Search by Committee*

James Albrecht
Georgetown University and IZA

Axel Anderson
Georgetown University

Susan Vroman
Georgetown University and IZA

October 2007

Abstract

We consider the problem of sequential search when the decision to stop searching is made by a committee. We show that a symmetric stationary equilibrium exists and is unique given that the distribution of rewards is log concave.

Committee members set a lower acceptance threshold than do single-agent searchers. In addition, mean preserving spreads in the distribution of rewards may lower each member's continuation value — an impossibility in the single-agent setting. If committee members are very patient *or* very impatient, expected search duration is lower than it would be for a single agent, but, for intermediate levels of patience, this comparison may be reversed.

Holding the fraction of votes required to stop fixed, expected search duration rises with committee size on patient committees but falls with committee size on impatient committees. Finally, we consider the effect of varying the number of votes required to stop, holding committee size constant. We show that the welfare-maximizing vote threshold increases in the rate of patience and that there is a finite bound on patience such that *unanimity* is welfare maximizing.

*We thank Lones Smith for particularly helpful comments. We also received helpful comments at the UPenn Search and Matching Workshop, at the IZA Search conference, at the Tinbergen Institute, and at Georgetown University.

"A committee is a group that keeps minutes and loses hours." (Milton Berle)

1 Introduction

In the classic sequential search problem, an individual makes one draw per period from an exogenous and known distribution. These draws are independently and identically distributed through time. After every draw, there is a decision to be made – to stop and accept the payoff given by the realization of the most recent draw or to continue searching. The benefit of further search is the expectation that a higher payoff will eventually be realized; the cost is that the searcher's enjoyment of this payoff will be delayed. Elaborations and applications of this optimal stopping framework abound in economics. See Lippman and McCall (1979), Mortensen (1986), and Rogerson, Shimer and Wright (2005) for a set of excellent surveys.

All of the search literature to date has a common feature, namely, that the stopping decision is made by a single agent. For many applications, this assumption is a good one, but often the decision to stop or to continue searching is made by a group of agents. Consider, for example, an academic department that brings a sequence of candidates to campus to interview for an open position, and suppose that after each visit, a decision is made whether to make an offer to the latest candidate or to continue the search. This decision is typically taken by a group of faculty. Or, consider a couple that is shown a sequence of rental properties. After each property is observed, the couple decides as a pair whether to accept the latest apartment they have seen or to continue their search. It is, of course, easy to think of other situations in which a group makes a stopping decision.

We model this group decision as a problem of *search by committee.* As in the single-agent problem, in each period, a group of agents (the committee) is presented with an option. The values that the committee members place on this option are draws from an exogenous and known distribution, and these draws are *iid* across committee members and over time. In each period, the committee votes whether to stop or to continue. Specifically, we consider a committee with N members and we suppose that at least M votes are required to stop. The voting game played by the committee aggregates its members' preferences. If at least M members find the current option acceptable, the search stops; otherwise, the search continues. Our approach thus combines two literatures, sequential search and private-values voting.

What do we learn from this combination? *First*, we show that the problem of search by committee is well posed. A symmetric stationary equilibrium exists and is unique given a log concavity assumption on the distribution of payoffs. *Second*, we compare the expected outcomes of committee and single-agent search. Suppose the committee and the single agent face the same environment, i.e., the same distribution of payoffs and the same cost of delay. We show that given the same environment, committee members are always *less picky* than a single agent in the sense that the acceptance threshold they set is always less than that set by the single agent. The fact that committee members set a lower acceptance threshold than a single agent need not imply, however, that expected duration of search is shorter for a committee than it is for the single agent. The comparison between the expected duration of search for a committee and for a single-searcher depends on the cost of delay, and it does so in an interestingly non-monotonic way. Specifically, so long as $M < N$, a committee can expect to end its search faster than a single agent does if the cost of delay is either sufficiently high or sufficiently low.

We also show that a standard comparative static result from the single-agent search literature, namely, that a single agent adjusts his or her acceptance threshold upward in response to a mean-preserving spread in the distribution of rewards, can be reversed in the search-by-committee problem. In this sense, committee search is *more conservative* than is single-agent search. The two results, *less picky* and *more conservative*, follow from two fundamental elements of the search-by-committee problem. First, committee members impose externalities on each other that are by definition absent in the single-agent problem. (See Börgers 2004 for a related point in a static voting game.) Second, the voting game played by its members defines a value function (the expected discounted payoff) for the committee. The value function in the single-agent problem is necessarily convex, but the committee value function is not.

Third, we provide comparative statics results for the search-by-committee problem. We begin by relating the acceptance threshold and expected search duration to committee size. Fix $\alpha \equiv M/N$, the fraction of votes required to stop searching. We prove a counterintuitive equilibrium relationship between acceptance thresholds and search duration; namely, as we vary N holding α fixed, expected search duration rises (falls) whenever acceptance thresholds fall (rise). For example, if we increase the committee size and committee members become *more* picky as a result, expected search duration must *fall*. Given that the acceptance threshold set by committee members is lower than that of a single-searcher, one might expect the threshold to be monotonic in committee size, but we show that committee members need not become less picky as the size of the committee increases. In fact, if committee members are sufficiently impatient (the cost of delay is high enough), the acceptance threshold must increase and expected search duration must decrease in N. On the other hand, if committee members are sufficiently patient, these comparative statics results are reversed.

Finally, we consider the effect of varying M, holding N fixed. We show that for low values of M, the acceptance threshold increases as the required number of votes increases. In this case, expected search duration is also increasing in M. Eventually, however, the effect of an increase in M on the acceptance threshold reverses. In short, the acceptance threshold is hump-shaped in M. This is the basis for a *mechanism design* result. We show that the welfare-maximizing choice of M increases as committee members are more patient and that unanimity ($M = N$) is optimal for high enough (but bounded) rates of patience. The idea that unanimity can be optimal is in contrast to a standard result (e.g., Feddersen and Pesendorfer 1998) from the common-values voting literature, albeit in a different context.

To make progress on a new problem, we make simplifying assumptions. On the search side, we restrict our attention to the stationary sequential problem, and we assume that once an option is discarded it is lost forever to the committee (no recall). These assumptions are close to those of McCall (1970). That is, we use a simple, one-sided search framework and do not embed the search-by-committee problem in a market environment, in which the distribution of payoffs is endogenously generated by the actions of agents on the other side of the market. On the voting side, we restrict our attention to the private-values case, in which the values that committee members place on the option at hand are independent draws. Thus, we do not allow for the possibility that voting can convey information. Most fundamentally, we restrict the strategies available to committee members in the voting game. We consider Markovian strategies in which each committee member's vote (stop or continue) depends only on the option at hand. Thus our voting model harks back to the pioneering work of Hotelling (1929) and Black (1948).[1]

In the next section, we describe our model and prove the existence of a unique symmetric stationary equilibrium. In Section 3, we compare search by committee to single-agent search and show that committee members are *less picky* and *more conservative*. We elaborate on the two crucial aspects of the search-by-committee problem that drive these results, *externalities* and *nonconvexities*, in the context of the simplest possible committee, namely, the case of $N = 2$. Section 4 presents the comparative statics results with respect to committee size and the number of votes required to stop. Section 5 presents our mechanism design result. In Section 6, we conclude.

2 The Model

Assumptions. A *committee* is a pair (M, N), where N is the number of members and M is the number of votes required to end the search. Time is discrete, and all committee members discount the future at common rate $\delta \in (0, 1)$. In each period, the committee is presented with an option. Each committee member then draws a value for the option from a continuous cdf $F : [0, 1] \mapsto [0, 1]$ with positive density f. For convenience, we assume $F(0) = 0$. These values can represent von Neumann-Morgenstern utility payoffs or monetary payoffs for risk neutral agents. We assume these draws are *iid* across time and across committee members. We rule out side payments, i.e., utility is non-transferable.

We assume that both $\int_0^z F(s)ds$ and $\int_0^z (1 - F(s))ds$ are log concave in z. To see how we use these assumptions, define the truncated means:

$$\mu_h(z) \equiv E[X|X \geq z] \quad \text{and} \quad \mu_\ell(z) \equiv E[X|X < z].$$

Burdett (1996) shows that the above log-concavity assumptions imply that $\mu_h'(z) \leq 1$

[1]We follow the terminology of Black (1948), who also calls a collection of voters a *committee*. In his model, the committee decides between a proposal and the status quo. In our model, the proposal is the current option and the status quo is continuing to search. In Black (1948), the status quo is exogenous, while the value of the "status quo" is endogenous in our model.

and $\mu'_\ell(z) \leq 1$.[2] We use these upper bounds on the two derivatives to establish uniqueness of equilibrium and also to sign some of our comparative statics results. Note that f log concave is sufficient but not necessary for log concavity of the two integrals. Many common distributions are log concave, e.g., uniform, normal, exponential, logistic, power, gamma, and Weibull.

Each period is divided into two stages. In the first stage, the option arrives and each committee member's value is realized. In the second stage, the committee decides whether to stop searching and accept the most recently observed option or to continue to the next period. By restricting voting to stopping with the most recently observed option or continuing to search we are ruling out *recall*; i.e., once an option is discarded it is forever lost to the committee.[3] We model this choice using a simple voting mechanism; i.e., the committee members simultaneously vote either to stop and accept the current option or to continue to search. The search ends in period t if and only if at least M committee members vote to stop. Each committee member seeks to maximize his or her own discounted payoff.

A strategy for committee member i is a sequence of functions $\sigma_i = \{\sigma_i(t)\}_t$, such that $\sigma_i(t)$ maps from possible histories through time t to the set {continue, stop}. Player i employs a Markov Strategy if $\sigma_i(t)$ is only a function of the most recently evaluated option. We restrict attention to symmetric stationary equilibria in which all players employ the same Markov Strategy.

We assume that the above description of the model is common knowledge among the committee members. Given our Markovian restriction, whether individual draws are private or public information is immaterial. Further, whether an agent knows in advance that he or she is pivotal is also immaterial.

[2] Similar results can be found in Goldberger (1983) and in Vroman (1985). Log concavity assumptions are common in many economic applications (search, signaling, mechanism design, etc.). See Bagnoli and Bergstrom (2005) for a survey of log concavity results and applications.

[3] In single-agent search, the no-recall assumption is without loss of generality. However, search by committee is a game, and thus a plethora of strategies can potentially be supported in equilibrium by conditioning on past history. With recall, the state variable is the entire past history of draws, so the no-recall assumption is important in the committee search problem. The assumption of Markov strategies, however, takes the bite out of the no-recall assumption in the committee search problem.

Equilibrium. It is convenient to work with *reverse order statistics*.[4] Let $X_1, \ldots, X_N$ be the *iid* draws of the N committee members. Let $Y^1 \geq \ldots \geq Y^N$ be the corresponding reverse order statistics, and let G^j be the cdf of Y^j. We seek a stationary symmetric equilibrium. Assuming (for now) that such an equilibrium exists, we define $\Psi(z; M, N, \delta)$ to be the expected continuation value starting just before draws are made given that each committee member sets threshold z for all *future* time periods. The committee votes to stop if M or more members draw a value of z or more, so the probability that the committee stops searching is $1 - G^M(z)$. The continuation value solves:

$$\Psi(z; M, N, \delta) = G^M(z)\delta\Psi(z; M, N, \delta) + (1 - G^M(z))\Omega(z),$$

where $\Omega(z) = E[X|Y^M \geq z]$, i.e., the expected payoff conditional on stopping. Rearranging, we have:

$$\Psi(z; M, N, \delta) = \frac{(1 - G^M(z))\Omega(z)}{1 - \delta G^M(z)}. \tag{1}$$

An agent's payoff is only affected by the cutoff chosen when he or she is pivotal, i.e., when exactly $M - 1$ of the remaining $N - 1$ agents vote to stop. Thus, best responses are determined by considering a pivotal voter, i, who can choose either to stop and accept the most recently drawn reward, x_i, or instead continue. The pivotal agent solves:

$$\max\{x, \delta\Psi(z; \cdot)\}.$$

Since the continuation value, $\Psi(z; \cdot)$ is a constant with respect to the current period's decision, the optimal strategy is an acceptance threshold, i.e., vote for an option *iff* its value exceeds some threshold. Thus, if all members set threshold z in the future, the stopping threshold for the pivotal voter equates the value of stopping and continuing, given that he or she is pivotal now, i.e., $x(z) \equiv \delta\Psi(z, \cdot)$. For an equilibrium to be stationary and symmetric the following *equilibrium condition* must be satisfied: $x^* = \delta\Psi(x^*; \cdot)$. In equilibrium, $\Psi(x^*; \cdot)$ is the expected payoff for each committee member. Equivalently, since $\Psi(x^*; \cdot)$ is proportional to x^*, the equilibrium acceptance threshold can be used as a measure of each committee member's welfare.

[4]Reverse order statistics are opposite to usual order statistics in that they index the draws of the random variable in descending rather than ascending order. Thus, reverse order statistic $Y^1 = Y_N$, the Nth usual order statistic. In general, $Y^J = Y_{N-J+1}$.

To establish existence and uniqueness and to characterize equilibrium, we need expressions for $G^M(z)$ and $\Omega(z)$. We can write

$$G^M(z) = \sum_{i=0}^{M-1} \binom{N}{i} (1 - F(z))^i F(z)^{N-i},$$

that is, the probability that $M - 1$ or fewer of the committee members draw a value of z or greater. Absent simplification, $\Omega(z)$, is a bit more complicated, and so, we give a simplified version as a weighted average of $\mu_h(z)$ and $\mu_\ell(z)$ and bound its derivative in the next Lemma. The proof is in the Appendix. Let $\alpha \equiv M/N$ to simplify notation.

Lemma 1 $\Omega(z) = (1 - (1 - \alpha)F(z))\mu_h(z) + (1 - \alpha)F(z)\mu_\ell(z)$ *and* $\Omega'(z) \leq 1$.

Having characterized $G^M(z)$ and $\Omega(z)$, we state the main result of this section.

Proposition 1 *An equilibrium exists and is unique.*

The details of the proof are in the Appendix. However, the basic idea is straightforward, as shown in Figure 1. Existence follows from $\Psi(0; \cdot) = E[X] > 0$, $\Psi(1; \cdot) = 0$, and $\Psi(z; \cdot)$ continuous in z. To establish uniqueness, we need to show that $\delta\Psi(z; \cdot)$ crosses the 45-degree line only once, i.e., that $\delta\Psi_z(z, \cdot) < 1$. This last condition follows from Lemma 1.

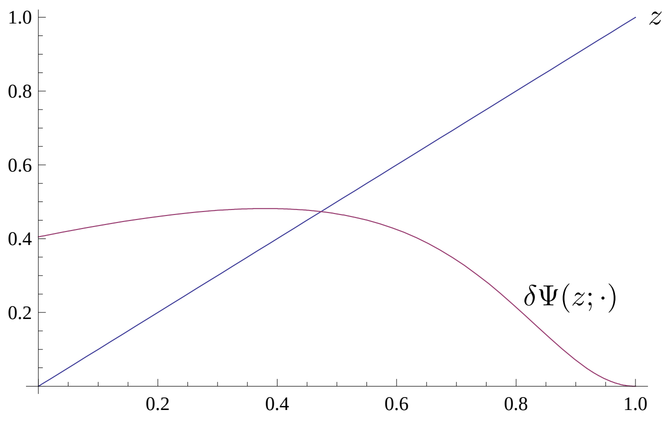


Figure 1: We have graphed $\delta\Psi(z; 2, 2, 0.9)$ and z for uniform F on $[0, 1]$ to illustrate our equilibrium: $x^* \approx 0.515$.

3 Comparison to Single-Agent Search

In this section, we compare the symmetric stationary committee search equilibrium outcome with the outcome in the case of a single-searcher. We first show that committee members are less picky than single-searchers; i.e., they set a lower acceptance threshold in equilibrium. We then show that committees conclude search more quickly for extreme (high or low) rates of patience but that the comparison can be reversed for intermediate rates of patience. Finally, we establish that committees are more conservative than single-searchers in the sense that a mean preserving spread in F can lower the equilibrium acceptance threshold.

Committees are Less Picky. For fixed δ and F, we say that committee members are *less picky* than a single agent if the equilibrium acceptance threshold of the committee members is lower than the acceptance threshold that a single-searcher would set, which we prove in the Appendix.

Proposition 2 (Less Picky) *Committee members are less picky than a single-searcher.*

The intuition for this result is quite simple. A single-searcher maximizes his or her continuation value, but a pivotal committee member cannot count on the committee doing so. More specifically, there are two potential negative externalities that committee members can impose on one another that do not arise in the single-searcher case. They can vote to stop when a committee member has drawn a low value or they can vote to continue when the committee member has drawn a high value. These externalities lead committee members to set a lower acceptance threshold than would a single-searcher.[5] Notice that this argument does not require independence of draws. Committees are less picky than single-searchers even with correlated values, as long as the correlation is not perfect, at which point the single-agent and committee problems are identical.

[5] In the Börgers (2004) model of costly committee voting and voluntary participation, there is a single negative externality that arises because an individual's decision to participate makes it less likely that other committee members are pivotal and thus imposes a cost on them. While Börgers's model is static and ours is dynamic, there is a similarity to the externalities in his model and ours.

Patient and Impatient Committees Conclude Search Faster. We know that committee members set an acceptance threshold that is below the corresponding threshold for a single-searcher (holding F and δ constant), but this does not necessarily imply that the committee has a shorter expected search duration. In fact, the committee may take longer to search, as the following example illustrates.

Example 1 *Let F be uniform on $[0,1]$ and $\delta = 0.8$. Then a single-searcher sets threshold 0.5, i.e., stops with probability 0.5. With $N = 5$ and $M = 4$, the equilibrium committee threshold is approximately 0.39, which yields a probability of stopping of approximately 0.36; i.e., the committee searches longer on average.*

The point illustrated by the above example is straightforward. We can, however, say something considerably less obvious about expected search duration. The comparison of expected search duration between a committee and a single-searcher has an interesting non-monotonicity in δ. While expected search duration must rise in δ for committees and for single-searchers, the rate of change differs between committees and single-searchers, so that the sign of the difference in expected search duration is not monotonic.

Proposition 3 *If $M < N$, then there exists $0 < \delta_L \leq \delta_H < 1$ such that expected search duration is lower for the committee whenever $\delta \notin (\delta_L, \delta_H)$. That is, committees conclude search more quickly than individuals for low* and *high rates of patience.*

Proof: Given in the Appendix.

To understand why committees conclude search faster for extreme rates of patience (and also why the comparison may be reversed for medium rates of patience), first note that expected search duration depends on both the acceptance threshold (the threshold effect) and the probability of stopping given any acceptance threshold (the vote aggregation effect). Since committee members are less picky, the threshold effect always pushes committees toward concluding search faster. Thus, committees can only search longer if the vote aggregation effect is dominant and has the opposite sign of the threshold effect.

However, when $M < N$, the binomial distribution has the property that $G^M(x) < F(x)$ for low enough x. Thus, whenever the committee and single-searcher acceptance thresholds are low enough, the vote aggregation effect reinforces the threshold effect, and the committee can expect to conclude search faster. Of course, low δ implies low acceptance thresholds, which implies that committees conclude search faster on average for low enough δ.

If δ is high, we cannot make the same argument. In fact, for high enough acceptance thresholds, the vote aggregation effect must have the opposite sign from the threshold effect (as long as $M > 1$). So the question becomes: which effect dominates? As $\delta \to 1$, the single-agent threshold goes to 1; thus, the probability of continuing goes to 1. The committee threshold is bounded away from 1 as $\delta \to 1$ (as long as $M < N$). This in turn implies that the committee's probability of continuing is bounded away from 1. altogether, for sufficiently high δ, the threshold effect must dominate the vote aggregation effect. For intermediate rates of patience, the vote aggregation effect can be of the opposite sign and dominant as in Example 1. However, this intermediate range need not exist. For example, if $N = 2$ and $M = 1$ and values are distributed uniformly on $[0, 1]$, then the committee has a lower expected search duration than a single agent for *all* discount rates.

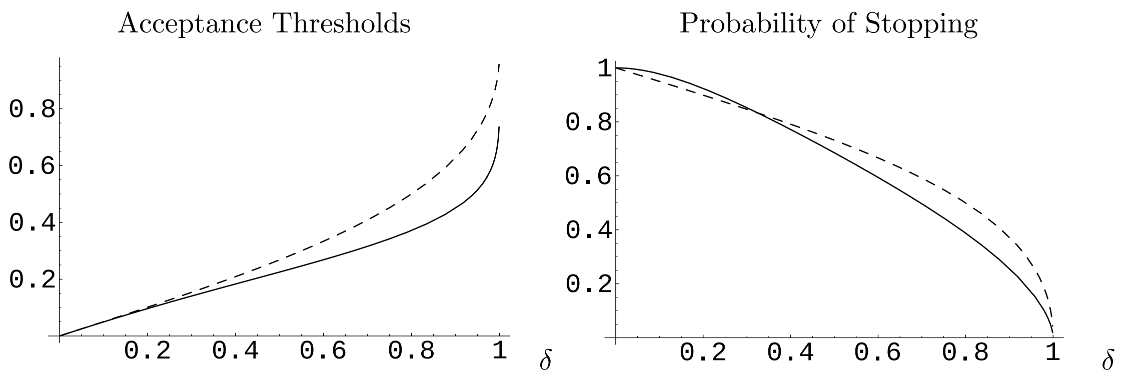


Figure 2: We compare the optimal single-agent solution to the committee equilibrium with $(N, M) = (5, 4)$ and F uniform on $[0, 1]$. In the graph on the left, we compare the single-agent acceptance threshold, $\tilde{x}$ (dashed line) to the committee threshold x^* (solid line). On the right, we compare the probability of stopping for a single agent, $1 - F(\tilde{x})$ (dashed line) to the probability of stopping for the committee, $1 - G^M(x^*)$ (solid line).

Committees are More Conservative. In the single-agent search problem, mean preserving spreads in the distribution of rewards, F, are always good news, increasing continuation values and raising acceptance thresholds. This comparative static follows from considering the single-searcher's value function ν which solves the recursion $\nu(x) = \max\{x, \delta E[\nu(X')]\}$. Since ν is the max of a constant and a linear function, ν is convex. As is well known, mean preserving spreads increase the expectation of a convex function, so that $E[\nu(X')]$ increases in mean preserving spreads. In turn, the acceptance threshold for single-searchers (where $\widetilde{x} = \delta E[\nu(X')]$) must rise in mean preserving spreads.

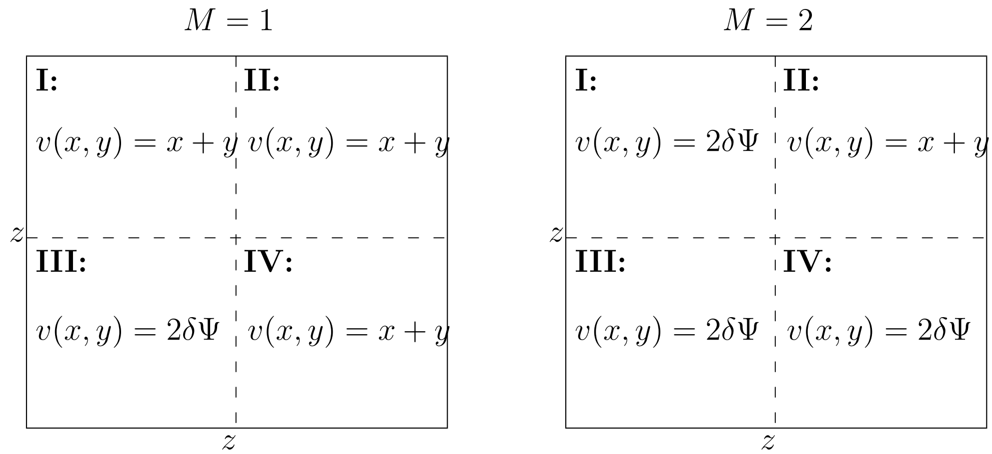


Figure 3: Joint value function. We show $v(x, y)$ in each region, given symmetric threshold z and continuation value Ψ.

Why does this simple logic not carry over to the committee problem? To provide insight we (temporarily) fix $N = 2$ and consider an analogous value function approach to the committee problem. Define $v(x, y)$ as the sum of the payoffs to the agents given that they both set threshold z and draw values (x, y). Our continuation value function is related to $v(x, y)$ as follows:

$$\Psi(z; \cdot) = \frac{1}{2} \int \int v(x, y) f(x) f(y) dx dy, \tag{2}$$

where $v(x, y) = x + y$ when the committee stops searching and $2\delta\Psi(z; \cdot)$ otherwise. In Figure 3, we consider the function v. Any symmetric threshold z divides the unit square into four regions. If $M = 1$, then $(x, y) < (z, z)$ is the continuation region in which $v(x, y) = 2\delta\Psi(z; \cdot)$, while $v(x, y) = x + y$ elsewhere. When $M = 2$, then $v(x, y) = x + y$ on the stopping region $(x, y) \geq (z, z)$, and $v(x, y) = 2\delta\Psi(z; \cdot)$ elsewhere.

Negative externalities obtain in Regions I and IV. For example, when $M = 1$, in Region I, member 2 forces a conclusion to the search problem despite relatively low draws by member 1. When $M = 2$, it is member 1 who imposes a negative externality on member 2 in Region I, forcing a continuation of search even though member 2 has drawn a high value.

Now consider how changes in F affect the continuation value and the threshold x^* by examining equation (2). As in the single-agent problem, continuation values and acceptance thresholds must be increasing in first order stochastic dominance changes in F since v is increasing in both x and y. What about mean preserving spreads in F? If v were convex in x and y, then we would get monotonicity in mean preserving spreads in F, as in the single-agent problem. However, it turns out that v is not convex in x and y.

To see the nonconvexities, fix $M = 1$ and again consider Figure 3. Take any three pairs $(x, y), (x', y'), (x'', y'')$ and any $\lambda \in (0, 1)$ such that $(x, y) = \lambda(x', y') + (1-\lambda)(x'', y'')$ and

$$x' < x < z < x'' \qquad \text{and} \qquad y'' < y < z < y',$$

i.e. (x, y) in Region III, (x', y') in Region I and (x'', y'') in Region IV. Then $v(x, y) = 2\delta\Psi = 2z$ (in equilibrium) and $\lambda v(x', y') + (1-\lambda)v(x'', y'') = \lambda(x', y') + (1-\lambda)(x'', y'') = x + y$. Finally, $2z > x + y$ since z exceeds x and y individually. Thus, when $M = 1$, v is not everywhere convex in (x, y). To see that v is not convex in (x, y) when $M = 2$, choose pairs such that (x', y') is in Region I, (x'', y'') is in Region IV, and (x, y) is in Region II. Then $v(x', y') = v(x'', y'') = 2z < v(x, y) = x + y$.

While v is not convex in (x, y), given our independence assumption, we could still prove monotonicity in mean preserving spreads in F if $v(x, y)$ were convex in one variable holding the other fixed (bi-convex). However, this weaker convexity condition is also not met. We show this in Figure 4, in which we graph $v(x, y)$ for $M = 1$ and $y < z$ (left) and $v(x, y)$ for $M = 2$ and $y > z$ (right). These nonconvexities have an intuitive externality interpretation. For example, consider the $M = 1$, $y < z$ case. At the boundary $x = z$, agent 1 is just indifferent between continuing and stopping, while agent 2 strictly prefers continuing ($y < z$). Thus, agent 1 confers a negative externality

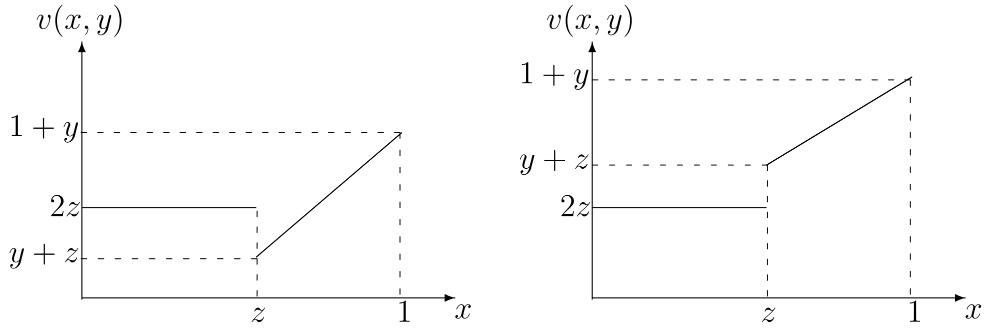


Figure 4: v is not bi-convex. We graph $v(x, y)$ as a function of x holding y fixed. In the graph on the left: $M = 1$ and $y < z$, while on the right: $M = 2$ and $y > z$.

on agent 2, and the joint payoff jumps down discontinuously. Similarly, when $M = 2$ and $y > z$, agent 2 strictly prefers to stop, so that agent 1 confers a negative externality on agent 2 by forcing continuation when $x < z$, and the joint payoff discontinuously jumps upward at $x = z$.

As mentioned above, the non-convexities in v can cause a reversal of the sign of the comparative static in mean preserving spreads compared to the single-agent problem.

Example 2 (Non Monotonicity in MPS) *Consider uniform draws on* $[\underline{x}, \overline{x}]$*. Let* $\delta = 0.65$ *and* $M = 2$*. If* $[\underline{x}, \overline{x}] = [1, 3]$ *then* $x^* \approx 1.25$*, while if* $[\underline{x}, \overline{x}] = [0.5, 3.5]$ *then* $x^* \approx 1.22$.

We have graphed $\Psi(z; \cdot)$ for this example in Figure 5.

4 Comparative Statics

In this section, we investigate the effect of committee size (N) and the plurality rule (M) on the equilibrium acceptance threshold and on expected search duration.

Committee Size. We examine the effects of changes in committee size holding the fraction of votes required to stop, $\alpha = M/N$, constant. We do so recognizing that the committee problem is only defined when αN is an integer. Since committee members impose negative externalities on one another, one might suppose that the larger the committee, the larger the associated externalities, leading to lower continuation values

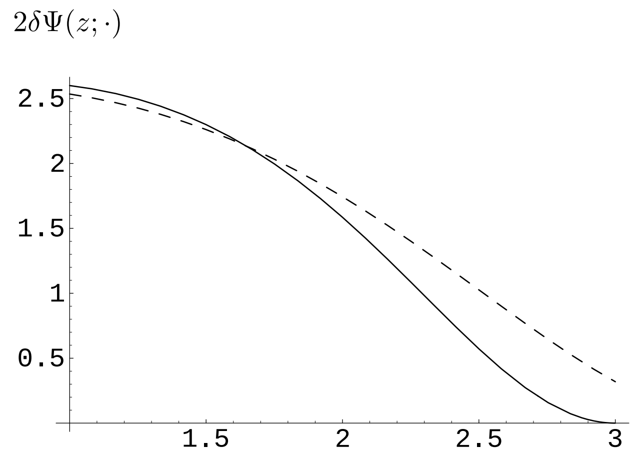


Figure 5: Thresholds may decrease in MPS. We have graphed $2\delta\Psi(z;\cdot)$ with $\delta = 0.65$ for two different uniform distributions. The solid line is drawn for support $[1, 3]$, while the dashed line is drawn for support $[0.5, 3.5]$. For low values of z the MPS causes a decrease in $\Psi(z;\cdot)$.

and lower acceptance thresholds in equilibrium. It turns out that this simple intuition is incomplete. In fact, acceptance thresholds can rise as the size of the committee increases.

Further, as we have shown above, simply knowing whether acceptance thresholds rise or fall in committee size is not enough to determine the effect of a change in N on expected search duration. There are two effects of varying N on expected search duration: the effect on x^* holding the function $G^{\alpha N}$ fixed and the effect on the function $G^{\alpha N}$ as N varies. These effects may work in opposite directions when $\alpha = M/N$ is held constant. *A priori,* the effect of committee size on expected search duration is thus unclear.

The key to resolving this difficulty is exploiting the equilibrium relationship between the acceptance threshold x^* and expected search duration. Equation 1 can be rewritten as:

$$\frac{x^*}{\Omega(x^*)} = \frac{\delta(1 - G^{\alpha N}(x^*))}{1 - \delta G^{\alpha N}(x^*)}.$$

The LHS of this equality increases with x^* and not independently in N. This follows from Lemma 1, which shows (i) $\Omega(x)$ is not a function of N (holding α constant) and (ii) $\Omega'(x) \leq 1$. Since the LHS rises in x^*, so too must the RHS. The RHS is decreasing in $G^{\alpha N}(x^*)$, so if x^* increases with N, then $G^{\alpha N}(x^*)$ must decrease with N. Finally, expected search duration ($\frac{1}{1-G^{\alpha N}(x^*)}$) decreases as $G^{\alpha N}(x^*)$ falls. That is, if x^* increases with N, holding α constant, then, via the above equilibrium condition, $G^{\alpha N}(x^*)$ must fall and so too must the expected duration of search.

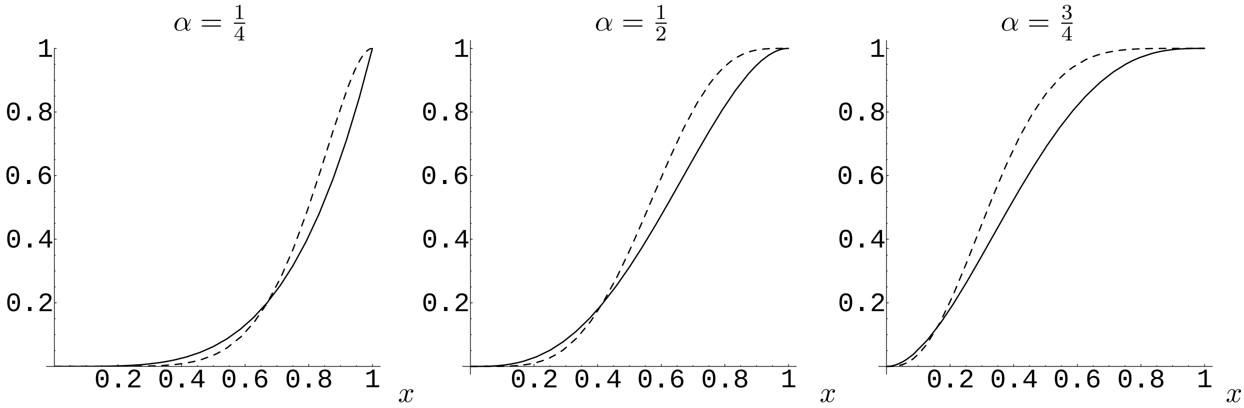
The key question then is how the acceptance threshold, x^*, varies with N when α is held constant.

Proposition 4 *Given $\alpha < 1$ and $N' > N$, there exists $0 < \delta^*(\alpha) \leq 1$ such that $\delta < \delta^*(\alpha)$ ($\delta > \delta^*(\alpha)$) implies that x^* is higher (lower) and expected search duration lower (higher) at N' than N.*

We prove Proposition 4 in the Appendix. To help understand this proposition, consider Figure 6. Given $\alpha < 1$, $G^{\alpha N'}(x) - G^{\alpha N}(x)$ satisfies a single crossing property. There is a threshold value of x, $\bar{x}(\alpha)$, such that the sign of the difference $G^{\alpha N'}(x) - G^{\alpha N}(x)$ is fully determined by $x - \bar{x}(\alpha)$. We show in the Appendix that this single crossing property is general. In the top portion of Figure 6 we have graphed $G^{N'}$ and G^{N} for 3 values of α to illustrate this single crossing property.

We also show in the Appendix that if $x^* < \bar{x}(\alpha)$, then x^* increases in N and that if $x^* > \bar{x}(\alpha)$, then x^* decreases in N. We know that x^* continuously increases in δ, starting from $x^* = 0$ for $\delta = 0$. Thus, for low enough δ, $x^* < \bar{x}(\alpha)$ so x^* rises in N and expected search duration falls. As long as $\alpha < 1$, $x^* < 1$ at $\delta = 1$, thus we can have $x^* < \bar{x}(\alpha)$ for all δ, implying $\delta^*(\alpha) = 1$. However, we can also have $\bar{x}(\alpha) < x^*$ at $\delta = 1$, so that $\bar{x}(\alpha) < x^*$ for high enough δ, implying $\delta^*(\alpha) < 1$. In the bottom portion of Figure 6 we evaluate the critical functions $\bar{x}(\alpha)$ and $\delta^*(\alpha)$ in one example.

Plurality Rule. In this subsection, we consider how x^* and expected search duration vary with M, holding N constant. Comparative statics with respect to M are of interest in their own right, but, in addition, we use these results when solving for the optimal voting rule in the next section. The following proposition states our comparative statics results.



α	$\bar{x}(\alpha)$	$x^*(\delta = 1)$	$\delta^*(\alpha)$
1/4	0.672	0.519	1
1/2	0.413	0.598	0.795
3/4	0.160	0.768	0.334

Figure 6: We have graphed $G^{\alpha N}(x)$ for $N = 4$ (solid line) and $N = 8$ (dashed line), with α increasing from left to right, specifically: $\alpha \in \{1/4, 1/2, 3/4\}$.

Proposition 5 *There exists a $k \in (0, N]$ such that:*

- *For all $M < k$, x^* is increasing in M, while for all $M > k$, x^* is decreasing in M.*
- *For all $M < k$, expected search duration rises in M.*

Proof of Part 1: Given in the Appendix.

Proof of Part 2. Expected search duration is given by $\frac{1}{1 - G^M(x^*)}$. When M increases, there are two effects on expected search duration. The first is a direct effect - increasing M decreases $1 - G^M(x^*)$. The second is an indirect effect: $1 - G^M(x^*)$ is inversely related to x^*. When $M < k$, these effects reinforce each other so that increasing M causes an increase in expected search duration. When $M > k$ these effects work at cross purposes so the effect of an increase in M on expected search duration is ambiguous. □

5 Mechanism Design: Optimal Plurality Rule

In this section, we consider setting the plurality rule M to maximize average expected welfare. Since $\Psi(x^*; \cdot)$ is a committee member's expected discounted equilibrium payoff just prior to any draw, and since $x^* = \delta\Psi(x^*; \cdot)$ in equilibrium, we can simply recast this as choosing M to maximize x^*. We show that the welfare-maximizing choice of M increases with δ and that unanimity is optimal for high enough (but bounded) δ.

Proposition 6 *The welfare-maximizing plurality rule, M^*, weakly increases in δ, and there exists a threshold $\delta^* < 1$ such that $M^* = N$ for all $\delta > \delta^*$.*

Proof: Given in the Appendix.

We can gain some intuition for this result by again considering an externality interpretation. Assume $N = 2$. Then when $M = 1$ there is only one type of externality: one member may choose to stop when the other would prefer to continue. If instead $M = 2$, there is again only one type of externality: one member may choose to continue when the other would prefer to stop. When $N > 2$, the same externalities are present. As M increases (with fixed N), the first externality becomes less important and the second more important. The relative cost of these externalities changes with δ. Specifically, the higher is δ, the higher is the cost of the first externality (being forced to stop on a low draw) and the lower is the cost of the second externality (being forced to continue on a high draw). Thus, a higher value of δ leads to a higher optimal M.

We can compare our welfare maximizing results to the extensive literature on voting with common values and private information, which began with Condorcet (1785). To fix ideas, assume there are two choices, $\{c_1, c_2\}$, and two states of the world, $\{\omega_1, \omega_2\}$. Everyone agrees that choice c_i is optimal in state ω_i, but everyone has private information about the state of the world (think of a jury that would like to convict *iff* the defendant is guilty). In this context Condorcet argued that for a "large population" (formally as $N \to \infty$), majority rule will yield the correct choice with probability 1 (the Condorcet Jury Theorem), although he assumed sincere voting. Austen-Smith and Banks (1996) proved the Condorcet Jury Theorem for strategic voting, i.e., assuming that pivotal voters correctly process the information contained in being pivotal. Feddersen and Pesendorfer (1998) showed that unanimity is the *uniquely* suboptimal rule. That is, requiring any fraction of the votes to agree will result in the correct action with probability arbitrarily close to 1 for high enough N, while if unanimity is required, the probability of a correct choice does not converge to 1.

Clearly, we have a very different model, one with private values and search externalities, while in the information aggregation literature, values are common and suboptimal decisions result from information externalities. Thus, we do not want to push the comparison too far. To the extent that the information aggregation literature's main message has been that requiring unanimity is (uniquely) suboptimal, we offer a contrasting message; namely, with search externalities, unanimity can be optimal.

6 Conclusion

In this paper, we analyze a new type of search problem, search by committee in which the decision to stop or to continue searching is made by a group of agents. First, we show that the problem is well posed in that a symmetric stationary equilibrium exists and is unique given a log concavity assumption on the distribution of payoffs. We then show that agents in a committee are less picky than they would be were they searching on their own; that is, they set a lower acceptance threshold. This does not necessarily imply a lower expected search duration. We find that the expected search duration of a committee versus that of a single agent varies with the discount rate in a non-monotonic way. We also show that committee members are more conservative than a single-agent searcher in the sense that a mean preserving spread in the distribution of returns may make them worse off.

In addition to comparing committee search with single-agent search, we also provide comparative statics results for the search-by-committee problem. We examine the effects of varying both committee size and the plurality rule. Finally, we consider the mechanism design problem of finding the optimal plurality rule. We find that the welfare-maximizing M increases with the discount rate and that unanimity is optimal for sufficiently high (but bounded) discount rates.

Since ours is the first analysis of the search-by-committee problem, we use a framework that is as close as possible in structure to the canonical single-agent sequential search problem. Of course, that canonical problem has been extended in many directions in the single-agent case. One could do the same in the search-by-committee problem. We have also chosen to model the strategic interaction among committee members in the simplest possible way. Again, this aspect of our model could also be extended. We leave these extensions for later research since our aim here is to introduce and analyze the search-by-committee model in its most basic form.

A Appendix

A.1 Proof of Lemma 1

We want to prove that

$$\Omega(z) = E[X|Y^M \geq z] = (1-(1-\alpha)F(z))\mu_h(z) + (1-\alpha)F(z)\mu_\ell(z)$$

and that $\Omega'(z) \leq 1$. Recall that

$$\mu_h(z) \equiv E[X|X \geq z] \quad \text{and} \quad \mu_\ell(z) \equiv E[X|X < z]$$

and that our log concavity assumptions imply $\mu_h'(z) \leq 1$ and $\mu_\ell'(z) \leq 1$.

Suppose *at least* M committee members vote to stop, i.e., $Y^M \geq z$. The probability that i additional committee members ($i = 0, ..., N-M$) also draw values of z or more is

$$\binom{N-M}{i}(1-F(z))^i F(z)^{N-M-i}.$$

Given i, the expected payoff to a randomly selected committee member is a weighted average of $\mu_h(z)$ and $\mu_\ell(z)$, specifically,

$$\frac{M+i}{N}\mu_h(z) + \frac{N-M-i}{N}\mu_\ell(z).$$

We can thus write $\Omega(z) = E[X|Y^M \geq z]$ as:

$$\begin{aligned}
\Omega(z) &= \sum_{i=0}^{N-M}\binom{N-M}{i}(1-F(z))^i F(z)^{N-M-i}\left[\frac{M+i}{N}\mu_h(z) + \frac{N-M-i}{N}\mu_\ell(z)\right] \\
&= \frac{M}{N}\mu_h(z) + \frac{N-M}{N}\mu_\ell(z) + (\mu_h(z) - \mu_\ell(z))\sum_{i=0}^{N-M}(\frac{i}{N})\binom{N-M}{i}(1-F(z))^i F(z)^{N-M-i} \\
&= \frac{M}{N}\mu_h(z) + \frac{N-M}{N}\mu_\ell(z) + (\mu_h(z) - \mu_\ell(z))\frac{N-M}{N}(1-F(z)) \\
&= (1-(1-\alpha)F(z))\mu_h(z) + (1-\alpha)F(z)\mu_\ell(z),
\end{aligned}$$

which establishes the form of $\Omega(z)$.

Next, differentiating with respect to z gives:

$$\begin{aligned}
\Omega'(z) &= (1-(1-\alpha)F(z))\mu_h'(z) + (1-\alpha)F(z)\mu_\ell'(z) - (1-\alpha)f(z)\left[\mu_h(z) - \mu_\ell(z)\right] \\
&\leq (1-(1-\alpha)F(z))\mu_h'(z) + (1-\alpha)F(z)\mu_\ell'(z) \leq 1,
\end{aligned}$$

where the last is a weighted average of $\mu_h'(z)$ and $\mu_\ell'(z)$. The result then follows from our log concavity assumptions. □

A.2 Proof of Proposition 1

Existence: We establish existence with a very simple argument, namely, $\Psi(z;\cdot)$ is continuous in z, $\delta\Psi(0;\cdot) = \delta\int xf(x)dx > 0$, and $\delta\Psi(1;\cdot) = 0$.

SINGLE CROSSING $\Rightarrow$ UNIQUENESS: Note that given $\delta\Psi(0,\cdot) > 0$ and $\Psi(z,\cdot)$ continuous in z, if there are multiple equilibria, it must be that $\delta\Psi_z(x^*,\cdot) \geq 1$ for at least one of them. We can thus prove uniqueness by showing $\delta\Psi_z(x^*,\cdot) < 1$ for any x^* that satisfies our equilibrium condition $x^* = \delta\Psi(x^*;\cdot)$. Instead we establish the stronger condition that $\delta\Psi_z(z;\cdot) < 1$ for all z. Note that if $\Psi_z(z;\cdot) \leq 0$ we are done, so henceforth we assume $\Psi_z(z;\cdot) > 0$.

To do this, we first show that $\Psi_z(z;\cdot) > 0$ implies that $\delta\Psi_z(z;M,N,\delta)$ is nondecreasing in δ, so that we can then establish our claim by showing $\Psi_z(z;M,N,1) \leq 1$. To see that $\delta\Psi_z(z;M,N,\delta)$ is nondecreasing in δ, we need to differentiate it with respect to δ. First, using equation (1) we find:

$$\frac{\partial(\delta\Psi(z;M,N,\delta))}{\partial\delta} = \frac{\Psi(z;M,N,\delta)}{1-\delta G^M(z)},$$

so that

$$\frac{\partial^2(\delta\Psi(z;M,N,\delta))}{\partial\delta\partial z} = \frac{\Psi_z(z;M,N,\delta)}{1-\delta G^M(z)} + \frac{\delta\Psi(z;M,N,\delta)g^M(z)}{(1-\delta G^M(z))^2}.$$

The second term is positive, and $\Psi_z > 0$ implies that the first term is positive: together $\Psi_{z\delta} \geq 0$. Thus, $\Psi_z(z;M,N,1) \leq 1$ yields $\delta\Psi_z(z;M,N,\delta) \leq 1$ for all δ. Direct computation yields $\Psi_z(z;M,N,1) = \Omega'(z) \leq 1$, where the final inequality was established in Lemma 1. □

A.3 Proof of Proposition 2

Let x^* be the common equilibrium acceptance threshold for the N committee members. Let $\widetilde{x}$ be the optimal acceptance threshold for the single-searcher. The single-searcher can achieve at least as high an expected payoff as a committee member by setting threshold x^* now and committing to the following strategy in the future: generate $N-1$ standard uniform random variables; stop whenever M of these exceed $1-F(x^*)$; continue if fewer than $M-1$ exceed $1-F(x^*)$; and employ threshold x^* otherwise. The single-searcher, however, can do better. For example, suppose the single-searcher draws $x < x^*$ and that M of the $N-1$ draws exceed x^*. The single-searcher can then increase his or her expected payoff by continuing. The expected payoff for a single-searcher thus exceeds the expected payoff for a committee member; equivalently, $\widetilde{x} > x^*$. That is, committee members are less picky than a single-searcher. □

A.4 Proof of Proposition 3

Let $x^*(\delta)$ be the threshold for a committee and $\widetilde{x}(\delta)$ the threshold for a single agent searching from the same F and given the same δ. The committee has a lower expected search duration *iff* $G^M(x^*(\delta)) < F(\widetilde{x}(\delta))$.

First, we establish that when $M = 1$, expected search duration is shorter for the committee than for the single-searcher for all δ. Clearly, $G^1(x) < F(x)\ \forall x \in (0,1)$. Since G^1 and F are strictly increasing functions and $x^*(\delta) < \widetilde{x}(\delta)\ \forall \delta \in (0,1)$ by Proposition 2, $G^1(x^*(\delta)) < F(\widetilde{x}(\delta))$, i.e., the committee has a shorter expected search duration.

Next, we establish the result for $M \in \{2, \ldots, N-1\}$.

Claim 3 *For each $M \in \{2, ..., N-1\}$, there exists an $\bar{x} \in (0,1)$ such that $G^M(x) < F(x)$ for all $x < \bar{x}$.*

Proof of Claim 3: Note first that $F(0) = G^M(0) = 0$, $F(1) = G^M(1) = 1$, and $F(x)$ and $G^M(x)$ are continuous. To prove our result, it then suffices to show that $f(0) > g^M(0)$ and $f(1) > g^M(1)$. To do this, it is convenient to use the relationship between the binomial and beta cdfs (see, e.g., Casella and Berger 2002, p. 82), namely:

$$\begin{aligned} G^M(x) &= \sum_{j=0}^{M-1} \binom{N}{j} (1-F(x))^j F(x)^{N-j} \\ &= \frac{N!}{(N-M)!(M-1)!} \int_0^{F(x)} t^{N-M}(1-t)^{M-1} dt. \end{aligned}$$

We then have:

$$g^M(x) = f(x) \frac{N!}{(N-M)!(M-1)!} F(x)^{N-M}(1-F(x))^{M-1}.$$

That is, $g^M(0) = g^M(1) = 0$ for each $M \in \{2, ..., N-1\}$. Since $f(0)$ and $f(1)$ are both positive, we have shown that there exists an $\bar{x} \in (0,1)$ such that $G^M(x) < F(x)$ for all $x < \bar{x}$. □.

Next, fix $M \in \{2, ...N-1\}$. Let $\bar{x}$ be the threshold value defined by Claim 3. Since $\widetilde{x}(\delta)$ is strictly increasing in δ, $\widetilde{x}(0) = 0$, and $\lim_{\delta \to 1} \widetilde{x}(\delta) = 1$, we can define $\delta_L(\bar{x})$ by $\widetilde{x}(\delta_L(\bar{x})) = \bar{x}$. That is, $\delta_L(\bar{x})$ is the discount rate that leads the single agent to choose an acceptance threshold of $\bar{x}$. Since we have just shown that $G^M(x) < F(x)$ for all $x < \bar{x}$, it follows from Proposition 2 that $x^*(\delta) < \widetilde{x}(\delta) \leq \bar{x}$ for all $\delta \leq \delta_L(\bar{x})$. Finally, $G^M(x)$ and $F(x)$ increasing establishes that $G^M(x^*(\delta)) < F(\widetilde{x}(\delta))$ for all $\delta < \delta_L(\bar{x})$.

The final step is to establish that there exists a $\delta_H < 1$ such that $G^M(x^*(\delta)) < F(\widetilde{x}(\delta))$ for all $\delta > \delta_H$. To see this, note that when $M < N$, even if every committee member were to set a threshold of $x = 1$, the expected draw conditional on stopping would be strictly less than 1. Thus, $x^*(\delta) < 1$, for all δ, which implies that $G^M(x^*(\delta)) < 1$ for all δ. However, for the single-searcher, $\lim_{\delta \to 1} \widetilde{x}(\delta) = 1$, which implies $\lim_{\delta \to 1} F(\widetilde{x}(\delta)) = 1$. Since both $G^M(x^*(\delta))$ and $F(\widetilde{x}(\delta))$ are continuous in δ, there exists a $\delta_H < 1$ such that $G^M(x^*(\delta)) < F(\widetilde{x}(\delta))$ for all $\delta > \delta_H$. $\square$

A.5 Proof of Proposition 4

The proof of Proposition 4 consists of 3 steps.

Claim 4 *Fix $N' > N$. For fixed $\alpha < 1$ there exists a cutoff $\bar{x}(\alpha) > 0$ such that $G^{\alpha N'}(x) - G^{\alpha N}(x)$ satisfies a single crossing property: negative for all $x < \bar{x}(\alpha)$ and positive for all $x > \bar{x}(\alpha)$.*

Claim 5 *For fixed α, consider the effect on x^* of an increase in committee size from N to N'. If the effect is to increase x^*, then expected search duration falls. If the effect is to decrease x^*, then expected search duration increases.*

Claim 6 *If $x^* < \bar{x}(\alpha)$ ($x^* > \bar{x}(\alpha)$) then x^* increases (decreases) in committee size, holding α constant, and thus expected search duration falls (rises).*

The proof follows from the above steps and the fact that x^* is increasing in δ. That $\delta^*(\alpha) > 0$ follows from $x^* = 0$ when $\delta = 0$. The weak inequality $\delta^*(\alpha) \leq 1$ follows from the fact that $x^* < 1$ even when $\delta = 1$, so long as $\alpha < 1$.

We now prove each of the above claims.

Proof of Claim 4: Consider $g^{\alpha N'}(x)/g^{\alpha N}(x)$. Using the expression for $g^M(x)$ developed in the proof of Proposition 3 and letting $u = F(x)$, this ratio is proportional to

$$h(u) = \frac{u^{(1-\alpha)N'}(1-u)^{\alpha N'-1}}{u^{(1-\alpha)N}(1-u)^{\alpha N-1}} = \left[u^{1-\alpha}(1-u)^{\alpha}\right]^{N'-N}.$$

Note that $\lim_{u\to 0} h(u) = \lim_{u\to 1} h(u) = 0$. Since u is continuous and monotonically increasing in x, $G^{\alpha N'}(x) < G^{\alpha N}(x)$ for x sufficiently close to zero, while $G^{\alpha N'}(x) > G^{\alpha N}(x)$ for x sufficiently close to one. Thus, by continuity, $G^{\alpha N'}(x) = G^{\alpha N}(x)$ for at least one $x \in (0,1)$. The fact that there is only one such crossing point follows from noting that $h(u)$ is continuous and unimodal on $[0,1]$. By the endpoint conditions, if $G^{\alpha N'}(x)$ and $G^{\alpha N}(x)$ cross more than once in $(0,1)$, they must cross at least 3 times. Call these three crossing points $0 < u_1 < u_2 < u_3 < 1$. Again, by the endpoint conditions, we must have $h(u_1) > 1$, $h(u_2) < 1$, and $h(u_3) > 1$. This, however, contradicts the fact that $h(u)$ is continuous and unimodal. □

Proof of Claim 5: Expected search duration equals $\frac{1}{1-G^{\alpha N}(x^*)}$, so to establish the effect of a change in x^* brought about by a change in committee size holding α constant on expected search duration, we need to establish the effect of such a change on $G^{\alpha N}(x^*)$. From Lemma 1, note that $\Omega(x)$ only depends on N through α; that is, holding α constant, $\Omega(x)$ does not depend on committee size. We take advantage of this fact by rewriting our equilibrium condition as:

$$k(x^*) \equiv \frac{x}{\Omega(x)} = \frac{\delta(1-G^{\alpha N}(x^*))}{1-\delta G^{\alpha N}(x^*)}.$$

We claim that $k'(x^*) > 0$ and that this implies the desired result. To see the latter, note that the right-hand side is decreasing in $G^{\alpha N}(x^*)$. Thus if $k'(x^*) > 0$, then $G^{\alpha N}(x^*)$ moves in the opposite direction of x^*. The implication then follows from expected search duration monotonically increasing in $G^{\alpha N}(x^*)$.

To see that $k'(x^*) > 0$, differentiate to get:

$$k'(x) = \frac{\Omega(x) - x\Omega'(x)}{\Omega(x)^2}.$$

Thus,

$$k'(x) > 0 \quad \Leftrightarrow \quad \Omega(x) > x\Omega'(x).$$

By Lemma 1, $\Omega'(x) \leq 1$, so that, $k'(x^*) > 0$ follows if $x^* < \Omega(x^*)$. This latter is implied by $\delta(1 - G^{\alpha N}(x^*))/(1 - \delta G^{\alpha N}(x^*)) < 1$. □

Proof of Claim 6: We have:

$$x^* = \frac{\delta(1 - G^{\alpha N}(x^*))\Omega(x^*)}{1 - \delta G^{\alpha N}(x^*)}.$$

Lemma 1 establishes that $\Omega(x)$ is constant in N (α held constant). Claim 4 establishes that for all $x < \bar{x}(\alpha)$, $G^{\alpha N}(x)$ is decreasing in N and for all $x > \bar{x}(\alpha)$, $G^{\alpha N}(x)$ is increasing in N. Thus, if $x^* < \bar{x}(\alpha)$ ($x^* > \bar{x}(\alpha)$) then x^* increases (decreases) in N. The result for expected search duration then follows from Claim 5. □

A.6 Proof of Part 1 of Proposition 5

Our equilibrium conditions can be written as a system of two equations in two unknowns:

$$\begin{aligned} \Psi(x; M, \cdot) &= G^M(x)\delta\Psi(x; M, \cdot) + (1 - G^M(x))\Omega(x) \\ x &= \delta\Psi(x; M, \cdot) \end{aligned} \tag{3}$$

Let $(\Psi^*(x^*; M, \cdot), x^*)$ be the solution to this system.

Claim 7 *Given our log concavity assumptions, the solution to the system of equations*

$$\begin{aligned} \phi(x; M, \cdot) &= G^M(x)x + (1 - G^M(x))\Omega(x) \\ x &= \delta\phi(x; M, \cdot) \end{aligned} \tag{4}$$

is equivalent to that of system (3) in the sense that $(\phi^*(x^*; M, \cdot), x^*)$ solves system (4) iff $(\Psi^*(x^*; M, \cdot), x^*)$ solves system (3).

Proof of Claim 7: Our equilibrium (the solution to system (3)) is unique. The solution to system (3) also solves system (4). To show equivalence, we thus only need to show that the solution to system (4) is unique. As before, this follows from single crossing, i.e., $\delta\phi_x(x^*; \cdot) \leq 1$. We have:

$$\begin{aligned} \phi_x(x^*, \cdot) &= g^M(x^*)\left[x^* - \Omega(x^*)\right] + G^M(x^*) + (1 - G^M(x^*))\Omega'(x^*) \\ &< G^M(x^*) + (1 - G^M(x^*))\Omega'(x^*), \end{aligned}$$

where the inequality follows from $x^* < \Omega(x^*)$. Thus, the derivative is bounded above by a $(G^M(x^*), 1 - G^M(x^*))$ weighted average of 1 and $\Omega'(x^*)$, and, as we have shown in Lemma 1, our log concavity assumptions imply $\Omega'(x) \leq 1$.

Having established that the solutions to the two systems are equivalent, we work with the simpler ϕ function.

Claim 8 *Let $b(M; N, 1 - F(x))$ be the (binomial) probability that exactly M out of N committee members draw values greater than or equal to x. Then,*

$$\phi(x; M + 1, \cdot) - \phi(x; M, \cdot) = b(M; N, 1 - F(x)) \left[x - \Delta(x, M)\right],$$

where

$$\Delta(x, M) \equiv \frac{M}{N} E[X'|X' \geq x] + \frac{N - M}{N} E[X'|X' \leq x],$$

is the expected payoff when exactly M members vote to stop and the acceptance threshold is x.

Proof of Claim 8: Given that we have fixed the "continuation value" and acceptance threshold at x in ϕ, the only time that requiring $M+1$ versus M votes to stop matters is when exactly M committee members draw values of x or more. The probability that this occurs is $b(M; N, 1 - F(x))$. If $M + 1$ votes are required to stop but only M votes are realized, the committee continues and each member receives "continuation value" x. If, instead, M votes are required to stop, then when exactly that many votes to stop are realized, each member's expected payoff is $\Delta(x, M)$. Thus, the change in value is $x - \Delta(x, M)$ times the probability that exactly M committee members draw values of x or more.

Claim 9 *The function $\Delta(x, M)$ has the following properties: $0 \leq \Delta_x(x, M) \leq 1$ and $\Delta(x, M + 1) > \Delta(x, M)$. Further, if $0 < M < N$, then $\Delta(0, M) > 0$ and $\Delta(1, M) < 1$. Thus, for $0 < M < N$, the difference $x - \Delta(x, M)$ satisfies a single-crossing property – negative for smaller values of x and positive for larger values of x.*

Proof of Claim 9: The fact that $0 \leq \Delta_x(x, M) \leq 1$ follows from log concavity. The other properties follow directly.

Claim 10 *Let* $x^*(M)$ *be the solution to system (4) (equivalently to (3)). Then*

$$\begin{aligned} x^*(M+1) &> x^*(M) \quad \Leftrightarrow \quad x^*(M) > \Delta(x^*(M), M) \text{ and} \\ x^*(M+1) &< x^*(M) \quad \Leftrightarrow \quad x^*(M) < \Delta(x^*(M), M) \end{aligned}$$

Proof of Claim 10: The fact that system (4) has a unique solution implies:

$$\begin{aligned} x^*(M+1) &> x^*(M) \quad \Leftrightarrow \quad \phi(x^*(M); M+1, \cdot) > \phi(x^*(M); M, \cdot) \text{ and} \\ x^*(M+1) &< x^*(M) \quad \Leftrightarrow \quad \phi(x^*(M); M+1, \cdot) < \phi(x^*(M); M, \cdot) \end{aligned}$$

The result then follows from Claims 8 and 9.

At this point, it is worth restating Part 1 of Proposition 5 in the notation that has been developed in Claims 7-10. Specifically, Part 1 of Proposition 5 states that there exists a $k \in (0, N]$ such that for all $M < k$, $x^*(M+1) > x^*(M)$, while for all $M > k$, $x^*(M+1) < x^*(M)$. This follows from two final claims, namely,

Claim 11 *Let* $x^*(0) = \delta E[X]$[6]*. Then* $x^*(1) > x^*(0)$*.*

Claim 12 *For* $0 < M < N$, $x^*(M+1) < x^*(M) => x^*(M+2) < x^*(M+1)$*.*

Simply stated, Claim 11 asserts that x^* is initially increasing in M, and Claim 12 asserts that once x^* begins to decrease in M, it continues to do so for higher values of M.

Proof of Claim 11: By inspection $x^*(0) > 0$, then:

$$x^*(0) > E[X'|X' \leq x^*(0)] = \Delta(x^*(0), 0).$$

Then by Claim 10, $x^*(1) > x^*(0)$.

Proof of Claim 12: By Claim 10

$$x^*(M) > x^*(M+1) \Rightarrow x^*(M) < \Delta(x^*(M), M).$$

[6]This follows from our definition of x^* equal to δ times the continuation value. When $M = 0$, the search stops with probability 1 and thus the continuation value is the expected value conditional on stopping which is $E[X]$.

Given that $x^*(M) > x^*(M+1)$, the single crossing of $x - \Delta(x, M)$ shown in Claim 9 implies that

$$x^*(M+1) < \Delta(x^*(M+1), M+1).$$

By Claim 10, we then have

$$x^*(M+1) > x^*(M+2).$$

That is, there exists a $k \in (0, N]$ such that for all $M < k$, $x^*(M+1) > x^*(M)$, while for all $M > k$, $x^*(M+1) < x^*(M)$. □

A.7 Proof of Proposition 6

Claim 10 established that increasing M will raise x^* and thus expected welfare *iff:*

$$\begin{aligned} x^*(M) &> \Delta(x^*(M), M) \equiv \frac{M}{N} E[X'|X' \geq x^*(M)] + \frac{N-M}{N} E[X'|X' \leq x^*(M)] \\ &= \frac{M}{N}\mu_h(x^*(M)) + \frac{N-M}{N}\mu_\ell(x^*(M)). \end{aligned}$$

We claim that if this inequality is satisfied at some δ, then it must also be satisfied at all $\delta' > \delta$, which establishes that the optimal M must increase in δ. First, recall that x^* increases in δ, which means that both sides of the inequality are increasing in δ. However, by log concavity $\Delta_x(x, M) \leq 1$, thus the LHS is increasing more rapidly.

Henceforth we assume $M = N - 1$ and then show that for δ high enough x^* can be increased by setting $M = N$, i.e. $x^*(N-1) > \Delta(x^*(N-1), N-1)$. The optimal threshold x^* is continuous in δ and bounded, so that its limit as $\delta \to 1$ exists. Call this limit x_1^*. We have $\Delta(x, N-1)$ continuous in x and x^* continuous in δ, so that, $x^* - \Delta(x^*, N-1)$ is continuous in δ. Thus $x_1^* > \Delta(x_1^*, N-1)$ is sufficient to establish the existence of a threshold δ^* such that setting $M = N$ maximizes x^*, for all $\delta > \delta^*$.

First suppose that $x_1^* = 1$. The we have:

$$\begin{aligned} \Delta(x_1^*, N-1) &= \frac{N-1}{N}\mu_h(x_1^*) + \frac{1}{N}\mu_\ell(x_1^*) \\ &= \frac{N-1}{N} + \frac{1}{N}\mu_\ell(x_1^*) < \frac{N-1}{N} + \frac{1}{N} = 1 = x_1^* \end{aligned}$$

Thus, for δ sufficiently close to 1: $x^* > \Delta(x^*, N-1)$ and welfare is increased by raising M to N.

Assume instead that $x_1^* < 1$. The $\lim_{\delta \to 1} G^M(x^*) < 1$, which implies

$$\lim_{\delta \to 1} \frac{\delta(1 - G^M(x^*))}{1 - \delta G^M(x^*)} = 1,$$

and, in the limit, our equilibrium condition simplifies to $x_1^* = \Omega(x_1^*)$, which from Lemma 1 may be rewritten:

$$\begin{aligned} x_1^* &= \Omega(x_1^*) = \left(1 - \frac{1}{N} F(x_1^*)\right) \mu_h(x_1^*) + \frac{1}{N} F(x_1^*) \mu_\ell(x_1^*) \\ &> \left(1 - \frac{1}{N}\right) \mu_h(x_1^*) + \frac{1}{N} \mu_\ell(x_1^*) = \Delta(x_1^*, N-1), \end{aligned}$$

where the strict inequality follows from $F(x_1^*) < 1$. Since $x_1^* > \Delta(x_1^*, N-1)$, we may again conclude that $x^* > \Delta(x^*, N-1)$ for δ sufficiently close to 1. □

References

[1] Austen-Smith, David and Jeffrey Banks (1996), Information Aggregation, Rationality, and the Condorcet Jury Theorem, *The American Political Science Review*, 90 (1), 34–45.

[2] Bagnoli, Mark and Ted Bergstrom (2005), Log-concave Probability and its Application, *Economic Theory*, 26 (2), 445–469.

[3] Black, Duncan (1948), The Decisions of a Committee Using a Special Majority, *Econometrica*, 16 (3), 245–261.

[4] Börgers, Tilman (2004), Costly Voting, *American Economic Review*, 94 (1) 57–66.

[5] Burdett, Kenneth (1996), Truncated Means and Variances, *Economics Letters*, 52 (3), 263–267.

[6] Casella, George and Roger Berger (2002), *Statistical Inference*, Second Edition, Pacific Grove: Duxbury.

[7] Condorcet, M.D. (1785), Essay on the Application of Analysis to the Probability of Majority Decisions, *Paris.*

[8] Feddersen, Timothy and Wolfgang Pesendorfer (1998), Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting, *American Political Science Review*, 92 (1), 23–35.

[9] Goldberger, Arthur (1983), Abnormal Selection Bias, in *Studies in Econometrics, Time Series, and Multivariate Statistics*, (eds.) S. Karlin, T. Amemiya, and L. Goodman, 67-84, New York: Academic Press.

[10] Hotelling, Harold (1929), Stability in Competition, *Economic Journal*, 39 (153), 41–57.

[11] Lippman, Steven and John McCall (1976), The Economics of Job Search: A Survey, *Economic Inquiry*, 14 (2), 155–189, 347–68.

[12] McCall, John (1970), Economics of Information and Job Search, *Quarterly Journal of Economics*, 84 (1), 113–126.

[13] Mortensen, Dale (1986), Job Search and Labor Market Analysis, in *Handbook of Labor Economics*, O. Ashenfelter and R. Layard, eds., Amsterdam: North-Holland, 849–920.

[14] Rogerson, Richard, Robert Shimer and Randall Wright (2005), Search-Theoretic Models of the Labor Market: A Survey, *Journal of Economic Literature*, XLIII (December), 959–988.

[15] Vroman, S.B. (1985), No-Help-Wanted Signs and the Duration of Job Search, *Economic Journal*, 95, 767-773.