Part-time Employment Traps.

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Abstract

We model educational investment, wages and employment status (full-time, part-time or non-participation) in a frictional world in which heterogeneous workers have different productivities, both at home and in the workplace. We investigate the degree to which there might be under-employment and distortions in investment in education. A central insight is that the ex-post participation decision of workers endogenously generates increasing marginal returns to education and this non-convexity generates a part-time employment trap. We show how childcare policy can be used to correct the ex post under-participation problem and to provide efficient incentives to invest optimally ex ante in education.

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1 Introduction

Childcare and work-family reconciliation policies have attracted much recent interest in developed countries. Reasons advanced to justify such policy intervention are that it will increase female participation, assist child development, and encourage greater investment by women in human capital, thereby improving their labor market opportunities. This discussion is premised on the notion that the labor market, left on its own, will produce a less than optimal outcome.[1] This paper considers optimal labor market policy within an equilibrium model of general human capital investment in an imperfectly competitive labor market. Its purpose is to identify circumstances under which employment subsidies, paid as childcare benefits, may generate large welfare gains.

The paper extends recent work on imperfect competition (see Bhaskar, Manning and To (2002) for a recent survey) to a labour market context where workers might substitute to home production. A central feature is that workers are heterogeneous, having different productivities both in the home and in the workplace.[2] Importantly, labor market distortions affect such workers differently. For example, the talented home-maker who has high home productivity but low workplace productivity is unaffected by imperfect competition in the labor market. Such homemakers earn their marginal product in the household and are largely unaffected by market failures in the workplace. In contrast, the incompetent parent with high workplace productivity is most affected by imperfect competition in the labor market - that worker participates in the labor market with probability one and may suffer a significantly reduced pay packet through imperfect wage competition. However the corresponding deadweight loss is small because the worker still chooses to participate in the labor market, which is the socially optimal outcome. The efficiency losses are instead greatest for those who are relatively talented in both dimensions. For these workers, a non-competitive labor market may lead to large substitution effects to home production.

Those deadweight losses are magnified by ex-ante education decisions. In the first phase of their lives, youngsters can increase their future workplace ability by investing in general skills that affect workplace productivity. Such investments may not necessarily improve future home productivity. For example, they might invest in a mathematics course or a qualification in information technology, imbuing them with

[1] For examples of discussion of childcare policy issues, see the websites of various US federal and state governments, and in the UK see http://www.number-10.gov.uk/output/page1430.asp

[2] Some may have very high levels of workplace productivity and low levels of domestic productivity - such as the talented physicist who cannot imagine that the Big Bang could be something happening to the kids in the kitchen. Others may be highly proficient in both workplace and home production: the so-called superwomen, such as Ma Baker and Sandra Day O'Connor in the US, and Cherie Blair in the UK. Others again might be characterized by rudimentary literacy and numeracy abilities, rendering them of low workplace productivity, but with a high level of competence at home-making. Other less fortunate individuals might be untalented in both dimensions.

expertise that is invaluable in the workplace but is unlikely to increase their skills at home-decorating or their patience at child-rearing. Of course if the labor market were perfectly competitive, so that workers were paid their full market value, each worker would invest in general human capital at the socially optimal level. But if a worker expects to receive less than the full marginal return to human capital investment, the worker not only underparticipates in the labor market ex-post but also underinvests in human capital ex-ante.

A central insight of the paper is that the participation decision of workers endogenously generates increasing marginal returns to education. Certainly the individual who does not intend participating in the workplace has a zero return to any investment in workplace skills. In the model, workers with greater workplace skills receive better wage offers and so are more likely to participate in the workplace. The critical point is that a higher participation probability raises the ex-ante expected marginal return to human capital investment. A second reason for increasing returns to education is that greater general human capital can make it worthwhile for the worker to switch from part-time to full-time employment. Rather than only spend a proportion of time $l^* < 1$ in the workplace, the switch to full time work increases the expected return to human capital investment by a factor of $1/l^*$.

Even with a competitive labor market, there are increasing marginal returns to education. But the assumed market imperfection (described below) generates a third increasing return to education. Wages in the model are determined using a Hotelling-type pricing structure.[3] That structure implies that wage competition becomes more intense as the worker's value of employment increases; i.e., wages rise more quickly with productivity as productivity increases, and rises one for one with productivity once the worker (endogenously) participates with probability one.

Increasing marginal returns to education implies workers specialize. Those with a comparative advantage in workplace production invest ex-ante in high levels of human capital and participate ex-post in full-time employment with probability close to one. Those with a comparative advantage in home production choose little education ex-ante and focus on home production ex-post, though possibly taking part-time employment (assuming decreasing returns to home production). But most interestingly, this non-convexity and an imperfectly competitive labour market generates a part-time employment trap. For youngsters in that trap, the social planner's optimum implies the youngster should choose a high level of education and ex-post enjoy a high participation probability in full time employment. But as workers are not paid their full value in the labor market, these youngsters substitute to home production. They make low skills investments ex-ante, and participate with low probability in the labor market ex-post - and only in part-time employment if they do choose to

[3] Bhaskar, Manning and To (2002) also note that transport costs can usefully summarize the variety of reasons for imperfect competition in the labour market - such as imperfect information about alternative jobs, mobility costs, and heterogeneous preferences over the non-wage characteristics of a job.

participate. The large substitution effect implies a correspondingly large deadweight loss.

The policy section establishes that an employment subsidy, paid to the worker, not only corrects the ex-post under-participation problem, but also corrects the ex-ante under-education problem. This section also examines empirically which sectors of society are characterised by low participation rates/part-time employment and relatively low education levels. As the title of the paper suggests, married women with children fall squarely into this category. The optimal employment subsidy can be targeted directly to this group as child care subsidies.

The next section describes the model and Section 3 determines equilibrium remuneration and participation rates of workers by productivity type. Section 4 examines the worker's optimal investment decision and shows how undertraining and under-participation is closely associated with part-time employment. Section 5 develops the implications for optimal childcare policies.

2 The Model

Each individual is productive both at home and in the workplace. A representative person is born in the first period with ability a and has expectations of future home productivity b. In the first period, the individual at cost $\phi(k)$ can invest in k units of general skills, whereupon the worker's second period productivity in the workplace is $\alpha = a + k$. Although one could argue that training in the first period might also improve home productivity skills, for example a university degree creates a more erudite parent, we simplify by assuming the worker takes the second period value of b as given. Assume ϕ is continuously diffentiable, strictly convex and $\phi(0) = \phi'(0) = 0$.

In the second period, the worker has a unit time endowment which is allocated between time spent in home production (h) and in the workplace (l), so that $h+l = 1$. To motivate the existence of part-time employment contracts, we assume there are diminishing marginal returns to home production. If the worker allocates time h to home production, assume the value of home output is $bx(h)$ where $x(.)$ is increasing, differentiable and concave with $x(0) = 0$.

There are constant marginal returns to labour in the workplace; a worker with workplace productivity α who supplies l units of labor to the workplace generates revenue αl. One could instead assume diminishing marginal returns to labour, but if the worker's output is small relative to the scale of the firm, the constant returns assumption seems a reasonable approximation. The critical ingredient for what follows is that this revenue function $R = \alpha l$ has increasing returns to scale in productivity and labour supply. It is worth noting that this feature is also consistent with a competitive labour market where, given competitive wage rate w, the worker earns income $E = wl$ by supplying l hours to the market. Given a competitive wage rate $w = w(\alpha)$ which is increasing in worker productivity, the corresponding earnings function $E = lw(\alpha)$ also exhibits increasing returns to scale where labour supply and human capital are complementary inputs.

To model imperfect competition in the labor market, we follow recent work by Bhaskar et al (2002). As pointed out in the conclusion, many of the results also occur if instead wages are determined by Nash bargaining. Both approaches imply equilibrium wage compression and hence underinvestment in general human capital (e.g. Acemoglu and Pischke(??)). The added insight of the Bhaskar et al (2002) approach, however, is that there is also equilibrium underparticipation. Underparticipation occurs as we assume the individual's disutility to workplace employment is private information. This implies each firm faces an upward sloping labour supply schedule where the higher the wage offered, the more likely the worker will accept the job offer. Standard monopsony arguments then imply firms offer wages below marginal revenue product and workers then underparticipate in the labour market.

Wages are determined in the model by Bertrand competition between n firms, $n \geq 2$. The worker's productivity α is the same in all firms and is common knowledge. Firms however are not fully informed on worker preferences. As an employer might accurately gauge that a mother with young children has higher home productivity than an 18 year old with a Game Boy, we simplify by assuming employers can infer home productivity. Firms cannot however infer a worker's relative preference for working in the home or participating in the workplace. If the worker accepts employment at firm i, we suppose she has an additional disutility $c_i \geq 0$ to working there. Note we assume this cost is a fixed cost to working at firm i; i.e. it is analogous to a transport or commuting cost. Although one could instead specify a disutility cost $c_i l$, where that loss is proportional to the amount of time spent working at the firm, this would then introduce screening issues - a firm posts a menu of contracts where part-time employment contracts are targeted to workers with high c_i and full time contracts for those with low c_i. The transport (fixed) cost approach adopted here abstracts from such issues. This structure is then analogous to a Hotelling pricing game with n competing firms.

Formally, then, each worker is characterised by productivities (α, b) which are observed, and employment disutilities c_i, $i = 1, .., n$ which are considered as i.i.d. draws from c.d.f. F and are private information. Assume F is twice differentiable and its density is decreasing over its support $[0, \bar{c}]$; i.e., F is concave.

Given observed productivities (α, b), each firm i simultaneously makes a contract offer (y^i, l^i), where y^i is the amount paid to the worker in return for providing l^i units of labor time. Given those contract offers, the worker either accepts one, say at firm i, and so obtains period 2 utility $U_2 = bx(1 - l^i) + y^i - c_i$, or rejects all and so obtains period 2 utility $U_2 = bx(1)$ through home production. Should the worker accept firm i's contract offer, firm i makes profit $\alpha l^i - y^i$, while the other firms obtain zero profit.

Throughout we shall only consider symmetric pure strategy equilibria. In the second period and given (α, b), each firm i chooses (y^i, l^i) to maximise expected profit, given the acceptance strategy of the worker and the offer strategies of the competing firms. The corresponding symmetric Nash equilibrium implies contract offers $(y^i, l^i) = (y^*(\alpha, b), l^*(\alpha, b))$. Given those equilibrium contract offers (y^*, l^*), we can then compute the worker's expected second period utility, denoted $U_2^*(\alpha, b)$. In the first period, the worker invests in skills k to maximise $-\phi(k) + U_2^*(\alpha(a, k), b)$.

In anticipation of the results below, it is useful to define the following.

Definition: Given (α, b), the value of workplace employment is

$$V(\alpha, b) = \max_{l \in [0,1]} [\alpha l - b[x(1) - x(1-l)]] ;$$

i.e. V is defined as the (maximised) value of workplace output net of foregone home production. Let $l^*(\alpha, b)$ denote the optimal labour supply decision (conditional on participation); i.e.

$$l^*(\alpha, b) = \arg\max_{l \in [0,1]} [\alpha l - b[x(1) - x(1-l)]] .$$

and note that the Envelope Theorem implies $l^* = \partial V / \partial \alpha$. Claim 0 describes their basic properties.

Claim 0. Characterisation of V, l^*.

(i) $l^* = 0$ and $V = 0$ for $\alpha \leq bx'(1)$;

(ii) $l^* \in (0, 1)$ and $V > 0$ are both strictly increasing in α and strictly decreasing in b for $\alpha \in (bx'(1), bx'(0))$;

(iii) $l^* = 1$ and $V = \alpha - b[x(1) - x(0)]$ for $\alpha \geq bx'(0)$; .

Claim 0 follows from standard optimisation theory. We shall refer to $\alpha = bx'(0)$ as the full time margin and productivities $\alpha \in [bx'(0), \infty)$ as the full-time employment region, noting that $l^* = 1$ is optimal for such α. We shall refer to $\alpha = bx'(1)$ as the part-time margin, and the interval $(bx'(1), bx'(0))$ as the part-time employment region as $l^* \in (0, 1)$ is optimal for such α. Note that $\alpha \leq bx'(1)$ implies there is no gain to trade as home productivity strictly dominates workplace productivity.

3 Equilibrium Wages

Given the set of contract offers $\{(y^i, l^i)\}_{i=1,..,n}$ and idiosyncratic utility costs c_i, the worker's second period payoff is

$$U_2 = \max_{i=1,..,n} \left[bx(1 - l^i) + y^i - c_i, bx(1) \right]$$

where the worker either accepts firm i's offer or rejects all. This section characterizes the (symmetric, pure strategy) Nash equilibrium where, given (α, b), the firms simultaneously make contract offers (y^i, l^i) to maximise expected profit, given the job acceptance strategy of the worker.

As productivities are observed, each firm's optimal contract offer implies $l^i = l^*$. Given the set of optimal contract offers, $\{(y^i, l^*)\}_{i=1,..,n}$, the worker's optimal job acceptance strategy is to accept employment at firm i if $y^i - c_i + bx(1-l^*) > \max_{j\neq i}\{bx(1), y^j - c_j + bx(1-l^*)]\}$. Note that firm i faces two margins; a participation margin, where the worker considers firm i's offer only if $y^i - c_i > b[x(1) - x(l^*)]$ (i.e. the job offer must fully compensate for foregone home production), and a poaching margin where firm i's offer must be preferred to all other offers; i.e. $y^i - c_i > y^j - c_j$ for all $j \neq i$. Theorem 1 now describes the symmetric Nash equilibrium to this contract posting game.

Theorem 1. Equilibrium Contract Offers.

For any (α, b) with $V > 0$, a pure strategy, symmetric contract posting equilibrium implies each firm offers contract (y^*, l^*) where

$$y^* = b[x(1) - x(l^*)] + s^*$$

with $s^* = s^*(V)$ given by

$$\frac{1}{n}\left[1 - [1 - F(s^*)]^n\right] = [V - s^*]\left[[1 - F(s^*)]^{n-1} f(s^*) + (n-1)\int_0^{s^*} [1 - F(c_1)]^{n-2}[f(c_1)]^2 dc_1\right]. \tag{1}$$

Proof is in the Appendix.

The equilibrium wage offer, y^*, fully compensates the worker for foregone home production and offers additional surplus s^*. The worker participates in the labour market (i.e. accepts a job offer) only if $y^* - c_i + bx(1 - l^*) > bx(1)$ for at least one i, which is equivalent to $c_i < s^*$ for at least one firm. Hence the worker's participation probability is

$$P(s^*) = 1 - [1 - F(s^*)]^n.$$

The surplus offered s^*, as defined in (1), depends on the value of workplace employment and on the number of competing firms. As n becomes arbitrarily large, competition between firms implies s^* converges to V; i.e. firms offer all the employment surplus in a competitive equilibrium. For finite n, however, firms shade those offers so that $s^* < V$. The equilibrium choice, described by (1), reflects the standard monopsony trade-off between lower wage offers and lower employment. The left hand side of (1) is the probability of employment (it is $P(s^*)/n$) and so describes the marginal loss in profit should, say, firm 1 offer slightly more surplus than the equilibrium offer. The right hand side describes the marginal increase in firm 1's profit by attracting more workers, where $f(s^*)[1 - F(s^*)]^{n-1}$ is the measure of workers who are marginally attracted from non-participation (i.e. workers whose $c_1 = s^*$ and $c_j > s^*$ for $j \neq 1$), and $(n-1)\int_0^{s^*} [1 - F(c_1)]^{n-2}[f(c_1)]^2 dc_1$ is the measure of workers who are marginally poached from a competing firm j, where the worker is indifferent between accepting firm 1's offer and a firm $j's$ offer (i.e. $c_1 = c_j < s^*$ and $c_k > c_1$ for $k \neq 1, j$), where this state potentially occurs with each of the $n-1$ competing firms. Optimality requires that these two margins are equal. Also note that (1) describes the optimal contract offer with pure monopsony, where $n = 1$, and there is no poaching margin.

The critical feature for what follows is that the equilibrium contract offer implies both wage compression and underparticipation in the labour market.

Claim 1. $s^*(V)$ is increasing and continuously differentiable in V with:
(i) $s^* = 0$ at $V = 0$;
(ii) $ds^*/dV < 1$ and $s^*(V) < \bar{c}$ for $V \in (0, \bar{c} + d)$,
(iii) $s^*(V) = V - d$ for $V \geq \bar{c} + d$ where

$$d = \frac{1}{n(n-1)\int_0^{\bar{c}}[1 - F(c)]^{n-2} f(c)^2 dc}. \tag{2}$$

Proof is in the Appendix.

It can be shown that the same properties of s^* occur when F is only log concave; i.e. when $F''F < F'^2$, but the proof is both long and tedious.[4] Formally the equilibrium outcome described in Theorem 1 corresponds to an n-buyer first price auction, but where the seller has private independent match values. Although assuming F is concave (or log concave) is sufficient to guarantee non-paradoxical comparative statics; i.e. more productive workers receive higher wage offers, establishing that a pure strategy symmetric equilibrium necessarily exists is less straightforward. The Technical Appendix describes the formal existence problem. In what follows, we simply assume a symmetric pure strategy equilibrium exists.

The next section describes optimal investment in the first period given workers anticipate contract offers as described in Theorem 1. Those results depend critically on the following market failures.

I. Equilibrium Wage Compression.

Given constant returns to labour in the workplace, a competitive labour market implies contract offers (y^c, l^c) with $l^c = l^*$ and $y^c = \alpha l^*$. Define the competitive offer, s^c, as

$$s^c = y^c - b[x(1) - x(l^c)]$$

and note the definition of V implies $s^c \equiv V$;i.e. the competitive outcome implies $s = V$.

Imperfect competition in the labour market implies firms offer surplus $s^* < V$. Claim 1 establishes that at low workplace productivities, where $0 < V(.) < \bar{c} + d$, then $ds^*/dV < 1$. Following Acemoglu and Pischke (??) we describe this outcome as wage compression; i.e., wage offers do not increase one-for-one with workplace productivity. An important feature for what follows is that wage compression disappears at high enough levels of workplace productivity. In particular, Claim 1 implies

[4] Establishing that $0 < \frac{ds^*}{dV} < 1$ in (8) in the Appendix requires showing

$$[1 - F(s^*)]^{n-1} f(s^*) + [V - s^*][1 - F(s^*)]^{n-1}[-F''(s^*)] > 0$$

where s^* is defined by (1). Using (1) to substitute out $(V - s^*)$ it is possible, but tedious, to show that log concavity of F, which implies $FF'' < F'^2$, is sufficient to imply the above inequality at s^*.

(i) there is wage compression for (α, b) satisfying $V < \overline{c} + d$ as $ds^*/dV < 1$ in that region, while
(ii) there is no wage compression for (α, b) satisfying $V > \overline{c} + d$ as $ds^*/dV = 1$.

To understand why there is no wage compression at high V note, as pointed out before, that a firm faces two oligopsony margins: a poaching margin and a participation margin. By offering higher wages, a firm might not only attract an employee from a competing firm - the poaching margin - but also attract a non-participant into the market sector.

The participation margin does not bind for workers with sufficiently high V that, in equilibrium, they accept a job offer with probability one. A useful analogy is the Hotelling pricing literature where we might interpret c_i as the worker's transport cost to work at firm i. The case "V sufficiently high that an offer is always accepted" is typically referred to as a "covered market". The equilibrium is that both firms offer a wage equal to the worker's value of output less 'price' $d > 0$. Equilibrium d reflects the marginal probability that a small increase in the offered wage will poach the worker away from the competing firm and, in a symmetric equilibrium, d depends only on the number of competing firms and the distribution of transport costs. The lump-sum deduction implies there is no wage compression.

In contrast, the participation margin binds for workers with V less than $\overline{c} + d$. Such workers include low workplace-productivity workers and intermediate productivity workers with high home productivities. An important property of the Hotelling pricing structure is that, as the value of employment increases, wage competition at the margin becomes more intense. In particular, (8) in the Appendix implies $ds^*/dV = 0.5$ at $V = 0$, $ds^*/dV < 1$ for $V < \overline{c} + d$ and $ds^*/dV \to 1$ as $V \to \overline{c} + d$. Hence wages rise more quickly with productivity as the participation margin peters out, where $ds^*/dV = 1$ for all $V \geq \overline{c} + d$.

II. Equilibrium Underparticipation

The worker's participation probability is $P(s^*) = 1 - [1 - F(s^*)]^n$. Given the competitive outcome implies $s^c = V$, Claim 1 implies:
(i) there is underparticipation for (α, b) satisfying $0 < V < \overline{c} + d$ as $P(s^*) < P(V)$ with $P(s^*) < 1$, while
(ii) there is efficient participation for (α, b) satisfying $V > \overline{c} + d$ as $P(s^*) = P(V) = 1$.

The underparticipation problem arises as worker preferences are not observed and firms offer less than full surplus. If the value of workplace productivity is sufficiently high, however, that the worker participates with probability one, then the privately optimal participation decision coincides with the socially optimal one.

In what follows, we shall find investment is efficient for α, b satisfying $V(\alpha, b) \geq \overline{c} + d$ as there is no wage compression and there is efficient participation. For given $b \geq 0$, define the efficiency frontier $\alpha = \overline{\alpha}(b)$ where

$$V(\overline{\alpha}, b) = \overline{c} + d$$

and note that $V(\alpha, b) \geq \overline{c} + d$ if and only if $\alpha \geq \overline{\alpha}(b)$. Claim 0 implies $\overline{\alpha}$ is strictly increasing in b.

4 The Worker's Optimal Education Decision

To identify the optimal investment decision in the first period, Claim 2 now computes expected second period utility, which is denoted $U_2^*(\alpha, b)$.
Claim 2. For any (α, b) and offers as described in Theorem 1:

$$U_2^*(\alpha, b) = bx(1) + \int_0^{s^*} [1 - (1 - F(c))^n] \, dc. \tag{3}$$

Proof is in the appendix.

Expected second period utility equals the option value of home production plus the expected surplus from employment, which depends on $V = V(\alpha, b)$ and labor market imperfections as $s^* = s^*(V)$.

In the first period given ability a and expected home productivity b, the worker's optimal investment decision solves:

$$\max_{\alpha \geq a} U_2^*(\alpha, b) - \phi(\alpha - a)$$

where the worker chooses second period productivity $\alpha \geq a$ at investment cost $\phi(k)$, where $k = \alpha - a$. The necessary condition for a maximum is

$$\partial U_2^* / \partial \alpha = \phi'(\alpha - a),$$

i.e., the worker sets the marginal return to education equal to its marginal cost, where (3) implies the marginal return to education, denoted MR, is

$$\begin{aligned} MR &= \frac{\partial U_2^*}{\partial \alpha} = [1 - (1 - F(s^*))^n] \frac{ds^*}{dV} \frac{\partial V}{\partial \alpha} \\ &= P(s^*) \frac{ds^*}{dV} \frac{\partial V}{\partial \alpha}. \end{aligned} \tag{4}$$

Note, MR depends on three components: $P(s^*)$ is the probability the worker participates in the labor market; ds^*/dV is the rate at which offered compensation s^* increases with V; and $\partial V / \partial \alpha$ describes how V increases with productivity α.

In a competitive labour market with revenue function $R = \alpha l$, the Envelope Theorem would imply marginal return to education $\partial R / \partial \alpha = l^*$, which is simply expected labour supply. The above expression is more complicated as there are labour market imperfections. Nevertheless the interpretation is the same. The definition of V and the Envelope Theorem imply $\partial V / \partial \alpha = l^*$. Hence $[P(.)][\partial V / \partial \alpha]$ together describe expected labour supply. The marginal return to education is expected labour supply times the marginal increase in wage through higher productivity (given a non-competitive labour market).

Figure 1 plots MR (with b fixed). Most importantly for what follows, note that there are increasing marginal returns. This occurs for three reasons:

(i) Participation effects: an increase in productivity implies firms offer better wages which increases the worker's participation probability; i.e. $P(s^*)$ increases as α increases. The higher participation probability increases directly the marginal return to education.

(ii) Increasing Labor Supply: $\partial V/\partial\alpha$ equals l^* and as an increase in workplace productivity implies an increase in labour supply l^* (Claim 0), this further increases the marginal return to education.

(iii) Increasing wage competiveness: as the value of employment V increases, firms at the margin bid more competitively for the worker's services. In particular, $ds^*/dV = 0.5$ at $V = 0$, while $ds^*/dV \to 1$ as $V \to \overline{c} + d$ (see Claim 1); i.e. wage compression decreases at higher productivity levels.

We know that $MR = 0$ for $\alpha < bx'(1)$ [Claim 0 implies $l^* = V = 0$ in this region and so $P(s^*) = 0$; i.e. there is no gain to trade]. Also MR has a zero slope at the part-time margin $\alpha = bx'(1)$ as $P(s^*) = \partial V/\partial\alpha = 0$ at that point.

Suppose $\overline{c}$ is relatively large; specifically $b[x'(0)-[x(1)-x(0)]] < \overline{c}+d$. This implies that a person at the full time margin, one with productivity $\alpha = bx'(0)$, has value of employment $V < \overline{c} + d$ and so does not necessarily participate in the labour market. It also implies $\overline{\alpha}(b) > bx'(0)$ as drawn in Figure 1 and so $MR = 1$ for $\alpha \geq \overline{\alpha}(b)$.

Figure 1 here

Although MR is continuous, its slope is not continuous at the full time margin (where $\alpha = bx'(0)$). In particular, labour supply $l^* \equiv \partial V/\partial\alpha$ is strictly increasing in α in the part-time employment region, where increasing labour supply generates increasing returns to education [see (ii) above]. At the full time margin, however, labour supply becomes constrained $l^* = 1$ and this source of increasing returns stops discontinuously at that point.

$\phi'(\alpha - a)$ is the marginal cost to skill accumulation and is denoted MC_a in Figure 1. The assumptions on ϕ imply $MC_a = 0$ at $\alpha = a$ and is strictly increasing in α. The optimal skills investment decision of a worker with ability a occurs where MC_a crosses MR, though this may not be a sufficient condition for a maximum as there are increasing marginal returns.

Note that an increase in ability implies a rightward shift in MC_a. This implies that, ceteris paribus, workers with higher ex-ante ability invest in education to a higher ex-post skill level α.

Now consider the ability type with $a = a_M$ as drawn in Figure 1. Such an individual is interesting as, given the two shaded areas are equal, this person is indifferent to investing to $\alpha = \alpha_2 > bx'(0)$ or investing to $\alpha = \alpha_1 < bx'(0)$. Workers with ability $a > a_M$ train so that $\alpha > \alpha_2 > bx'(0)$; such workers have high V ex-post, have relatively high participation probabilities and work full time (choose $l^* = 1$). In contrast those with ability $a < a_M$ invest so that $\alpha < \alpha_1 < bx'(0)$. Such workers have low V ex-post, low participation probabilities and will only consider part-time employment. Increasing returns to education potentially lead to discontinuous investment decisions around the part-time margin.

To see that this discontinuity generates large deadweight losses, consider the optimal investment and participation decisions in a competitive labour market. Recall that the private marginal return to investment is

$$MR = P(s^*)\frac{ds^*}{dV}\frac{\partial V}{\partial \alpha} = P(s^*)\frac{ds^*}{dV}l^*.$$

As previously explained the competitive outcome implies $s = V$ and so the social return to education, denoted SR,is

$$SR = P(V)\frac{\partial V}{\partial \alpha} = P(V)l^* \tag{5}$$

which is expected labour supply. Hence $MR < SR$ if there is underparticipation, $P(s^*) < P(V)$,or if there is wage compression $ds^*/dV < 1$.

It follows that MR=SR for very low productivities, where $\alpha < bx'(1)$, in which case $V = 0$ and so $MR = SR = 0$ (there is no gain to trade). It also follows that $MR = SR$ for very high productivities, where $\alpha > \overline{\alpha}(b)$, as there is efficient participation and no wage compression. For intermediate productivities, however, we have $MR < SR$ due to underparticipation and wage compression.

Figure 2 here.

Note, both MR and SR have a zero slope at the part-time margin and both have discontinuous slopes at the full-time margin. Claim 1 implies $SR > MR$ for all $\alpha \in (bx'(1), \overline{\alpha}(b)]$.

Recall that the worker with ability a_M is indifferent between investing to α_1 or α_2. The shaded area describes the deadweight loss associated with the low investment decision. The socially optimal decision is that the worker invests to α_s. If the worker invests to α_2, the resulting deadweight loss corresponds to the Harberger triangle labelled DWL_2 in Figure 2. If the worker instead invests to α_1, the large substitution effect implies deadweight loss DWL_1 which is clearly much larger.

Increasing returns to education and an imperfectly competitive labour market can therefore lead to a part-time employment trap. Workers with ability $a < a_M$ invest in skills where $\alpha < \alpha_1$. Having low V, they have low participation probabilities, and only participate in part-time employment. But the socially optimal decision for these workers may be that they invest to skills $\alpha_s > bx'(0)$ and participate in full time employment with a high participation probability. The discontinuity in investment behaviour leads to a large deadweight loss.

Figure 1 describes a_M for a particular value of home productivity b. More generally for any b, let (a_M, b) denote the worker who is indifferent to investing to high α and working full-time, or investing low α and working part-time with a low probability.

As the value of employment V depends on b, then a_M varies with b. The following characterises $a_M = a_M(b)$.

An increase in b does not affect the MC curve. Now consider how an increase in b affects the MR curve. First note that a (small) increase in b implies an increase in $\overline{\alpha}(.)$ and a right shift in the part-time and full time margins. Second, fix an $\alpha \in (bx'(1), \overline{\alpha}(b))$. A (small) increase in b implies lower labour supply l^* (strictly lower in the part-time region), strictly lower V (Claim 0) and as $P(s^*) < 1$ in this region, MR falls in this region. Figure 3 draws two MR curves, denoted MR,MR$'$ corresponding to two different home productivities b, b' with $b < b'$.

Figure 3.

An increase in b to b' implies a fall in MR as drawn in Figure 3. The marginal worker as depicted in Figure 1, the one with ability $a = a_M(b)$ and home productivity b, now strictly prefers to choose low skills $\alpha < \alpha_1$ should home productivity increase to $b' > b$. Hence $a_M(b') > a_M(b)$; *i.e.* the part-time employment trap is increasing in home productivity. It also follows that if home productivity is sufficiently small that $a_M < 0$, then the part-time employment trap disappears.

Of course, the above applies if the marginal cost curve, MC_a, is relatively flat. If the marginal cost curve is steep enough, then the part-time employment trap does not exist. Figure 4 depicts this case.

Figure 4 here.

As in the previous cases, the investment and participation decisions are distorted for those with intermediate ability. Those with very low workplace ability and high home productivity do not invest in general human capital and focus purely on home production. Those with very high workplace ability invest fully in skills, where $MC = 1$, and participate with a high probability in full time employment. The imperfect labour market distorts market behaviour for those with with intermediate participation probabilities. Although there are increasing marginal returns to education,a steep marginal cost curve (implying education choices are inelastic relative to endowed ability) implies relatively small substitution effects and the efficiency loss corresponds to standard Harberger triangles.

5 Policy and Discussion

It is well known that an imperfectly competitive labour market may lead to wage compression and underinvestment in general human capital. The key insight of our analysis is that an imperfectly competitive labour market also generates underparticipation. Further with heterogeneous workers and increasing returns to education, the corresponding welfare losses are largest for a particular subset of workers - those whose home productivity is sufficiently high that they have low participation rates and, if they do participate, are more likely to take part-time rather than full time employment. For workers in the so-called part-time employment trap, the social planner's optimum implies the worker should make large investments in human capital and have a high participation probability in full time employment. But as workers do not receive the full return to those investments, they instead substitute to home production – they make low skills investments and participate with low probability in part-time employment. The trap is increasing in home productivity: thus workers with intermediate ability are more likely to be caught in this trap if their home productivity is relatively high.

Table 1 describes male and female participation rates, and type of employment contract, by demographic group in the U.K.. The data source is the British Household Panel Survey (BHPS). Columns [1] and [3] report male and female participation rates, while Columns [2] and [4] report the proportions of the total male and female working populations respectively that are employed part-time. Notice that women have similar participation rates to men in each demographic group except for those who are married and, more significantly, those who have kids between 0-16 years. Furthermore, within this latter group, over 50% of women who participate take part-time employment, while 98% of men who participate take full time employment.

Table 1 near here.

Table 1 demonstrates that, on average, women take the brunt of child care, which in the context of this paper might be interpreted as relatively high home productivity. Such women are over-represented in part-time employment.

The theory implies that individuals caught in the part-time employment trap will be characterised by much lower education levels. Table 2 demonstrates that women with kids in the part-time employment sector have surprisingly low education levels.

Table 2 here.

Table 2 summarizes the ex-ante education decisions, and ex-post participation rates, of working-age men and women who have kids up to 16 years old living at home. First consider the full-time employment row. For each sex and conditional on having kids up to 16 years old, the full time employment row describes the proportion of workers who have a particular level of education. It is striking that the composition of education is very similar between the two sexes in full-time work. This is consistent with the theory developed above, which argues that skill levels are not distorted for the very highest workplace ability types (who typically work full time and presumably organise private childcare).

The distortion becomes evident as we consider the part-time employment and non-participation rows in Table 2. These establish that, for women with kids who choose to work part-time, 62% have no qualification higher than GCSE, while the corresponding figure for men is 37%. Similarly, of women with kids who choose not participate, 65% have no qualification higher than GCSE, while the corresponding figure for men is 53%. Together these facts suggest that women with kids who take part-time employment are ex-post undereducated. Assuming a non-competitive labour market, young women, when making their education choices, may well be caught in the part-time employment trap.

Optimal policy requires increasing the return to participation in the labor market relative to non-participation. The obvious approach is either (i) tax non-participants with a home production tax, or (ii) subsidise participation with an employment subsidy. The first approach, a tax on non-participation, is unlikely to be politically feasible and so we focus on the latter.

Suppose the government observes the worker's productivity parameters α, b and offers an employment subsidy $x = x(V)$ to workers who participate in the labor market, where $V = V(\alpha, b)$ as defined before. Repeating the analysis as before and given $x \geq 0$, it is straightforward to show that, in a pure strategy symmetric equilibrium, the equilibrium surplus offered by firms is $s^*(V + x) - x$. In other words, the firms extract the employment subsidy from the worker (the $-x$ term), but their equilibrium offers then reflect that the value of workplace employment is $V + x$. Given such offers, workers obtain net surplus $s^*(V + x)$.

To identify the optimal subsidy note that the competitive outcome implies $s^c = V$. Hence the optimal employment subsidy, x^*, satisfies

$$s^*(V + x^*) = V,$$

which is an implicit function for x^*, where $s^*(.)$ is defined by (1) in Theorem 1.
Claim 3. The optimal employment subsidy $x^*(.)$ satisfies $x^* = 0$ at $V = 0$,
(i) for $V \in (0, \overline{c})$, $x^*$is strictly increasing with $x^*(\overline{c}) = d$, and
(ii) for $V \geq \overline{c}$, $x^* = d$.
Proof: follows from Claim 1 and the equation for x^*.

Claim 3 establishes that, in order to guarantee efficient participation **and** efficient training, the government needs to compensate for the oligopsony rents extracted by firms. Of course for many types the welfare gains through such a scheme may be small (and is zero for types who ex-post invest $\alpha \geq \overline{\alpha}(b)$ and participate ex-post with probability one). In contrast, workers with the most distorted participation rates are those found in the part-time employment trap. The evidence suggests that individuals most likely to be caught in this trap are young women who expect to have kids. An obvious employment subsidy which targets precisely this group is a state-subsidised childcare scheme. Such an employment subsidy may generate large welfare gains as it not only corrects the ex-post underparticipation distortion, but also encourages women to invest more in education when young.

6 Conclusion

This paper has considered an imperfectly competitive labour market where worker preferences on the disutility of workplace employment are private information. This information asymmetry not only leads to equilibrium wage compression and underinvestment in general human capital, but also to equilibrium underparticipation. An important insight is that the worker participation decisions generate increasing returns to education - those with low participation probabilities have a low ex-ante return to skills investment, while those with high participation probabilities who take full-time employment have a high return to skills investment. Given the more highly skilled are paid more and so are more likely to participate in the labour market, this generates increasing returns to education.

Increasing returns to education and a non-competitive labour market can imply large substitution effects, and hence large deadweight losses. The paper identifies a part-time employment trap for those located at the non-participation/part-time employment margin. The data clearly suggest that it is young women who are most likely to be caught in this trap and optimal corrective policy is an employment subsidy which can be appropriately targeted as a childcare benefit.

A popular alternative model of an imperfectly competitive labour market assumes instead search frictions and that wages are determined by Nash bargaining. In particular given (α, b) and free entry of firms, the axiomatic Nash bargaining approach would imply the firm negotiates profit π and labour supply l as

$$\max_{\pi,l}[\pi]^{1-\gamma}[\alpha l - \pi + bx(1-l) - bx(1)]^{\gamma}$$

where $\gamma \in [0,1]$ is the worker's bargaining power, $bx(1)$ is the worker's threatpoint [i.e. the value of home production] and the firm's threatpoint is zero in a free entry equilibrium. By definition of V in the text, this reduces to

$$\max_{\pi}[\pi]^{1-\gamma}[V-\pi]^{\gamma}$$

and Nash bargaining implies worker remuneration y^* satisfies $dy^*/dV = \gamma$. As in Claim 1, this implies equilibrium wage compression. The Nash bargaining approach identifies the wage compression issue but does not identify the underparticipation problem. Our approach shows that underparticipation and wage compression generate mutually reinforcing distortions on human capital investment: wage compression implies workers tend to underinvest in workplace skills, lower skills imply a lower participation probability which further reduces the expected return to human capital accumulation.

Manning (2003) provides extensive evidence of oligopsonistic wage setting for the two countries he analyses, the US and the UK. However even if one were to insist that the labor market is competitive, the same part-time employment trap arises if the government taxes labor income but does not tax home production (as is typically the case). Workers then substitute to home production and increasing marginal returns to education generate the same part-time employment trap. In fact we know from the optimal taxation literature that tax rates should be highest on those goods that are traded inelastically. An income tax on the incompetent parent who has high workplace ability is not going to distort that worker's participation probability. But it will distort the labor market decision of the intermediate ability worker with relatively high home productivity.

The paper has assumed that youngsters know their future home productivity b. One might expect, in the modern world, that the parent who can earn the highest wage in the workplace will take full time employment while the other parent focusses on childcare (or organises private childcare). When young, the privately optimal education decision then depends on the education decision of one's, as yet unknown, future partner. Given increasing returns to education, this implies a co-ordination problem in education choices and family organisation. For example, societies might organise family structures where men always work in the workplace and women always work in the home. Given increasing returns to education, the efficient education allocation may then be to focus training resources on boys. The obvious inefficiency is that high ability girls, who could be very productive in the workplace, are not trained. On the other hand, more flexible family structures may lead to highly trained individuals spending a lot of their time on childcare and so realise a relatively low return to their costly education. Comparing the efficiency of such outcomes within an equilibrium matching environment is an interesting issue which is left for future research.

References

[1] Apps PF and R Rees (1997). 'Collective Labor Supply and Household Production', *Journal of Political Economy*, 105, pp. 178-190.

[2] Becker, GS (1965). 'A Theory of the Allocation of Time', *The Economic Journal*, 75, pp. 493-517.

[3] Becker, GS (1981). 'The Division of Labor into Households and Families'. Chapter 2 in *A Treatise on the Family*, Harvard University Press, Cambridge Ma., pp.14-37.

[4] Bhaskar, V., Alan Manning and Ted To (2002). 'Oligopsony and Monopsonistic Competition in Labor Markets'. *Journal of Economic Perspectives,* 16(2), pp.155-174, Spring.

[5] Gronau, R (1977). 'Leisure, Home Production and Work – the Theory of the Allocation of Time Revisited', *Journal of Political Economy*, 85(6), pp 1099-1124, December.

[6] Manning, Alan (2003). *Monopsony in Motion: Imperfect Competition in Labor Markets.* Princeton, NJ: Princeton University Press.

[7] OECD *Employment Outlook* June 1999. Paris: OECD

[8] OECD *Employment Outlook* June 2001. Chapter 4: "Balancing Work and Family Life: Helping Parents into Paid Empoyment". Paris: OECD.

[9] OECD *Employment Outlook* June 2002. Chapter 2: "Women at work: Who are they and how are they faring?" Paris: OECD.

[10] Sandmo, Agnar (1990). "Tax Distortions and Household Production", *Oxford Economic Papers,* New Series, Vol. 42, No. 1, Special Issue on Public Economics. (January), pp. 78-90.

[11] Stevens, Margaret (1994). "A Theoretical Model of On-the-job Training with Imperfect Competition", *Oxford Economic Papers*, Vol. 46, pp. 537-62.

7 Technical Appendix

Proof of Theorem 1.

Consider a symmetric equilibrium where all firms post contract (y^*, l^*). Suppose firm 1 considers a deviating (but optimal) contract (y^1, l^*). Given the worker's optimal job acceptance strategy (as defined in the text), firm 1's expected profit by offering y^1, denoted π_1, is

$$\pi_1 = P(y^1 - c_1 \geq \max_{j \neq 1}[b[x(1) - x(l^*)], y^* - c_j])[\alpha l^* - y^1],$$

where $P(.)$ is the probability that the worker accepts firm 1's job offer,[5] whereupon the firm makes profit $\alpha l^* - y^1$.

To compute this probability, note that for each c_1 satisfying $y^1 - c_1 \geq b[x(1) - x(l^*)]$; i.e. for $c_1 \leq y^1 - b[x(1) - x(l^*)]$, the worker prefers employment at firm 1 rather than pure home production. Further for such c_1, the worker also prefers firm 1's employment offer to firm $j's$ offer as long as $y^* - c_j \leq y^1 - c_1$; i.e. as long as $c_j \geq y^* - y^1 + c_1$ which occurs with probability $1 - F(y^* - y^1 + c_1)$. Hence integrating over such c_1, the probability the worker accepts firm 1's contract offer is

$$\int_0^{y^1 - b[x(1) - x(l^*)]} [1 - F(y^* - y^1 + c_1)]^{n-1} f(c_1) dc_1.$$

Hence firm 1's expected profit is

$$\pi_1 = [\alpha l^* - y^1] \int_0^{y^1 - b[x(1) - x(l^*)]} [1 - F(y^* - y^1 + c_1)]^{n-1} f(c_1) dc_1.$$

Now define $s^* = y^* - b[x(1) - x(l^*)]$ and so

$$y^* = b[x(1) - x(l^*)] + s^*.$$

y^* is decomposed as full compensation for foregone home production plus additional surplus s^*. Similarly define $s^1 = y^1 - b[x(1) - x(l^*)]$. Substituting out y^1, y^* in the above and using the definition of $V(\alpha, b)$, firm 1's profit reduces to

$$\pi_1(s^1, s^*; \alpha, b) = [V - s^1] \int_0^{s^1} [1 - F(s^* - s^1 + c_1)]^{n-1} f(c_1) dc_1 \tag{6}$$

with $V = V(\alpha, b)$. Hence given s^*, firm 1's best response for s^1 is defined by the first order condition $\partial \pi_1 / \partial s^1 = 0$ where the above implies

[5] As there are no mass points in F, by assumption, we can assume a weak inequality.

$$\frac{\partial \pi_1}{\partial s^1} = -\int_0^{s^1} [1 - F(s^* - s^1 + c_1)]^{n-1} f(c_1) dc_1 \tag{7}$$
$$+[V - s^1] f(s^1)[1 - F(s^*)]^{n-1}$$
$$+[V - s^1] \int_0^{s^1} (n-1) \left[f(s^* - s^1 + c_1)[1 - F(s^* - s^1 + c_1)]^{n-2} \right] f(c_1) dc_1.$$

A pure strategy, symmetric equilibrium requires firm 1's best response $s^1 = s^*$, and so the above condition implies

$$\int_0^{s^*} [1 - F(c_1)]^{n-1} f(c_1) dc_1 = [V - s^*] f(s^*)[1 - F(s^*)]^{n-1}$$
$$+[V - s^*] \int_0^{y^*} (n-1)[1 - F(c_1)]^{n-2} f(c_1)^2 dc_1$$

is a necessary condition for a pure strategy symmetric equilibrium. The left hand side is integrable and this equation simplifies to (1). This completes the proof of the Theorem.

Proof of Claim 1. (1) immediately implies $s^*(0) = 0$. Differentiating (1) w.r.t. V and rearranging yields:

$$\frac{ds^*}{dV} = \frac{[1 - F(s^*)]^{n-1} f(s^*) + (n-1) \int_0^{s^*} [1 - F(c)]^{n-2} f(c)^2 dc}{2[1 - F(s^*)]^{n-1} f(s^*) + (n-1) \int_0^{s^*} [1 - F(c)]^{n-2} f(c)^2 dc + [V - s^*][1 - F(s^*)]^{n-1}[-F''(s^*)]} \tag{8}$$

Putting $s^* = V = 0$ implies part (i).

Noting $V > 0$ implies $s^* < V$ [a firm never offers $s^* > V$ as it implies a negative profit] then F concave over its support implies $0 < \frac{ds^*}{dV} < 1$ while $0 < s^* < \overline{c}$. As F is twice differentiable, ds^*/dV is continuous for $s^* < \overline{c}$ and note $s^* \to \overline{c}^-$ implies $ds^*/dV \to 1$. Putting $s^* = \overline{c}$ in (1) implies $V = \overline{c} + d$ where d is defined in the Claim. Finally (1) implies $s^* = V - d$ for $s^* \geq \overline{c}$. This completes the proof of the Claim.

Proof of Claim 2. Theorem 1 implies

$$U_2^*(\alpha, b) = E_{c_i} \max[bx(1), y^* - c_i + bx(1 - l^*)]$$
$$= bx(1) + E_{c_i} \max[0, s^* - c_i].$$

Let $c = \min[c_1, c_2, .., c_n]$ and note this random variable has c.d.f. $G = 1 - (1 - F)^n$. As

$$U_2^*(\alpha, b) = bx(1) + \int_0^{s^*} [s^* - c] dG(c),$$

integration by parts now implies the claim.

The Existence Problem.

Each firm offers a wage which fully compensates for home production and offers additional surplus s^* which depends on the value of workplace employment V. To address the existence issue, suppose each firm $j \neq 1$ announces $s^*$and suppose firm 1 deviates by announcing s. Let

$$L(s, s^*) = \int_0^y [1 - F(s^* - s + c_1)]^{n-1} f(c_1) dc_1$$

which is the probability the worker accepts firm 1's job offer. Hence

$$\pi_1 = L(s, s^*)[V - s].$$

Note that $\pi_1 \equiv 0$ for $s \leq s^* - \bar{c}$ (as $L = 0$) and $\pi_1 \leq 0$ for $s \geq V$. Hence define $\Gamma(V) = [\max[0, s^* - \bar{c}], V] \subseteq [0, V]$ where $s^* = s^*(V)$ is defined by (1). Note that Claim 1 implies $s^* \in \Gamma(V)$ and so Γ is non-empty. Without loss of generality we can restrict attention to $s \in \Gamma(V)$ - all other offers yield negative profit. As π_1 is not concave in s over this domain, a sufficient condition for existence of a pure strategy symmetric equilibrium is that π_1 is single peaked; i.e. that at any $s \in \Gamma(V)$ where $\partial\pi_1/\partial s = 0$, then $\partial^2\pi_1/\partial s^2 < 0$. Using the above definition of π_1, a sufficient condition is that

$$L\frac{\partial^2 L}{\partial s^2} - 2\frac{\partial L}{\partial s}^2 < 0 \text{ for all } s \in \Gamma(V). \tag{9}$$

Given the definition of L, (9) describes a restriction on F which guarantees existence of a symmetric, pure strategy Nash equilibrium (where Claim 1 implies s^* always exists). Unfortunately computing these terms yields long and unwieldy expressions. Although the restriction to F log concave (or the stronger condition that F is concave) guarantees sensible comparative statics, we have been unable to show it is sufficient to guarantee single peakedness as defined in (9).

It is well known in the Hotelling framework with linear transport costs that pure strategy equilibria may not exist. The problem there is that demand is discontinuous - a small price cut can imply a jump in demand. Such demand discontinuities do not arise here - idiosynchratic match values imply demand $L(.)$ is continuous in s. We believe the pure strategy symmetric equilibrium exists when F is log concave but have not been able to prove this formally.