Does product market competition improve the labour market performance? *

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November 21, 2006

Abstract

Many empirical and theoretical studies have recently emphasized the role of product market competition on labour market outcomes. In this paper, I construct a general equilibrium model where the labour market exhibits search frictions, whereas Cournot competition is assumed in the goods market. The properties of the long run free-entry equilibrium show that a loosely regulated product market fosters employment growth, a result in accordance with most of the literature. However, from a normative viewpoint, after having amended the search externalities by the so-called Hosios condition (stating that workers' bargaining power must be equal to elasticity of the expected duration of filling a vacancy), both the level of employment and the degree of competition tend to be inefficiently high. Numerical results based on Belgian data conclude that workers' bargaining power should be at least 50 per cent higher than the elasticity of the expected duration of filling a vacancy in order to bridge the gap between the optimal and the *laissez faire* employment rate.

Keywords: product market competition; search-matching equilibrium; barriers to entry.

*I thank Fabien Postel-Vinay, Bruno Van der Linden, Etienne Lehmann, Amina Mahdi, and the participants to the DMMG conference in the University of Algarve for useful discussions and comments. A grant of the Belgian National Bank is gratefully acknowledged .

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1 Introduction

The interactions between product market (de)regulation and labour market performance have been the objective of many empirical and theoretical works in recent years. Does tougher competition in the goods market increase the level of employment in the labour market? According to most of the literature, the answer seems a qualified yes. At a theoretical level, more agents competing in the product market implies a lower mark-up that can be chosen by the single firm and a larger aggregate quantity produced in equilibrium. This in turn raises labour demand, for any given level of wages. Such theoretical prediction seems to be confirmed by recent empirical studies. For instance, according to the OECD Employment Outlook (2006), liberalization in goods market is one decisive factor that helps to explain why some countries (Ireland, Austria, Scandinavia and the Netherlands) experience high employment rates even if their labour markets remain very regulated.

Less attention, however, has been devoted to the welfare implications of product marked (de)regulation on the labour market. The objective of this paper is twofold. First, I want to verify what are the effects of tougher competition in the goods market on employment, wages and hours worked when the labour market present frictions and efficient bargaining is assumed between workers and firms. Second, turning to the normative analysis, I wonder what is the optimal level of competition and employment in such economy.

To perform this task, I construct a general equilibrium model with Cournot competition in the goods sector and matching frictions *à la* Pissarides (2000) in the labour market. The choice of Cournot competition is due to two reasons. First, differently from other papers (for instance Blanchard and Giavazzi, 2003 and Ebell and Haefke, 2006), I am considering a framework in which the number of firms producing in a market is not constant but varies at the equilibrium, so that any firm' strategy depends not only on the actual level of competition, but also on the probability that new competitors will enter the market. The properties of the Cournot equilibrium as the number of players changes are well-known (see Frank, 1965), and it seems therefore an appropriate choice for this kind of analysis. Second, this paper focuses on the long run free-entry equilibrium, where Cournot models are not subject to the critiques sometimes moved to other settings (for instance, free-entry in a monopolistic competition set-up is simply modeled as a change in the elasticity of substitution in the utility function, a parameter that should remain fixed).

More in detail, I consider an economy with a finite and exogenous number I of intermediate sectors, each of them composed by L (employed and unemployed) workers, and only one final consumption good. In the final good sector perfect competition is assumed, whereas firms compete *à la* Cournot in the intermediate sectors. To produce in the intermediate market, any firm needs first to find a worker by posting a vacancy in the labour market. Keeping the assumption of one firm-one job present in standard Pissarides models, the level of employment, and consequently the degree of competition in the product market, can vary between a monopoly, when only one firm is active, and L, when the sector is in full employment. The only margin that matters for the production is the number of hours worked, that, together with the wage, is the result of a bargaining between firms and workers. In a free-entry equilibrium, firms post vacancies as long as they earn positive expected profits. The creation and the destruction of jobs in each market follow a continuous time Markov Chain with a discrete number of states. The probability that one more job is created in one sector is endogenous and depends on the level of unemployment and the number of vacancies posted in that sector. In addition, at a certain exogenous rate, a new intermediate product, replacing an existing one, is invented in the economy and all the jobs present in the "old" sector are destroyed.

The equilibrium properties of the model confirm the results obtained by most of the literature. Lower entry costs or a reduction in workers' bargaining power raises the level of labour market tightness (defined as the number of vacancies per unemployed worker) in each sector. On the other hand, from a normative viewpoint, the conclusion reached is that, in a free-entry equilibrium where the so-called Hosios (1990) condition[1] holds (so that the search externalities are amended), the number of job vacancies created is always inefficiently high. Such excessive number of vacancies tend to increase the level of employment in the aggregate economy and the degree of competition in each intermediate sector, as shown by the numerical results. In other terms, a social planner would select a lower number of firms competing in the goods market, and so a lower employment rate.

This result can appear striking. Actually, it depends on the two kinds of externalities present in the economy. Any firm deciding to enter the market lowers both the probability for other firms to fill their vacancy and, by making the market more competitive, their (expected) profits. These two effects are not taken into account by the single firm, so that entry is more desirable to the entrant than it is to society. In a standard matching model, where product market is assumed to be perfectly competitive, only the former externality is present and the Hosios condition is sufficient to restore the efficiency of the decentralized equilibrium. In this model, the presence of the imperfect competition inefficiency still makes entry too attractive for the entrants, even when the Hosios condition holds. Too much entry implies too many firms competing and the employment level tends therefore to be too high.

The model is finally calibrated on the basis of Belgian data during the period 1997-

[1] It states that workers' bargaining power should be equal to the elasticity of the expected duration of filling a vacancy with respect to tightness.

1998. In the simulation part, I consider how to shorten the gap between the *laissez faire* and the optimal employment rate. A policy aimed at reducing the cost of entering the labour market (in the model represented by the cost of opening a job-vacancy) does not better the performance of the decentralized economy. On the contrary, an increase in workers' bargaining power, by squeezing firm's profits and making entry less desirable to the firms, results effective. Workers' bargaining power should be at least 50 per cent higher than the elasticity of the expected duration of filling a vacancy in order to bridge the gap between the optimal and the *laissez faire* employment rate.

2 Related Literature

The theoretical papers more closely related to this one are Blanchard and Giavazzi (2003), Amable and Gatti (2004), and Ebell and Haefke (2006). Blanchard and Giavazzi consider an economy with monopolistic competition in the goods market and efficient bargaining in the labour market. They conclude that a decrease in the entry costs, enhancing the number of competitors in the long run, raises not only employment but also the real wage, via a reduction in firms' mark-up and, consequently, in the consumption good price. On the other hand, a lower workers' bargaining power reduces the real wage in the short run, but, in the long run, by inducing more firms to enter, it decreases unemployment and restores the wage to the initial pre-deregulation level, because the short-run reduction in the bargaining power is totally offset by the decrease in firms' mark-up. The conclusions they reach are therefore similar to those obtained in this paper. The papers differ in that Blanchard and Giavazzi do not conduct any welfare analysis, ignoring the issue about the optimal degree of competition and the optimal level of employment, whereas I neglect the short-run equilibrium properties of the model.

A different approach is explored by Amable and Gatti. They again build up a general equilibrium model with monopolistic competition, but, in contrast with the other papers, an efficiency wage mechanism is adopted in the labour market. Real wage rigidities impose competing firms to adjust to shocks by varying employment and not prices. More turnover is therefore generated when shocks occur. When an efficiency wage criterion is assumed in the negotiation, the impact of turnover raises wage pressures and may eventually enhance unemployment.

Finally, Ebell and Haefke construct a general equilibrium model where labour market exhibits search frictions, while monopolistic competition is assumed in the goods market. Such a model of imperfect competition is static, in the sense that the level of competition is constant at the steady-state equilibrium. They show first that a decrease in the entry costs or a higher elasticity of substitution between goods has a positive effect on employment, confirming the theoretical predictions of Blanchard and Giavazzi. They calibrate and simulate the model in order to show how much of the performance in the the European and U.S. labour market can be explained in terms of product market deregulation. Finally they turn to the normative analysis, showing that, provided that the Hosios condition holds, the level of employment is inefficiently low, a result in contrast with that obtained in this paper. Such opposite result depend on the different way the social welfare function has been formulated in the two papers. Ebell and Haefke's social planner has to select the quantity produced by a single firm, but not the number of firms that can be active in the market. This choice is then compared with the short-run decentralized equilibrium, where free-entry is not allowed. In such a case, monopolistic competition induces each firm to produce less than the optimal level, in order to secure itself a higher mark-up. So, firms hire less workers than in a competitive optimal framework[2]. In this paper, on the contrary, the social planner problem consists on choosing not only the optimal quantity produced by a single firm, but also the optimal number of firms that must compete in each sector. Such results are then compared with the free-entry long run equilibrium. As in Ebell and Haefke's paper, each firm produce less than the optimum level (so the number of hours worked is inefficiently low), but, in addition, free-entry leads to an excessive number of competitors, so that the level of employment tends to be too high. Moreover, numerical results show that the aggregate volume of work (i.e the amount of hours worked multiplied by the level of employment) is also inefficiently high.

It must be stressed that such excess of entry result is in line with the conclusion exposed by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) about free entry and social inefficiency. Mankiw and Whinston prove that imperfect competition models with an homogeneous good and a fixed cost of entry deliver an inefficiently high level of competition, exactly because of the "business stealing" effect explained above.

Turning to the empirical evidence, several studies conducted by the OECD have emphasized the importance of product market policies in order to understand the relative poor labour market performance of some countries (i.e. Italy, France, Portugal and Greece) with respect to other ones (U.S., Britain and New Zealand). In this avenue, the most recent work is performed by Nicoletti and Scarpetta (2005)[3]. They consider a

[2]Actually, in Ebell and Haefke's framework, also a hiring externality - opposite in sign - emerges. Since the wage is proportional to the marginal revenues, that are decreasing in a monopolistic set-up, firms will be induced to hire more than the optimal level in order to reduce the wage paid to all the workers. Such strategic behaviour has been first studied by Stole and Zwiebel (1996) and extended to matching models by Cahuc and Wasmer (2001). In their model, Ebell and Haefke show that the first, monopolistic effect prevails and firms hire less than in a competitive framework, unless workers' bargaining power is extremely high.

[3]A detailed survey about the cross-country evidence of the impact of product market regulation on employment, growth and innovation is in Schiantarelli (2005).

panel of some OECD countries over the past two decades. The objective is not only to verify a positive causal relationship between product market reforms and the employment rate, but also to check if product and labour market policies are complementary, studying which are the effects of the interaction of such policies. In this kind of estimation, multicollinearity problems can arise, as many policy ans institution variables are often closely correlated. To address this issue, the authors construct three synthetic indicators summarizing the characteristics of the labour market in each country. They also control for the presence of unobservable country specific effects correlated with the explanatory variables, by running a fixed-effect estimation. The conclusion that the authors reach are the following. First, regulations that curb competition and entry have substantially reduced the employment rates in OECD countries over the past two decades. Second, the negative impact of such product market rigidities on employment is much costlier, the more regulated is the labour market. Therefore, product market reforms should induce larger gains in term of employment in countries whose labour market is more rigid.

The rest of the paper is organized as follows. Section 3 presents the model. Section 4 characterizes the decentralized equilibrium. Section 5 analyzes the welfare problem. Section 6 shows the quantitative results obtained. Finally, section 7 concludes.

3 The model

3.1 Preferences and technology

I consider an economy with one final consumption good (the numeraire) and I intermediate goods. The final good market is perfectly competitive, whereas Cournot competition is assumed within each intermediate sector. The final good production function is given by $Y = \sum_{i=1}^{I} F(Q_i)$, where Q_i is the amount of intermediate good i used by the production process of the final good. A linear final good production function is a simplifying hypothesis that allows to concentrate only on the strategic behaviour within the intermediate markets and not among the intermediate markets[4]. Let p_i be the real price of the intermediate good i. Profit maximization in the final good firm leads to $p_i = F'(Q_i)$. I assume that $\frac{\partial^2 F}{\partial^2 Q_i} < 0$ and that the Inada conditions hold.

Time is continuous. In each intermediate sector there are L infinitely-lived and risk-neutral workers; they can be employed only in that industry, so there are I perfectly

[4]Alternatively, one could construct a model with monopolistic competition among the intermediate markets but perfect competition within each market. Ebell and Haefke (2006) perform this task. Assuming imperfect competition both among the sectors and within would complicate the model without adding too much insight.

segmented labour markets. Each firm is made of a (filled or vacant) job, as usual in matching models (see Pissarides, 2000). This means that any intermediate firm can hire only one worker and the only margin that matters for the production is the amount of hours worked. The I labour markets are not competitive but present some unexplained frictions that make the trading process between firms and workers not instantaneous. Therefore, to produce and compete in one sector, a firm has to post a job vacancy, meet a worker and bargain with him about the wage and the number of hours worked. The intermediate firm production function is identical in each sector and is given by $f(l)$, where l is the amount of hours worked supplied by the employee. Function $f(l)$ is assumed to be increasing and concave. The total amount produced of good i at time t is equal to $Q_{i,t} = \sum_j f(l_{i,j}(t))$, the subscript j denoting a generic firm operating in sector i at time t.

On the other side of the market, workers' instantaneous utility is denoted by $\phi(1-l) + vl$, with $\phi(1-l)$ being the utility of leisure ($\phi(1-l)' > 0$ and $\phi''(1-l) < 0$) and v the hourly wage. When unemployed, the worker enjoys an instantaneous utility $\phi(1) - d$, the value of devoting all your time to leisure net to the (constant) cost d of searching for a job.

3.2 The Stochastic Environment

The creation and destruction of jobs in each intermediate market i follows a continuous time Markov chain $X \equiv \{X(t), t \geq 0\}$ that takes values in the set $L = \{0, 1, 2, ...L\}$. The q-matrix $Q \equiv (q_{x,y},\ x, y \in L\ )$ of the chain is given by:

$$\begin{aligned} q_{x,x+1} &= M_x, \quad q_{x,0} = \delta, \quad q_{x,x} = -(M_x + \delta), \quad \text{for} \quad x > 0 \\ q_{x,y} &= 0, \quad y - x > 1, \qquad \text{and} \quad q_{0,1} = M_1, \; q_{0,0} = -M_1. \end{aligned} \tag{1}$$

Following Karlin and Tavaré (1982) and Van Doorn and Zeifman (2005), I refer to a process of this type as a birth process with killing, with M_x and δ respectively being the birth (i.e. the creation of one more job) and the killing (i.e. the destruction of all the jobs in the sector) rate. The birth rate is endogenous. More precisely, $M_x = m(V_{x_i}, L - x_i)$ denotes the rate at which one match is created in sector i when x firms are already active in that market and during a small interval of time dt. V_{x_i} is the number of job-vacancies and $L - x_i$ the number of unemployed workers in sector i. The function $m(.,.)$ is assumed to be identical in every sector, homogeneous of degree one, and increasing in both arguments. As usual in equilibrium matching models, M_x is a sort of black box, capturing the presence of frictions in the labour market.

Note that in a text-book Pissarides model, it is assumed an unique labour market and the measure $M = m(V, U)$ represents the measure of matches produced at each moment in the aggregate economy. The law of motion of employment in such a framework is therefore given by $dE/dt = M_t - E_t\delta$. In this paper, on the contrary, I consider a large number of small and distinct labour markets and $M_{x_i} = m(V_{x_i}, L - x_i)$ represents the *rate* at which a new match is created in a generic labour market i. This approach is preferred to the standard one since this paper studies the dynamic behaviour of firms subject to Cournot competition. Any firm computing its optimal strategy has to consider both the number of competitors present in the market and the rate at which new players will enter. Such stochastic process, where the number of possible entrants in each intermediate market cannot be greater than one in a small interval of time dt allows to model firms' dynamic behaviour, while keeping the model as tractable as possible.

Let define the level of tightness in the labor market as $\theta_{x_i} \equiv \frac{V_{x_i}}{L - x_i}$. So, knowing that $M_{x_i} = (L - x_i)\theta_{x_i} q(\theta_{x_i})$, the rate at which a single worker finds a job when x firms are already active in market i is given by $M_{x_i}/(L - x_i) = \theta_{x_i} q(\theta_{x_i})$ and the rate at which a single firm fills its vacancy is equal to $M_{x_i}/V_{x_i} = q(\theta_{x_i})$. I also define $\eta \equiv \frac{d\,(1/q(\theta))}{d\,\theta} \cdot \theta\, q(\theta)$, the elasticity of the expected duration of filling a vacancy with respect to tightness.

The killing rate δ is assumed exogenous for simplicity. More in detail, I consider that at a Poisson exogenous arrival rate δ a new product is invented in the economy. Such product replaces an existing one, so that the number of sectors I in the economy remains constant over time. All the the jobs in the “old” intermediate good sector are destroyed and massive layoffs occur. To keep the model as simple as possible, I also assume that all the L workers of the sector destroyed start searching for a job in the new one. Such hypothesis about a sector-specific destruction rate wants to be an (admittedly simplified) approximation of a product life-cycle. The economy is subject to a “creative destruction” force that allows the creation of new products but makes the existing ones obsolete. Indeed, as stressed in many marketing studies, the final stage of a product life-cycle does not not necessarily take the form of a slow decline in time[5]. Sometimes, the rise of new goods (often but not always technologically more advanced) makes the decline more steady or even transform it in a “collapse” [6].

Intermediate sectors are identical *ex-ante*, having the same number of workers L, and the same matching and production technology. So I can remove the subscript i. Let

[5] Consider for instance the analysis about “disruptive innovation” pioneered by Christensen (1997).

[6] In a standard Pissarides model, the destruction rate is job-specific, meaning that every match faces a probability of being destroyed. I do not consider this hypothesis for simplicity. A job-specific separation rate would make the asset price equations even more difficult to manage with, since every firm would have to consider both the probability that the sector evolves by one unit and the probability that it decreases by one unit. However, this paper is about the two-ways relationship between imperfect competition in the goods market and search rigidities in the labour market: different assumptions about the destruction rate do not affect the outcomes of such relationship, especially if tractability imposes to consider such rate as exogenous.

$\pi_{x,t}$ be the probability that a time t there are x active firms in a generic intermediate market. Then:

$$\begin{aligned} \pi_{x,\,t+dt} &= [\,1-\delta dt - M_x dt\,] \cdot \pi_{x,t} \;+\; M_{x-1}dt \cdot \pi_{x-1,\,t} \quad \forall x \in [1,2,...,L], \\ \pi_{0,\,t+dt} &= [\,1 - M_0 dt\,] \cdot \pi_{0,t} \;+\; \delta dt \cdot \sum_{x=1}^{L} \pi_{x,\,t}. \end{aligned} \tag{2}$$

One can look for a steady-state probability distribution, where $\pi_{x,\,t+dt} = \pi_{x,t}, \; \forall t$. Expressing π_x in terms of π_{x-1} and knowing that $\sum_{x=1}^{L} \pi_x = 1 - \pi_0$ yields:

$$\begin{aligned} \pi_x &= \frac{M_{x-1}}{M_x + \delta} \cdot \pi_{x-1} \quad \text{with } x \in [\,1,2,.,L\,], \\ \pi_0 &= \frac{\delta}{\delta + M_0}. \end{aligned} \tag{3}$$

Finally, solving backward, one obtains:

$$\pi_x = \prod_{n=0}^{x-1} \frac{M_n}{M_{n+1} + \delta} \cdot \pi_0 \quad = \quad \frac{\delta}{M_x + \delta} \cdot \prod_{n=0}^{x-1} \frac{M_n}{M_n + \delta} \tag{4}$$

The probability π_x that in one intermediate sector x firms compete in the market depends on L, δ and the endogenous probabilities $\theta_n q(\theta_n)\ \forall n \in [0,\, 1,\, 2, \ldots x]$.

If I is sufficiently large, I can apply the law of large numbers and define the aggregate level of employment as:

$$E = \sum_{x=0}^{L} x \cdot \pi_x \cdot I. \tag{5}$$

Of course, the level of unemployment is given by: $U = \sum_{x=0}^{L} (L - x) \cdot \pi_x \cdot I$.

Before turning to the Bellman equations of the model, a last remark must be highlighted. Assuming matching frictions in labour markets composed by a relatively small number of workers L may appear odd; usually, equilibrium matching models are adopted to mimic the behaviour of aggregate labour markets. However, this does not mean in principle that search frictions should be negligible if the number of potential traders in the market is small. Indeed, the assumption of constant returns to scale for matching functions implies that the magnitude of frictions (trade costs, asymmetry of information, geographical distances) in the economy does not depend on the number of people searching for a job or firms opening a vacancy.

3.3 Asset price equations

At each moment, the timing of decisions is by assumption the following:

1. Intermediate firms enter the market by posting vacancies. This costs a fixed amount h per unit of time. Jobless workers search for a job.

2. At a certain endogenous rate, a firm meets a worker and both the wage and the number of hours worked are bargained.

3. If an agreement is reached, production occurs in the intermediate-goods sector. Intermediate firms compete *à la* Cournot to sell their goods to the final sector. Total surplus is shared between the worker and the firm.

4. An exogenous fraction δ of new sectors emerge in the economy and, consequently, δI existing ones become unproductive. All the jobs in these "old" sectors are destroyed. A worker employed in a sector destroyed enter unemployment and start searching for a job in the new one. As it will soon be clear, workers have no incentive to quit.

Individuals have no access to capital markets. Let r be the discount rate common to all agents. The expected lifetime income for an unemployed worker in a sector with x competitors, $W_U(x)$ solves the following equation:

$$\begin{aligned} rW_U(x) \;&=\; \phi(1) - d \,+\, \theta_x q(\theta_x)\left[W_E^*(x+1) - W_U(x)\right] + \\ &+ (L - x - 1)\,\theta_x q(\theta_x)\left[W_U(x+1) - W_U(x)\right] + \delta\left[W_U(0) - W_U(x)\right], \end{aligned} \tag{6}$$

with $x \in [0, 1.., L-1]$. Being unemployed when the level of employment is equal to x is like holding an asset that pays you a dividend of $\phi(1) - d$ and at a rate $\theta_x q(\theta_x)$ it can be transformed into employment (hence, $x+1$ jobs are active in that market). The superscript * indicates that $W_E^*(x+1)$ is the result of a optimal negotiation between workers and firms. In addition, the value of the asset can also change because at a rate $(L-x-1)\,\theta_x q(\theta_x)$ some other unemployed worker can find a job. In that case, the value of being unemployed shifts from $W_U(x)$ to $W_U(x+1)$. Finally, at a rate δ that sector can become obsolete in the economy. All the workers employed there lose their job and start their unemployment spell in the new sector. The capital gain will be equal to $W_U(0) - W_U(x)$.

Consider the probability that another worker but you is hired and so employment increases by one unit. This event is taken into account by every agent, for one more firm in the market changes the quantity produced (and so the price) in the Nash equilibrium of the Cournot game. In a standard matching model, on the contrary, firms and workers are price takers in the product market and such price variation is ignored by the single agent computing his expected lifetime income.

Similarly, the asset price equation for a worker employed in a sector with x competitors is equal to:

$$\begin{aligned} rW_E^*(x) \;=\; & v_x^* l_x^* \;+\; \phi(1-l_x^*) \;+\; \delta\left[W_U(0) - W_E^*(x)\right] + \\ & + (L-x)\theta_x q(\theta_x)\left[W_E^*(x+1) - W_E^*(x)\right] , \end{aligned} \tag{7}$$

with $x \in [1, 2, ..L]$.

On the other side of the market, the value of an active firm with $x-1$ competitors takes the following form:

$$\begin{aligned} rJ_E^*(x) \;=\; & p(Q_x^*)\, f(l_x^*) - v_x^* l_x^* \;-\; \delta\left[J_E^*(x) + J_V(0)\right] \\ & + (L-x)\theta_x q(\theta_x)\left[J_E^*(x+1) - J_E^*(x)\right] , \end{aligned} \tag{8}$$

with $x \in [1, 2, ..L]$. The term $p(Q_x^*)$ denotes the equilibrium price when x firms are competing in the market, while $J_V(0)$ is the value of a vacancy when the sector is destroyed. More precisely, $p(Q_x^*) = F'(Q_x^*)$. Moreover, I define $p'(Q_x) \equiv \partial p(Q_x)/\partial f(l_x)$.

4 Equilibrium

4.1 Bargaining and Cournot-Nash equilibrium

Firms and workers bargain over wages and hours worked. I assume continuous renegotiation, meaning that every employer-employee pair renegotiates the level of the wage and the numbers of hours worked every time a change in the demand occurs because a new competitor enters the market[7]. An axiomatic Nash solution is considered. I impose that the threats points for workers and firms in the Nash program are not their options outside the match (respectively, W_U and J_V), but their utilities of remaining together and producing nothing, until the level of competition changes. I make this choice for two reasons. The first one is tractability. Assuming, as in a standard Pissarides model, that the threats points are the outside options does not rule out the existence of an equilibrium, but makes the model less tractable (details are available on request).

[7]Assuming continuous renegotiation seems more “rational” than imposing that the wage and the hours worked remain constant whatever the conditions in the goods market. If this were the case, then, for instance, a worker would receive a really high wage even when the product market is very competitive only because he was hired when there was an monopoly. In other terms, wages and hours worked should instantaneously adjust to changes in firms’ profits.

Second, in this framework with imperfect competition in the goods market, adopting the values of remaining together without producing as threat points seem a more convenient and realistic choice. Rosen (1997) and Hall and Milgrom (2006) pursued a bargaining game very similar to this one. The arguments they put forward are convincing: in reality, workers (or unions) and firms negotiate without seriously considering either permanent resignations or discharging employees as an option. A disagreement over wages and hours worked usually implies a delay in the production, strikes, not massive lay-offs or quits. Moreover, besides the points advanced by Hall and Milgrom, another argument can be raised with respect to this model. Instantaneous renegotiation implies that each firm-worker pair bargains wages and hours worked every time a new job is formed in that market. In other words, wages and hours worked are bargained not only by workers (respectively, firms) that have justed ended their unemployment (resp. vacancy) spell, but also by existing pairs that have to change their strategy in the Cournot game. It seems more appropriate, especially for such workers and firms, to assume that in the case of failure of an agreement they decide not to leave. One can think for instance that workers are on strike and nothing is produced. Only a change in the demand conditions, induces firms and workers to come back to the negotiation.

I assume therefore that the threats points for an employee and an employer when the negotiation fails and are respectively given by:

$$r\bar{W_E}(x) \;=\; \phi(1) \;+\; \delta\left[W_U(0)-\bar{W_E}(x)\right] \;+\; (L-x)\theta_x q(\theta_x)\left[W_E^*(x+1)-\bar{W_E}(x)\right]. \quad (9)$$

$$r\bar{J_E}(x) \;=\; -\delta\left[\bar{J_E}(x)-J_V(0)\right] \;+\; (L-x)\theta_x q(\theta_x)\left[J_E^*(x+1)-\bar{J_E}(x)\right]. \quad (10)$$

If an agreement is not concluded, the worker remains employed, he does not receive any wage and enjoys an instantaneous utility of $\phi(1)$. Still, at a rate δ that sector becomes unproductive. If the number of competitors in that market changes, a new bargaining process starts. Similarly, the firm that has not reached an agreement does not produce, does not pay the wage and it is subject to the same probability events of the worker.

I define $w \equiv v \cdot l$ and solve the Nash maximization problem with respect to $\{w, l\}$ instead of $\{v, l\}$:

$$\begin{aligned} w_x^*,\, l_x^* \;&=\; \text{argmax}\left[W_E(x)-\bar{W_E}(x)\right]^{\beta}\left[J_E(x)-\bar{J_E}(x)\right]^{1-\beta} \\ &\text{s.t.} \\ W_E(x) \;&>\; W_U(x-1) \\ J_E(x) \;&>\; J_V(x-1) \quad \text{with}\, x\in[1,2,..L]. \end{aligned} \quad (11)$$

$J_V(x-1)$ represents the expected discounted value of a vacancy when $x-1$ firms compete in the market. In Appendix 1, I show that the solution of (11) coincides with the equilibrium of an extensive form game with workers and firms alternating each other in making offers in the limit case in which parties have only one instant to make their bargain. The constraints imposed in the maximization imply that the worker (the firm) always has the possibility to abandon the negotiation and become unemployed (an idle vacancy) if this choice makes him (it) better off. I assume, as Rosen (1997) and Hall and Milgrom (2006) do, that such constraints are not binding: no player has an incentive to quit the negotiation and this holds $\forall x \in [1, 2, ..L]$.

Computing the F.O.C.s yields :

$$\beta \left[J_E^*(x) - \bar{J}_E(x)\right] = (1-\beta)\left[W_E^*(x) - \bar{W}_E(x)\right].$$

$$\beta \frac{\phi'(1-l_x^*)}{W_E^*(x) - \bar{W}_E(x)} = (1-\beta) \frac{f'(l_x^*)\left[p'(Q_x^*)f(l_x^*) + p(Q_x^*)\right]}{J_E^*(x) - \bar{J}_E(x)},$$

$\forall x \in [1, 2, ..L]$. By using equations (8), (7), (10) and (9), I get the following equilibrium equations of wages and hours worked :

$$w_x^* = \beta\, p(Q_x^*)\, f(l_x^*) + (1-\beta)\left[\phi(1) - \phi(1-l_x^*)\right] \quad (12)$$

$$f'(l_x^*)\left[p'(Q_x^*)f(l_x^*) + p(Q_x^*)\right] = \phi'(1-l_x^*). \quad (13)$$

$\forall x \in [1, 2, ..L]$. For every x, equations (12) and (13) define the equilibrium values of l_x and w_x. Equation (12) has a straightforward interpretation. The wage is a weighted average of the total revenues obtained in the intermediate sector ($p_x f(l_x)$) and the opportunity cost of employment in terms of hours worked ($\phi(1) - \phi(1-l_x)$). The weights are given by the bargaining power of workers and firms, β and $1-\beta$. If the worker has no bargaining power, he receives an instantaneous utility from being employed exactly equal to $\phi(1)$ and so his expected lifetime income is equal to $r\bar{W}_E(x)$, the threat point in the Nash program. On the other hand, when $\beta = 1$, the firm has no bargaining power and all the profits earned in the market accrue to the employee. Its expected discounted value is then equivalent to $r\bar{J}_E(x)$.

Equation (13) looks very similar to a standard solution of a $x-$players Cournot game. I restrict the attention only on symmetric equilibria, where all firm-worker pairs in the market produce exactly the same quantity $f(l_x^*)$. Each worker-firm pair maximizes its profit, given the optimal strategy of the other players. In equilibrium, the marginal revenue for a firm must be equal to the marginal utility of leisure for a worker[8].

[8]Differentiating equation (13) with respect to l_x, I obtain:

$$f''(l_x)\left[p'(Q_x)f(l_x) + p(Q_x)\right] + f'(l_x)^2\left[p''(Q_x)f(l_x) + 2p'(Q_x)\right] + \phi''(1-l_x).$$

4.1.1 Equilibrium wages and hours worked

I henceforth drop the superscript * to simplify the notation. Computing the derivative of w with respect to l, I get:

$$\frac{dw_x}{dl_x} = \beta f'(l_x)\,[p'(Q_x)f(l_x) + p(Q_x)] + (1-\beta)\phi'(1-l_x) = \phi'(1-l_x) > 0.$$

In l_x/w_x space, equation (13) is a vertical line, whereas (12) is a monotonically increasing function (see Figure 1). Moreover, some standard properties of Cournot models are fulfilled: As the number of competitors x increases, the quantity produced by a single firm, $f(l_x)$, decreases, whereas the aggregate quantity Q_x increases[9]. So the total wage decreases as x increases:

$$w_{x+1} - w_x = \beta[p(Q_{x+1})f(l_{x+1}) - p(Q_x)f(l_x)] + (1-\beta)[\phi(1-l_x) - \phi(1-l_{x+1})] < 0,$$

Of course, since $f(.)$ in an increasing function, also hours worked l_x at single firm level go down when competition gets tougher. Therefore, people employed in more competitive sectors get lower wages but more leisure time. The former effect outweighs the latter: Using (12) and ignoring for a moment that x is an integer, the instantaneous utility of an employed worker, $w + \phi(1-l)$, is decreasing in x:

$$\frac{d\,[w_x + \phi(1-l_x)]}{d\,x} = \beta\left\{\frac{d\,p(Q_x)}{d\,Q_x}\frac{d\,Q_x}{d\,x}f(l_x) + \frac{d\,l_x}{d\,x}\,[p(Q_x)f'(l_x) - \phi'(1-l_x)]\right\} < 0.$$

The first term inside the graph is negative because Q_x is increasing in x; the second term is also negative since l_x decreases in x, while the expression inside the square brackets is positive by (13). Figure 2 provides a graphical explanation of these results.

4.2 Free-entry in vacancy creation

To close the model and find the equilibrium values of θ_x, a free-entry condition in vacancy creation is introduced. Firms enter one intermediate market as long as the expected return of posting a vacancy is non negative. This means that:

$$r J_V(x) = 0 \quad \forall\, x \in [0, 1, ..L-1] \tag{14}$$

As usual in this kind of models (see Tirole, 1988), a sufficient condition for this equation to be negative is $p''(Q_x) < 0$.

[9]The necessary assumptions to prove such properties are satisfied (demand twice differentiable and tending to 0 for Q_x sufficiently large, cost function increasing and twice differentiable, profit function strictly concave). For the complete proof, I refer to Frank (1965).

The expected discounted value of a job when $x+1$ agents are active in a market must be equal to the expected cost of filling a vacancy:

$$J_E(x+1) = \frac{h}{q(\theta_x)} \quad \forall x \in [0,1,2,..L-1] \tag{15}$$

Finally, using (8), (14), and (15) one gets:

$$\frac{h}{q(\theta_{x-1})} = \frac{p(Q_x)f(l_x) - w_x + (L-x)\theta_x h}{r + \delta + (L-x)\theta_x q(\theta_x)} \qquad \forall x \in [1,2,..L]. \tag{16}$$

The RHS represents the expected duration of filling a vacancy when $x-1$ firms are already active in the market. At the LHS, the expected profits are composed of two terms: profits attained at state x (that is $p(Q_x)f(l_x) - w_x$) and all the profits that can be earned with more than x competitors weighted by the rate M_x (since $(L-x)\theta_x h = M_x \frac{h}{q(\theta_x)} = M_x \cdot J_E(x+1)$).

The L equations in (16) represent the equilibrium system of the model, with a vector $[\theta_0, \theta_1, ..., \theta_{L-1}]$ of unknown variables. Knowing the values of θ_x $\forall x$, one can derive the steady-state probability distribution of states, $[\pi_0, \pi_1, ..., \pi_L]$, and the aggregate level of employment through equations (4) and (5).

Note that for $x = L$ we have:

$$\frac{h}{q(\theta_{L-1})} = \frac{p(Q_L)f(l_L) - w_L}{r + \delta} \tag{17}$$

Labour market tightness θ_{L-1} does not depend on other values of θ. The endogenous variables l_L and w_L are uniquely defined by the F.O.C.s (12) and (13) evaluated at $x = L$. I can therefore solve the system in (16) "backward", starting from θ_{L-1} and going back to θ_{L-2}, θ_{L-3}, ..., θ_0. The system in (16) has therefore a unique equilibrium in tightness levels, $[\theta_0, \theta_1, ..., \theta_{L-1}]$.

4.2.1 Properties of labour market tightness

I am interested to know how the equilibrium value of tightness θ_x changes with x. The following lemma summarizes the results:

Lemma 1 $\theta_x < \theta_{x-1}$, $\forall x \in [1,2,..L-1]$. *Hence,* $M_x < M_{x-1} \forall x \in [1,2,..L-1]$.
.

Proof. First, let denote for simplicity $P_x \equiv p(Q_x)f(l_x) - w_x$ $\forall x \in [1,2,..L]$ and recall that P_x is decreasing in x (firms' revenues decrease with competition). Knowing by (17) that $r+\delta = \frac{P_L q(\theta_{L-1})}{h}$, equation (16) can be written as:

$$\frac{1}{q(\theta_{x-1})} = \frac{P_x + h\,(L-x)\theta_x}{P_L\, q(\theta_{L-1}) + h\,(L-x)\theta_x q(\theta_x)}.$$

Multiplying both sides by $q(\theta_x)$, one gets:

$$\frac{q(\theta_x)}{q(\theta_{x-1})} = \frac{P_x\, q(\theta_x) + h\,(L-x)\theta_x q(\theta_x)}{P_L\, q(\theta_{L-1}) + h\,(L-x)\theta_x q(\theta_x)} \qquad \forall\, x \in [\,1,..L\,]. \tag{18}$$

Consider the case $x = L-1$. Equation (18) evaluated at $x = L-1$ implies that $q(\theta_{L-1}) > q(\theta_{L-2})$ if and only if $P_{L-1} > P_L$. This is always the case, since firms' revenues P_x decrease with competition.
Now consider the case $x = L-2$. Again, equation (18) evaluated at $x = L-2$ implies that $q(\theta_{L-2}) > q(\theta_{L-3})$ if and only if $P_{L-2}\, q(\theta_{L-2}) > P_L\, q(\theta_{L-1}) = h(r+\delta)$. This yields:

$$\begin{aligned}
\frac{P_{L-2}}{r+\delta} > \frac{h}{q(\theta_{L-2})} &= \frac{P_{L-1} + h\theta_{L-1}}{r+\delta+\theta_{L-1}q(\theta_{L-1})} \iff \\
(r+\delta)\,P_{L-1} + h\,(r+\delta)\,\theta_{L-1} &< (r+\delta)\,P_{L-2} + P_{L-2}\,\theta_{L-1}q(\theta_{L-1})
\end{aligned}$$

Since $P_{L-2} > P_{L-1}$, a sufficient condition for the last inequality to hold is:

$$\begin{aligned}
h(r+\delta) &< P_{L-2}\, q(\theta_{L-1}) \iff \\
\frac{h}{q(\theta_{L-1})} &< \frac{P_{L-2}}{r+\delta} \iff \\
\frac{P_L}{r+\delta} &< \frac{P_{L-2}}{r+\delta}
\end{aligned}$$

The last inequality is always verified since P_x is decreasing in x. So $q(\theta_{L-2}) > q(\theta_{L-3})$ holds.
With $x = L-3$, by (18), one gets that $q(\theta_{L-3}) > q(\theta_{L-4})$ if and only if $P_{L-3}\, q(\theta_{L-3}) > P_L\, q(\theta_{L-1}) = h(r+\delta)$. Following the same steps, one gets:

$$\begin{aligned}
\frac{P_{L-3}}{r+\delta} > \frac{h}{q(\theta_{L-3})} &= \frac{P_{L-2} + 2h\theta_{L-2}}{r+\delta+2\theta_{L-2}q(\theta_{L-2})} \iff \\
(r+\delta)\,P_{L-2} + 2h\,(r+\delta)\,\theta_{L-2} &< (r+\delta)\,P_{L-3} + P_{L-3}\,2\theta_{L-2}q(\theta_{L-2})
\end{aligned}$$

A sufficient condition for the last inequality to hold is:

$$\begin{aligned}
h(r+\delta) &< P_{L-3}\, q(\theta_{L-2}) \iff \\
\frac{h}{q(\theta_{L-2})} &< \frac{P_{L-3}}{r+\delta}
\end{aligned}$$

The last inequality always holds, since we have just shown that $\frac{h}{q(\theta_{L-2})} < \frac{P_{L-2}}{r+\delta}$ and $P_{L-3} > P_{L-2}$. Therefore $q(\theta_{L-3}) > q(\theta_{L-4})$.

The same steps can be undertaken for any other value of x. So, $\theta_x < \theta_{x-1}$, $\forall\, x \in [\,1, ..L\,]$. See figure 3.

■

What Lemma 1 simply states is that the number of vacancies posted decrease as competition gets tougher. This makes sense, since a more competitive product market squeezes firms' profits, dampening the incentives in vacancy creation. Such negative effect on the supply side of the labour market outweighs the reduction in the number of unemployed workers as x goes up, so that $\theta_x \equiv V_x/(L-x)$ is decreasing in x. Equation (17) implies that expected discounted profits are equal to zero for any given level of competition in the goods market. A trade-off arises: in less competitive markets firms can attain higher revenues but stand in a longer queue to fill their vacancies.

4.3 The Effects of the Deregulation in Products and Labour Markets

To simplify the analysis, I assume henceforth a Cobb-Douglas matching function, $M_x = a\,(L-x)^{\eta} \cdot V_x^{1-\eta}$, in line with the results of Pissarides and Petrongolo (2001). The results are summarized in the following Proposition:

Proposition 1 *If* $\frac{\theta_{x+1}}{\theta_x} > (1-\eta)^{\frac{1}{\eta}} \quad \forall\, x \in [\,0, 1, ..L-1\,]$*, then a decrease in workers' bargaining power* β *or the cost of opening a vacancy* h *raises* $\theta_x \;\; \forall\, x \in [\,0, 1, ..L\,]$.

Proof. Consider the case of a decrease in β (the proof for h is identical). By equation (12), one gets $\frac{dw_x}{d\beta} > 0\; \forall x$.[10] Then, looking at (17), $\frac{d\theta_{L-1}}{d\beta} < 0$. In the case $x \neq L$, one gets:

$$\frac{d\theta_{x-1}}{d\beta} = \frac{\partial\theta_{x-1}}{\partial\beta} + \frac{\partial\theta_{x-1}}{\partial\theta_x}\frac{\partial\theta_x}{\partial\beta} + \frac{\partial\theta_{x-1}}{\partial\theta_x}\frac{\partial\theta_x}{\partial\theta_{x+1}}\frac{\partial\theta_{x+1}}{\partial\beta} + ... + \frac{\partial\theta_{x-1}}{\partial\theta_x}\frac{\partial\theta_x}{\partial\theta_{x+1}} \cdot ... \cdot \frac{\partial\theta_{L-1}}{\partial\beta} =$$
$$= \frac{\partial\theta_{x-1}}{\partial\beta} + \frac{\partial\theta_{x-1}}{\partial\theta_x} \cdot \frac{d\theta_x}{d\beta}$$

[10]Differentiating (12) with respect to w_x, we have:

$$\frac{dw_x}{d\beta} = p(Q_x)f(l_x) + \phi(1-l_x) - \phi(1) > 0.$$

Of course, this term is assumed to be positive $\forall x$. Otherwise, firms' profits and workers' utility would be negative and production would never occur.

From (16), it it straightforward to see that the partial derivative $\frac{\partial \theta_{x-1}}{\partial \beta}$, capturing the direct effect of β on θ_{x-1}, is negative $\forall x$, via the positive impact of β on w_x. Moreover (computations are presented in Appendix 2):

$$\frac{d\,\theta_{x-1}}{d\,\theta_x} = \frac{(L-x)\ [\,(1-\eta)q(\theta_x)\ -\ q(\theta_{x-1})\,]\ q(\theta_{x-1})}{[\,r\ +\ \delta\ +\ (L-x)\theta_x q(\theta_x)\,]\ q'(\theta_{x-1})}.$$

Such derivative is positive if the term inside the square brackets is negative. This implies:

$$\frac{d\,\theta_{x-1}}{d\,\theta_x} > 0 \ \text{ if } \frac{q(\theta_x)}{q(\theta_{x-1})} < \frac{1}{1-\eta}.$$

Finally, with a Cobb-Douglas matching technology, such inequality becomes $\frac{\theta_x}{\theta_{x-1}} > (1-\eta)^{\frac{1}{\eta}}$.

Therefore, if $\frac{\theta_{L-1}}{\theta_{L-2}} > (1-\eta)^{\frac{1}{\eta}}$, then $\frac{d\theta_{L-2}}{d\beta} < 0$. Reasoning backward, one gets $\frac{d\theta_x}{d\beta} < 0 \ \ \forall x \in [\,0, 1, ..L-1\,]$. A lower bargaining power or a reduction in the cost of opening a vacancy raises labour market tightness.

■

A lower bargaining power for workers reduces the wage and raises firms' expected profits. So, more competitors will enter the labour market by posting a vacancy. A similar effect occurs by lowering the cost of opening a vacancy h. If we consider $\eta = 0.5$, the value mostly adopted in the literature (see again Pissarides and Petrongolo), the inequality $\frac{\theta_x}{\theta_{x-1}} > (1-\eta)^{\frac{1}{\eta}}$ implies that θ_x must not be four times larger than θ_{x+1}. It does not seem a restrictive condition: intuitively, the decrease in firms' profits caused by one more entrant in the market must not be so large to induce a great reduction in vacancy creation.

The effects of a change in θ_x on the probability distribution $[\pi_0, \pi_1, \pi_2, ..., \pi_L]$ are not obvious. The reason is that π_x is a decreasing function of θ_x and an increasing function of θ_m, $m \in [0, 1, 2, L-1]$, $m \neq x$. In Appendix 3, I study the properties of the distribution when δ is close to 0. Even in this simplified case, the only two probabilities whose derivatives can be easily computed are π_L and π_{L-1}. A lower θ_x $\forall x$ reduces π_L and raises π_{L-1}. When more vacancies are posted the probability of reaching full employment goes up. In order to have a clearcut conclusion about the effects of larger θ_x on the probability distribution and aggregate employment, a numerical simulation is performed and exposed in Section 6. The results predict that a lower β or h unambiguously raises aggregate employment.

5 Optimality

In this section, I wonder what is the optimal level of product market competition and labour market tightness in this economy. Consider a centralized economy where a social planner has to choose the optimal number of vacancies and hours worked in any sector and for any given level of employment. Following Shimer (2004), the social welfare function for a representative sector takes the following recursive form:

$$\begin{aligned} rS_x &= \max_{\theta_x, l_x} F(Q_x) + x\phi(1-l_x) + (L-x)\phi(1) - h(L-x)\theta_x + \\ &\quad + (L-x)\, a\, \theta_x^{1-\eta}\, [S_{x+1} - S_x] + \delta\, [S_0 - S_x] \end{aligned} \tag{19}$$

$$\text{s.t. } Q_x = x \cdot f(l_x). \qquad \forall x \in [0, 1, 2, ...L]$$

The social surplus at x level of employment is given by the total amount of the intermediate good produced and the utility of leisure of the (employed and unemployed) workers, net to the cost of opening a vacancy. Moreover, at a rate $M_x = (L-x)\, a\, \theta_x^{1-\eta}$ the level of employment increases by one unit, causing a change of the surplus from S_x to S_{x+1}, and at a rate δ the sector is destroyed and another one is instantaneously created. The constraint in (19) reminds that, differently from the *laissez faire* economy, the social planner considers *ex ante* a symmetric solution, where every firm uses the same amount of hours worked.

The solutions (θ°, l°)s to problem (19) verify the following F.O.Cs:

$$(1-\eta)\, a(\theta_x^\circ)^{-\eta} \cdot [S_{x+1} - S_x] = h \tag{20}$$

$$F'(xf(l_x^\circ)) \cdot f'(l_x^\circ) = \phi'(1-l_x^\circ) \tag{21}$$

The intuition of the above equations is the following. At the social optimum, the cost of marginal increase in θ_x, h, must be equal to the marginal gain, given by $(d\,\theta_x q(\theta_x)/d\,\theta_x)\,[S_{x+1} - S_x] = (1-\eta)\, a(\theta_x^\circ)\, [S_{x+1} - S_x]$. Moreover, the optimal level of hours worked l_x° is such that the increase in production must be equal to the opportunity cost in terms of leisure.

Comparing (13) with (21) one obtains $l_x^\circ > l_x^* \; \forall x$. This inequality holds since $\phi'(1-l)/f'(l)$ is increasing in l and $F'(xf(l_x^\circ))$ is always greater than $p(Q_x^*)+p'(Q_x^*)f(l_x^*)$. So the level of hours worked in equilibrium is always inefficiently low. Notice also that equation (21) would coincide with the outcome of a worker-firm negotiation, were the good market perfectly competitive (since $p(Q_x) = F'(Q_x)$)[11].

[11]Such result hinges on the assumption of workers' additive utility function. In this case, it is like the two parties use their bargaining power only for one component of the negotiation (the wage) and behave in an utilitarian way for the second one (the hours worked). That is also the reason for which β is absent in (13).

Using (20) and (21) and subtracting the optimal solution S_x from S_{x+1} yields:

$$\frac{(r+\delta)h}{a(1-\eta)}(\theta_x^\circ)^\eta = F(Q_{x+1}^\circ) + (x+1)\phi(1-l_{x+1}^\circ) - F(Q_x^\circ) - x\phi(1-l_x^\circ) - \phi(1) + \\ + \frac{\eta}{1-\eta}h\left[(L-x-1)\theta_{x+1}^\circ - (L-x)\theta_x^\circ\right].$$

and, finally, defining

$$\Delta F(Q_{x+1}^\circ) + \Delta(x+1)\phi(1-l_{x+1}^\circ) \equiv F(Q_{x+1}^\circ) + (x+1)\phi(1-l_{x+1}^\circ) - F(Q_x^\circ) - x\phi(1-l_x^\circ),$$

one gets:

$$\begin{aligned} &\frac{r+\delta}{a}(\theta_x^\circ)^\eta + \eta(L-x)\,\theta_x^\circ = \\ &= \frac{1-\eta}{h}\left[\Delta F(Q_{x+1}^\circ) + \Delta(x+1)\,\phi(1-l_{x+1}^\circ) - \phi(1)\right] + \eta(L-x-1)\,\theta_{x+1}^\circ, \end{aligned} \tag{22}$$

A comparison of (22) with the free-entry equilibrium condition (16) delivers the following result:

Proposition 2 *Suppose that the Hosios condition $\beta = \eta$ holds. If the demand function is not too concave, then in the decentralized equilibrium the level of tightness θ_x is inefficiently high $\forall x$.*

Proof. Using the wage equation (12) and the Hosios condition $\eta = \beta$, the decentralized equilibrium condition (16) can be written as:

$$\begin{aligned} &\frac{r+\delta}{a}(\theta_x^*)^\eta + (L-x-1)\,(\theta_{x+1}^*)^{1-\eta}(\theta_x^*)^\eta - (1-\eta)(L-x-1)\,\theta_{x+1}^* = \\ &= \frac{1-\eta}{h}\left[p(Q_{x+1}^*)f(l_{x+1}^*) + \phi(1-l_{x+1}^*) - \phi(1)\right] + \eta(L-x-1)\,\theta_{x+1}^*. \end{aligned} \tag{23}$$

I proceed now in three steps. First, I show that, for all x, the term inside the square brackets in (23) is always larger than the terms inside the square brackets in (22), provided that the demand for the intermediate goods is not too concave. Then I show that $\theta_{L-1}^* > \theta_{L-1}^\circ$. Finally, I prove that $\theta_x^* > \theta_x^\circ$, $\forall x$.

STEP 1 : $p(Q_{x+1}^*)f(l_{x+1}^*) + \phi(1-l_{x+1}^*) > \Delta F(Q_{x+1}^\circ) + \Delta(x+1)\phi(1-l_{x+1}^\circ)$

For the proof, see Appendix 4.

STEP 2 : $\theta_{L-1}^* > \theta_{L-1}^\circ$.

When $x = L - 1$, equations (22) and (23) respectively become:

$$\begin{aligned} \frac{r+\delta}{a}(\theta^{\circ}_{L-1})^{\eta} + \eta\, \theta^{\circ}_{L-1} &= \frac{1-\eta}{h}\left[\Delta F(Q^{\circ}_L) + \Delta\, \phi(1-l^{\circ}_L) - \phi(1)\right] \\ \frac{r+\delta}{a}(\theta^{*}_{L-1})^{\eta} &= \frac{1-\eta}{h}\left[p(Q^{*}_L) f(l^{*}_L) + \phi(1-l^{*}_L) - \phi(1)\right]. \end{aligned}$$

From Step 1, the RHS in the decentralized equilibrium equation is larger than the RHS in the welfare equation. Then, looking at the LHS, $\theta^{*}_{L-1} > \theta^{\circ}_{L-1}$.

STEP 3 : $\theta^{*}_x > \theta^{\circ}_x\ \forall x \in [0, 1, 2, ...L-2]$.

Having shown that $\theta^{*}_{L-1} > \theta^{\circ}_{L-1}$ I can proceed backward and consider the case $x = L-2$. It is then clear that the RHS in (23) is larger than the RHS in (22), since the inequality between the terms in the square brackets has been proved in Step 1 and $\eta(L-x-1)\,\theta^{*}_{L-1} > \eta(L-x-1)\ \theta^{\circ}_{L-1}$ in Step 2. So, the LHS in (23) is larger than the LHS in (22). Consider now the LHS in (23). If:

$$\begin{aligned} &\eta(L-x)\theta^{*}_x \geq \\ &(L-x-1)(\theta^{*}_{x+1})^{1-\eta}(\theta^{*}_x)^{\eta} - (1-\eta)(L-x-1)\theta^{*}_{x+1} \qquad \forall x \end{aligned} \tag{24}$$

then,

$$\frac{r+\delta}{a}(\theta^{*}_x)^{\eta} + \eta(L-x)\,\theta^{*}_x > \frac{r+\delta}{a}(\theta^{\circ}_x)^{\eta} + \eta(L-x)\ \theta^{\circ}_x, \qquad \forall x \tag{25}$$

since the LHS in (25) is larger than the RHS in (23), that in turn is larger than the RHS in (25). But (24) implies:

$$\left(\frac{\theta^{*}_x}{\theta^{*}_{x+1}}\right)^{\eta} - \eta\,\frac{L-x}{L-x-1}\cdot\frac{\theta^{*}_x}{\theta^{*}_{x+1}} - 1 + \eta \leq 0.$$

Such inequality is always verified, provided that $\theta^{*}_x > \theta^{*}_{x+1}$[12]. Then, with (25) being always true, $\theta^{*}_x > \theta^{\circ}_x\ \ \forall x \in [0, 1, 2, ...L-1]$.

■

In the decentralized economy, there are two departures from the competitive framework, namely frictions in the labour market and imperfect competition in the goods market. First of all, Cournot competition leads to an inefficiently low level of hours worked: Each firm, for any given number of competitors x, tends to produce a quantity $f(l_x)$ smaller than the optimal one in order to keep the market price higher.

In addition, the presence of frictions in the labour market makes search externalities emerge, as any firm deciding to post a vacancy fails to consider the decrease in other

[12] When $\theta_x = \theta_{x+1}$, the LHS is negative. Moreover, the function is decreasing in θ_x/θ_{x+1} when $\theta^{*}_x > \theta^{*}_{x+1}$, $\forall\, 0 < \eta \leq 1$.

firm's probability of finding a worker. Such externalities can be amended by the well-known Hosios condition, stating that workers' bargaining power must be equal to the elasticity η. In this economy, however, firm's decision to post or not a vacancy creates another source of inefficiency, absent in standard matching models. Namely, any firm deciding to enter or not the market also fails to consider the reduction in other firms' profits caused by the increase in competition. For this reason the Hosios condition does not guarantee the efficiency of the decentralized equilibrium: it eliminates only the distorsions in entry behaviour caused by search externalities, not those caused by the imperfect competition in the goods market[13]. So, labour market tightness is lower in the social optimum than in a decentralized economy with $\eta = \beta$ because, in posting a vacancy, a social planner also considers the reduction of profits of the incumbent firms. For it, he prefers to have a lower level of competition. In the numerical simulation, it is shown that such excessive number of vacancies posted unambiguously produces an inefficiently high level of employment and competition.

This excess of entry result is in line with the findings of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). Both papers prove that imperfect competition models where firms can enter the market paying a fixed cost deliver an inefficiently high level of competition. Indeed, this paper can be framed in the same environment: It assumes an imperfectly competitive good market where firms can enter only by involving in a costly search in the labour market. What for Mankiw and Whinston is a fixed cost, in this paper corresponds to the expected cost of filling a job vacancy, $h/q(\theta_x)$. Were the labour market perfectly competitive, an infinite number of firms would enter and produce, ensuring perfect competition even in the goods market. The social optimum would then coincide with the decentralized outcome. Simulation results (presented in the next sections) try to quantify the order of magnitude in terms of employment of such excess of entry inefficiency.

6 Quantitative Results

6.1 Calibration

I take the month as unit of time. Data refer to the 1997-1998 period where the stocks were fairly stable in Belgium. To calibrate the model, I make use of various surveys, the quantitative results obtained in Cardulllo and Van der Linden (2006), other statistics collected for the purpose of this study, and results found in the literature. As in

[13] In other words, following Cooper (1999), it can be said that two kinds of strategic substitutabilities emerge in this economy. The entry decision of a single agent negatively affects other agents' decisions because both the probability of filling a vacancy and the profits that can be attained in the goods market go down.

the previous sections, I assume the following Cobb-Douglas matching function $M_x \equiv a(L-x)^{\eta}V_x^{\eta}$. The elasticity η is imposed equal to 0.5, the value mostly adopted in the literature (see Petrongolo and Pissarides, 2001). In Cardullo and Van der Linden, the calibrated value for workers' bargaining power β is 0.5 for the high-skilled sector and 0.56 for the low-skilled one. I set it equal to 0.5, so that the Hosios condition holds. Making use of the zero profit condition in vacancy creation, I calibrate the cost of opening a vacancy h so that the expected duration of unemployment is in line with the findings of Dejemeppe (2005).[14] Parameter a is a scaling factor for h and it is set equal to 0.125, so that the expected duration of filling a vacancy is around 3 months, a value slightly larger than that estimated by Dejemeppe. The discount rate is fixed at 0.004 (5% on an annual basis). The number of workers in each intermediate sector is assumed equal to 20. Of course it is an arbitrary value, chosen in order to have a sufficiently large degrees of product competition. The single firm production function is assumed equal to $f(l) = l^{\epsilon}$, with $0 < \epsilon < 1$. Moreover, the final good production function is given by $Y = \sum_i p_0 Q_i + Q_i^{\lambda}$, so that the demand in each sector i is $p_0 + \lambda Q_i^{\lambda-1}$. Parameters λ and ϵ are respectively set to 0.9 and 0.5, in order to have a ratio $\phi'(1-l)/f'(l)$, increasing and concave in the Cournot F.O.C for hours worked. I assume that hours worked l are in an interval between 0 and 2. Workers' utility of leisure is given by $2^{\gamma} - l^{\gamma}$. The parameter γ is calibrated in order to have to have an average wage in the economy of 1348 (a value in accordance with the results obtained in Cardullo and Van der Linden). In absence of precise estimations about the sector specific destruction rate δ, a value of 0.005 is taken.

6.2 Simulation Results

Figures 5 and 6 show that labour market tightness θ_x is decreasing in x while the steady-state distribution π_x is an increasing function both in the *laissez faire* economy and in the centralized one.

The simulation results are summarized in Table 2 and Table 3. I first evaluate the impact of a decrease on the cost of opening a vacancy h on the average values of the following variables: the wage, the rate of employment ($e = E/L$), the number of hours worked[15], and the volume of work (defined as the total number of hours worked in the economy over their total potential amount, $\sum_{x=0}^{L} l_x \, \pi_x \, / \, L \cdot 2$). In the the first column of Table 2 it is shown the main result: the employment rate in the free-entry

[14] From her analysis of unemployment dynamics in Belgium, the average unemployment duration in 1992 was equal to 2 years in the South of Belgium and to 1.5 years in the North. On the other side of the market, the expected duration of filling a vacancy was estimated to 2.5 months.

[15] The average wage and the average number of hours worked are defined respectively as: $(1/E) \cdot \sum_{x=1}^{L} w_x \, x \, \pi_x$ and $(1/E) \cdot \sum_{x=1}^{L} l_x \, x \, \pi_x$.

equilibrium is higher than the optimal one, the difference being around 9 per cent. In terms of volume of work such discrepancy is around 4 per cent. In the other columns of Table 2, there are listed the effects of a decrease in the cost of opening a vacancy h. Such reduction almost has no impact both on the wage and on the share of time spent working, whereas it slightly raises the employment rate and the volume of work. The discrepancy between the optimal and the decentralized employment level remains fairly stable. A reduction by one and half of the vacancy cost (from 14.500 to 7250 euros) is needed in order to shorten such employment gap by 1.15 per cent. The reason is the following. A lower h decreases the inefficiency cost of one more vacancy created, but at the same time induces more firms to post vacancies. In other terms, the negative externality a single firm creates when entering the market has a lower cost for the society, but more firms generate such externality. The first effect tends to shorten the gap between the optimal and the *laissez faire* outcomes, the second tends to widen it. Bringing down vacancy costs does not better the performance of the decentralized economy.

In table 3, I consider the effects of a change in workers' bargaining power. Keeping the assumption of a matching function elasticity $\eta = 0.5$, I wonder for which value of β the welfare inefficiency can be close to 0. Differently from h, the parameter β does not appear in the welfare function, since the social planner cares only on the total surplus and not on how it is shared between workers and firms. So, a higher β, by squeezing firms' profits and making entry less attractive, could (partially) offset the excess of entry inefficiency. Indeed, with $\beta = 0.75$, the difference between optimal and decentralized employment rate is around 1 per cent and the volume of work gap is almost nil. So, as far as the value of 0.5 can be considered a good proxy of the elasticity in the matching technology, β should be at least 50 per cent larger of η to set to zero the inefficiency gap. In the sensitivity analysis section, I evaluate the performance of the model for different values of η.

6.3 Sensitivity analysis

A sensitivity analysis has been conducted on some parameters of the model. The results are listed in Table 4 and 5. In Table 4, I consider a change in the production function parameters in the intermediate sector ϵ and in the final sector λ, as well as in the workers' utility parameter γ. Such variations do not change the main conclusion of the original model, that is a difference around 9 per cent between the optimal and the decentralized employment rate.

In Table 5, I consider different values for the matching elasticity η. The level of wages and the amount of hours worked barely change, since these variables are chosen via the bargaining process and β is kept equal to 0.5. Employment increases with η. By equation (16), the higher η the larger the expected cost of filling a vacancy ($1/q(\theta_x) = a^{-1}\,\theta_x^{\eta}$) and the lower the discounting factor of future profits ($\theta_x q(\theta_x) = a\,\theta_x^{1-\eta}$). The second effect is stronger: more vacancies are created, raising the employment rate. Keeping workers' bargaining power equal to 0.5, the employment inefficiency gap decreases with η. This is because even the social planner, when η goes up, selects more vacancies for any given level of $L - x$. Such increase is slightly more stronger than in the *laissez faire* equilibrium. In the last row of Table 5 I compute for any η the value of β such that the difference between the decentralized and the optimal employment rate is less than 1 %. Since the inefficency gap decreases with η, a lower β/η ratio is needed to be close to the optimum. With $\eta = 0$, β must be equal to 0.75, 50% larger; with $\eta = 0.7$, β must be set to 0.9, around 30% more.

7 Conclusions

In this paper, the two-way relationship between product market competition and labour market performance has been studied both from a positive and from a normative viewpoint. As far the positive analysis is concerned, it is shown that a lower cost in vacancy creation or a reduction in workers' bargaining power raise aggregate employment, by making entry more attractive for the firms. Such result is in accordance with most of the theoretical and empirical literature on the subject. Turning to the welfare analysis, however, the conclusion reached is that, if the search externalities are corrected, in the decentralized economy, too many vacancies are created and employment is inefficiently high. A "business stealing" effect is at work in such framework: Any single firm deciding to enter the market fails to consider the reduction both in other firms' expected profits and in their probability of finding a worker. Simulation results predicts that, in order be close to the optimal level of employment, workers' bargaining power must be larger than the elasticity η in the matching function. If the latter is imposed to be 0.5, then β must be around 0.75.

Some caveats must be advanced about the model specification. Imposing perfectly segmented labour markets is undoubtedly a major restriction. Workers are locked in their sector unless a new product is invented; then, they shift to such new one. Allowing for cross-sectors search by workers would be a more realistic extensions. Moreover, a job-specific destruction rate, and not only a sector-specific one, should be considered. Finally, it would be also interesting to study the dynamic evolution of the model and not focusing only on the steady state distribution. All these extensions are left for future research.

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Appendix 1: The bargaining game

The bargaining process I pursue is very close to Hall and Milgrom (2006); their model, in turn, is an adapted version of Binmore et al. (1986). The maximization problem in (11) can be seen as a limit case of an extensive form bargaining game of offers and counter-offers. More precisely, consider a bargaining process that takes place over time and where firms and workers alternate in making proposals about the wage and the numbers of hours worked. After a proposal of the counterpart, a player has three options. He can abandon the bargaining (and so get an utility of either J_V or W_U, the outside options of the employer an the employee), disagree and make a counter-offer, accept the offer. Binmore et al. (1986) show that the subgame perfect equilibria of two bargaining games beginning with a proposal either by the employer or the worker are unique. So the value of rejecting an offer and continuing to bargain is uniquely defined.

When the worker (respectively, the firm) decides to reject the other player's offer and make a counter-proposal, he receives an utility flow equal to $\phi(1)$ (resp. to zero), his utility of leisure. I also introduce an hazard rate, s, that the agreement is no longer convenient. In this case, I assume, differently from Hall and Milgrom (2006), that the firm-worker pair is not broken, but they choose to not produce anything until the market conditions change (in this Cournot set-up, that means a variation in the number of competitors). Then, the pair starts a new negotiation. The expected discounted values for a an employer and an employee in the case the production opportunity disappears and x firms active, are given respectively by $\bar{J_V}(x)$ and $\bar{W_U}(x)$ (equations 10 and 9). Consider for instance a negotiation over the wage when employment in that market is equal to x [16]. The time period separating one offer from the next one is τ. Since the value of rejecting an offer and continuing to bargain is uniquely defined[17], the worker's equilibrium strategy is to accept any offer that makes him at least as well-off than both continuing the bargaining and abandoning it. There exists, therefore, a lowest wage w' that makes the worker indifferent between such options and, symmetrically, there exists a highest wage w'' that makes the firm indifferent. It is then clear that the optimal strategy for a worker is to offer always w'' and for a firm to offer always w'. The equations governing the equilibrium are the following:

$$\begin{aligned} W_E(x, w') &= \max\left\{W_U(x-1),\ \phi(1)\tau + e^{-r\tau}\left[\left(1-e^{-s\tau}\right)\bar{W_E}(x) + e^{-s\tau}W_E(x, w'')\right]\right\} \\ J_E(x, w'') &= \max\left\{J_V(x-1),\ e^{-r\tau}\left[\left(1-e^{-s\tau}\right)\bar{J_E}(x) + e^{-s\tau}J_E(x, w')\right]\right\} \end{aligned} \tag{26}$$

I assume, as Hall and Milgrom and Rosen (1997), that neither workers nor firms have

[16]The case of a negotiation over wages and hours worked is similar but more lengthy.

[17]For the proof, I refer to Binmore et al. (1986).

an incentive to abandon the negotiation. In other terms, the constraints in (11) are never binding[18]. Therefore, the system (26) becomes:

$$\begin{aligned} W_E(x,w') &= \phi(1)\tau + e^{-r\tau}\left(1-e^{-s\tau}\right)\bar{W}_E(x) + e^{-(r+s)\tau}W_E(x,w'') \\ J_E(x,w'') &= e^{-r\tau}\left(1-e^{-s\tau}\right)\bar{J}_E(x) + e^{-(r+s)\tau}J_E(x,w') \end{aligned} \tag{27}$$

In equilibrium, $w' = w'' = w$. So, $W_E(x,w') = W_E(x,w'') = W_E(x)$ and $J_E(x,w'') = J_E(x,w') = J_E(x)$. Moreover, letting τ, the period separating one offer from the next, approach 0, I get:

$$(W_E(x) - J_E(x)) = \frac{\phi(1)}{r+s} + \frac{s}{r+s}\left(\bar{J}_E(x) - \bar{W}_E(x)\right) \tag{28}$$

This equation is very similar to equation (17) in Hall and Milgrom (2006). If I assume $s \to 0$, that is the parties have only an instant to make their bargain, the surplus sharing rule will become:

$$W_E(x) - \bar{W}_E(x) = J_E(x) - \bar{J}_E(x). \tag{29}$$

It coincides with the F.O.C. for w_x of the maximization problem in (11) when $\beta = 0.5$.[19] The threats points for an employer and employee are given respectively by $\bar{J}_E(x)$ and $\bar{W}_E(x)$. Using equations (8) and (7), I get:

$$\begin{aligned} &\beta\,\frac{w_x^* + \phi(1-l_x^*) + \delta W_U(0) + (L-x)\theta_x q(\theta_x) W_E(x+1)}{r+\delta+(L-x)\theta_x q(\theta_x)} - \bar{W}_E(x) = \\ &(1-\beta)\,\frac{p(Q_x^*)f(l_x^*) - w_x^* + \delta J_V(0) + (L-x)\theta_x q(\theta_x) J_E(x+1)}{r+\delta+(L-x)\theta_x q(\theta_x)} - \bar{J}_E(x). \end{aligned} \tag{30}$$

Finally, using (9) and (10), I obtain:

$$w_x^* = \beta p_x f(l_x^*) + (1-\beta)\left[\phi(1) - \phi(1-l_x^*)\right].$$

[18] This is possible for h and d sufficiently high.

[19] Assuming a probability β that Nature selects the worker as first mover in the game yields the generalized Nash solution.

Appendix 2: Computations for the Comparative statics analysis

Consider the equilibrium equation (16):

$$\Delta \equiv \frac{h}{q(\theta_{x-1})} - \frac{p(Q_x)f(l_x) - w_x + (L-x)\theta_x h}{r + \delta + (L-x)\theta_x q(\theta_x)} = 0.$$

I want to differentiate such equation with respect to θ_x and θ_{x+1}. Knowing that $\frac{d\,\theta q(\theta)}{d\,\theta} = (1-\eta)q(\theta)$, with $\eta \equiv \frac{d\,(1/q(\theta))}{d\,\theta} \cdot \theta\, q(\theta)$, I get:

$$\frac{d\,\Delta}{d\,\theta_x} = \frac{-(L-x)h[r+\delta+(L-x)\theta_x q(\theta_x)] + (1-\eta)(L-x)q(\theta_x)\,[p(Q_x)f(l_x) - w_x + (L-x)h\theta_x]}{[r + \delta + (L-x)\theta_x q(\theta_x)]^2} =$$

$$= -\frac{(L-x)\,h}{r + \delta + (L-x)\theta_x q(\theta_x)} + \frac{(1-\eta)\,h\,(L-x)q(\theta_x)}{[r + \delta + (L-x)\theta_x q(\theta_x)]\,q(\theta_{x-1})} =$$

$$= \frac{(L-x)\,h\,[(1-\eta)q(\theta_x) - q(\theta_{x-1})]}{[r + \delta + (L-x)\theta_x q(\theta_x)]\,q(\theta_{x-1})}$$

Moreover:

$$\frac{d\,\Delta}{d\,\theta_{x-1}} = -\frac{h\;q'(\theta_{x-1})}{q^2(\theta_{x-1})}$$

Therefore:

$$\frac{d\,\theta_{x-1}}{d\,\theta_x} = \frac{(L-x)\;[\,(1-\eta)q(\theta_x) - q(\theta_{x-1})\,]\;q(\theta_{x-1})}{[\,r + \delta + (L-x)\theta_x q(\theta_x)\,]\;q'(\theta_{x-1})}.$$

Appendix 3: Properties of the steady-state distribution when δ is close to 0

Look at (3) and (4). When δ is close to 0, $\pi_x = \frac{M_{x-1}}{M_x}\,\pi_x$ and $\pi_x = \frac{M_0}{M_x}\,\pi_0$, with $M_x = (L-x)\theta_x q(\theta_x)$. Knowing that $\pi_0 = 1 - \sum_{m=1}^{L} \pi_m$, we get:

$$\pi_x = \frac{\frac{M_0}{M_x}}{1 + \frac{M_0}{M_1} + \frac{M_0}{M_2} + ... + \frac{M_0}{\delta_L}} = \frac{\frac{M_0}{M_x}}{1 + M_0\left(\frac{1}{\delta_L} + \sum_{m=1}^{L-1}\frac{1}{M_m}\right)} \quad \forall x \in [1, ..., L-1],$$

$$\pi_0 = \frac{1}{1 + \frac{M_0}{M_1} + \frac{M_0}{M_2} + ... + \frac{M_0}{\delta_L}} = \frac{1}{1 + M_0\left(\frac{1}{\delta_L} + \sum_{m=1}^{L-1}\frac{1}{M_m}\right)},$$

$$\pi_L = \frac{\frac{M_0}{\delta_L}}{1 + \frac{M_0}{M_1} + \frac{M_0}{M_2} + ... + \frac{M_0}{\delta_L}} = \frac{\frac{M_0}{\delta_L}}{1 + M_0\left(\frac{1}{\delta_L} + \sum_{m=1}^{L-1}\frac{1}{M_m}\right)}.$$

Suppose a contemporaneous increase in $[M_1, M_2..., M_L]$. Then:

$$\frac{d\,\pi_x}{d\,M_0} + \frac{d\,\pi_x}{d\,M_x} + \sum_{x\neq m=1}^{L-1} \frac{d\,\pi_x}{d\,M_m} = \frac{\frac{1}{M_x} - \frac{M_0}{M_x^2}\left[1 + M_0\left(\frac{1}{\delta_L} + \sum_{x\neq m=1}^{L-1}\frac{1}{M_m}\right)\right] + \frac{M_0^2}{M_x}\left[\sum_{x\neq m=1}^{L-1}\frac{1}{M_m^2}\right]}{\left[1 + M_0\left(\frac{1}{\delta_L} + \sum_{m=1}^{L-1}\frac{1}{M_m}\right)\right]^2}$$

$$\frac{d\,\pi_0}{d\,M_0} + \sum_{m=1}^{L-1} \frac{d\,\pi_0}{d\,M_m} = \frac{-\left[\frac{1}{\delta_L} + \sum_{m=1}^{L-1}\frac{1}{M_m}\right] + M_0\left[\sum_{m=1}^{L-1}\frac{1}{M_m^2}\right]}{\left[1 + M_0\left(\frac{1}{\delta_L} + \sum_{m=1}^{L-1}\frac{1}{M_m}\right)\right]^2}$$

$$\frac{d\,\pi_L}{d\,M_0} + \sum_{m=1}^{L-1} \frac{d\,\pi_L}{d\,M_m} = \frac{\frac{1}{\delta_L} + \frac{M_0^2}{\delta_L}\left[\sum_{m=1}^{L-1}\frac{1}{M_m^2}\right]}{\left[1 + M_0\left(\frac{1}{\delta_L} + \sum_{m=1}^{L-1}\frac{1}{M_m}\right)\right]^2} > 0$$

The only derivative that can easily signed is the last one: a generalized increase in M_x enhances π_L. There is a higher probability that sectors can attain full employment. Consider the derivative of π_0 with respect to M_0, and M_m $\forall m \in [1, 2, ..., L-1]$. The numerator can be written as:

$$\sum_{m=1}^{L-1}\left[\frac{1}{M_m}\left(\frac{M_0}{M_m} - 1\right)\right] - \frac{1}{\delta_L}$$

The first term inside the square brackets is always positive $\forall m \in [1, 2, ..., L-1]$ since, for Proposition 1, θ_m (and so M_m) is decreasing in m. However, $\frac{1}{\delta_L}$ is a negative number, so we cannot sign the derivative of π_0. About the derivative of π_x when $x \neq 0, L$, notice that the numerator can be written as:

$$\frac{1}{M_x}\left[1 - \frac{M_0}{M_x} - \frac{M_0^2}{M_x\delta_L}\right] + \frac{M_0^2}{M_x} \cdot \sum_{x\neq m=1}^{L-1}\left[\frac{1}{M_m^2} - \frac{1}{M_m\,M_x}\right].$$

The first term inside the square brackets is negative. On the other hand, the term $\frac{1}{M_m^2} - \frac{1}{M_m\,M_x} < 0$ if and only if $M_x < M_y$. In the case $x = L-1$, $M_{L-1} < M_m, \forall m \in [0, 1, 2, ...L-2]$, $x \neq y$ and, consequently, a contemporaneous increase in $[M_1, M_2..., M_L]$ makes π_{L-1} decrease. When $x < L-1$, the sum written above will be formed by negative terms (for $y < x$) and positive ones (for $x > y$). In this case, the sign of the derivative of π_x with respect to $[M_1, M_2..., M_L]$.

Appendix 4: Details of the proof of Proposition 2

I prove the first step of the proof of Proposition 2: $p(Q^*_{x+1})f(l^*_{x+1}) + \phi(1 - l^*_{x+1}) > \Delta F(Q^\circ_{x+1}) + \Delta(x+1)\phi(1 - l^\circ_{x+1})$
Since $F(Q_{x+1})$ is homogeneous of degree γ, for Eulero's formula $F(Q_{x+1}) = \frac{1}{\gamma}(d\,F/d\,Q_{x+1})\cdot Q_{x+1}$, with $0 < \gamma < 1$. Notice that:

$$\begin{aligned} \Delta F(Q^\circ_{x+1}) + \Delta(x+1)\phi(1 - l^\circ_{x+1}) = \frac{1}{\gamma}p(Q^\circ_{x+1})f(l^\circ_{x+1}) + \phi(1 - l^\circ_{x+1}) + \\ + x \cdot \left[\frac{1}{\gamma}p(Q^\circ_{x+1})f(l^\circ_{x+1}) + \phi(1 - l^\circ_{x+1}) - \frac{1}{\gamma}p(Q^\circ_x)f(l^\circ_x) - \phi(1 - l^\circ_x)\right], \end{aligned} \tag{31}$$

Consider first the term outside the square brackets. Recall from the bargaining problem that $l^*_{x+1} = \text{argmax}\,[\,p(Q_{x+1})f(l_{x+1}) + \phi(1 - l_{x+1})\,]$. So:

$$\frac{1}{\gamma}p(Q^\circ_{x+1})f(l^\circ_{x+1}) + \phi(1 - l^\circ_{x+1}) < p(Q^*_{x+1})f(l^*_{x+1}) + \phi(1 - l^*_{x+1}),$$

if γ is not too small (the function is not too concave). It is then sufficient to show that the term in the second line in (31) is negative to prove the inequality of Step 1. But this is the case if $\frac{1}{\gamma}p(Q^\circ_x)f(l^\circ_x) - \phi(1 - l^\circ_x)$ is decreasing in x. Ignoring for simplicity the integer problem, I get:

$$\frac{d\left[\frac{1}{\gamma}p(Q^\circ_x)f(l^\circ_x) - \phi(1 - l^\circ_x)\right]}{d\,x} = \frac{1}{\gamma}\frac{d\,p(Q^\circ_x)}{d\,Q^\circ_x}f(l^\circ_x)\frac{d\,Q^\circ_x}{d\,x} + + \frac{d\,l^\circ_x}{d\,x}\left[\frac{1}{\gamma}p(Q^\circ_x)f'(l^\circ_x) - \phi'(1 - l^\circ_x)\right] < 0$$

The first term is negative, given the concavity of $F(Q_x)$. By totally differentiating (21), it is easy to see that $\frac{d\,l^\circ_x}{d\,x}$ is also negative. Finally, the term inside the square brackets is positive, for the F.O.C. (21) and $0 < \gamma < 1$.

Figures and Tables

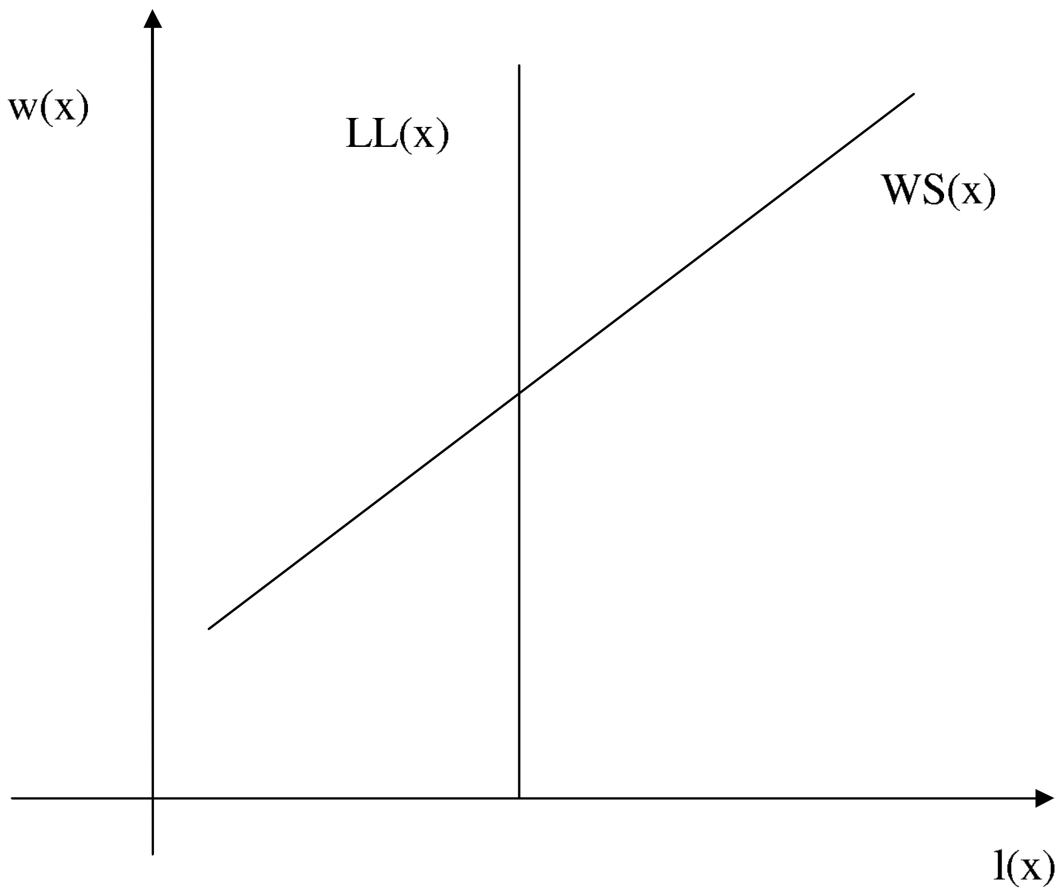

Figure 1: Equilibrium equations for wage and hours worked

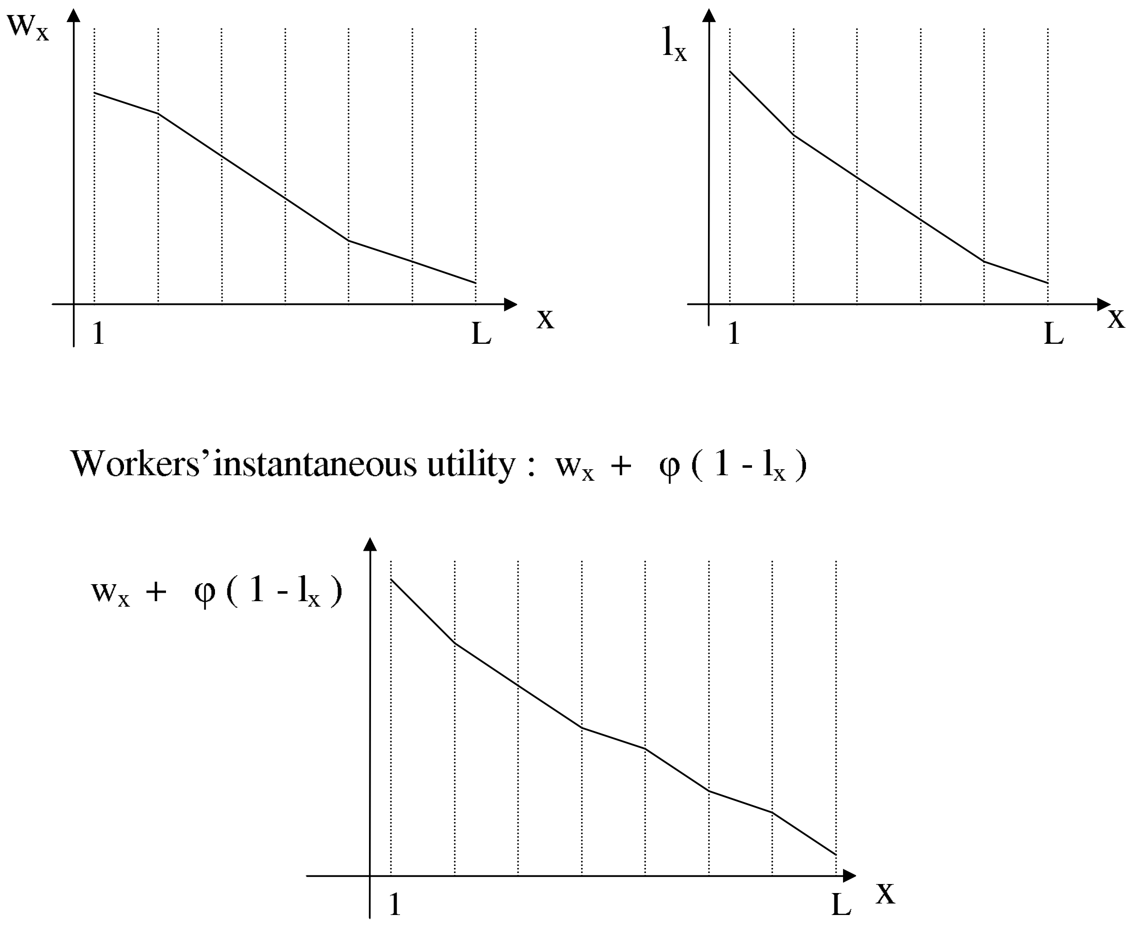

Figure 2: Wages and Hours worked.

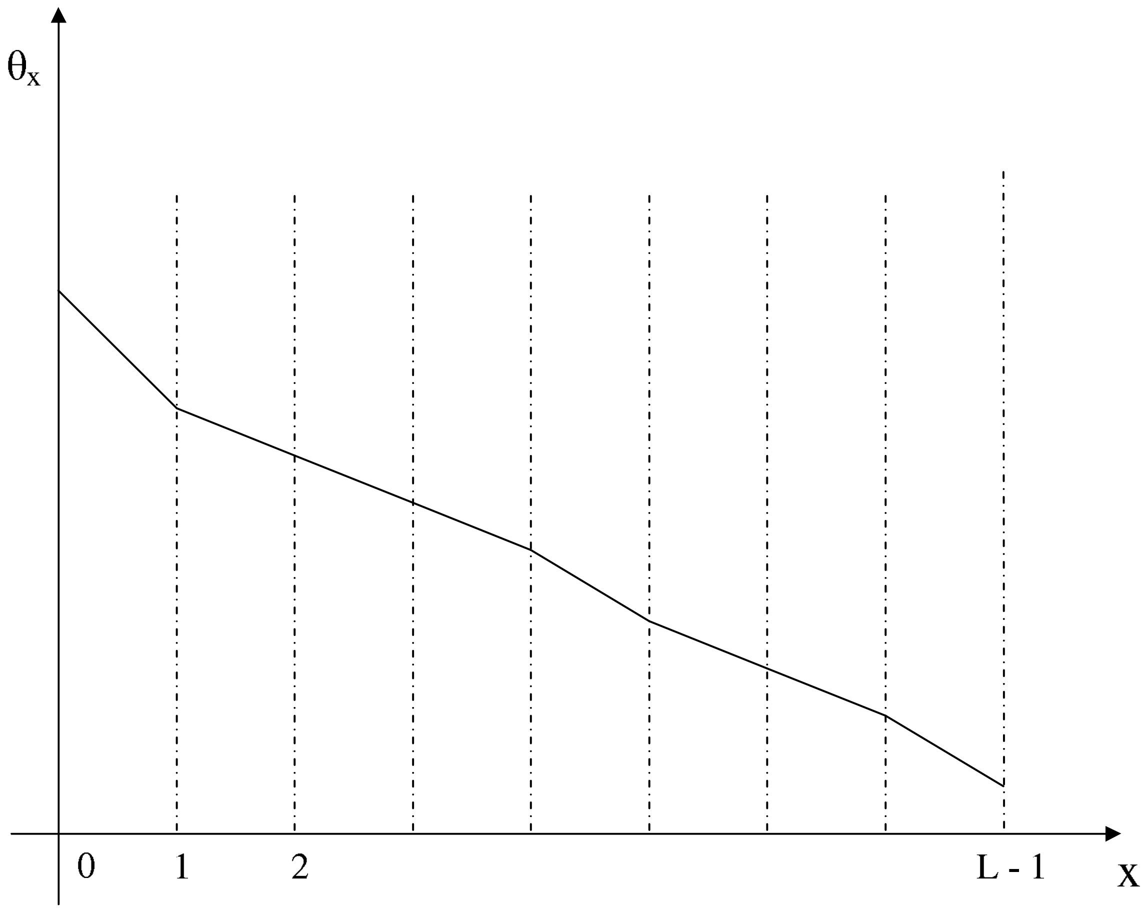

Figure 3: Labour market tightness.

Parameters	
r	0.004
δ	0.005
ϵ	0.5
γ	0.9
λ	0.9
h (Euro/month)	14500
β	0.5
η	0.5
L	20
a	0.125
p_0	1.1
Endogenous var. (average)	
θ	0.14
$1/\theta q(\theta)$ (months)	21.6
$1/q(\theta)$ (months)	2.9
w (Euro/month)	1348

Table 1. Calibration: Parameters and levels of endogenous variables in steady state.

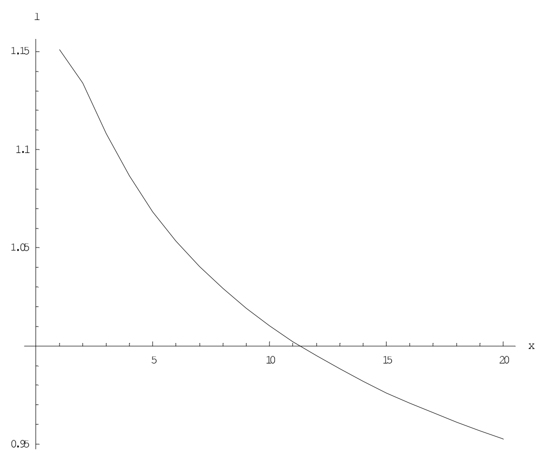

Figure 4: Simulation results: Hours Worked $l \in [0, 2]$.

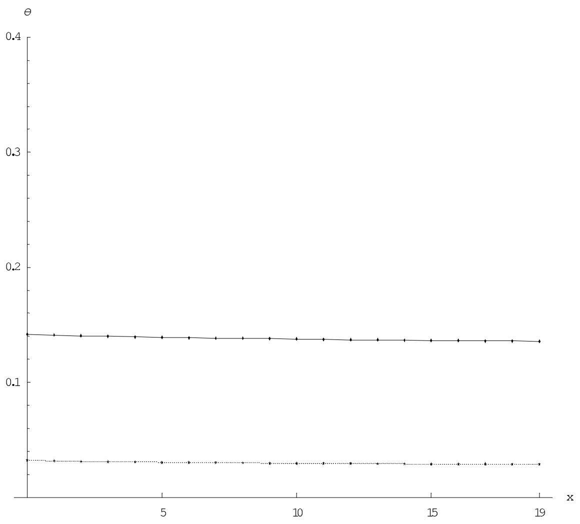

Figure 5: A comparison of the optimal level of labour market tightness (dotted line) with the decentralized one (continuous line)($\beta = \eta = 0.5$).

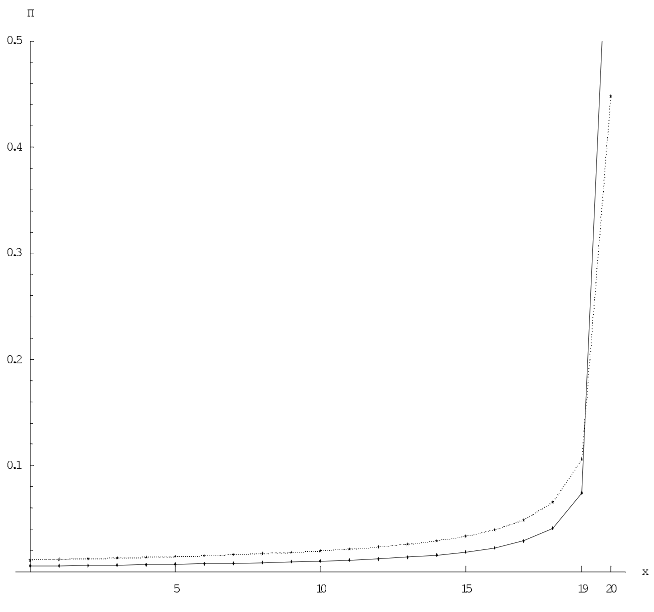

Figure 6: A comparison of the optimal steady state distribution (dotted line) with the decentralized one (continuous line) ($\beta = \eta = 0.5$).

Variables	h = 14500	h = 13500	h = 12000	h = 7250
w (euros /month)	1348	1348	1347	1345
Employment rate e^* (per cent)	90.2	90.8	91.8	94.8
Share of hours worked (per cent)	47.8	47.8	47.8	47.7
Volume of work V^* (per cent)	43.2	43.5	43.9	45.3
$e^* - e^\circ$ (per cent)	9.04	8.96	8.80	7.91
$V^* - V^\circ$ (per cent)	3.87	3.83	3.76	3.36

Table 2. Simulation Results. Variation in the cost of opening a vacancy. Superscript * denotes the free-entry equilibrium values, while superscript $^\circ$ the optimal ones.

Variables	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$	$\beta = 0.75$
Employment rate e^* (per cent)	90.2	88.2	84.9	82.4
Volume of work V^* (per cent)	43.2	42.3	40.8	39.7
$e^* - e^\circ$ (per cent)	9.04	6.93	3.62	1.16
$V^* - V^\circ$ (per cent)	3.87	2.92	1.42	0.30

Table 3. Simulation Results. Variation in workers' bargaining power β when $\eta = 0.5$.

Parameters	Benchmark	1° case	2° case	3° case
ϵ	0.5	0.4	0.6	0.5
γ	0.9	0.8	0.9	0.7
λ	0.9	0.9	0.8	0.7
Variables				
θ	0.14	0.14	0.06	0.04
e^* (per cent)	90.2	90.5	86.8	82.8
w (euros per month)	1348	1184	1351	1180
Share of hours worked (per cent)	47.8	37.1	54.3	48.7
Volume of work V^* (per cent)	43.2	33.6	47.2	40.3
$V^* - V^\circ$ (per cent)	3.87	2.99	3.87	2.70
$e^* - e^\circ$ (per cent) if $\beta = \eta = 0.5$	9.04	9.04	9.03	8.88
$e^* - e^\circ$ (per cent) if $\beta = 0.75$	1.16	1.28	-0.07	-2.7

Table 4. Sensitivity analysis.

Parameter	Benchmark	1° case	2° case	3° case
η	0.5	0.4	0.6	0.7
Variables				
e^* (per cent)	90.2	85.1	92.8	94.2
w (euros per month)	1348	1366	1346	1345
Share of hours worked (per cent)	47.8	48.0	47.8	47.8
Volume of work V^* (per cent)	43.2	40.9	44.4	45.0
$V^* - V^\circ$ (per cent)	3.87	3.41	3.34	2.54
$e^* - e^\circ$ (per cent) if $\beta = \eta$	9.04	8.01	7.94	6.11
β/η s.t $e^* - e^\circ < 1\,\%$	1.5	1.62	1.42	1.29

Table 5. Sensitivity analysis: change in the matching elasticity η.