. (2001), Pollack (1990)). Other research has explored the effects of the marriage market household behavior. While Aiyagari, Greenwood and Guner (2000) have focused on the link between the marriage market and parental investments on children and intergenerational mobility, Fernandez et al. (2004) have studied the implication of marital sorting on household income inequality.

Micro analyses such as Browning et al. (2003), Seitz (1999), and Igiyun and Walsh (2004) have explored aspects of household formation that precede marriage to merge household models with marital sorting in order to explore the implications of spousal matching on household allocations. While the objective of these papers is mainly to identify sharing rules and consider efficiency implications for household allocations, we use marital sorting to investigate *what* type of interaction is most consistent with observed outcomes.

The plan of the paper is as follows. Section 2 contains the description of the model and the bilateral matching algorithm. In Section 3 we explore econometric issues, which are quite straightforward for the most part. Empirical results are presented in Section 4, and Section 5 contains a brief conclusion.

## 2 Model

A focus of our attention will be household formation. Without loss of *empirical* generality (as we shall see below), we will assume the following simple determination of household utility in a static context. We assume a Stone-Geary utility function for spouse  $i$  of the form

$$u_i(l_i, K) = \alpha_i \ln(l_i - \lambda_i) + (1 - \alpha_i) \ln(K - \underline{K}_i), \quad i = 1, 2,$$

where  $l_i$  is the leisure of spouse  $i$ ,  $\lambda_i$  is their leisure “subsistence level,”  $K$  is a public good that is produced within the household,  $\underline{K}_i$  is the subsistence level of the public good for spouse  $i$ , and  $\alpha_i$  is the preference weight attached to “discretionary” leisure. The household good  $K$  is produced according to a Cobb-Douglas technology

$$K = \tau_1^{\delta_1} \tau_2^{\delta_2} M,$$

where  $\tau_i$  is the time input of spouse  $i$  in household production,  $\delta_i$  is the elasticity of  $K$  with respect to time input  $\tau_i$ , and  $M$  is total income of the household, or

$$M = w_1 h_1 + w_2 h_2 + y_1 + y_2,$$

where  $w_i$  is the wage rate of spouse  $i$ ,  $h_i$  is their hours of work, and  $y_i$  is the nonlabor income of spouse  $i$ . We assume that each of the production elasticities  $\delta_1$  and  $\delta_2$  is strictly positive, so that there are increasing returns to household production. We have chosen not to impose constant returns to scale in this function for purposes of conducting the matching analysis conducted below. The “physical” time endowment of each spouse is  $T$ , and

$$T = l_i + h_i + \tau_i, \quad i = 1, 2.$$

It will be convenient to think of there being a “notional” time endowment specific to each individual in the population. This notational time endowment is equal to  $\tilde{T}_i \equiv T - \lambda_i$ , where  $\lambda_i$  can be positive, negative, or zero.

Each individual has their own value of market productivity, with the value of their time in the market given by  $w_i$ . Moreover, each individual has a nonlabor income level of  $y_i$ . Both of these quantities are determined outside of the model.

Within our framework, all households in the population share the same preference and household production structure. The population is, however, characterized by heterogeneity in all of the parameters that appear in the functions defined above. The population consists of two types of agents,

males (husbands) and females (wives). Each subpopulation is characterized by a distribution of characteristics particular to that type. The cumulative distribution function of characteristics of individuals of gender  $i$  is

$$G_i(\alpha_i, \delta_i, \tilde{T}_i, \underline{K}_i, w_i, y_i).$$

Then a household is defined by the vector of state variables

$$S = (\alpha_1, \delta_1, \tilde{T}_1, \underline{K}_1, w_1, y_1) \cup (\alpha_2, \delta_2, \tilde{T}_2, \underline{K}_2, w_2, y_2).$$

Given a value of  $S$ , the household determines equilibrium time allocations and the resultant welfare distribution in the household according to some rule  $R$ . Thus  $R$  is a mapping from  $S$  into a vector of observable household choices, in our case given by the vector

$$C = (h_1, h_2, \tau_1, \tau_2).$$

Thus

$$C = R(S). \tag{1}$$

We will discuss specific properties of the mapping  $R$  below, but for now we assume that  $R$  assigns an unique value  $E$  to any vector  $S \in \Omega_S$ , where we will think of  $\Omega_S$  as the parameter space of household characteristics.

## 2.1 Noncooperative Behavior

We begin our investigation of the time allocation decision of the household with the case of Nash equilibrium. Later we will turn our attention to cooperative models of household behavior.

To simplify the notational burden we assume

$$\underline{K}_1 = \underline{K}_2 = 0$$

for all members of the male and female populations.<sup>2</sup> For reasons to be made precise below, we will say that this assumption is without loss of (empirical) generality (*WLOG*). Then the reaction function for spouse 1 in a household characterized by  $S$  is given by

$$\begin{aligned} (h_1, \tau_1)^*(h_2, \tau_2; S) &= \arg \max_{h_1, \tau_1} \alpha_1 \ln(\tilde{T}_1 - h_1 - \tau_1) \\ &\quad + (1 - \alpha_1)[\delta_1 \ln \tau_1 + \delta_2 \ln \tau_2 + \ln(y + w_1 h_1 + w_2 h_2)]. \end{aligned}$$

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<sup>2</sup>Which implies that the marginal distribution of  $\underline{K}_i$  is degenerate. As a result, it is admitted when discussing the distribution function  $G_i$  in what follows.

Assuming an interior solution for  $h$ ,<sup>3</sup> the solutions are given by continuously differentiable functions

$$\begin{aligned} h_1^* &= h_1^*(h_2, \tau_2; S) \\ \tau_1^* &= \tau_1^*(h_2, \tau_2; S). \end{aligned}$$

An analogous pair of reaction functions exists for the second individual. Under our specification of preferences and the production technology, there exists a unique Nash equilibrium

$$\begin{aligned} h_1^{**} &= h_1^*(h_2^{**}; \tau_2^{**}; S) \\ \tau_1^{**} &= \tau_1^*(h_2^{**}, \tau_2^{**}; S) \\ h_2^{**} &= h_2^*(h_1^{**}, \tau_1^{**}; S) \\ \tau_2^{**} &= \tau_2^*(h_1^{**}, \tau_1^{**}; S). \end{aligned}$$

Insuring that  $h_1^{**}$  and  $h_2^{**}$  are both greater than zero requires restricting the parameter space  $\Omega_S$ . We will provide further discussion on this point More on this in the econometric section which follows.

Associated with the Nash equilibrium is a welfare pair  $(V_1^{NE}(S), V_2^{NE}(S))$ . These values will be used as outside options in the Nash Bargaining part of the analysis. After considering the marital sorting process, we will justify the use of these values as threat points.<sup>4</sup>

## 2.2 Nash Bargaining

We consider the case of symmetric Nash bargaining, once again, without any loss of (empirical) generality. Denote the outside options of the husband and wife by  $Q_1(S, Z_1)$  and  $Q_2(S, Z_2)$ , where  $Z_i$  represents environmental characteristics for individual  $i$  that influence the value of the alternative to behaving cooperatively within marriage  $S$ . Then the Nash bargained household time allocation is

$$\begin{aligned} &(h_1^{NB}, \tau_1^{NB}, h_2^{NB}, \tau_2^{NB})(S, Z_1, Z_2) \\ &= \arg \max_{h_1, \tau_1, h_2, \tau_2} (U_1(h_1, \tau_1, h_2, \tau_2; S) - Q_1(S, Z_1)) \times (U_2(h_1, \tau_1, h_2, \tau_2; S) - Q_2(S, Z_2)), \end{aligned}$$

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<sup>3</sup>Whenever  $\alpha_1 > 0$  and  $\delta_1 > 0$ , an interior solution for  $\tau_1$  is assured by the Inada condition.

<sup>4</sup>We will consider the case in which there are an equal number of males and females in the population. In the marriage equilibrium we define all agents will have the possibility of being married to an individual of the opposite sex. We find that the value of marriage exceeds the value of living alone for all population members in equilibrium, so the correct outside option will be the value of noncooperative marriage.

where  $U_i(h_1, \tau_1, h_2, \tau_2; S) = \alpha_i \ln(\tilde{T}_i - h_i - \tau_i) + (1 - \alpha_i)[\delta_1 \ln \tau_1 + \delta_2 \ln \tau_2 + \ln(y_1 + y_2 + w_1 h_1 + w_2 h_2)]$ ,  $i = 1, 2$ . Given our soon to be justified assumption that  $Q_i(S, Z_i) = V_i^{NE}(S)$ , we will dispense with the variables  $(Z_1, Z_2)$ , and write

$$\begin{aligned} & (h_1^{NB}, \tau_1^{NB}, h_2^{NB}, \tau_2^{NB})(S) \\ = & \arg \max_{h_1, \tau_1, h_2, \tau_2} (U_1(h_1, \tau_1; S) - V_1^{NE}(S)) \times (U_2(h_2, \tau_2; S) - V_2^{NE}(S)) \end{aligned}$$

We note that since we restrict the parameter space  $\Omega_S$  so as to produce noncooperative time allocations that are strictly positive, the choices made under Nash bargaining, with the noncooperative equilibrium values serving as outside options, ensures that the Nash bargaining choices of time allocations will be strictly positive as well.

### 2.3 Single Agent Welfare

Single agents must produce their own household goods and as a result receive no “subsidy” from a partner in terms of time contributions to production<sup>5</sup> or money contributions through earnings and nonlabor income. Then the production technology the single individual  $i$  faces is

$$K = \tau_i^{\delta_i} (y_i + w_i h_i), \quad (3)$$

where we have used the convention  $0^0 = 1$  in eliminating the missing spouse’s time contribution.<sup>6</sup> Then the single agent has a utility yield of

$$\begin{aligned} V_i^0(S_i) = & \max_{h_i, \tau_i} \alpha_i \ln(\tilde{T}_i - h_i - \tau_i) \\ & + (1 - \alpha_i)[\delta_i \ln \tau_i + \ln(y_i + w_i h_i)], \end{aligned}$$

where  $S_i \equiv (\alpha_i, \delta_i, \tilde{T}_i, w_i, y_i)$ .

### 2.4 Marital Sorting

The subpopulation distributions  $G_1$  and  $G_2$  are assumed to exogenously determined. The marriage model equilibrium which matches males and females

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<sup>5</sup>Our specification of household production and utility could lead to “negative” subsidies if the spouse provides less than 1 unit of time to household production. Income externalities could be zero but never negative.

<sup>6</sup>That is, the missing spouse has an associated  $\delta$  equal to 0 and supplies 0 amounts of time to household production.

produces an endogenous joint distribution of  $S$ , which we denote by  $H(S)$ , of which  $G_1$  and  $G_2$  are appropriately defined marginals.

We consider the case of a closed population in which there exists a total of  $2N$  individuals, equally divided between males and females. Male  $i$  is defined by his vector of characteristics

$$m_i = (\alpha_{1i}, \delta_{1i}, \tilde{T}_{1i}, w_{1i}, y_{1i}),$$

while female  $i$  is defined by her characteristics vector

$$f_i = (\alpha_{2i}, \delta_{2i}, \tilde{T}_{2i}, w_{2i}, y_{2i}).$$

Following GS, we consider the simple case in which there exists a marriage market in which individuals from the different subpopulations are matched one-to-one, all individual characteristics are perfectly observable, and the market clears instantaneously. Each male has preferences over possible mates, with the preference ordering of male  $m_i$  given by  $P(m_i)$ .

Similarly, the preference ordering of woman  $j$  is given by  $P(f_j)$ . In each case, the preference ordering amounts to a sequence of potential mates ranked in descending order, and may include ties. In addition, remaining single may dominate being married to certain individuals of the opposite sex. The value of this state we shall denote by  $f_0$  to a male (that is, the “null” female) and  $m_0$  if we are describing the preference ordering of a female. For example, with  $N = 5$ , we could have

$$P(m_4) = f_3, f_1, f_2, f_5, f_4.$$

That is, male 4’s first choice as a mate is female 3, followed by 1, 2, 5, and 4. The preferences of female 2 might be represented by

$$P(f_2) = m_4, m_1, m_3, m_0. \tag{4}$$

In this case, she prefers male 4 to male 1 to male 3, and would rather live alone than be married to either male 2 or male 5. As soon as we hit the “null” individual in the preference ordering, the ordering stops.

A marriage market is defined by  $(M, F; P)$ , where

$$P = \{P(m_1), \dots, P(m_N); P(f_1), \dots, P(f_N)\}$$

is the collection of preferences in the population,  $M = \{m_1, \dots, m_N\}$ , and  $F = \{f_1, \dots, f_N\}$ . Then we have the following:



**Definition 1** A matching  $\mu$  is a one-to-one correspondence from the set  $M \cup F$  onto itself of order 2 (that is  $\mu^2(x) = x$ ) such that  $\mu(m) \in F$  and  $\mu(f) \in M$ . We refer to  $\mu(x)$  as the mate of  $x$ .

The notation  $\mu^2(x) = x$  is read as  $\mu(\mu(x))$ , and just means that the mate of individual  $x$ 's mate is individual  $x$ .

**Definition 2** The matching  $\mu$  is individually rational if each agent is acceptable to his or her mate. That is, a matching is individually rational if it is not blocked by any (individual) agent.

This is a weak concept, particularly in our application, since matched individuals will almost invariably be better off than unmatched individuals no matter what the quality level of their mate. A stronger notion is one of stability. Say that a matching  $\mu$  has resulted in  $\mu(m_i) = f_j$  and  $\mu(f_k) = m_l$ , but that male  $i$  strictly prefers  $f_k$  to  $f_j$  and female  $k$  strictly prefers  $m_i$  to  $m_l$ . Then the pair  $(m_i, f_k)$  can deviate from the matching assignment  $\mu$  and improve their welfare. Such a match is unstable in the terminology of Gale and Shapley (1962).

**Definition 3** A matching  $\mu$  is stable if it is not blocked by any individual or any pair of agents.

The main achievement of GS was to set out an algorithm for finding an equilibrium of the marriage game that was decentralized and constructive in the sense of establishing that at least one stable matching equilibrium exists. They assumed that preferences of agents were public information and a convention regarding the meeting and offering technology. Roth and Sotomayer (1990) devote considerable attention to the design of mechanisms that elicit truthful revelation of preference orderings when preferences are not public information, and also explore alternative meeting and proposal technologies. These important issues will be of less importance to us here given the nature of the application and the econometric and empirical focus of our analysis.

In our application a male individual  $i$  is characterized by the vector  $m_i = (\alpha_{1i}, \delta_{1i}, \tilde{T}_{1i}, w_{1i}, y_{1i})$ . His induced preference ordering over the females  $f_1, \dots, f_N$  is determined by  $R$  in the following manner. If  $m_i$  and  $f_j$  are matched, then the household is characterized by

$$S_{i,j} = m_i \cup f_j. \tag{5}$$

Then equilibrium time allocations in the household are given by

$$C_{ij}(R) = R(S_{ij}). \quad (6)$$

Given our assumptions regarding the form of the “payoff” functions to  $i$  and  $j$ , we can define the value to  $m_i$  of being matched with  $f_j$  under  $R$  as

$$\begin{aligned} V_i(j; R) &= \alpha_{1i} \ln(l_1^*(S_{ij}; R)) + (1 - \alpha_{1i}) \ln(\tau_1^*(S_{ij}; R)^{\delta_{1i}} \tau_2^*(S_{ij}; R)^{\delta_{2j}}) \\ &\quad \times (w_{1i} h_1^*(S_{ij}; R) + w_{2j} h_2^*(S_{ij}; R) + y_{1i} + y_{2j}). \end{aligned}$$

Then given behavioral mode  $R$ , the preference ordering of  $i$  is given by

$$P(m_i|R) = f_{[1]}^i(R), f_{[2]}^i(R), \dots, f_{[N]}^i(R),$$

where

$$V_i(f_{[1]}^i(R); R) > V_i(f_{[2]}^i(R); R) > \dots > V_i(f_{[N]}^i(R); R).$$

Given knowledge of  $m_i$ ,  $f_j$ , and  $R$ , the preference ordering of all population members is determined. This implies the following.

**Definition 4** *A marriage market is defined by  $(M, F; R)$ .*

An equilibrium assignment is a function of marriage market characteristics. Then the set of stable matchings is determined by the characteristics vectors  $M$  and  $F$  and the behavioral model  $R$ , or  $\Theta(M, F; R)$ . Now there may exist, and generally do exist, multiple stable assignment equilibria. Among this set of equilibria, attention has focused on the two “extreme” stable matchings, the one that is most beneficial to men and the one most beneficial to women.<sup>7</sup> The GS matching algorithm, which they termed “deferred acceptance,” enables one to determine at least these two, of the many possible, equilibria in a straightforward manner. We describe the computation of the male-preferred equilibrium. In a given round,

1. Each male not tentatively matched with a female makes a marriage proposal to the woman he most prefers among the set of women who have not rejected a previous proposal of his. If he prefers the state of being single to any of the women in his choice set, he makes no offer.
2. Each woman (tentatively) accepts the proposal that yields the maximum payoff to her from the set of offers made to her during the round

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<sup>7</sup>When there is a unique equilibrium, these stable matchings are identical, of course.

plus the value of the match with the offer she carries over from the previous round (she may reject one or more proposals because the option of remaining single dominates them). Any man whose offer is refused in the period cannot make another marriage proposal to the woman rejecting him in future rounds.

3. The process is repeated until no man makes a marriage proposal to any woman.

The female preferred stable matching equilibrium is found in the identical way after reversing the roles of two sexes as proposers and responders.

There may well exist other stable matchings besides these two. Given the generality of the preference structure, the size of the individual characteristic space, and the number of individuals in the marriage market in our empirical analysis (877), it is not possible to attempt to enumerate all possible equilibria. We have computed the predicted marriage assignments using estimates of the state vectors  $m_i$  and  $s_j$  under both Nash equilibrium and We found that the same pairs were matched in over 96 percent of the case in the male preferred and female preferred equilibria. As a result, we use pairings from the male preferred equilibria only in all of the empirical work that follows. The reader should bear in mind that other equilibria exist, even if they are not so different in metrics of concern to us in this exercise.

### 3 Econometrics

We consider estimation of the marriage market equilibrium in sequence. We begin with the issue of the estimation of  $(M, F)$ , the distribution of gender types. In this paper we do not treat the difficult censoring issues that arise when not all household members supply time to the labor market. Furthermore, under our model, and for the most part in the data, both household members supply positive amounts of time to household production. In this case we are able to posit that the entire vector

$$A_k = (h_{1k}, h_{2k}, \tau_{1k}, \tau_{2k}, w_{1k}, w_{2k}, y_{1k}, y_{2k}), \quad k = 1, \dots, N,$$

is observable by the analyst, where we have constructed the male and female indexing so that in the data male  $i$  is married to female  $i$ ,  $i = 1, \dots, N$ . It will be useful to partition this vector into two subvectors,

$$\begin{aligned} A_k^1 &= (h_{1k}, h_{2k}, \tau_{1k}, \tau_{2k}), \\ A_k^2 &= (w_{1k}, w_{2k}, y_{1k}, y_{2k}), \end{aligned}$$

with  $A_k^1$  representing the (endogenous) time allocations of the household and  $A_k^2$  the state As it now stands, each spouse is characterized by the unobserved characteristics  $(\alpha_{si}, \tilde{T}_{si}, \delta_{si})$ , since we have already normalized  $\underline{K}_{si} = 0, \forall (s, i)$ . As will become apparent soon, we will require further restrictions on the variability in the unobservable characteristics if we are to be able to nonparametrically identify the model. For the moment, we will treat  $\alpha_{si}$  as having no variation within the population of males and females (individually), so that

$$\begin{aligned}\alpha_{1i} &= \alpha_1, \\ \alpha_{2i} &= \alpha_2, \quad i = 1, \dots, N.\end{aligned}$$

Assume that the values  $\alpha_1$  and  $\alpha_2$  are known, for now. We will denote the remaining unobserved household characteristics by

$$A_k^3 = (\tilde{T}_{1k}, \tilde{T}_{2k}, \delta_{1k}, \delta_{2k}).$$

The data used in the empirical work discussed below are drawn from the Panel Study of Income Dynamics (PSID). In keeping with the static setting of the model, we use data pertaining to household characteristics and time allocation decisions in one year, 2000. We chose this year because information on the time spent in household tasks is widely available for both spouses in this year.

We assume that the PSID is randomly drawn from the population distribution of married households in this year (which is an unlikely situation, admittedly). We think of there being a continuum of men and women in the population, with each male matched to a female. The characteristic vectors defining males and females,  $m$  and  $f$ , both have absolutely continuous distributions in the population, so that associated with the respective distribution functions  $G_1$  and  $G_2$  there exist probability density functions  $g_1$  and  $g_2$ . Since we have a random sample of households, we also have a random sample of household members *given* the marriage assignment rule.

Using a random sample of  $N$  households, the first task is to estimate the distribution functions  $G_1$  and  $G_2$ . For household  $k$ , we can restate (1) as

$$A_k^1 = R(A_k^2 \cup A_k^3).$$

**Proposition 5** *Assume all households in the population behave according to  $R$ , and that  $R$  is invertible in the sense that there is a unique value of  $A_k^3$  such that*

$$A_k^3 = R^{-1}(A_k^1 \cup A_k^2) \tag{7}$$

for all values of  $A_k^1 \cup A_k^2$ . Then the distributions  $G_1$  and  $G_2$  are nonparametrically identified and can be consistently estimated.

*Proof:* Given knowledge and invertibility of  $R$ , then  $R^{-1}$  is a known function. If  $A_k^1$  and  $A_k^2$  are observed without error, then the vector  $A_k^3$  is observable as well. Since the vectors  $A_k^1$  and  $A_k^2$  are observed for a random sample of households, then  $A_k^3$  is as well. Define the vectors

$$\begin{aligned} X_k &= (A_k^3, w_{1k}, w_{2k}, y_{1k}, y_{2k}), \\ X_k^1 &= (\tilde{T}_{1k}, \delta_{1k}, w_{1k}, y_{1k}), \\ X_k^2 &= (\tilde{T}_{2k}, \delta_{2k}, w_{2k}, y_{2k}). \end{aligned}$$

The vector  $X_k^1$  is an i.i.d. draw from  $G_1$  and  $X_k^2$  is an i.i.d. draw from  $G_2$ . Then define

$$\begin{aligned} \hat{G}_1^N(x) &= N^{-1} \sum_{k=1}^N \chi(X_k^1 \leq x), \\ \hat{G}_2^N(x) &= N^{-1} \sum_{k=1}^N \chi(X_k^2 \leq x). \end{aligned}$$

Since  $\{X_1^1, \dots, X_N^1\}$  and  $\{X_1^2, \dots, X_N^2\}$  are both random samples from their respective populations, we know that

$$\text{plim}_{N \rightarrow \infty} \hat{G}_i^N(x) = G_i(x), \quad i = 1, 2,$$

at all points of continuity of  $G_1$  and  $G_2$ . Since we have assume that the population distributions are absolutely continuous, this means that the distributions can be consistently estimated everywhere on their support. ■

The following important implication immediately follows.

**Proposition 6** *Let  $\mathfrak{R}$  be the set of equilibrium rules that determine time allocations in the household that are invertible in the sense of (7). Then all  $R \in \mathfrak{R}$  are equivalent descriptions of sample information.*

*Proof:* Consider a household  $k$  in the sample. We observe four household choices  $D_k^1 = (h_{1k}, h_{2k}, \tau_{1k}, \tau_{2k})$  and we have four unobservable characteristics of the spouses. Thus given any  $D_k^2 = (w_{1k}, w_{2k}, Y_k)$  and any  $R \in \mathfrak{R}$ , there exists a unique vector of characteristics  $(\tilde{T}_{1k}, \tilde{T}_{2k}, \delta_1, \delta_2)$  that generate  $D_k^1$ , or

$$D_k^1 = \Gamma(\tilde{T}_{1k}(R), \tilde{T}_{2k}(R), \delta_{1k}(R), \delta_{2k}(R) | D_k^2, R).$$

Then for any two  $R, R' \in \mathfrak{R}$ ,  $R \neq R'$ ,

$$\begin{aligned} & \Gamma(\tilde{T}_{1k}(R), \tilde{T}_{2k}(R), \delta_{1k}(R), \delta_{2k}(R) | D_k^2, R) \\ &= \Gamma(\tilde{T}_{1k}(R'), \tilde{T}_{2k}(R'), \delta_{1k}(R'), \delta_{2k}(R') | D_k^2, R'), \end{aligned}$$

which describes a correspondence between  $(\tilde{T}_{1k}, \tilde{T}_{2k}, \delta_{1k}, \delta_{2k})(R)$  and  $(\tilde{T}_{1k}, \tilde{T}_{2k}, \delta_{1k}, \delta_{2k})(R')$ .

Consider any distance function

$$\mathbb{Q}(D_k^1, \hat{D}_k^1(\tilde{T}_{1k}, \tilde{T}_{2k}, \delta_{1k}, \delta_{2k} | D_k^2, R)),$$

where  $\hat{D}_k^1$  is the predicted value of the household time allocations given the characteristics  $(\tilde{T}_{1k}, \tilde{T}_{2k}, \delta_{1k}, \delta_{2k})$ ,  $D_k^2$ , and  $R$ . But given invertibility

$$\begin{aligned} & (\tilde{T}_{1k}(R), \tilde{T}_{2k}(R), \delta_{1k}(R), \delta_{2k}(R) | D_k^2, R) \\ &= \arg \min \mathbb{Q}(D_k^1, \hat{D}_k^1(\tilde{T}_{1k}, \tilde{T}_{2k}, \delta_{1k}, \delta_{2k} | D_k^2, R)) \end{aligned}$$

and

$$\begin{aligned} \mathbb{Q}(D_k^1, \hat{D}_k^1((\tilde{T}_{1k}(R), \tilde{T}_{2k}(R), \delta_{1k}(R), \delta_{2k}(R) | D_k^2, R) | D_k^2, R)) &= 0, \\ \forall R \in \mathfrak{R} \end{aligned}$$

■

Because of the flexible parameterization of spouses in terms of their types, if  $\mathfrak{R}$  contains more than one element there are multiple ways to “reparameterize” the data, in essence. The cardinality of  $\mathfrak{R}$  depends on assumptions made regarding the functional form of the utility and household production functions and the features of the data. Since the proof is not especially instructive, we simply state the following.

**Proposition 7** *For Stone-Geary utility functions and the Cobb-Douglas home good production technology and for a population in which both household members supply time to the market, the Nash equilibrium and the symmetric Nash bargaining behavioral rules both belong to  $\mathfrak{R}$ .*

This proposition carries the important implication that it is not possible to determine whether household members (in the general population) operate under Nash equilibrium or Nash bargaining rules of behavior by observing only within household behavior. This “impossibility” result mainly results from the flexible specification of population heterogeneity. Clearly, by restricting the variability of these underlying parameters in the population, it will generally be possible to develop tests pitting the two forms of behavior against one another, but the outcome of such a test will be heavily dependent upon the parameter restrictions adopted.

### 3.1 Marital Sorting

Flexible specifications of population heterogeneity reduce the analyst’s ability to derive different empirical implications from various behavioral modes of behavior. However, they do provide possibilities for developing tests based on marital sorting patterns. We explore the construction of such tests in this subsection.

We have assumed that our PSID sample of married individuals is drawn from an indefinitely large population of married couples. Given marriage equilibrium we are using, and side-stepping uniqueness issues, we know that households in our  $N$  household sample consist of husbands and wives who were matched under the G-S deferred acceptance algorithm in the marriage market defined over all population members. Then we have the following result.

**Proposition 8** *Let  $\Theta_{MP}(M, F; R)$  denote the male-preferred equilibrium in the population. Define a random sample of  $N$  households matched in this equilibrium by  $M^N$  and  $F^N$ . Then the set of male-preferred stable matchings in the random sample matches male  $i$  with female  $i$ ,  $i = 1, \dots, N$ .*

The import of this is that adding households to the sample does not change the matches predicted under the model given the behavioral rule  $R$ . This enables us to invoke standard asymptotic properties.

The key to using marital sorting to test the nature of intrahousehold behavior is the dependence of the preference orderings  $P(m_i)$  and  $P(f_j)$  on  $R$ . Under certain configurations of male and female characteristics  $M$  and  $F$ , changes in  $R$  produce different sorts in the male-preferred stable matching equilibrium. Our focus in this application is on the comparison between the matches predicted under Nash equilibrium and symmetric Nash bargaining, only. Our empirical strategy is as follows.

1. Under rule  $R$ , determine the vector  $\tilde{T}_{si}(R), \delta_{si}(R), s = 1, 2; i = 1, \dots, N$ . Then form the characteristics vectors

$$(\tilde{T}_{si}(R), \delta_{si}(R), w_{si}, Y_{si}), s = 1, 2; i = 1, \dots, N. \quad (8)$$

2. Form the preference orderings of each male and female in the PSID sample using the characteristics vectors generated under  $R$  and under the assumption that all households behave according to  $R$ .
3. Apply the male-preferred deferred acceptance algorithm of GS using these preference orderings, and let the equilibrium matches be given by  $\Theta_{MP}(\hat{M}(R), \hat{F}(R); R)$ .

We complete steps (1)-(3) for  $R = NE$  and  $R = NB$ . The idea of the exercise is to determine which behavioral concept is most consistent with observed marital sorts. It is reasonably obvious that our informal test relies on a number of extremely stringent assumptions. Perhaps the most critical is that the GS algorithm leads to predicted sorting patterns that are consistent with observed matches. We will see below that the observed sorting patterns are not well predicted by our competing behavioral models, though from this we cannot say with certainty that there is not some set of  $R_S \subset \mathfrak{R}$  that is. In fact, if it were possible to analytically describe the set  $\mathfrak{R}$ , an interesting abstract estimation exercise is to find an  $\hat{R}$  such that

$$\hat{R} = \arg \min_{R \in \mathfrak{R}} \mathbb{Q}(\Theta_0, \Theta_{MP}(\hat{M}(R), \hat{F}(R); R)), \quad (9)$$

where  $\mathbb{Q}$  is some distance function. This ambitious task is well beyond the goals of the current exercise. We simply want to stress the fact that there may exist an  $\hat{R}$  such that the fit between predicted and observed sorts is close. The static and highly structured nature of the sorting algorithm does not in itself rule out the possibility of it being a good predictor. It will turn out in this exercise that neither  $R = NE$  or  $NB$  leads to particularly good predictions.

Other critical assumptions are that the observed conditioning variables and time allocation choices observed in the PSID are measured without error. If there is error in these measures, then our rankings are erroneous as well, leading to poor predictions of equilibrium matches. We incorporate measurement error in wages into the model in the following subsection, and this element of randomness implies stochastic marital sorts.<sup>8</sup> This randomness allows us to compute the relative probability that the observed marital sorts are generated by  $NE$  as opposed to  $NB$ .

In empirical analyses such as these, objections are often raised as to the sensitivity of results to functional form assumptions. Our assumption of Stone-Geary utility functions and a Cobb-Douglas home good production technology were heavily exploited in backing out the parameters characterizing males and females in the marriage market. The “estimates” we obtain are, in fact, only interpretable under these assumptions. While our within-household analysis has stressed the point that all modes of behavior

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<sup>8</sup>One might say that even without measurement error the sorting equilibrium is nonunique in the sense there can exist multiple equilibria. This situation is not properly thought of as stochastic, however, so that randomness in some measured or inferred state variables is required to produce randomness in marital sorts, conditionally or unconditionally on the existence of multiple equilibria at any realization of the state variables for the marriage market.



$R \in \mathfrak{R}$  are equally perfect descriptions of the time allocation data under our functional form assumptions, similar results could be derived for alternative specifications of individual utilities and household production technologies. This source of arbitrariness should be borne in mind when interpreting our results, though one should remember that all econometric formulations are subject to this same fundamental nonuniqueness issue.

Perhaps the biggest limitation of the current setup is its static nature. Cohabitation, marriage, separation, and divorce decisions take place in a dynamic setting, one that is perhaps best modeled allowing for search frictions, imperfect information, and complicated institutional constraints (for one such framework, see Brien et al. (2004)). The same criticism applies to our static labor supply model, though perhaps a bit less forcefully.<sup>9</sup> Properly incorporating marriage market and labor supply dynamics into this analysis adds an order of magnitude of complexity and requires introducing other sets of arbitrary assumptions when specifying the dynamic environment. That is not to imply that such a task would not be worthwhile, merely that it is beyond the scope of this paper.

### 3.2 Choosing Between Alternative $R$

We look at the ability of either  $R$  to predict in-sample matches using two methods. The first is purely descriptive, and involves computing the rank order correlation between the predicted marriage partners under the behavioral rules and the actual marriage partners. Since the model does not contain any random elements, if we restrict our attention to the  $NE$  and  $NB$  rules, one of them should fit perfectly and neither does. Now the setup we have developed may still be able to produce a perfect correspondence between the observed and observed matches if there exists an  $\hat{R} \in \mathfrak{R}$  in equation (9) such that  $\mathbb{Q}(\Theta_0, \Theta_{MP}(\hat{M}(\hat{R}), \hat{F}(\hat{R}); \hat{R})) = 0$ . Since it seems difficult to constructively characterize the set  $\mathfrak{R}$ , this does not appear to be a promising direction to follow.

Instead, we consider the likely event that the data contains measurement error. While there is undoubtedly measurement error in all of the information available to us from the PSID, for simplicity we will only consider measurement error in the wages of the spouses. We assume “classical”

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<sup>9</sup>It is reasonable to criticize the basic neoclassical labor supply framework as untenable, of course, even when formulated as a dynamic model. Dey and Flinn (2005) and Garcia-Perez and Rendon (2004) have attempted to jettison this framework altogether by looking at household employment decisions using a joint spousal search model.

measurement error in the logarithm of wages, or

$$\ln w_s^* = \ln w_s + \varepsilon_s, \quad s = 1, 2,$$

where  $w_s^*$  is the true wage of spouse  $s$ ,  $w_s$  is the reported wage, and  $\varepsilon_s$  is independently and identically distributed across households and spouses. In order to generate “true” wages based on the observed wage rates, it is necessary for us to make a functional form assumption regarding the distribution of  $\varepsilon_s$ , and, as is common, we assume normality. One of the principle reasons we have chosen to add measurement error in wages is the availability of high quality estimates of the measurement error variance in the logarithm of wages in the PSID. Using a special validation survey performed in the 1980s that involved administering the standard PSID survey instrument to a group of workers at a large factory in the Detroit area, Bound et al (1994) were able to get reasonably precise estimates of measurement error in wage reports by comparing subject responses with payroll records. In line with estimates of the variance of  $\varepsilon_s$  they obtained (see their Table 3), we set  $\sigma_s^2 = .13$  for both husbands and wives. We denote the distribution of measurement errors for spouse  $i$ 's wages by  $B_i$ . Note that since we are working in wage levels, we have

$$\begin{aligned} w_s^* &= \exp(\ln w_s + \varepsilon_s) \\ &= w_s \exp(\varepsilon_s). \end{aligned}$$

Since  $\varepsilon_s$  is distributed as a mean 0 normal with variance 0.13, the measurement error in wages has a lognormal distribution with mean 1.067 and variance 0.158.

Given an  $N$  married household sample, there exist  $2N$  measurement errors associated with all of the measured wage rates. Given independence of these shocks across households as well as across spouses, it is conceptually straightforward to express the probability that a given observed pattern of sorts was generated under any of our alternative behavioral models  $R$ . To simplify notation, let

$$\Theta_{MP}(M, F, R|\varepsilon) \tag{10}$$

denote the marital sorting pattern given measured characteristics  $M$  and  $F$ , behavioral rule  $R$ , and measurement errors  $\varepsilon$ . The observed marital sorting pattern is given by  $\Theta_0$ . Then the probability that the observed marriage pattern is generated by  $R$  is

$$p(R) = \int \cdots \int \chi[\Theta_0 = \Theta_{MP}(M, F, R|\varepsilon)] dB_1^N(\varepsilon_1) dB_2^N(\varepsilon_2). \tag{11}$$

It is not immediately apparent that a given  $\Theta_0$  can be generated by *any* draw of  $\varepsilon$  given  $(M, F, R)$ . In this case,  $p(R) = 0$  and no further consideration of the rule  $R$  is warranted.

Now we will generate  $p(NE)$  and  $p(NB)$  using the model. We will then say that the odds that the marital sorting pattern was generated by  $NE$  are given by

$$LR = \frac{p(NE)}{p(NB)}. \quad (12)$$

In computing  $p(R)$  we face a computational problem, mainly stemming from the fact that there is no closed form expression for the integral in (11). We adopt a Monte Carlo integration approach, in which we take  $N$  draws from the distributions  $B_1$  and  $B_2$  over  $M$  replications. Our estimate of  $p(R)$  is then given by the proportion of the  $M$  replications that resulted in the observed distribution of marital sorts. More formally, let the  $m^{\text{th}}$  draw of  $N$  measurement errors from the distributions of  $B_1$  and  $B_2$  be given by  $\varepsilon^m$ . Then

$$\hat{p}_M(R) = M^{-1} \sum_{m=1}^M \chi[\Theta_0 = \Theta_{MP}(M, F, R|\varepsilon^m)]. \quad (13)$$

Computation of this quantity is conceptually and numerically straightforward. However, the size of  $M$  required to adequately approximate  $p(R)$  will depend critically on the size of the married population in the sample. For example, say  $M$  is set at 10000. If  $N = 10$ , we may expect to observe a nontrivial number of correspondences between the predicted matches under  $R$  and the observed marriage sorts if  $R$  is indeed the correct behavioral rule. However, even if households behave according to  $R$ , we would expect the likelihood that a sample of  $M$  draws yields the observed sorts to be arbitrarily close to 0 if  $N$  is equal to 10 million. We circumvent this problem by subsampling our group of 877 households in the following manner.

From the original sample of  $N$  households, randomly select a subset of size  $n$ , and denote the  $j^{\text{th}}$  subsample by  $A_j$ . For each of the  $J$  subsamples, take  $M$  draws of size  $n$  from each of the distributions  $B_1$  and  $B_2$ . Denote the  $m^{\text{th}}$  draw of  $\varepsilon$  in subsample  $j$  by  $\varepsilon^m(j)$ . Then define a modified estimator of  $p(R)$  by

$$\hat{p}_{M,n,J}(R) = J^{-1} \sum_{j=1}^J M^{-1} \sum_{m=1}^M \chi[\Theta_0 = \Theta_{MP}(M, F, R|\varepsilon^m(j))]. \quad (14)$$

We experiment with a few different settings of  $(M, n, J)$  to determine sensitivity of the odds ratio to these parameters. In the empirical results reported

below, we set the subset size  $n = 4$ . Since the subsets are randomly selected, and we have a fairly large number of them, we consider the distribution of the  $\hat{p}_{M,n,J}(R)$  to be represent the sampling distribution of the proportion of correct predictions. From the sampling distribution of the differences ( $\hat{p}_{M,n,J}(NE) - \hat{p}_{M,n,J}(NB)$ ) we can get an idea of the mean difference in the quality of prediction relative to the sampling variability in the difference. While we could try to push things further by making probabilistic assessments regarding the true underlying difference, the results we have obtained seem sufficient to draw relevant conclusions.

### 3.3 Computation of $\alpha_s$

To this point we have assumed that the preference weight on leisure varies only by gender (i.e., all individuals of the same gender share the same value of  $\alpha_s$ ) and we have treated it as known. The four first order conditions uniquely determine the four unobserved characteristics of the husband and wife conditional on a behavioral rule  $R$  and  $\alpha_1$  and  $\alpha_2$ . We determine values of  $\alpha_s$  after adopting a particular normalization.

To stress the dependence of the implied values of the time endowments in the household on the preference weights  $\alpha_1$  and  $\alpha_2$ , write the implied time endowment for individual of gender  $s$  in household  $i$  as

$$\tilde{T}_{si}(R; \alpha). \quad (15)$$

There are 168 hours in a week. We define the values of  $\hat{\alpha}_s$  as those that result in the *average* time endowment in the sample being equal to 168, or

$$168 = N^{-1} \sum_{i=1}^N \tilde{T}_{1i}(R; \hat{\alpha}) \quad (16)$$

$$168 = N^{-1} \sum_{i=1}^N \tilde{T}_{2i}(R; \hat{\alpha}). \quad (17)$$

The use of the average is admittedly somewhat arbitrary, and an argument could be made for using the median, for example, instead. Nonetheless, given the parameterization of the model adopted, some such normalization is required if we are to “estimate” the two values  $\alpha_1$  and  $\alpha_2$ .

## 4 Empirical Results

The empirical work is performed using a sample of married couples taken from the PSID. The data refer to household characteristics in 2000 that were

collected in the 2000 and 2001 survey years. To be included in the sample, the household must have been headed by a married couple, both of whom were between the ages of 25 through 49, inclusive. All information on time allocations within the household must have been available for both spouses; this consists of the average amount of time spent in the labor market per week in 2000 as well as average hours spent in housework per week. Because household production activities change so markedly when young children are present, we excluded all households in which there was a child less than six years of age.

We also excluded any household in which one of the spouses made more than \$150 an hour or who reported more than 80 hours of market work per week. We also required that the household not receive more than \$1000 per week in nonlabor income. A few households reported negative total income for the year, and these were excluded.

The (almost) final selection criterion imposed was that both spouses spend time in the labor market and in home production. This, of course, is a substantive restriction that is imposed so that we can invert four first order conditions for each household to obtain four values of the unobserved characteristics of the spouses (two for each spouse). Approximately 18 percent of the sample was eliminated by insisting that both spouses report supplying time to the market in the previous year. Some spouses were reported to have supplied zero time to household production; for these individuals we assumed that the actual amount of time spent in housework was 1 hour per week.<sup>10</sup> During the process of estimation we found that data from 9 households in our “final” sample produced problematic values for the four unobserved household characteristics. We excluded these from all further analyses. The total sample size with which we work is  $N = 877$ .

Table 1 contains descriptive statistics for our sample. We think of the decision period as the week. The unit of time is the week, and all monetary units are expressed in terms of current (year 2000) dollars. For now we focus only on the means and variances of variables taken directly from the data.

The average wage of husbands is about 40 percent greater than that of their wives. They work about 20 percent more hours per week than their wives in the market, while their wives supply about twice as much time in housework. It is interesting to note that the average total time spent in the labor market and performing household tasks is essentially identical for

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<sup>10</sup>It would be interesting to look at the distribution of responses to these housework questions as a function of the identify of the respondent. We hazard the conjecture that, conditional on observable characteristics, respondents are likely to over-emphasize their contributions to the household workload while under-emphasizing the spouses.

husbands and their wives.

The average nonlabor income per household is 120 dollars per week, with a large standard deviation. Nonlabor income of less than 100 dollars is reported by two-thirds of the households in the final sample. Recall that we have excluded households in which nonlabor income exceeded 1000 dollars per week.

The first task performed was to back out the implied values of  $(\tilde{T}_{si}, \delta_{si}, \alpha_s)$ ,  $s = 1, 2$ ,  $i = 1, \dots, N$ , under  $NE$  and  $NB$ . The means and standard deviations of these characteristics are presented in Table 1. We see that the preference weights on leisure are far greater under  $NB$ . This is to be expected since cooperative behavior will lead to greater supply of time to the market and household tasks for a given set of household characteristics. Thus to be consistent with the same observed time allocations, the leisure weights under Nash bargaining must be greater than those computed under Nash equilibrium. The normalization of the mean time endowments results in this value being equal to 168 for both sexes and under either behavioral mode. The large standard deviation of  $\tilde{T}_s$  indicates substantial heterogeneity in this parameter in the sample.

The average value of efficiency in household production varies across the genders and the modes of behavior. For the same reason that  $NB$  led to higher imputed preference weights on leisure, it also leads to lower values of the household production elasticities for both sexes. For both sexes, the average value of the Cobb-Douglas parameter under  $NB$  is about one-third of its average value under  $NE$ .

There are large changes in the means of  $\alpha_s$  and  $\delta_s$  when moving from  $NE$  to  $NB$ , and in the standard deviations of  $\tilde{T}_s$  and  $\delta_s$ . Nevertheless, as Figures 1 and 2 and Table 2 illustrate, the imputed values of  $(\tilde{T}_{si}, \delta_{si}, \alpha_s)$  computed under  $NB$  are linear transformations of the values computed under  $NE$ .<sup>11</sup> In spite of this extreme dependence of the parameter values computed under the two behavioral rules, the preference orderings and resulting marital sorts can be very different, as we shall see below.

It may be of some interest to investigate the gains to cooperative behavior and “rational” marriage sorts starting from the noncooperative baseline. We perform an experiment that utilizes our parameter estimates under  $NE$  and first computes the welfare gains to existing households if they switched their behavior to  $NE$ . We then look at the change in welfare that would result

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<sup>11</sup>The small deviations from this claim that we see in Table 2 result from numerical inaccuracies involved in performing the inversion of the first order conditions in the Nash bargaining case.

if all households continued to behave noncooperatively, but were matched according to the GS algorithm.

Table 3 and Figure 3 contain the results of this exercise. By definition, when existing households switch to Nash bargaining there is a welfare gain for all husbands and wives in the sample. However, the welfare gains are small, raising the utility levels of husbands by less than 1 percent and those of wives by 1.2 percent. When we resort individuals according to the GS (male-preferring) algorithm, the average welfare gains are identical for husbands but about one-half as much for wives. Roughly speaking, the scope for welfare improvements is about as great for marital reshuffling as it is for moves to cooperative behavior.

We have now reached the most important part of the empirical analysis. Which behavioral assumption is most consistent with the observed patterns of marital sorts? The short answer is that neither fits the data very well, in large part for the reasons given in the previous section. Table 4 contains the rank order correlations between observed, *NE*, and *NB* equilibrium sorts. First note that even though there exists a linear mapping between unobserved parameters characterizing individuals computed under *NE* and *NB*, there is only a correlation of 0.028 in the rank order correlation of the marital sorts under these two models. While the correlation between the observed sorts and that predicted under *NE* is only 0.015, the correlation between observed sorts and those predicted under *NB* is a relatively strong and “perverse” -0.063.

We can perform slightly more formal “tests” between the two behavioral modes using the measurement error in wages specification discussed above. By dividing our 877 observations into groups of size 4, we created 219 groups ( $n$ ). The distribution of correct predictions across the  $n$  groups for the two behavioral modes is presented in Figure 4. The average proportion of correct predictions across the groups under *NE* was 0.135, while under *NB* it was 0.131. The mean difference was thus 0.004, and the standard deviation of the differences was 0.091. Thus the mean difference is less than 5 percent of 1 standard deviation of the sampling distribution. There is no strong evidence to support the claim that either mode of behavior dominates the other in terms of its ability to explain marital sorting patterns.

## 5 Conclusion

In this paper we have attempted to make the point that there is no general nonparametric test to distinguish between modes of household behavior

when individual heterogeneity in unobservable characteristics is not severely restricted. This general specification means that within household behavior is not useful in distinguishing modes of behavior, which is the bad news from the analysis. The good news is that this heterogeneity does give the potential for different preferences over potential mates, and hence different equilibrium marriage patterns, under competing behavioral assumptions. Using the Gale-Shapley bilateral matching algorithm, we found no basis to claim that either behavioral mode dominated as an explanation of observed marital sorting patterns.

The general point we wish to make is reminiscent of the general problem of model over-fitting. We adopted a model framework that was capable of perfectly fitting the data (i.e., the mapping from the data space to the parameter space was 1 to 1) under an entire class of behavioral rules  $\mathfrak{R}$ . In order to “test” one specification against another, some restrictions have to be imposed on the parameterization to make the mapping no longer 1 to 1, and to raise the possibility that one of the elements of  $\mathfrak{R}$  fits better than another. Of course the test results we obtain in the end are a function of sample realizations and the restrictions we have placed on the parametric specification. It is seldom possible to claim that one parameterization should be preferred over another on theoretical grounds.

Given this inherent arbitrariness, we have moved the test to a different playing field - one that is “out of sample,” so to speak. The richness of the specification of individual heterogeneity leads to zero power in testing one element of  $\mathfrak{R}$  against another using only time allocation data, but has the potential to produce the implication of very different marital sorts - an empirical phenomenon that is not used in backing out the individual characteristics. In this application neither of the elements of  $\mathfrak{R}$  that we considered was very successful in explaining the observed marriage patterns, but the hope remains that by considering other elements of  $\mathfrak{R}$  or enriching the modeling framework to include dynamic time allocation and marriage decisions more striking test results could be obtained.



**Table 1**  
**Means and (Standard Deviations) of Individual Characteristics**  
**N = 877**

<i>Characteristic</i>	<i>Husband</i>		<i>Wife</i>	
	<i>NE</i>	<i>NB</i>	<i>NE</i>	<i>NB</i>
$\alpha$	0.563	0.715	0.467	0.655
$\tilde{T}$	168.000 (58.637)	168.000 (50.532)	168.000 (70.130)	168.000 (57.139)
$\delta$	0.101 (0.097)	0.027 (0.031)	0.139 (0.109)	0.045 (0.037)
$w$	21.522 (13.655)		15.206 (9.434)	
$h$	45.707 (8.421)		38.202 (10.569)	
$\tau$	7.853 (6.878)		15.323 (9.672)	
$Y$	120.455 (183.175)			

**Table 2**  
**Correlation Between Imputed Parameters**

<i>Nash Equilibrium</i>	<i>Nash Bargaining</i>			
	$\tilde{T}_1$	$\tilde{T}_2$	$\delta_1$	$\delta_2$
$\tilde{T}_1$	1.000	-0.172	-0.137	0.070
$\tilde{T}_2$	-0.175	1.000	0.160	-0.255
$\delta_1$	-0.166	0.141	0.993	0.097
$\delta_2$	0.066	-0.256	0.108	0.998

**Table 3**  
**Changes in Average Welfare Values from**  
**NE Baseline Behavior and Observed Matches**  
**(Proportionate Gain from Baseline)**

	Husbands	Wives
Baseline	6.103	6.396
NB Behavior	6.159 (0.009)	6.473 (0.012)
NE Marriage	6.158 (0.009)	6.431 (0.005)

**Table 4**  
**Correlations Between Marriage Sorts**

	<i>Actual</i>	<i>Nash Equilibrium</i>	<i>Nash Bargaining</i>
<i>Actual</i>	1.000	0.015	-0.063
<i>Nash Equilibrium</i>		1.000	0.028
<i>Nash Bargaining</i>			1.000

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Figure 1.a  
Histogram of  $T_1$   
Nash Equilibrium

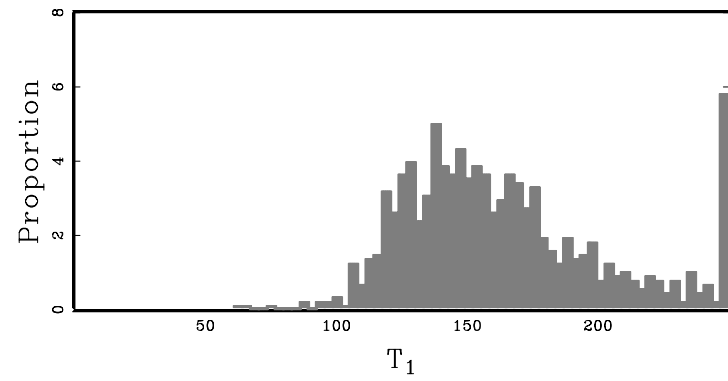


Figure 1.b  
Histogram of  $T_2$   
Nash Equilibrium

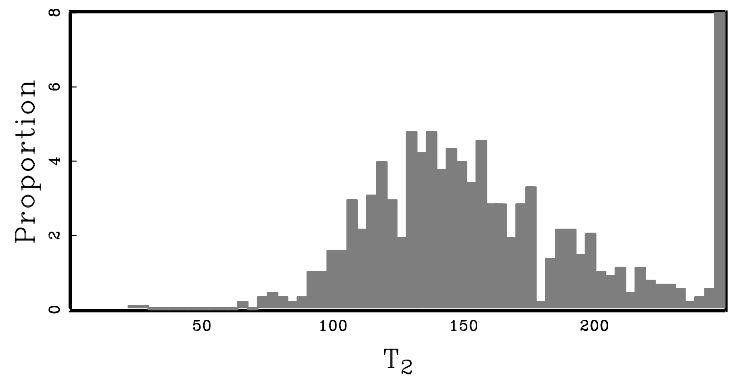


Figure 1.c  
Histogram of  $\delta_1$   
Nash Equilibrium

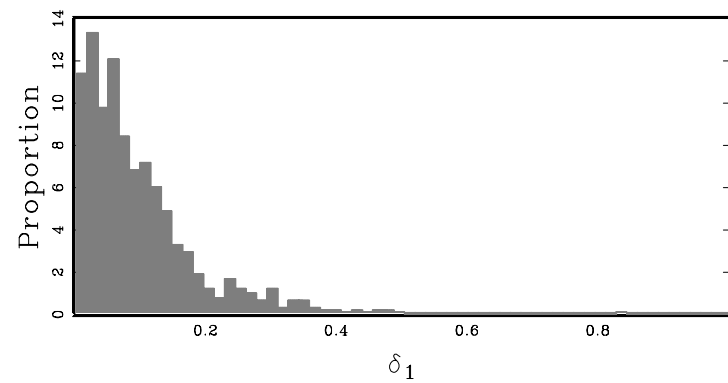


Figure 1.d  
Histogram of  $\delta_2$   
Nash Equilibrium

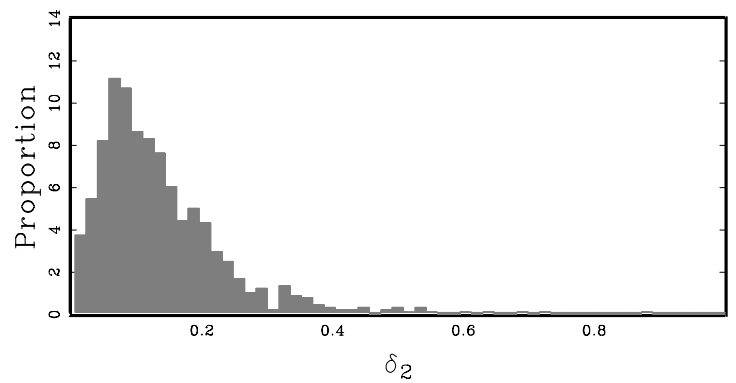


Figure 1.e  
Histogram of  $w_1$

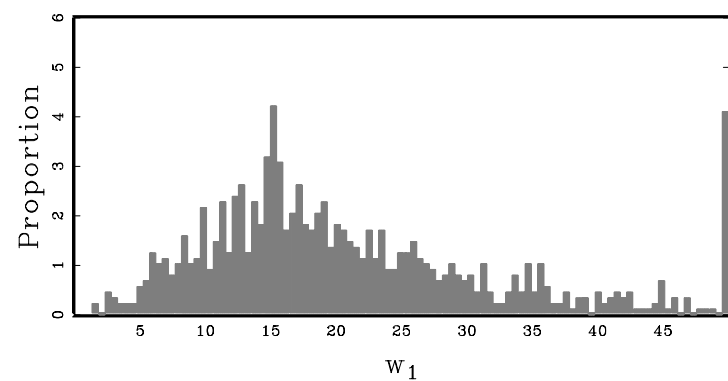


Figure 1.f  
Histogram of  $w_2$

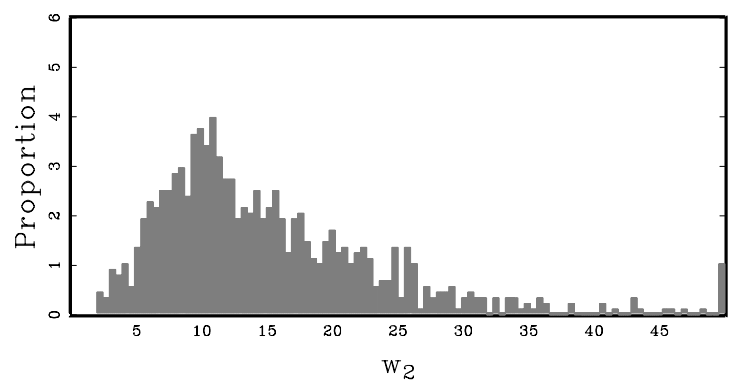


Figure 2.a  
Histogram of  $T_1$   
Nash Bargaining

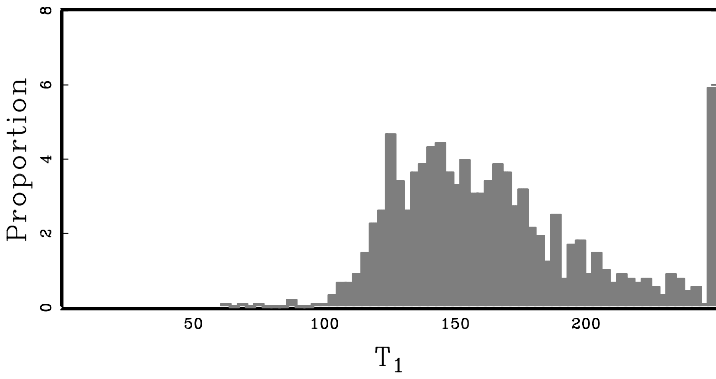


Figure 2.b  
Histogram of  $T_2$   
Nash Bargaining

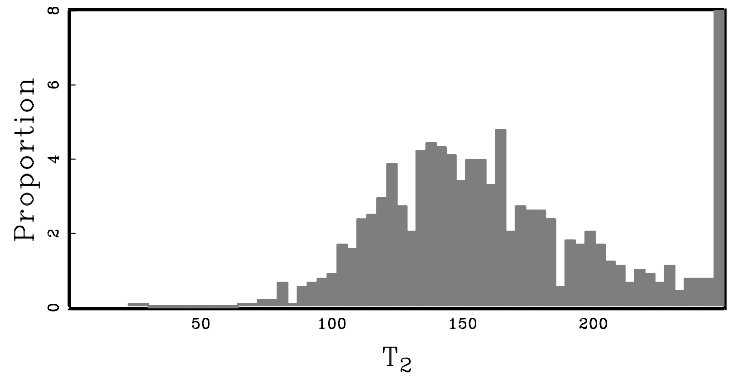


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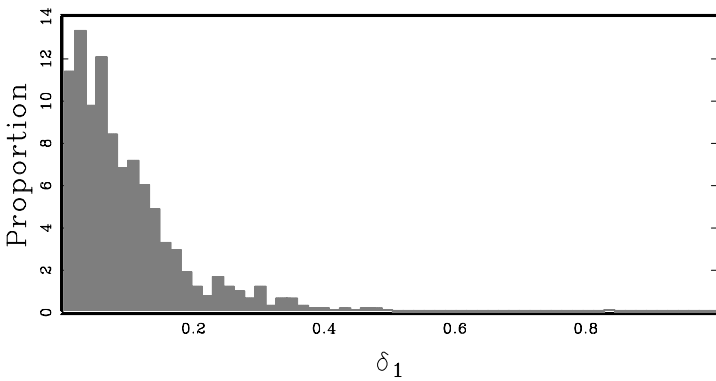


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Histogram of  $\delta_2$   
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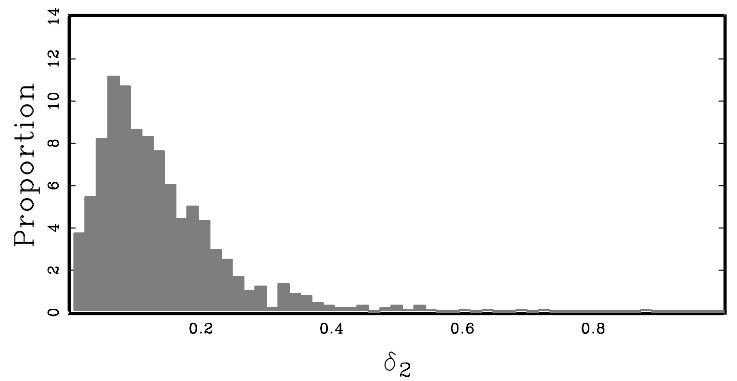


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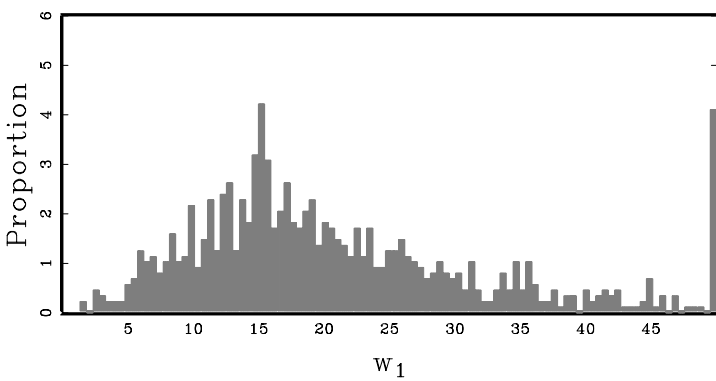


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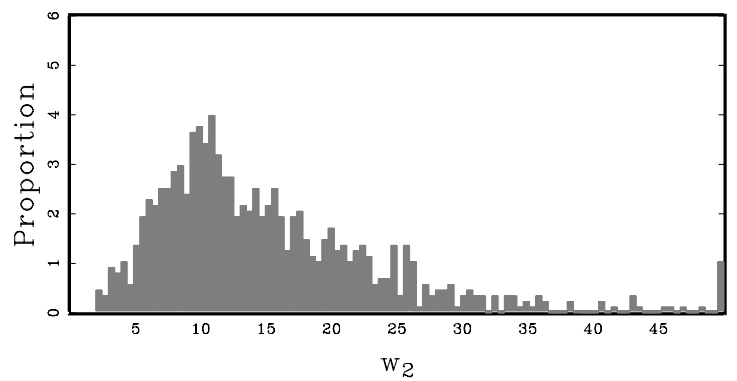




Figure 3.a  
Histogram of  $V_1$   
Nash Equilibrium

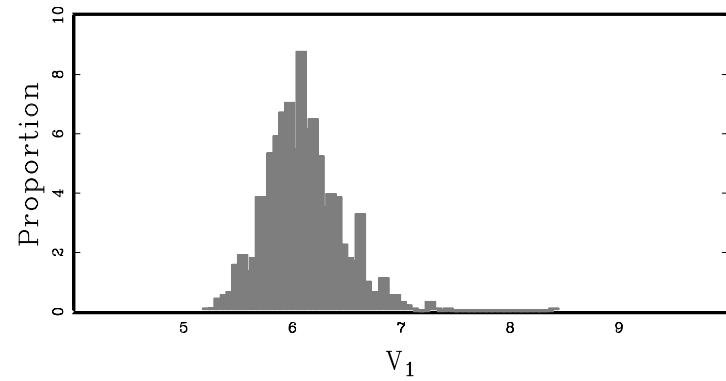


Figure 3.b  
Histogram of  $V_2$   
Nash Equilibrium

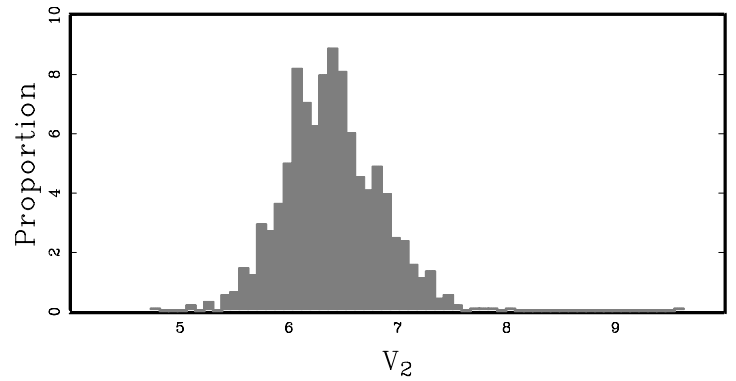


Figure 3.c  
Husbands' Proportionate Welfare Gain  
Nash Bargaining

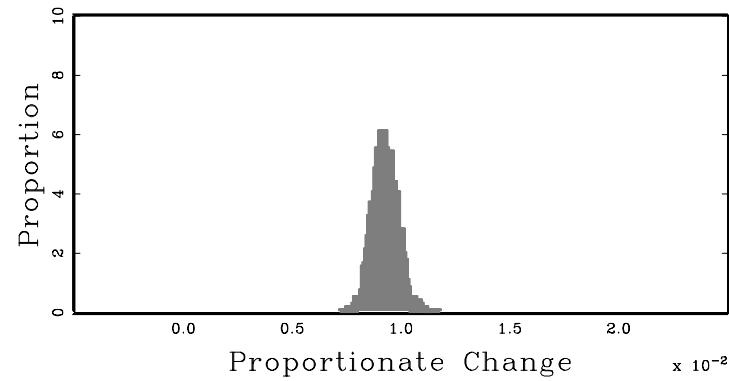


Figure 3.d  
Wives' Proportionate Welfare Gain  
Nash Bargaining

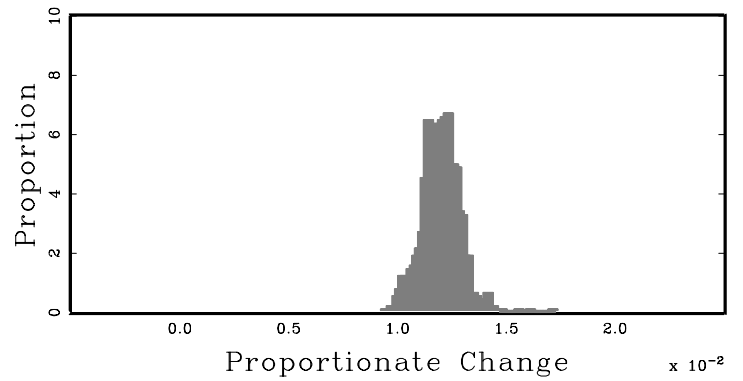


Figure 3.e  
Husbands' Proportionate Welfare Gain  
Equilibrium Marriage

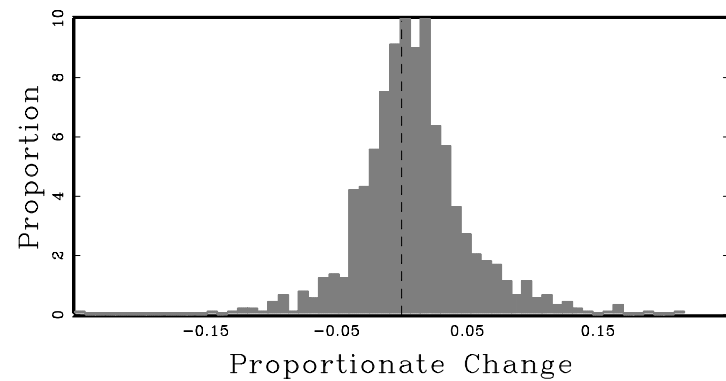


Figure 3.f  
Wives' Proportionate Welfare Gain  
Equilibrium Marriage

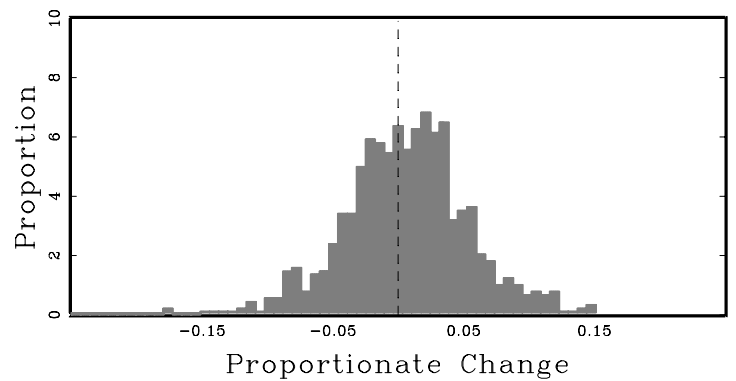


Figure 4.a  
Proportion of Correct Marriage Predictions  
Nash Equilibrium

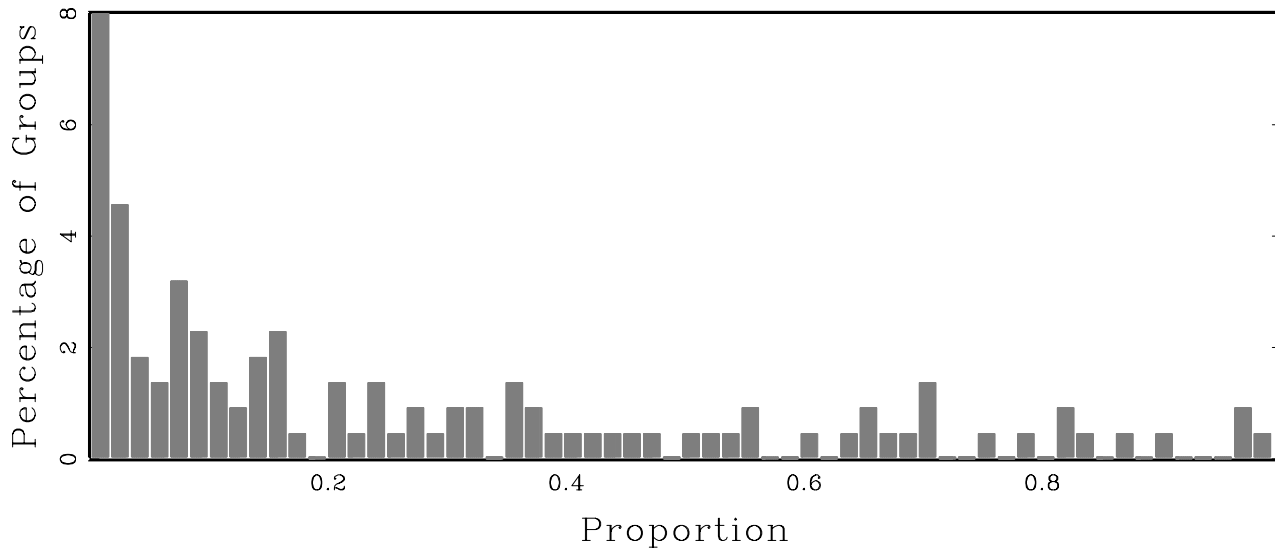


Figure 4.b  
Proportion of Correct Marriage Predictions  
Nash Bargaining

