

# Distributional effects in household models: separate spheres and income pooling.\*

Martin Browning<sup>†</sup> Pierre-André Chiappori<sup>‡</sup> and Valérie Lechene<sup>§</sup>

June 6, 2005

## Abstract

We derive distributional effects for a non-cooperative alternative to the unitary model of household behaviour. We consider the Nash equilibria of a voluntary contributions to public goods game. Our main result is that, in general, the two partners either choose to contribute to different public goods or they contribute to at most one common good. The former case corresponds to the separate spheres case of Lundberg and Pollak (1993). The second outcome yields (local) income pooling. A household will be in different regimes depending on the distribution of income within the household. Any bargaining model with this non-cooperative case as a breakdown point will inherit the local income pooling. We conclude that targetting benefits such as child benefits to one household member may not always have an effect on outcomes.

JEL codes: D10, C71, C72

Keywords: Nash equilibrium, Nash bargaining, collective models, intra-household allocation, local income pooling, separate spheres.

---

\*Some of the results in this paper appeared in an earlier paper entitled "Caring and sharing" which should now be considered superseded. Browning thanks the Danish National Research Foundation for support through its grant to CAM.

<sup>†</sup>CAM, Institute for Economics, Copenhagen.

<sup>‡</sup>Department of Economics, University of Chicago

<sup>§</sup>Wadham College, Oxford and IFS, London.

# 1 Introduction

Using policy to channel resources towards certain types of individuals within households, thereby exogenously altering the intra-household distribution of income, is an instrument often used by governments, typically to further the welfare of children. Such transfers are usually put in the hands of women, on the basis of the belief that additional resources to mothers, over and above the level of income they generate by choice, leads to additional power in the allocation of these resources and to additional resources going to children. In contrast with an abundance of evidence from sociology and anthropology, economic evidence on these phenomena is relatively scarce. Two reasons can be invoked to justify this. On the one hand, scarcity of data has hampered progress on this issue, and on the other, suitable conceptual tools have been developed only relatively recently. In the standard ‘unitary’ approach to household behavior, for instance, income is pooled at the household level and the identity of the recipient is irrelevant. Thus the issue of ‘targeting’ benefits to one household member can only be analyzed *outside* the unitary framework.

Non unitary models can be classified into two broad categories, depending on whether they assume cooperation (hence Pareto efficiency) or non cooperation. Blundell, Chiappori and Meghir (2005) have recently analyzed the ‘targeting’ issue in a cooperative context. In the non-cooperative framework, two main avenues have been explored. One relies on non-cooperative Nash equilibrium and private provision of public goods. An alternative approach, introduced by Lundberg and Pollak (1993) relies on a ‘separate spheres’ approach, whereby in the absence of cooperation each individual within the household specializes into specific tasks (for instance, those that are ‘traditionally’ assigned to their gender). While intuitively appealing, the notion of ‘separate spheres’ has not been given a sound theoretical underpinning.

The main goal of the present contribution is to extend existing results, and to clarify the links between the ‘Nash equilibrium’ and ‘separate spheres’ approaches. In doing this we provide a framework which contains all current suggestions as special cases. We consider a model in which agents decide on the provision of several public goods; in this context, we analyze the Nash equilibrium with voluntary contributions. We first extend the ‘local income pooling’ result of Warr (1983) and Bergstrom *et al* (1986) to this context and show that when both partners are contributing to at least one public good, household demands *for all goods* are independent of individual incomes and only depend on aggregate household resources. This suggests that, in a non-cooperative context, the scope of policies targeting a specific member as the main (or only) recipient of a given benefit may be narrow, since this choice may fail to influence the household’s decision.

Second, and more interestingly, we show that in general there is *at most one* public good to which both agents contribute. Hence all public commodities but possibly one are exclusively provided by one agent only. Furthermore, in a wide set of circumstances, the sets of public goods to which each person contributes are disjoint; i.e., each person chooses to contribute to a different

set of public goods than the other. Hence the important conclusion that the ‘separate spheres’ model of Lundberg and Pollak (1993) is but a sub-case of the general, non-cooperative ‘ approach. Individual specialization, in this context, need not be assumed initially; rather, it emerges endogenously as an equilibrium feature. Finally, the definition of the individual ‘spheres’ can be endogenised; we show how it is determined by individual preferences and the within household distribution of income.

Two additional remarks can be made. First, our result has a wider bearing than intra-household allocation, as the Nash equilibrium is often used to represent situations involving large number of agents and of goods, for instance the provision of public goods in society. Second, the scope of our conclusions is not limited to non-cooperative models. Several existing contributions consider cooperative models based on Nash bargaining, with individual outside options modeled as stemming from non-cooperative solutions (either non-cooperative or separate spheres). Since bargaining outcomes inherit the features of the underlying non-cooperative outcomes, the local income pooling result applies to this context as well.

## 2 Nash equilibrium with voluntary contributions to the public goods

### 2.1 Framework

We consider a two person ( $I = A, B$  with  $A$  being a ‘she’ and  $B$  being a ‘he’) household which faces fixed prices (which we normalise to unity) and allocates a given income between different goods. Agent  $J$  has income  $Y^J$ , and  $Y = Y^A + Y^B$  denotes the household’s total income. We assume in all that follows that goods are either public or private<sup>1</sup> and that each person has a representable preference ordering over the within household allocation of goods. Denote person  $I$ ’s  $n$ -vector of their private good by  $\mathbf{q}^I$  and let the  $m$ -vector of public goods be denoted  $\mathbf{Q}$ . Let  $\mathbf{q} = \mathbf{q}^A + \mathbf{q}^B$  be the vector of household consumption of the private good.

The household budget constraint is:

$$\mathbf{Q}'\mathbf{e} + \mathbf{q}'\mathbf{e} = Y \tag{1}$$

where  $\mathbf{e}$  is the appropriately sized unit vector. Preferences are assumed to be egoistic, in the sense that each person’s utility function is defined over public goods and the individual’s private consumption,  $v^I(\mathbf{q}^I, \mathbf{Q})$ .<sup>2</sup>

---

<sup>1</sup>We could allow that goods have the possibility of having both a private and a public nature; this complicates the notation without adding anything of substance.

<sup>2</sup>The analysis below can easily be extended to the case in which there is caring. We shall return to this in the conclusion.

## 2.2 Definition and existence

In the Nash equilibrium with voluntary contributions to the public goods, each individual chooses how to allocate their income between the private goods and the amounts they contribute to the public goods, given the level of contributions of the other individual to the public goods. The household's expenditure on a public good is the sum of the individual contributions to that public good.<sup>3</sup> A solution in this problem is a vector of contributions to the public goods  $(\mathbf{g}^{A*}, \mathbf{g}^{B*})$  such that each individual's belief are confirmed in equilibrium, that is such that  $(\mathbf{q}^{J*}, \mathbf{g}^{J*})$ ,  $J = A, B$  are solutions of the programs:

$$\begin{cases} \text{Max}_{\mathbf{q}^J, \mathbf{g}^J} U^J(\mathbf{q}^J, \mathbf{g}^A + \mathbf{g}^B) \\ \mathbf{q}^{J'}\mathbf{e} + \mathbf{g}^{J'}\mathbf{e} = Y^J \\ \mathbf{g}_i^J \geq 0, i = 1, \dots, m \end{cases} \quad (2)$$

This program can be rewritten equivalently in terms of private goods and of public goods for the household. For  $A$ :

$$\begin{cases} \text{Max}_{\mathbf{q}^A, \mathbf{Q}} U^A(\mathbf{q}^A, \mathbf{Q}) \\ \mathbf{q}^{A'}\mathbf{e} + \mathbf{Q}'\mathbf{e} = Y^A + \mathbf{g}^{B'}\mathbf{e} \\ \mathbf{Q} \geq \mathbf{g}^B \end{cases} \quad (3)$$

One can write a similar program for  $B$ , with  $B$  choosing the quantity of private goods he consumes and the quantity of public goods the household consumes in equilibrium.

**Proposition 1** *Assume that  $U^A$  and  $U^B$  are continuously differentiable and strictly quasi concave. Then a Nash equilibrium exists.*

**Proof.** *The proof relies on a standard fixed point argument. ■*

## 2.3 Properties of the Nash equilibrium

We now study the features of Nash equilibria in this context. We say that member  $A$  *contributes* to public good  $j$  if  $\mathbf{g}_j^{A*} > 0$ . We then have the following result:

**Proposition 2** (TYPES OF EQUILIBRIUM). *Let  $m^A$  be the number of public goods to which  $A$  contributes, and  $m^B$  the number of public goods to which  $B$  contributes. In general,  $m^A + m^B \leq m + 1$ ; that is, there is at most one public good to which both contribute. If all public goods are bought, either  $m^A + m^B = m$  or  $m^A + m^B = m + 1$ .*

---

<sup>3</sup>Or equivalently, a function of the sum of the contributions. A more general assumption would be to allow for the household's consumption of public goods to be a function of the individual contributions. This would make it possible to capture for instance semi-publicness for some goods. We do not allow for this.

**Proof.** The first order conditions (2) give a set of  $(n + m^A - 1)$  equalities for  $A$ . A similar argument applied to agent  $B$  generates another set of a set of  $(n + m^B - 1)$  equalities. With the budget constraint, we thus have

$$(2n + m^A + m^B - 1)$$

equalities in  $(2n + m)$  unknowns. Since these equations are algebraically independent, they cannot in general be simultaneously fulfilled unless  $m^A + m^B - 1 \leq m$ . Finally, if  $m^A + m^B < m$  then there must be a commodity to which no one contributes. ■

Note that the result is only ‘general’ (or ‘generic’), in the sense that it is ‘almost always’ satisfied. Still, it is possible, for arbitrary preferences, that it is violated at specific points (but then typically these points are locally unique). Also, one can find preferences such that the result is violated upon an open set; this is the case, for instance, when public goods are separable and (sub)preferences over the public goods are identical across individuals, a case studied in Lechene and Preston (2005). Note, however, such preferences are not robust to local perturbations. A precise statement of these ‘genericity’ conditions would require some heavy mathematical apparatus (transversality theory) that is outside the scope of this paper; instead, a detailed example is provided below.

The next proposition states explicitly an implication for the case  $m^A + m^B = m + 1$ .

**Proposition 3** (LOCAL INCOME POOLING) *When there is one public good to which both household members contribute ( $m^A + m^B = m + 1$ ), redistributions of income between household members which do not exceed the amount of each individual’s contributions to the jointly contributed public good have no effect on households expenditures.*

**Proof.** Assume, without loss of generality, that both agents contribute to commodity 1, while  $A$  is an exclusive contributor for commodities  $2, \dots, p$  and  $B$  is an exclusive contributor for commodities  $p + 1, \dots, m$  (where  $2 \leq p \leq m$ ). Then the  $(2n + m)$  vector  $(q^A, q^B, Q)$  satisfies  $n + p - 1$  first order conditions for  $A$ ,  $n + m - p$  first order conditions for  $B$  plus the global budget constraint. None of these conditions depend on individual incomes (all but the last do not depend on incomes at all and the last only depends on aggregate income). Hence the set of solutions does not depend on individual incomes. ■

Proposition 3 generalizes the remarkable result first obtained by Warr (1983) in the case of one public good, then also by Kemp (1984), and Bergstrom, Blume and Varian (1986) for several public goods.<sup>4</sup> It shows, in particular, that a given household may or may not pool income; in fact, pooling behavior

---

<sup>4</sup>Kemp’s method of proof relies on counting equations and unknowns, but assumes that all agents contribute to all public goods, which is impossible in general. Bergstrom, Blume and Varian use a revealed preference argument. Lechene and Preston (2005) give a proof in the same spirit as the proof provided here, but at an interior equilibrium, that is under restrictions on preferences so that all agents contribute to all public goods.

obtains endogenously as the outcome of non-cooperation for certain ranges of the relevant parameters. A heterogeneous sample of households may therefore contain “pooling” and “non-pooling” households, a fact which has important implications for empirical work.

Proposition 2 is more original. It states that in a non-cooperative setting, individuals specialize in funding public goods, so that all public goods but maybe one, are exclusively funded by one individual. Proposition 2 can thus be interpreted as a ‘separate spheres’ result: in practice, each publicly consumed commodity (but maybe one) belongs exclusively to the ‘sphere of influence’ of one of the household members.

Two remarks are however in order. First, in contrast with Lundberg and Pollak’s approach, respective spheres are *endogenously* determined. Specifically, for any equilibrium vector  $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ , for any public good  $j$  which is consumed, we have the following characterization (assuming that both people buy the first private good):

- either  $\frac{\partial U^A/\partial Q_j}{\partial U^A/\partial q_1^A} < 1$ , then  $\frac{\partial U^B/\partial Q_j}{\partial U^B/\partial q_1^B} = 1$  and  $B$  is an exclusive contributor (so that commodity  $j$  belongs to  $B$ ’s sphere)
- or  $\frac{\partial U^A/\partial Q_j}{\partial U^A/\partial q_1^A} = 1$  and  $\frac{\partial U^B/\partial Q_j}{\partial U^B/\partial q_1^B} < 1$ , then  $A$  is an exclusive contributor (so that commodity  $j$  belongs to  $A$ ’s sphere)
- or  $\frac{\partial U^A/\partial Q_j}{\partial U^A/\partial q_1^A} = \frac{\partial U^B/\partial Q_j}{\partial U^B/\partial q_1^B} = 1$ , in which case both  $A$  and  $B$  contribute; from Proposition 2, this can will usually only happen for one commodity In this case we have income pooling.

In other words, for all public goods but possibly one, the marginal willingness to pay (out of private consumption) of one of the partners is smaller than the marginal cost (here normalized to one).

The second remark is that the definition of the ‘spheres’ is not fixed; it depends on individual incomes. For instance, when a member’s income is low enough, this member will in general contribute to no public good. This implies that any change in income distribution that affect the member’s respective incomes may change the definition of the spheres. A precise illustration is given below in a specific example.

Before turning to the example, we discuss briefly the possible implications of this analysis if agents use a bargaining models that takes the non-cooperative outcome as a breakdown point. In this case the bargaining outcomes inherit some of the features of the non-cooperative outcomes. In particular, the same segments of local pooling will hold for all goods.

## 2.4 An example

We now study an example with one private good and two public goods (denoted  $G$  and  $H$  for simplicity). Individual preferences are Cobb-Douglas:

$$\begin{aligned} u^A(q^A, G, H) &= q^A G^a H^\alpha \\ u^B(q^B, G, H) &= q^B G^b H^\beta \end{aligned}$$

We denote by  $A$ 's income by  $\rho$  and  $B$ 's by  $(1 - \rho)$ . We are particularly interested in analyzing changes in demand resulting from variations in the income share  $\rho$  (keeping total income constant at unity). We assume, as a normalization, that

$$\frac{a}{\alpha} > \frac{b}{\beta}$$

so that  $A$  cares (relatively) more for commodity  $G$  and  $B$  cares (relatively) more for commodity  $H$ .

The first order conditions give:

$$\begin{aligned} a \frac{q^A}{G} &\leq 1, \alpha \frac{q^A}{H} \leq 1 \text{ for } A, \text{ and} \\ b \frac{q^B}{G} &\leq 1, \beta \frac{q^B}{H} \leq 1 \text{ for } B. \end{aligned}$$

with an equality when the agents contributes to the commodity under consideration. Now assume, first, that  $A$  is contributing to  $G$  and that  $B$  is contributing to  $H$ . Then

$$a \frac{q^A}{G} = \beta \frac{q^B}{H} = 1$$

If, moreover,  $A$  also contributes to  $H$ , then  $\alpha \frac{q^A}{H} = 1$ , hence  $\frac{H}{G} = \frac{a}{\alpha}$ . Similarly, if  $B$  also contributes to  $G$ , then  $b \frac{q^B}{G} = 1$ , hence  $\frac{H}{G} = \frac{b}{\beta}$ . It follows that  $A$  and  $B$  cannot simultaneously contribute to both public goods unless  $\frac{a}{\alpha} = \frac{b}{\beta}$ . This is exactly the meaning of the 'in general' qualification in the statement of Proposition 2: such a condition is 'almost never' satisfied, and when it is the situation is 'knife-edge' and not robust to infinitesimal perturbations (here, infinitesimal changes in the parameters).

The exact solutions as a function of  $\rho$  are given by:

- if

$$\rho \leq \frac{b}{a + ab + a\beta + b}$$

then  $A$  contributes to no public good,  $B$  contributes to both public goods and

$$q^A = \rho, q^B = \frac{(1 - \rho)}{1 + b + \beta}, G = \frac{b(1 - \rho)}{1 + b + \beta}, H = \frac{\beta(1 - \rho)}{1 + b + \beta}$$

- if

$$\frac{b}{a + ab + a\beta + b} < \rho \leq \frac{b(a+1)}{a + ab + a\beta + b}$$

then  $A$  contributes to  $G$ ,  $B$  contributes to  $G$  and  $H$ , and

$$\begin{aligned} q^A &= \frac{b}{a + ab + a\beta + b}, q^B = \frac{a}{a + ab + a\beta + b}, \\ G &= \frac{ab}{a + ab + a\beta + b}, H = \frac{a\beta}{a + ab + a\beta + b} \end{aligned}$$

Note that, in that case, demand does not depend on  $\rho$ , as stated in Proposition 3, since both agents contribute to  $G$ .

- if

$$\frac{b(a+1)}{a + ab + a\beta + b} < \rho \leq \frac{\beta(a+1)}{\alpha + \alpha\beta + a\beta + \beta}$$

then  $A$  contributes to  $G$ ,  $B$  contributes to  $H$ , and

$$\begin{aligned} q^A &= \frac{\rho}{a+1}, q^B = \frac{(1-\rho)}{\beta+1}, \\ G &= \frac{a\rho}{a+1}, H = \frac{\beta(1-\rho)}{\beta+1} \end{aligned}$$

This is the ‘pure separate sphere’ case, in which each public good is funded by one agent.

- if

$$\frac{\beta(a+1)}{\alpha + \alpha\beta + a\beta + \beta} < \rho \leq \frac{\beta(\alpha + a + 1)}{\alpha + \alpha\beta + a\beta + \beta}$$

then  $A$  contributes to  $G$  and  $H$ ,  $B$  contributes to  $H$ , and

$$\begin{aligned} q^A &= \frac{\beta}{\alpha + \alpha\beta + a\beta + \beta}, q^B = \frac{\alpha}{\alpha + \alpha\beta + a\beta + \beta}, \\ G &= \frac{a\beta}{\alpha + \alpha\beta + a\beta + \beta}, H = \frac{\alpha\beta}{\alpha + \alpha\beta + a\beta + \beta} \end{aligned}$$

and again demand does not depend on  $\rho$ .

- finally, if

$$\frac{\beta(\alpha + a + 1)}{\alpha + \alpha\beta + a\beta + \beta} < \rho$$

then  $A$  contributes to  $G$  and  $H$ ,  $B$  contributes to no public good, and

$$q^A = \frac{\rho}{1 + a + \alpha}, q^B = (1 - \rho), G = \frac{a\rho}{1 + a + \alpha}, H = \frac{\alpha\rho}{1 + a + \alpha}$$



It is simple to show that the outcomes are inefficient for all values of  $\rho \in (0, 1)$ . At the endpoints the outcomes are efficient since then one or other person is a dictator. If we take a bargaining model which has the non-cooperative outcomes as a breakdown point then the household demands for both public goods will be higher than in the non-cooperative case (with equality at the endpoints). Importantly, the bargaining outcomes will then have intervals of income pooling over the same values of  $\rho$  as the non-cooperative case.

These results are summarized in figure 1, in which the horizontal axis represents the values of  $\rho$  and the vertical axis the expenditures on the two public goods. We take values of

$$\{a, \alpha, b, \beta\} = \{5/3, 8/9, 15/32, 1/2\}$$

which gives ‘join’ points at  $\rho = \{1/8, 1/3, 1/2, 2/3\}$ . In interval *I* person *A* does not contribute to either public good. As income is transferred to her she spends it on her private good and *B* cuts back expenditures on both public goods and his private good. Thus we see that expenditure on good *G* falls even though person *A* cares relatively more for this good. As an example, transferring income from father to mother will not necessarily lead to higher expenditures on children even if the mother cares more for the children. At the value  $\rho = 1/8$  person *A* starts to contribute to good *G* and we enter an interval of income pooling (*II*). As even more income is transferred to *A* we reach a point ( $\rho = 1/3$  in this case) at which *B* stops contributing to good *G*. This gives the pure separate spheres interval *III*. Intervals *IV* and *V* are obvious counterparts to *II* and *I* respectively. This figure shows many of the important features of our model:

- Local income pooling will hold for some values of the distribution of income but not for others.
- The household demands for public goods are not necessarily monotone in the distribution of income, even if one partner cares relatively more for one good than the other.
- The join points for the different regimes are the same across all goods (we do not show the private good expenditures but this property holds for them as well; see the conditions above). This is potentially important for empirical work; without this property, the chances of successfully devising powerful tests for the patterns displayed here would be remote. On the other hand, if preferences vary across women (different *a* and  $\alpha$  for different households) and across men (different *b* and  $\beta$  for different households) then the join points themselves will be heterogeneous which will have to be taken into account.

### 3 Conclusion

We have considered a model of household allocation to different goods. We have shown that if preferences are egoistic then the voluntary contributions

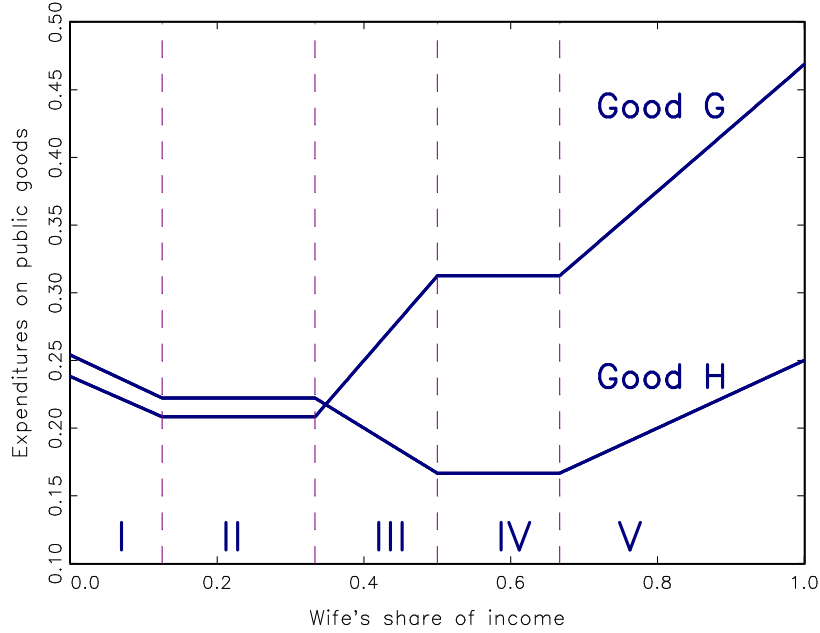


Figure 1: Household demands for public goods.

game gives two distinct regimes for household behaviour. In the first regime there is one public good to which both partners contribute and we have local income pooling. In the second regime, the two partners contribute to distinct sets of public goods (separate spheres) and a local re-distribution of income will lead to a change in household demands. One important corollary of the latter is that a reallocation to  $A$  may lead to a decrease in the household demand for the public good that  $A$  values most. This analysis also has implications for bargaining models if the household uses the non-cooperative outcomes suggested here as a breakdown point. In that case the bargaining outcomes will inherit the local income pooling from the non-cooperative model. Moreover, the analysis also implies the strong restriction that the only distribution factor<sup>5</sup> is relative income

Allowing for sharing (so that  $A$ 's preferences are represented by a weighted sum of her utility function and his utility function and similarly for  $B$ ) leaves the analysis unchanged, except that we add flat (local income pooling) segments to the demands for all goods at extreme values of the household distribution of income (that is,  $\rho$  close to zero or unity). This follows since at such values the high income and caring person will be effectively transferring resources to the

<sup>5</sup>A distribution factor is a household variable that enters the household demand functions but does not influence the preferences of the two partners.

low income partner and any local re-distribution is simply undone. Once again, any bargaining model will inherit these flats at extreme values of the within household income distribution.

The positive and policy implications of our analysis are quite sharp: even if households do not have a common utility function (the unitary model) they may exhibit local income pooling for some values of the within household distribution of income. When they do not pool income locally it must be that they are contributing to separate spheres. The exact importance of the local income pooling intervals and the separate spheres intervals will depend on preferences. For some values of preferences we may have that the household will almost never respond to changes in the within household allocation of income and that targeting income will have little effect. We end by emphasising that we stress the ‘may’ in the preceding sentences. We do not make the general claim that we believe redistribution or targeting does not matter in general - alternative theories to the ones we have analysed here will give more or less income pooling and potential for targeting. Whether or not households pool income (locally or globally) is, in the end, an empirical issue.

## References

- [1] Bergstrom, Theodore, Lawrence Blume and Hal Varian (1986), “On the private provision of public goods”, *Journal of Public Economics*, 29, 25-49.
- [2] Blundell, R., P.-A. Chiappori and C. Meghir (2004), “Collective Labor Supply With Children”, mimeo, Institute for Fiscal Studies, London.
- [3] Kemp, M (1984), “A note on the theory of international transfers”, *Economic Letters*, 14, 259-262.
- [4] Lechene, Valérie and Ian Preston (2005), “Household Nash Equilibrium with Voluntarily Contributed Public Goods”, Oxford Department of Economics Working Paper.
- [5] Lundberg, Shelly and Robert A. Pollak (1993), “Separate spheres bargaining and the marriage market”, *Journal of Political Economy*, 101, 988-1010.
- [6] Warr, Peter (1983), “The private provision of a public good is independent of the distribution of income”, *Economic Letters*, 13, 207-11.