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The Demographic Transition and the Sexual Division of Labor*

Abstract

This paper presents a theory where increases in female labor force participation and reductions in the gender wage-gap are generated as part of the same process of demographic transition that leads to reductions in fertility. The paper suggests a pathway linking changes in mortality to transformations in the role of women in society that has not been noticed before in the literature. Mortality reductions affect the incentives of individuals to invest in human capital and to have children. Particularly, gains in adult longevity reduce fertility, increase investments in market human capital, increase female labor force participation, and reduce the wage differential between men and women. Child mortality reductions, though reducing fertility, do not generate this same pattern of changes. The model reconciles the increase in female labor market participation with the timing of age-specific mortality reductions observed during the demographic transition. It generates changes in fertility, labor market attachment, and the gender wage-gap as part of a single process of social transformation, triggered by reductions in mortality.

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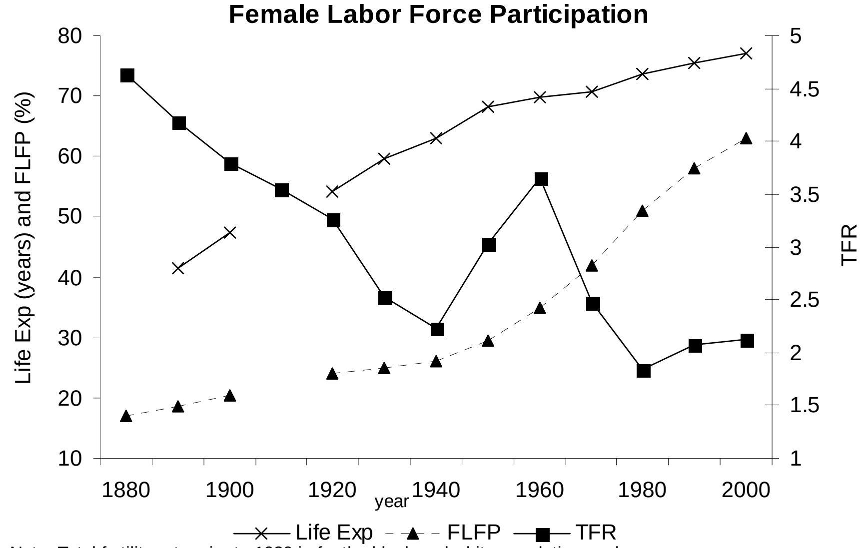
1 Introduction

This paper presents a theory where increases in female labor supply and reductions in the gender wage-gap arise as by-products of the classical process of demographic transition, characterized by increases in life expectancy and reductions in fertility. In the theory, gains in adult longevity raise the returns to human capital and reduce fertility, reducing the demand for household production. As women are initially specialized in the household sector, these changes lead to increases in the fraction of the productive lifetime that women allocate to the market, and, through changes in human capital accumulation, to reductions in the wage differential between men and women. Reductions in child mortality, though reducing fertility, do not generate this same pattern of changes. Since reductions in child mortality increase the return to investments in children, and therefore the return to time spent at home, their overall impact on female labor force participation is smaller than that of reductions in adult mortality. The different impact of adult and child mortalities in our model reconciles economic theory with the timing of changes in age-specific mortality and female labor force participation during the process of demographic transition. Our theory highlights a link between mortality and the changing role of women in society that has never before been identified in the literature. At the same time, this link generates a pattern of social transformation that is remarkably consistent with the timing of events during the historical experiences of demographic transition.

There have been profound changes in the labor supply of women in the last decades, both in developed and developing countries. In the United States, female participation in the paid labor force changed drastically in the course of the 20^{th} century. In 1880, only 17% of all American women at working ages participated in the labor market. By 2000, this number had risen to more than 60%. At the same time, individuals and families were changing in other important ways. Figure 1 shows the evolution of life expectancy, fertility and female labor force participation in the United States between 1880 and 2000. Throughout the 20^{th} century, life expectancy at birth rose by 30 years, from 47 to 77. The total fertility rate dropped from above 4 to 2.1.[1] These changes in life expectancy and fertility reflect trends that were observed at least since the beginning of the 19^{th} century, when the total fertility rate was above 7 points and life expectancy at birth was below 40 years.

[1] The temporary increase in fertility in the post-war period is the well-known "baby-boom" phenomenon. Our focus here is on the long term trend of fertility decline, which was only temporarily interrupted by the increased fertility rate of the 1950's and 1960's.

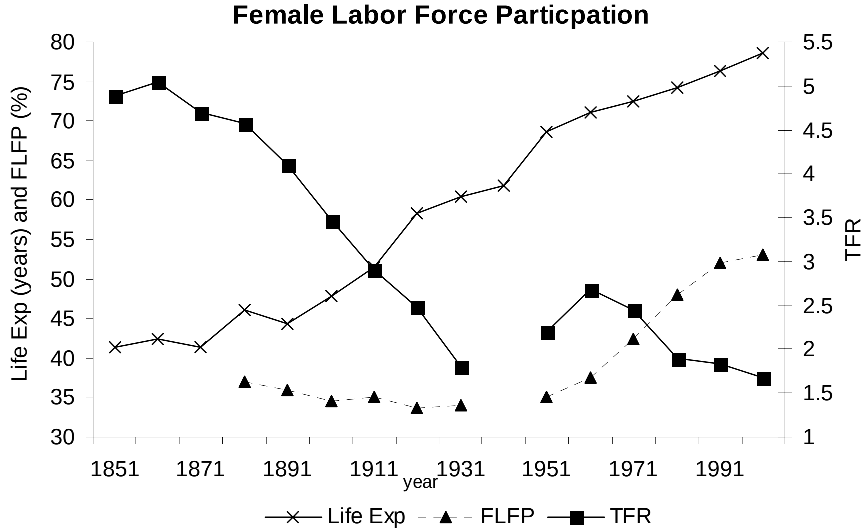
Figure 1: United States - Life Expectancy, Fertility and Female Labor Force Participation



Note: Total fertility rate prior to 1920 is for the black and white populations only.

Source: Costa (2000), National Center for Health Statistics (2003), and US Census Bureau (1975).

Figure 2: Great Britain - Life Expectancy, Fertility and Female Labor Force Particpation



Source: Wrigley et al (1997), Keyfitz and Fieger (1968), Costa (2000), World Bank (2004) and Human Mortality Database (2006).

Over this same period, similar trends were observed in other developed countries (the case of Great Britain is illustrated in Figure 2). These reductions in mortality and fertility were part of the process of demographic transition, which spread through most of the world in the post-war period. The empirical pattern characterizing the transition has been widely documented and discussed in the demographic literature. It is usually understood as being marked by an initial reduction in child mortality, followed by reductions in adult mortality and fertility (see, e.g., Heer and Smith, 1968, Cassen, 1978, Kirk, 1996, and Mason, 1997).[2] Maybe less acknowledged is the fact that changes in female labor force participation have also reached several of the "latecomers" of the demographic transition. Though data in these cases are more recent and sparse, it is already possible to notice the trends. Figure 3, for example, illustrates the demographic changes experienced by Brazil in the period between 1960 and 2000. In this forty-year interval, Brazil went through radical demographic changes: life expectancy at birth rose from 55 to 68 years, while total fertility rate declined from 6 to 2.2. Though data on female labor force participation is not available before the mid 1970's, the pattern of the series suggests that modest increases were being observed before 1975. But it is really only after 1980 that the increase in female labor force participation gained momentum. In the twenty five years between 1975 and 2000, women's labor market participation in Brazil increased from 39% to 58%. This pattern is not particular to Brazil. In the recent experience of the developing world, the fraction of the labor force composed by women increased in almost every single country where significant reductions in fertility were observed (see World Bank, 2004).

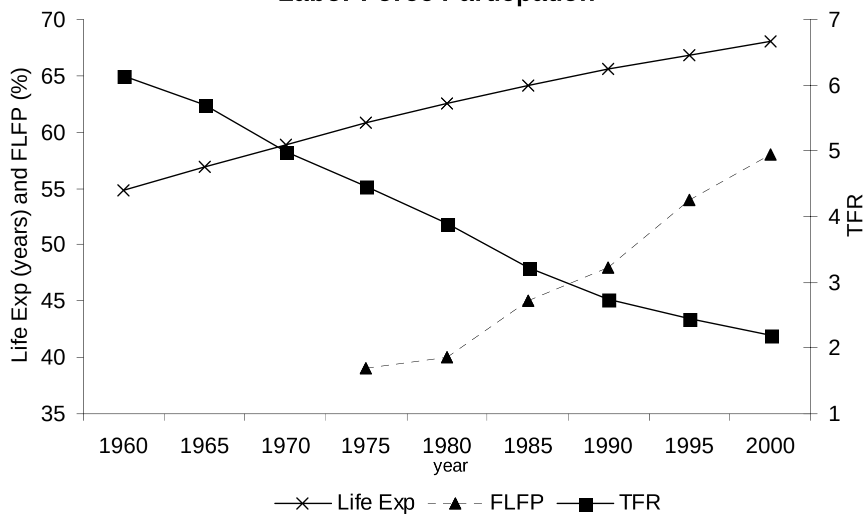
In addition, the wage differential between men and women has also been shrinking in virtually every documented history of increased female labor force participation. This is a general phenomenon, irrespective of cultural tradition or level of economic development (see Blau and Kahn, 2000). For the cases discussed before, only between the end of the 1970's and the end of the 1990's, the gap between female and male earnings was reduced by 12.3 percentage points in Great Britain, 13.8 percentage points in the United States, and 15.3 percentage points in Brazil (Blau and Kahn, 2000 and Simão et al, 2001).

The implications of the increased attachment of women to the labor market encompass issues such as the bargaining power of husband and wife within the household, the enhanced role of women in modern society, and the availability of parents to invest in children. On the one hand, this change has been linked to cultural transformations within society, which led to a change in the role of women in the household and in the labor market. On the other hand, economists have linked it to rising wages and falling fertility. Though these closer explanations are relevant, they do not identify the ultimate determinants of the observed trends. What is the underlying reason behind society's change of attitude toward women? Why did fertility decline and, even with rising labor supply, why did women's wages increase? It is not well understood either whether there is any theoretical reason to believe that reductions in mortality should be causally related to

[2] There is still some controversy regarding the early experience of the first Western European countries to go through the demographic transition. Nevertheless, this sequence of events is accepted as an accurate description of reality in the vast majority of cases.

increases in female labor force attachment and reductions in the gender wage-gap. Nevertheless, the evidence suggests that increased female labor force participation follows a quite general pattern, which, therefore, cannot be entirely explained by technological or institutional changes that are particular to each specific country.

Figure 3: Brazil - Life Expectancy, Fertility and Female Labor Force Particpation



Source: Soares and Izaki (2002) and World Bank (2003)

This paper suggests that changes in the role of women in society can be partly understood as a consequence of the impact of reductions in mortality on household decisions. In this context, increased female labor force participation and a narrowing gender wage-gap could be seen as later developments of the same process of demographic transition characterized by increases in life expectancy and reductions in the size of families. Our model incorporates all these dimensions into a unified theory of demographic change, which does not resort to exogenous changes in culture, habits or preferences. Though deep cultural transformations have certainly been an important component of the social changes observed during the last century, the model has the goal of highlighting one potentially important factor that has been entirely ignored in the literature. Therefore we abstract completely from changes in technologies (not related to health), culture and social norms, and concentrate our analysis on the impacts of mortality.

We argue that exogenous reductions in mortality – driven by technological progress in medical and biological sciences[3] – have two fundamental effects: they reduce the return from large families

[3] Our theory has exogenous reductions in mortality as the main driving force behind all other demographic changes. Though there are several dimensions on which individuals can invest in their own health, our focus here is

and increase the return from investments in human capital. We develop a model where households composed by two members (female and male) decide on their allocation of time, number of offspring, and human capital investments in children and adults. Since women are assumed to be marginally more productive at child raising, the initial allocation is one where females are at least partially specialized in household production. In this situation, we show that gains in adult longevity reduce fertility and increase the returns to investments in market-oriented human capital. Since women are initially responsible for child raising, reduced demand for household production leads to an increase in investments in female human capital and labor supply that is more than proportional to the increase in longevity and larger than the change observed for men. The differential change across genders translates into increased labor force participation of women and narrowing of the gender wage-gap.

In addition, and contrary to the superficial intuition, the model does not generate increases in female labor force participation or narrowing of the gender wage-gap as results of reduced child mortality. Reductions in child mortality do not increase the returns to market attachment directly, but they do increase the returns to investments in children. In our setup, when women initially allocate part of their time to household production, the increased return from investments in children guarantees that female labor force participation and investments in market human capital do not rise as child mortality is reduced. Therefore, the paper reconciles the theory with the timing of changes in fertility and female labor force participation after the demographic transition. Fertility reductions are observed soon after the onset of expressive gains in life expectancy – when reductions in child mortality still play a prominent role, while increases in female labor force participation appear only later on – as adult mortality gains relative importance.

This setup shows that there is an economic force linking together mortality, fertility, female labor force participation, and the gender wage-gap, and suggests that this connection may be important for understanding of the consequences of recent social changes for future generations. Most notably, increased female labor force participation has raised concerns about the possibility of negative impacts on the quality children, due to reduced presence of parents in the household.[4]

As a by product, the theory proposed here suggests that this question should be analyzed in light of the more general process of social change of which it is part.

The remainder of the paper is organized as follows. Section 2 reviews the relevant literature on

on changes in mortality brought about by technological advances or diffusion of previously existing technologies. An extensive literature has pointed to the fact that recent changes in mortality have been largely unrelated to income and living conditions, and are to a great extent exogenous to individuals and countries. This issue is discussed in Preston (1975, 1980), Becker et al (2005) and Soares (2005), and will not be explored in further detail here.

[4] This issue has been the topic of a series of recent presidential addresses to the Population Association of America (see, for example, Preston, 1984, Bianchi, 2000, and McLanahan, 2004), and is still subject of debate (e.g. Sayer et al, 2004, Gauthier et al, 2004, and James-Burdumy, 2005).

the demographic transition and the recent changes in female labor force participation. Section 3 presents the basic framework of the model. Section 4 discusses the effect of adult longevity gains. Section 5 analyzes the impact of child mortality reductions. Section 6 discusses the pattern of historical changes generated by the model and the related evidence. Finally, section 7 concludes the paper.

2 Related Literature

There are numerous empirical papers on female labor force participation, mainly within the literature on the determinants of labor supply. Typically, these papers concentrate on the analysis of the proximate determinants of female labor supply. For example, the first generation of models took lifetime wage profiles or fertility decisions as given, and analyzed their impact on labor supply decisions (see, e.g., Mincer, 1962, Gronau, 1973, or the survey in Heckman, 1978). More recent papers acknowledge the joint household decision regarding number of children, investments in human capital and labor supply of women. This literature usually tries to instrument for one of these dimensions of choice and analyze its impact on the other dimensions (Hotz and Miller, 1988, Rosenzweig and Schultz, 1985, and Angrist and Evans, 1998). Overall, this line of research is mainly focused on explaining the determinants of labor supply at a point in time, given the skills accumulated by women and the wages they face on the market. The objective is not to explain the joint historical evolution of these variables or the long term pattern of change.

From a theoretical perspective, few papers have focused on the structural determinants of the changing labor market participation of women and the narrowing of the gender wage-gap. In this respect, Jones et al (2003) is the paper closest in spirit to the labor supply literature. These authors calibrate a general equilibrium model of household decisions regarding labor supply and human capital accumulation, and argue that the narrowing wage-gap alone is enough to explain a large part of the recent changes in female labor force participation. In a similar vein, Olivetti (2001) claims that there are specific changes in the life-cycle profile of female labor force participation that cannot be accounted for by the narrowing wage-gap alone. She then argues that technologically induced changes in the return to one specific productive attribute – labor market experience – can account for these patterns. Both these papers take the time path of wages – or returns to productive attributes – as given, and try to understand the changes in female labor force participation from that. As argued before, our goal is to go one step further, and also understand what determines the changes in the market wages faced by women.

With a different perspective, Greenwood et al (2004) suggest that a major force determining the liberation of women from household chores was the technological revolution in household production induced by the birth of electricity. The introduction of electricity in the household – followed by a series of new durable goods – liberated women from household production and allowed increased labor force participation, without implying a reduction in the consumption of home produced goods. Goldin and Katz (2002), on the other hand, suggest that a technological innovation of a different nature was the main factor determining the changes in women's professional choices. They argue that the introduction of the oral contraceptive in 1960 was the driving force behind the changes in women's career and marriage decisions in the US.

Without denying the importance of these factors to the extent and timing of changes in female labor force participation observed in modern economies, our goal here is to point out that there is an additional economic force linking together the changing role of women in society to the previous process of demographic transition. Both these changes were observed in various parts of the world, under different levels of development and at different points in time. The generality of this pattern, therefore, cannot be entirely attributed to technological or institutional transformations that are typically country-specific, even though these factors are certainly important to explain the particularities of each different experience.

In this respect, the theory proposed here is closest in spirit to the work of Galor and Weil (1996). These authors analyze the process of increased participation of women in the labor market as a joint consequence of the reduced demand for children induced by economic growth. In their model, women are assumed to have comparative advantage in mental labor, as opposed to physical labor. In addition, capital is assumed to be complementary to mental labor, so that growth induced by capital accumulation tends to increase the relative return to mental labor and, therefore, the relative wage of women. In a situation where the capital stock is small, and the return to physical capital relatively high, women specialize in raising children. But capital accumulation increases the relative wage of women, therefore increasing the opportunity cost of children, reducing fertility, and raising female labor force participation.

We share the same basic goal of Galor and Weil (1996), but key aspects of our theory differ from theirs, and we also extend the analysis in new directions. First, we focus on the reductions in mortality that characterize the onset of demographic transition as the only reason behind all the changes observed thereafter, while Galor and Weil (1996) do not incorporate mortality changes as part of the transition. Second, the only difference between men and women in our model is that women are marginally more productive at raising children (childbearing, breast-feeding, etc.), while they rely on differential productivity of men and women in physical and mental labor. And third, we explore the impact of these changes on the quality of children, while they do not incorporate investments in children in their model.

Experiences of demographic transition in the vast majority of cases have been characterized by initial reductions in mortality that are, after some delay, followed by reductions in fertility (see, e.g., Heer and Smith, 1968, Cassen, 1978, Kirk, 1996, and Mason, 1997). This has taken place in different areas of the world at very different development levels, so that to link the changes exclusively to economic growth and to ignore the major role played by mortality during the demographic transition does not seem fully satisfactory.

This paper follows the literature that stresses the interaction between fertility and investments in human capital as one of the distinguishing features of modern economies, and as the main determinant of the differential behavior of population before and after the demographic transition (as Becker et al, 1990). As Meltzer (1992) and Soares (2005), the paper explores the particular way in which health affects this interaction in order to shed some light on the determinants of the behavior of society in the long run. The potential role of this mechanism in determining changes in female labor force attachment and the gender wage-gap has never been identified in the literature. The evidence discussed later on suggests that, together with the other channels mentioned in this section, it may be an important component of a more complete and satisfactory explanation of the recent social changes observed throughout the world.

3 The Model

Consider an economy inhabited by families that live for a deterministic amount of time. Each family is composed by a male and a female (denoted by subscripts m and f, respectively), who jointly decide on the allocation of time of each member towards investments in adult human capital, work, and raising children. As in Galor and Weil (1996), fertility is realized in terms of couples who grow up to be a household, so that we abstract from questions of matching and the formation of families. Each member of the family is endowed with a level of basic human capital determined from the previous generation's decision (both members receive the same level of basic human capital). This basic human capital determines the productivity of individuals in acquiring market human capital and raising children. Market human capital, on its turn, determines the productivity of each unit of time allocated to labor supply, and goods produced in the market may be used for consumption or investment in children.

In such context, where we incorporate the effects of mortality, quality of children cannot be understood simply as productive human capital. Evolutionary considerations lead to a formulation where parents derive utility not only from the number of children and human capital of each child, but also from the child mortality rate and adult longevity faced by the children. In addition, these considerations imply that parents regard life expectancy and fertility in similar ways. This formulation requires the recognition that adult longevity can be seen as an additional dimension of offspring quality, justified by the fact that survival into adulthood affected evolutionary fitness in earlier hunter-gatherer populations (due to the necessity of providing to offspring until the early teenage years). It also implies the existence of a biological trade-off between quantity and quality of offspring (for a detailed discussion, see Robson and Kaplan, 2003, Robson, 2004, or Soares, 2005). In this case, natural selection imposes a trade-off between life expectancy and number of offspring (fertility) that, if recognized by preferences, implies a dominant evolutionary strategy. This is the logic underlying a recent model developed by Robson (2004), in which preferences that aim at maximizing the product of life expectancy (quality) and fertility (quantity) arise as evolutionarily dominant in the long run. In his model, the relation between life expectancy and fertility is relevant for purely biological reasons. But their interaction in the induced preferences is exactly the same as assumed here: parents' utility regards number and life expectancy of offspring in similar ways. This same idea is implicit in traditional arguments relating child mortality to fertility, where parents are assumed to derive utility from the number of surviving children. Here we follow Soares (2005) and extend this idea to later ages, by assuming that parents also care about the adult longevity of their offspring. This is a natural extension once we consider that parents are concerned not only with the immediate survival of their children, but also with the continuing survival of their lineage. In this case, families should care about whether their children would live long enough to have and raise their own offspring, therefore guaranteeing the long term survival of descendents.

For these reasons, we adopt a simplified version of the formulation proposed in Soares (2005). Households derive utility from consumption in each period of life ($c(t)^{\sigma}/\sigma$, over T years of life) and from children. We assume that parents derive utility from the basic human capital of children (h_c^{α}/α), and that this utility is affected by the number of children (n), the child mortality rate (β), and the lifetime that each child will enjoy as an adult (T). The discount factor applied to basic human capital is assumed to be a concave and increasing function $\rho(.)$ of the total expected lifetime of the children ($nT(1-\beta)$), or the total of "child-years."[5] With this formulation, the household utility function is given by[6]

[5] Soares (2005) shows that the main results generated by this functional form are also present under a more general formulation for the $\rho(.)$ function. Specifically, if the altruism function assumes the general form $\rho(n,T,\beta)$, and $\varepsilon(n,T,\beta) = \frac{\rho_n(n,T,\beta)n}{\rho(n,T,\beta)}$ denotes its elasticity in relation to n, the condition needed is $sign\{\varepsilon_n(n,T,\beta)\} = sign\{\varepsilon_T(n,T,\beta)\} = -sign\{\varepsilon_\beta(n,T,\beta)\}$, where the subscripts denote partial derivatives. We adopt the alternative formulation because it relates directly to the endogenous preferences results from the evolutionary literature (Robson, 2004). In addition, it is intuitively more appealing and simpler.

[6] We investigate the effect of technologically induced mortality changes, which are perceived as permanent by agents. This is why we do not distinguish between parent's and children's adult longevity in this formulation. As long as the changes are permanent, the long-run effects are the ones discussed here, even if they apply only from the

$$\int_{0}^{T} \exp\left(-\theta t\right) \frac{c\left(t\right)^{\sigma}}{\sigma} dt + \rho\left(nT\left(1-\beta\right)\right) \frac{h_c^{\alpha}}{\alpha} \tag{1}$$

where $c(t)$ is consumption at instant t, θ is the subjective discount factor and $0 < \alpha, \sigma < 1$. The first term is the utility that parents derive from consumption in each period of life, and the second term is the utility that they derive from their children.[7]

Production of basic human capital for children (h_c) can use either one of two inputs. The first input (x_T) is produced with time invested by adult members of the household, while the second input (x_y) is a good purchased with income on the market (at a fixed price p). For simplicity, we assume that these two inputs are perfect substitutes in the production function of h_c:

$$h_c = ax_T + (1-a)\, x_y, \tag{2}$$

where a is a constant between zero and one.[8] The household produced input (x_T) makes use of the time of parents according to the following production function:

$$x_T = (Bb_f + Cb_m)h_p, \tag{3}$$

where h_p is the basic human capital of parents and B and C are constants. In this context, the idea that women are more productive at raising children can be translated into $B > C$, such that each unit of time invested in a child by a woman generates more of the input x_T than the same unit invested by a man. This hypothesis is maintained throughout the paper. It is the only intrinsic difference between men and women in our model.

Basic human capital of each parent is also used, together with time invested in adult education, to produce market (productive) human capital, according to a formulation similar to that used by Becker (1985) to analyze specialization within the household:

$$H_i = Ah_pe_i. \tag{4}$$

children's generation on. In this case, we could understand our exercise as comparing households two generations apart.

[7] Though we do not make this distinction formally in the model, the relevant dimension of change in T refers to the length of productive life. So extensions in life during years when individuals are not able to work should not bring together the effects of changes in adult longevity stressed by our theory. We discuss the empirical relevance of changes in mortality during productive years later on in the paper.

[8] In reality, when parents purchase investments in children through the market, a large part of these investments is composed of other people's time (baby-sitters, teachers, etc.). As long as the changes being analyzed here are economy-wide, any impact on parents' productivity and wages should also be reflected on the wages of these service providers, and therefore should minimize the differential change on household production *vis-à-vis* market purchase of inputs. In order to simplify the framework and to focus on the wedge between household production and market purchase, we take the price of x_y as given. Generally, similar results would hold as long as the price of x_y increased less than proportionally with the productivity and the market wages of parents (or as long as other people's time were not the only input used to produce x_y).

This is the human capital that is actually used to produce goods. The market human capital of each member determines the productivity of each unit of time used as labor. The total amount of market goods produced by the household is, therefore,

$$y = l_m H_m + l_f H_f, \tag{5}$$

where l_i indicates the labor supply of agent i. The distinction between basic and market human capital highlights the different types of human capital acquired at different points in the life cycle. Basic human capital (h) refers to basic skills (language, motor ability, etc.) and general knowledge accumulated during early stages of life, and while investments decisions are still taken by parents. Market human capital (H) refers to the accumulation of skills related to a specific occupation or profession, and when investment decisions are already taken by the individuals themselves. Since the model is unable to capture all the subtleties of human capital investments, we understand market human capital very broadly as referring to any type of human capital investment specific to a particular task, including college and graduate education, professional training, and the investment dimension of on-the-job training and learning-by-doing. The distinction between these two types of human capital turns out to be key in identifying the different effects of changes in child mortality and adult longevity.

The total amount of goods produced by the household is allocated between consumption and raising children. Borrowing from future generations and bequests are not allowed, so that the budget constraint is

$$y \geq \int_0^T \exp(-rt)\, c(t)\, dt + \exp(-r\tau)\, npx_y, \tag{6}$$

where r is the interest rate, x_y is the goods investment in the basic human capital of each child,[9] and all the children are assumed to be born in period τ.

Each adult member of the family also faces a time constraint. Her/his adult lifetime has to be allocated between studying, working and, possibly, raising children. The time constraint of agent i is

$$T = l_i + e_i + nb_i. \tag{7}$$

In order to simplify the problem and concentrate the analysis on the intergenerational behavior of the economy, we abstract from life-cycle considerations and set discount rates and interest rates to zero. In this context, once we substitute for y in the budget constraint and for h_c in the utility function, the problem of the household can be written in a simpler form:

[9] The model could also incorporate a parameter f capturing a fixed goods cost of having children. But since the economy analyzed here displays long-run growth, this parameter would be asymptotically irrelevant and would not affect the long-run behavior of the economy.

$$\begin{aligned} \max V &= T\frac{c^{\sigma}}{\sigma}+\rho\left(n\left(1-\beta\right)T\right)\frac{\left[ax_T+\left(1-a\right)x_y\right]^{\alpha}}{\alpha} \qquad (8)\\ \text{subject to } l_m A h_p e_m + l_f A h_p e_f &\geq Tc+pnx_y, \\ T &= l_i+e_i+nb_i, \text{ for } i=f,m, \text{ and} \\ x_T &= (Bb_f+Cb_m)h_p. \end{aligned}$$

In this framework, there are two forces working toward specialization. First, since women's time is relatively more productive in household activities, simple comparative advantage considerations would lead to women's partial specialization in household production. Second, the possibility of increasing market productivity through human capital investments generates increasing returns to the total amount of time allocated to market related activities (both investments in adult education and labor supply together).[10] The presence of increasing returns to market related activities enhances the tendency toward specialization generated by any minor difference in comparative advantage, and exacerbates *ex-post* differences.

Therefore, there can be no equilibrium where both agents share their time between market activities and household production. Comparative advantage and investments in education generate an incentive toward specialization, and at least one agent will always be completely specialized in some activity. When agents spend some amount of time investing in children, there can be only three possible equilibria: (a) m specializes in the market and f works both in the market and in the household; (b) m works in the market and in the household and f specializes in the household; or (c) m specializes in the market and f specializes in the household. This has to be the case because if both agents share their time between market and household activities, the household can always increase its total production by increasing the market time of m (there are increasing returns to the total amount of time dedicated to the market). In addition, since $B > C$, if only one agent works in the household, it must be f. It is also possible that both agents end up spending all of their time investing in market human capital and working, in which case investments in children are made using goods purchased on the market.

In what follows, we concentrate the discussion on the cases where women spend at least part of their time on the market, and men are completely specialized on market production. Since women labor force participation has been positive in modern economies for most of the recent

[10] If $\tilde{t}_i$ denotes the total amount of time allocated to market related activities by agent i ($\tilde{t}_i = e_i + l_i$), the optimal allocation of time between human capital investments and labor supply is $e_i = l_i = \tilde{t}_i/2$. Substituting both back into the production functions, this would imply a total production of $Ah_p\tilde{t}_i^2/4$ for agent i. So, there are increasing returns to the total amount of time $\tilde{t}_i$ dedicated to market related activities. This is the type of return to specialization discussed in Becker (1985).

past, this seems to be the relevant equilibrium from an empirical perspective (the other equilibria, and the effects of mortality on the transition between different equilibria, are briefly described in the Appendix). Early stages of the process of economic development and of the movement of households out of subsistence agriculture may be better characterized as a movement of men from household activities to the market. This is a possibility that deserves further thought and discussion, but we do not deal with it here. Our main focus is on the increased female labor force participation that characterizes most industrial societies and also many less developed countries that have already experienced the demographic transition.

4 The Effect of Longevity Gains

From the first order conditions for an optimum (see Appendix), it is easy to show that, when the woman shares her time between market and non-market activities, the optimal allocation of time of m and f is characterized by

$$\begin{aligned} e_m &= l_m = \frac{T}{2},\ b_m = 0, \\ \text{and } e_f &= l_f = \frac{T - nb_f}{2}. \end{aligned}$$

Using the first order conditions for n, b_f, and x_y (see Appendix):

$$\frac{\rho' nT(1-\beta)}{\rho} = \alpha \tag{9}$$

This expression determines the response of n to exogenous changes in longevity (T) and child mortality (β). Particularly:

$$\frac{dn}{dT} = -\frac{n}{T} < 0. \tag{10}$$

This is the same relationship found in Soares (2005), and it reflects the interaction between n and T inside the function ρ.

Since mother's time (b_f) and market goods (x_y) are substitutes in the production function for children's basic human capital (h_c), only the one with the higher relative return will be used. Analyzing the first order conditions, one can see that there are two possible choices: one where the investment in basic human capital is done using the domestic technology ($b_f > 0$ and $x_y = 0$, or Equilibrium A), and another where this investment is done using goods purchased on the market ($b_f = 0$ and $x_y > 0$, or Equilibrium B). In other words, in Equilibrium A the woman shares her time between market and household activities, while in Equilibrium B both man and woman spend all their time on the market, and investments in children are undertaken with goods purchased on the market.

Longevity affects the investment in children's human capital through two channels. First, it affects the choice of the technology to be used in this investment (extensive margin). And second, it affects the total amount of investment undertaken, possibly through both income and substitution effects (intensive margin).

Equilibrium A

We first analyze the effect of T on the intensive margin, when women use part of their time to invest in children. In Equilibrium A, the fraction of time that the woman dedicates to the market (human capital investments plus labor supply) increases in T. The fraction of time spent in the household declines, but the net effect on the absolute value of b_f (investment in each child) is ambiguous, since fertility is also reduced (see Appendix). The impact on consumption is positive. These effects are described by

$$\begin{aligned} \frac{d\left(e_f/T\right)}{dT} &= \frac{d\left(l_f/T\right)}{dT} > 0, \ \frac{dc}{dT} > 0, \\ \text{and } \frac{d\left(nb_f/T\right)}{dT} &< 0, \text{ but } \frac{db_f}{dT} \gtrless 0. \end{aligned} \tag{11}$$

When women spend part of their time in the household, an increase in T increases overall incentives to invest in market human capital, because the time over which the returns can be enjoyed is longer. Therefore, the opportunity cost of time (and fertility) increases. In addition, the gain in longevity itself also reduces the benefits from larger families, and these two forces together determine a reduction in fertility. The result is that the total amount of time spent on market related activities increases more than proportionally with the gain in life-span, reducing the fraction of time allocated to the household. In the end, the fraction of f's productive lifetime allocated to labor supply (l_f/T) – the female labor force participation – rises, and the time invested in each child may respond positively or negatively, depending on the relative responses of fertility and the total time allocated to the household.

Understanding the market wage as the productivity of one unit of time allocated to labor supply, we can define the wage rate as $w_i = H_i = Ah_p e_i = Ah_p \tilde{t}_i/2$, where $\tilde{t}_i$ denotes the total amount of agent i's time allocated to the market. Since gains in longevity lead to more than proportional increases in the time women allocate to the market, while increases in men's time are just proportional to T, the gender wage-gap $(1 - w_f/w_m)$ is reduced as the process described in the previous paragraph takes place.

In relation to the quality of children, the response of investment per child (b_f) to changes in longevity, though indeterminate, follows a very specific pattern. For lower levels of female labor force participation (below 40% irrespective of the parameters of the model), increases in longevity bring both increases in female labor force participation and improvements in the quality of children. For intermediary values of b_f, and conditional on other parameters, investments in children may decrease as longevity and female labor force participation increase. But for sufficiently high levels of female labor force participation, irrespective of the parameters of the model, increases in longevity are again accompanied by increased quality of children. Therefore, for a given set of parameters, the relation between female labor force participation and investment per child is either positive or non-monotonic. When the relation is non-monotonic, it starts as positive for low levels of female labor force participation, turns into negative for intermediary levels, and becomes positive again at high levels. Even in these cases, the quality of children increases with longevity for the vast majority of initial levels of female labor force participation.

The claims from the previous paragraph are proven in the Appendix, but here we give the intuition for the results. Three forces relevant to decisions regarding investments in children are at work when longevity increases. First, the increase in longevity itself relaxes the resources constraint and increases full lifetime income. Second, the reduction in fertility that accompanies the gain in longevity reduces the relative price of investments in children. And third, due to increasing returns to market related activities, longevity increases the opportunity cost of time used in the household. The first two forces work toward increased investments in children, while the third works against it. For low levels of female labor force participation, the effect of increasing returns is relatively weak, so the first two forces dominate. At the other extreme, for high levels of female labor force participation, consumption is very high and fertility is very small, so the income effect is strong and it takes relatively little time to increase the quality of children. Also in this case, the first two forces dominate. The only situation where the third force may dominate is the intermediary one, where increasing returns kick in strongly and, to take advantage of them, women shift their time abruptly toward the market. In any case, it is not generally guaranteed that this third force will be strong enough to overcome the first two and, even when it does, it is only in a relatively short interval.

Equilibrium B

In the case of Equilibrium B (market investments in children), the expressions derived before fully describe the household's allocation of time and its response to longevity changes. Both adult members of the household allocate their total lifetime to market related activities, sharing it equally between investments in human capital and labor supply. Contrary to before, the effect of longevity gains on the quality of children is unambiguously positive ($dx_y/dT > 0$), but the effects on consumption are ambiguous ($dc/dT \gtrless 0$). In this case, changes come from the income effect and from the fact that increases in longevity reduce the returns from larger families. Since fertility is reduced as a result of the latter, the shadow cost of investments in children goes down, so that parents end up investing more in each child (there are no time costs in this case, and this effect can be seen as a positive substitution effect toward the quality of children). Whether consumption increases or not depends on how strong the income and substitution effects are.

Extensive Margin Choice

The extensive margin choice of the technology used to invest in children is also affected by longevity. For a large enough T, returns to market human capital are so high that both members of the household spend their entire time on market related activities, and make their investments in children through the market. For lower levels of T, this may not be the case, and women may share their adult lifetime between market activities and raising children. The intuition for this is clear: for lower T, the returns to market human capital and the cost of time are lower, the family is poorer, and, therefore, it is cheaper to spend time investing in children instead of buying this investment through the market. As longevity increases, returns to human capital (and family income) rise, so that women increase their level of education and the opportunity cost of time. When this change is large enough, it becomes cheaper for the family to make its investments in children through the market, and allocate all the available time of its members to investments in adult education and labor supply.

Long-Run Growth

Differences in investments in children in this model translate into long-run differences in growth rates, since basic human capital determines the productivity of later investments in market specific human capital. In steady-state, the growth rate between generations is determined by the evolution of basic human capital between parents and children.[11] Changes in women's labor force participation may affect the accumulation of basic human capital in different ways, depending on the equilibrium that characterizes the economy. In the case of Equilibrium A, the effect is ambiguous, but tends to be positive. This comes directly from the fact that longevity has ambiguous effects on the quality of children, which nevertheless are positive for relatively low and high levels of female labor force participation. In terms of growth rates,

[11] As shown in Soares (2005), a steady-state only exists in this type of economy when $\alpha = \sigma$. We implicitly make this assumption whenever talking about steady-states. To keep notation to a minimum, we are not indexing by generations, and are distinguishing parent's and children's basic human capital by the subscripts p and c. These are obviously related across generations. If we let i index different generations, $h_{p,i+1} \equiv h_{c,i}$. So the growth rate of basic human capital from one generation to the next is $h_{c,i}/h_{p,i} = h_{p,i+1}/h_{p,i} = h_{c,i}/h_{c,i-1}$.

$$
\begin{aligned}
1+g &= \frac{h_c}{h_p} = aBb_f, \text{ and} \qquad (12)\\
\frac{d(1+g)}{dT} &= aB\frac{db_f}{dT} \gtreqless 0.
\end{aligned}
$$

In Equilibrium B, investments in children are purchased through the market. Also, in this situation, increases in longevity reduce fertility, relax the budget constraint, and increase investments in children:

$$
\begin{aligned}
1+g &= \frac{h_c}{h_p} = (1-a)\frac{x_y}{h_p}, \text{ and} \qquad (13)\\
\frac{d(1+g)}{dT} &= \frac{(1-a)}{h_p}\frac{dx_y}{dT} > 0.
\end{aligned}
$$

It is therefore possible to observe an intermediary period when the growth rate of the economy is reduced as women increase their attachment to the market. This is due to reduced investments in children during this transition period. But eventually, the quality of children and the growth rate start rising again as women intensify their attachment to the market. This tendency is reinforced after women enter fully into the job market, and investments in children are bought outside of the household. From this point on, the cost of time becomes irrelevant in determining the quality of children and the long-run growth of the economy.

5 The Effect of Child Mortality Reductions

The impact of child mortality changes is much simpler in nature than that of adult longevity. The effects of child mortality on fertility can be seen, as before, from the expression $\rho' n(1-\beta)T/\rho = \alpha$. This relation implies that reductions in child mortality will be accompanied by reductions in fertility:

$$\frac{dn}{d\beta} = \frac{n}{(1-\beta)} > 0. \qquad (14)$$

This is the only direct effect of child mortality in the model, and all subsequent changes follow from how fertility affects other margins of the household decision. In Equilibrium A (woman sharing her time between market and non-market activities), the household allocation of time between domestic and market related activities is also affected by the change in fertility. The reductions in fertility and child mortality imply a reduction in the shadow price of investments in children (or an increase in the rate of return to these investments). In the presence of increasing returns to market activities, the marginal cost of market goods in terms of time is decreasing in total production (or, alternatively, the marginal productivity of goods is increasing in the total amount of time allocated to the market). This non-linear time cost tends to magnify responses to price changes. For this reason, the reduction in the shadow price of child quality represented by the fertility decline is strong enough to increase the total amount of time allocated to children (see Appendix). This happens here, among other things, because there are increasing returns to market activities. Otherwise, the reduction in fertility would generate the typical price response of a normal good, where consumption (b_f or h_c) necessarily increases, but total expenditures (nb_f) may not. Since there is a reduction in the total amount of time allocated to the market, female educational attainment and labor force participation are actually reduced by reductions in child mortality, and the wage differential between men and women rises.

Still, the result that female labor force participation is unequivocally reduced by reductions in child mortality is not general, and depends partly on the specific functional forms adopted in the model. Nevertheless, it does highlight an economic principle that is quite general and is particularly strong in our theory. Changes in child mortality are, in nature, similar to changes in the price of investments in children (due to reduced fertility, together with higher survival probability of each child). Changes in adult longevity, on the other hand, are similar to changes both in the price of investments in children (due to reduced return to large families) and in the return to labor market attachment (due to longer productive life as an adult). Therefore, in the case of gains in adult longevity, there are two forces working toward increased labor supply (increased productive life and reduced fertility), while in the case of reductions in child mortality there is only one (reduced fertility), which is at least partially offset by the increased return to time spent at home (from reduced child mortality). This distinction is general in nature, and should lead to an effect of child mortality relatively modest in magnitude when compared to the effect of adult longevity, irrespective of the particular functional forms adopted. Contrary to common belief, the general result suggested by our theory is that female labor force participation should be more closely related to adult than to child mortality.

In Equilibrium B (investments in children through the market), the reduction in child mortality and fertility is reflected on higher investments in children, as the simple response of a normal good to price changes. But the choice between the two equilibria is also affected by child mortality. In order to be optimum for the household to allocate part of the woman's time to the market, it must be the case that $p\frac{a}{(1-a)}\frac{B}{A} > e_f$. Since the reduction in child mortality tends to reduce female investments in market human capital, reductions in child mortality tend to move the economy away from Equilibrium B and toward Equilibrium A.

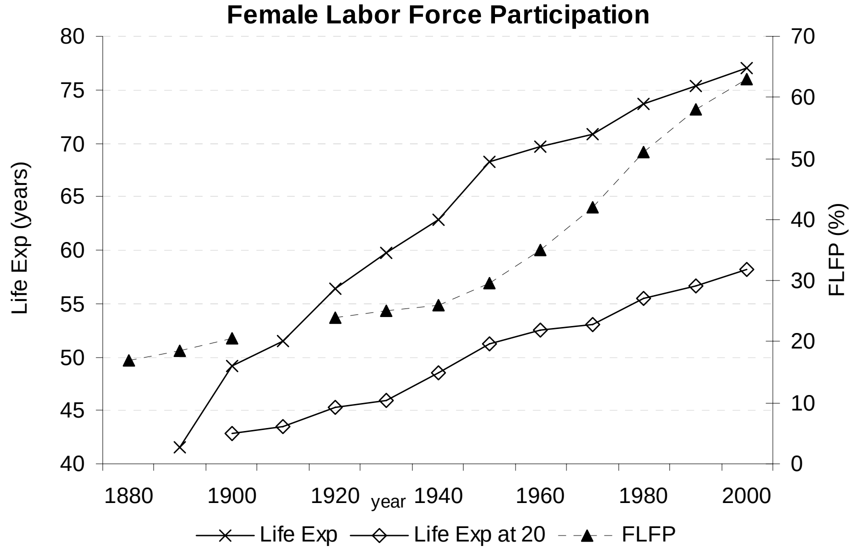
In both equilibria, increased investments in children lead to higher growth rates and, in the long run, higher consumption (this may take place at the expense of reductions in present consumption). But, most important, reductions in child mortality cannot generate increased female labor force participation nor a narrowing gender wage-gap. In fact, the model generates exactly the opposite result when women spend part of their time on domestic activities: reductions in child mortality lead to reduced participation of women in the labor market and to a widening wage differential between men and women. This result highlights the key position occupied by adult longevity in our theory. Specifically, reductions in fertility are not enough to generate the typical change in women's labor supply. Increased return to market specific human capital is an additional feature that is essential in explaining the observed trends.

6 Predictions of the Model and Related Evidence

The theory proposed here generates a close link between female labor force participation, the gender-wage gap, and adult longevity. On the other hand, child mortality does not seem to a particularly important determinant of female labor force participation, even though its effect on fertility is an important one. These predictions are in line with historical evidence, since female attachment to the labor force only increases significantly at later stages of the demographic transition, when changes in adult longevity gain importance. This would explain the lag between the initial increase in life expectancy during the onset of the demographic transition and the later increase in female labor force participation, as Figures 1, 2, and 3 show.

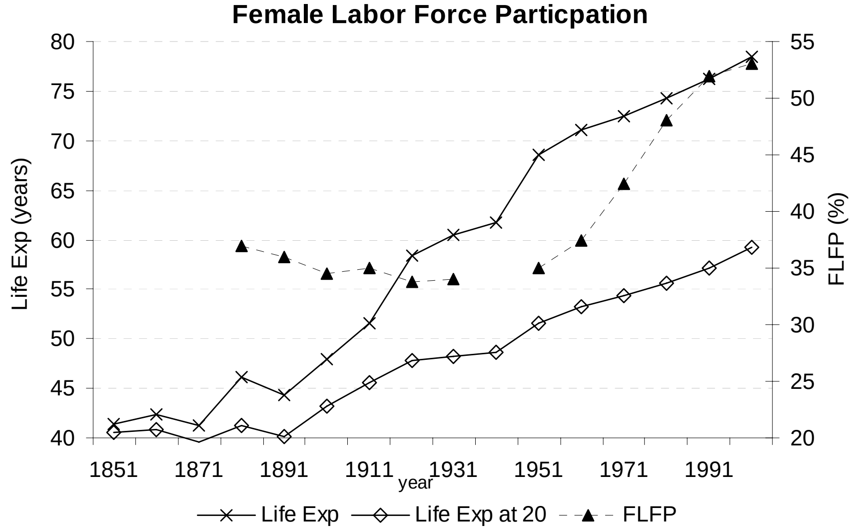
This point is further illustrated in Figures 4 and 5, where together with life expectancy at birth and female labor force participation, we present life expectancy at age 20 for the US and Great Britain (similar numbers are not available as a time series for Brazil). Two points in these figures deserve attention. First, female labor force participation seems to be much more closely related to life expectancy at age 20 than to life expectancy at birth. Second, both countries experienced very expressive gains in life expectancy at age 20 in the recent past. For example, just between 1940 and 2000, the US and Great Britain gained roughly 10 years in life expectancy at age 20. In developed and developing countries, a large part of the recent gains in life expectancy is due to reductions in mortality during productive years. For example, in the case of the US and Great Britain, female mortality between ages 15 and 60 was reduced by 35% between 1960 and 2000. For Brazil, the analogous number was 39%, while it was around 50% or above for countries as diverse as Australia, Austria, Chile, China, Colombia, Germany, India, Indonesia, Ireland, Korea, Mexico, Spain, and Thailand, among others (World Bank, 2004). In absolute terms, these changes ranged from reductions in the probability of death of 5 percentage points for the wealthiest countries, to up to 30 or 50 percentage points for countries such as China, Indonesia and Korea.

Figure 4: United States - Life Expectancy, Fertility and Female Labor Force Participation



Source: Costa (2000), National Center for Health Statistics (2003), and US Census Bureau (1975).

Figure 5: Great Britain - Life Expectancy, Fertility and Female Labor Force Particpation



Source: Wrigley et al (1997), Keyfitz and Fieger (1968), Costa (2000), World Bank (2004) and Human Mortality Database (2006).

One of the main issues that traditionally made it difficult to relate female labor force participation to the demographic transition was precisely the fact that increased labor supply of women starts only considerably after significant reductions in mortality and fertility are observed. Our theory overcomes this problem by showing that reductions in fertility are not enough to generate the movement of women out of household work. The latter is only guaranteed when reduced fertility is accompanied by increased return to market related activities, which does not happen until later stages of the demographic transition.

This close association between female labor force participation and adult longevity, as opposed to child mortality, is somewhat at odds with the accepted common knowledge in the profession. Typically, female labor force participation is seen as closely linked to fertility, which in turn is thought to depend to a great extent on child mortality. Therefore, one might think, child mortality should exhibit a negative correlation with female labor force participation. In addition, the simple fact that development brings together modernization and health improvements might mean that this same correlation should be reinforced, even if not because of a strictly causal relationship between the two variables.

The historical evidence discussed before shows that this is not necessarily the case. Also in a cross-country context, and contrary to common belief, this is not the typical pattern of correlations observed. Table 1 presents a set of cross-country panel regressions of the fraction of the labor force composed by women on different sets of variables. The different specifications include as independent variables various combinations of child mortality (before age 5), adult female and male mortalities (between ages 15 and 60), income per capita adjusted for terms of trade, average educational attainment in the population above 15, total fertility rate, and country and time fixed effects.[12] Income per capita is from the Penn World Tables version 6.1, educational attainment is from the Barro and Lee Dataset, and all the other variables are from the World Bank's World Development Indicators. These regressions are not intended to imply any causal relationship, as this would require an empirical effort that is beyond the scope of this paper. They are seen here simply as a descriptive tool, intended to reveal the pattern of conditional correlations observed across countries. The goal of the table is to show that this pattern of simple correlations is quite different from what is commonly thought and, maybe surprisingly, is strikingly consistent with the theory proposed here.

Since the measure of labor force participation available is a relative one, we control for adult male mortality in all regressions. The table shows that higher female mortality is significantly related to lower female labor force participation in all specifications. At the same time, in the simplest specification, child mortality does not show a significant correlation with female labor supply. But when country fixed-effects and/or other controls are added, child mortality starts

[12] In order to make consistent comparisons, the table keeps the same sample across the different specifications. Qualitative results are the same when the sample is allowed to include all the observations available for each different specification. The main qualitative results are also the same when child mortality before age 1 is used instead of child mortality before age 5.

having a positive and significant correlation with female labor supply, meaning that reductions in child mortality are associated with reductions in female labor force participation. Notice that, as specification (3) shows, this correlation is not due exclusively to the relation between child mortality and fertility. Though fertility is endogenous to the problem, the correlation between child mortality and female labor supply remains positive and significant even after fertility and educational attainment are included in the regression. This should be expected if the driving force behind these correlations were the change in investments in children, as the model would suggest. The inclusion of income per capita and country fixed effects also shows that these correlations are not driven by the association between the different variables and economic development, nor by any country-specific cultural or institutional characteristic.

Table 1: Regressions for the Fraction of the Labor Force Composed by Women, Cross-country, 1960-2000

	1	2	3	4	5
Child Mort	-0.0051	0.0436	0.0381	0.0339	0.0461
	0.0088	0.0068	0.0068	0.0099	0.0072
	0.5652	0.0000	0.0000	0.0007	0.0000
Female Mort	-0.0486	-0.0358	-0.0285	-0.0265	-0.0206
	0.0127	0.0089	0.0089	0.0122	0.0086
	0.0002	0.0001	0.0014	0.0304	0.0171
Male Mort	0.0745	0.0218	0.0171	0.0629	0.0105
	0.0122	0.0094	0.0090	0.0106	0.0085
	0.0000	0.0210	0.0570	0.0000	0.2219
ln(Income PC)			2.2791	-1.7530	1.6727
			0.7820	0.8044	0.7690
			0.0038	0.0298	0.0303
Fertility				-2.2714	-1.0464
				0.4347	0.3106
				0.0000	0.0008
Education				1.1937	0.5731
				0.2464	0.3256
				0.0000	0.0793
Country f.e.	no	yes	yes	no	yes
N Obs	453	453	453	453	453
R^2	0.32	0.92	0.92	0.42	0.93

Obs.: Standard errors and p-values below the coefficients. Dependent variable is the fraction of the labor force composed by women (WDI). All regressions include a constant and year fixed-effects. Child Mort is mortality under age 5 (per 1,000 live births, WDI); Female Mort is female mortality between ages 15 and 60 (per 100,000, WDI); Male Mort is male mortality between ages 15 and 60 (per 100,000, WDI); Income PC is income per capita adjusted for terms of trade (PWT 6.1, rgdptt), Fertility is the total fertility rate (WDI); Education is average educational attainment in the population aged 15 and above (Barro & Lee). Data in five-year intervals between 1960 and 2000; 97 countries included in the sample.

Across countries, female labor supply tends to be positively associated with adult female survival, and negatively associated with child survival. Even though these are simply descriptive patterns, it is still true that, at the same time as they are different from what is commonly believed, they are entirely consistent with our theory. Further research is needed in order to establish whether these correlations indeed reflect the causal links suggested by the model.

As a final note, the model also has implications regarding the relationship between female labor force participation and the quality of children. In our theory, there can be either a positive or a non-monotonic relationship between female participation in the labor market and child quality. For intermediary levels of longevity and income, increases in female labor force participation may reduce child quality, while the opposite certainly happens for sufficiently low or high levels of labor supply. Recent demographic evidence regarding the relationship between these two variables has not yet reached a consensus. In some circumstances, a non-monotonic relationship between female labor supply and time spent with children has been found. In others, positive, negative, and non-significant effects of mother's labor supply on different dimensions of child quality have also been registered. Gauthier et al (2004), for example, present evidence for the US, UK and Canada indicating that average time spent on child-care activities was reduced up to the end of the 1970's, after when it started rising again. They also show that since 1960 average time spent on housework by mothers was reduced by more than 30%. Overall, time spent by women in housework and child-care together was reduced by roughly 1.2 hour a day. Sayer et al (2004) also find the same non-monotonic trend in terms of time allocated to direct child-care by mothers, with reductions observed between 1965 and 1975, and increases afterward. In addition, they find that, even though reductions in fertility have reduced the total amount of time allocated to child-care, activities involving direct and deeper interaction with children have increased in relative importance.

The literature that tries to estimate the effect of mother's employment and hours worked on children's educational attainment, behavior, test performance and teacher's rating is reviewed by James-Burdumy (2005). The results from this literature are largely mixed, with negative, positive, and quantitatively irrelevant effects all being common. James-Burdumy (2005) addresses some of the problems common to the previous papers – absence of control for family specific effects and instruments for mother's labor force participation – and finds only modest effects of mother's labor supply on child performance, both positive and negative (depending on the test score analyzed and on the age of the child when the mother worked). So rather than settling the debate, the results from James-Burdumy (2005) add to the controversy.

Our theory suggests that the relationship between female labor supply and investments in children should be looked at from a broader perspective. It may be the case that families under different economic circumstances will display different patterns of correlation between mother's labor supply and the quality of children. In this respect, contradictory results in studies looking at different groups of the population, or weak quantitative results in studies looking at broad groups, might be exactly what should be expected, given the possibility of a non-monotonic relationship between female labor force participation and quality of children. The theory suggests that family heterogeneity and the possibility of a non-monotonic relationship may be fruitful directions for future empirical research in the area.

7 Concluding Remarks

This paper presents a model where, ultimately, longevity gains are solely responsible for the increased participation of women in the labor market and the narrowing of the gender wage-gap. Though the direct link between these two phenomena may seem obscure, the intuition becomes clear once other dimensions of the demographic transition are brought into the analysis. Increased longevity increases the returns to investments in market oriented human capital. Higher returns to investment in human capital increase the cost of time and shift the quantity-quality trade-off toward fewer and better educated children. In addition, lower mortality reduces the return to large families, representing an additional force toward reduced fertility. With higher returns to human capital and fewer children, women increase their investments in human capital and their attachment to the market. Since women are initially specialized in the household sector, this takes place at a rate more than proportional than the one observed for men, so that the gender wage-gap is reduced. The predictions generated by this framework suggest a single process of demographic transition, triggered by reductions in mortality, that is consistent with several social changes observed in the course of the last century. Though other technological and cultural changes are certainly important to help explain the transformation in the role of women in society, we highlight one important economic force that has been entirely ignored in the literature.

A Appendix

A.1 First Order Conditions

Let Ψ, λ_f, λ_m, and Π denote sequentially the multipliers for the constraints in the maximization of equation 8. Substituting for h_c and x_T in the objective function, the first order conditions for c, n, e_m, l_m, b_m, e_f, l_f, b_f and x_y are given by, respectively:

$$\begin{aligned} Tc^{\sigma-1} &= \Psi T, \\ \rho' T (1-\beta) \frac{h_c^{\alpha}}{\alpha} &= \Psi p x_y + \lambda_f b_f, \\ A h_p l_m \Psi &= \lambda_m, \end{aligned}$$

$$
\begin{aligned}
Ah_p e_m \Psi &= \lambda_m, \\
\rho h_c^{\alpha-1} aCh_p &< \lambda_m n \ , \ = \text{ if } b_m > 0 \\
Ah_p l_f \Psi &= \lambda_f, \\
Ah_p e_f \Psi &= \lambda_f, \\
\rho h_c^{\alpha-1} aBh_p &< \lambda_f n \ , \ = \text{ if } b_f > 0 \text{ and} \\
\rho h_c^{\alpha-1}(1-a) &< \Psi pn, \ = \text{ if } x_y > 0.
\end{aligned}
$$

A.2 The Choice of the Technology for Investments in Children

When investments make use of the mother's time, $b_f > 0$ and we have that $\rho' T (1-\beta) \frac{h_c^\alpha}{\alpha} = \frac{\rho h_c^{\alpha-1} aBh_p}{n} b_f$. And when investments make use of income, $x_y > 0$ so that we can write $\rho' T (1-\beta) \frac{h_c^\alpha}{\alpha} = \frac{\rho h_c^{\alpha-1}(1-a)}{n} x_y$.

Proposition 1 *There is a T^* such that, for every $T < T^*$, investments in children are done domestically (using the mother's time), and, for every $T \geqslant T^*$, investments are done with goods purchased on the market.*

Proof. The rate of return to investments using the domestic technology (marginal productivity divided by opportunity cost) is given by $RRb_f = \frac{\rho h_c^{\alpha-1} aBh_p}{\lambda_f n}$. The rate of return to investments using income is $RRx_y = \frac{\rho h_c^{\alpha-1}(1-a)}{\Psi pn}$. The household will choose to use time whenever $RRb_f \geqslant RRx_y$, or $\frac{aBh_p}{\lambda_f} > \frac{(1-a)}{\Psi p}$. Substituting for λ_f, this inequality can be rewritten as $e < p\frac{a}{(1-a)}\frac{B}{A}$. For T sufficiently low (in particular, for $T < p\frac{a}{(1-a)}\frac{B}{A}$) $RRb_f > RRx_y$ and investments in children are done domestically. In addition, since e increases at least proportionately with T, there is a T large enough so that $e \geqslant p\frac{a}{(1-a)}\frac{B}{A}$. In this case, $RRb_f \leqslant RRx_y$ and investments in children are done through the market. ■

A.3 The Equilibrium with Time Investments in Children

When investments in children make use of the domestic technology, it is convenient to rewrite the problem in terms of the shares of total lifetime (T) dedicated to each different activity. Define the new variables $e_i^* = e_i/T$, $l_i^* = l_i/T$ and $d_i = nb_i/T$ as the shares of total lifetime allocated to, respectively, investments in adult human capital, labor supply, and domestic activities (raising children). Incorporating $x_y = 0$, the original problem can be rewritten as

$$\max_{e_i^*, l_i^*, d_i, n, c} T\frac{c^\sigma}{\sigma} + \rho[(1-\beta)Tn]\frac{h_c^\alpha}{\alpha}$$

subject to

$$\begin{aligned}
\left(Ah_p e_m^* l_m^* + Ah_p e_f^* l_f^*\right) T^2 &\geq Tc, \\
1 &\geq e_i^* + l_i^* + d_i \text{ , with } i = m,\ f, \\
h_c &= \frac{T}{n} a \left(Bd_f + Cd_m\right) h_p,\ \text{and} \\
e_i^*, l_i^*, d_i &\geq 0.
\end{aligned}$$

Let λ be the multiplier on the first constraint, and λ_i the multiplier on the second constraint for agent i. In the equilibrium where men specialize in market activities and women share their time between market and domestic activities, first order conditions can be written in terms of the new variables as

$$\begin{aligned}
c^{-\sigma} &= \lambda, \\
\rho'(1-\beta)T\frac{h_c^\alpha}{\alpha} &= \rho h_c^{\alpha-1}\frac{T}{n^2} a \left(Bd_f + Cd_m\right) h_p, \\
\lambda T^2 Ah_p l_m^* &= \lambda_m, \\
\lambda T^2 Ah_p e_m^* &= \lambda_m, \\
\rho h_c^{\alpha-1}\frac{T}{n} aCh_p &< \lambda_m, \\
\lambda T^2 Ah_p l_f^* &= \lambda_f, \\
\lambda T^2 Ah_p e_f^* &= \lambda_f,\ \text{and} \\
\rho h_c^{\alpha-1}\frac{T}{n} aBh_p &= \lambda_f.
\end{aligned}$$

Therefore, $e_m^* = l_m^* = 1/2$, $d_m = 0$, and $l_f^* = e_f^*$. For each individual, the time spent with investments in adult human capital and labor supply is always the same. Therefore, to save on notation, we write t_i as the proportion of total lifetime allocated to market related activities (investments in human capital plus labor supply), so that $l_i^* = e_i^* = t_i/2$ ($t_i = \tilde{t}_i/T$, where $\tilde{t}_i$ was defined before as the total amount of agent i's time allocated to the market).

Rewriting the problem after substituting for the budget constraint directly into the utility function, and considering the equilibrium where $t_m = 1$, and d_f, $t_f \in [0,1]$:

$$\max_{t_f, d_f, n} T^{1+\sigma}\left(\frac{Ah_p}{4}\right)^\sigma \left[1 + t_f^2\right]^\sigma / \sigma + \rho[(1-\beta)\,Tn]\frac{h_c^\alpha}{\alpha}$$

subject to

$$\begin{aligned}
1 &\geq t_f + d_f,\ \text{and} \\
h_c &= \frac{T}{n} aBh_p d_f,
\end{aligned}$$

and to the additional constraint that $t_f, d_f \geq 0$. In this form, first order conditions become

$$\begin{aligned}
(1-\beta)T\rho'\frac{h_c^\alpha}{\alpha} &= \rho h_c^{\alpha-1}\frac{T}{n^2}aBh_p d_f, \\
T^{1+\sigma}\left(\frac{Ah_p}{4}\right)^\sigma \left[1+t_f^2\right]^{\sigma-1} 2t_f &= \lambda_f, \text{ and} \\
\rho h_c^{\alpha-1}\frac{T}{n}aBh_p &= \lambda_f.
\end{aligned}$$

The objective function is concave on d_f and convex on t_f, so we cannot, in principle, guarantee unicity of the internal solution nor trust on the Hessian to verify that a point of maximum is reached. The issue in question and the optimal solution to the problem can be better understood with the help of a figure. Define

$$\begin{aligned}
f(t) &= T^{1+\sigma}\left(\frac{Ah_p}{4}\right)^\sigma \left[1+t_f^2\right]^\sigma/\sigma, \text{ and} \\
g(d) &= \rho[(1-\beta)Tn]\frac{\left(\frac{T}{n}aBh_p d_f\right)^\alpha}{\alpha}.
\end{aligned}$$

Conditional on being on this equilibrium, and given the choice on the number of children (n),the optimal allocation of the woman's time is the solution to the following problem.

$$\max_{t_f, d_f} f(t_f) + g(d_f)$$

subject to $t_f + d_f = 1$ and with $t_f, d_f > 0$.

The optimum is characterized by $f'(t_f) = g'(d_f)$, where these derivatives are given by the left hand side of the last two first order conditions above. One can show that

$$\begin{aligned}
f''(t_f) &= 2T^{1+\sigma}\left(\frac{Ah_p}{4}\right)^\sigma \left[1+t_f^2\right]^{\sigma-2}\left[1+t_f^2+2(\sigma-1)t_f^2\right] > 0, \\
g''(d_f) &= (\alpha-1)\rho[(1-\beta)Tn]\left(\frac{T}{n}aBh_p\right)^\alpha d_f^{\alpha-2} < 0, \text{ and} \\
f'''(t_f) &= 2T^{1+\sigma}\left(\frac{Ah_p}{4}\right)^\sigma \left[1+t_f^2\right]^{\sigma-3}\left[6(\sigma-1)t_f + 2(\sigma-1)^2 t_f^3 + 2\sigma t_f^3(\sigma-1)\right] < 0,
\end{aligned}$$

where the inequality comes from the fact that $t_f > t_f^3$. In addition, these functions are characterized by the following properties: $g'''(d_f) > 0$, $\lim_{x\longrightarrow 0} g'(d_f) = \infty$, $\lim_{x\longrightarrow 1} g'(d_f) = \text{constant} > 0$, $\lim_{t\longrightarrow 0} f'(t_f) = 0$, and $\lim_{t\longrightarrow 1} f'(t_f) = \text{constant} > 0$. So we can plot the functions f' and g' against the fraction of time allocated to market and non-market activities (see Figure A.1).

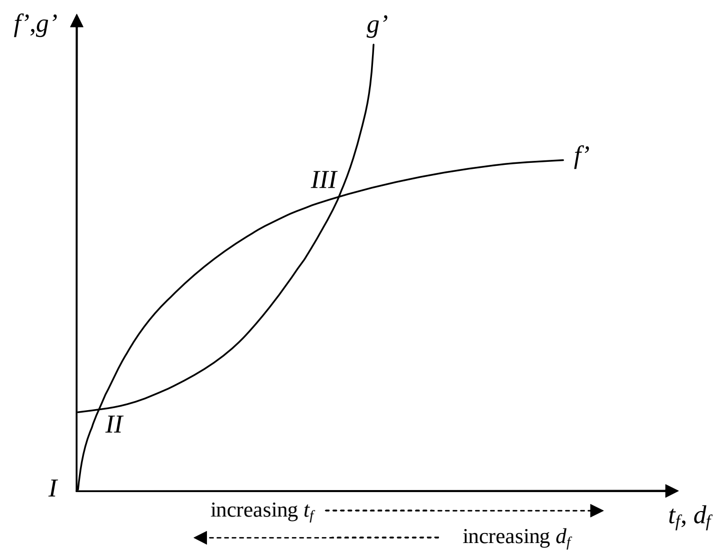
In the figure, we assume that the two curves intersect. If they did not, the origin would be the optimal choice. Points II and III are the ones that satisfy the first order conditions.

Proposition 2 *Point III is preferrable to point II and, therefore, is the solution to the household problem.*

Proof. Starting from point II and moving to the right, t_f increases while d_f is reduced. The gains in terms of utility are given by f', while the losses are given by g'. Since $f' > g'$ in (II, III), III is preferrable to II. ■

Points I and III are not comparable on a strictly graphical basis. More rigorously, their ordering would depend on the value of the integral $\int_0^c (f' - g')\, dt$. Point I corresponds to the equilibrium where there is total specialization within the household (man on the market and woman in the household). For lower values of T, the integral above is negative, and the optimal choice is point I. For a higher T, the value of the integral increases and eventually becomes positive. This is what characterizes the first movement of women into the labor market.

Figure A.1: Characterization of First Order Conditions



Generally, the comparative statics of the problem can be analyzed from the impact of changes in T on the curves f' and g'. As T increases, f' moves vertically at a rate $(1 + \sigma)$, and g' moves vertically at a rate 2α.[13] In order for a steady state to exist in this economy, we must have $\alpha = \sigma$ (see Soares, 2005). In this case, f' shifts at a faster rate and, therefore, point III moves to the right, corresponding to a higher t_f and a lower d_f.

[13] This result uses the fact that, in equilibrium, $(1 - \beta)nT$ is constant.

A.4 The Quality of Children in the Equilibrium with Time Investments

With the first order conditions and after some tedious algebra, it can be shown that

$$\frac{db_f}{dT} = \frac{D}{\left(2D + (1-\sigma)\left[(1-d_f) + (1-d_f)^3\right]\right)} \frac{b_f}{T},$$

where $D = -(1-\alpha)(1-d_f) - (1-\alpha)(1-d_f)^3 + d_f + d_f(1-d_f)^2 - 2(1-\sigma)(1-d_f)^2 d_f$ and $d_f = nb_f/T$. This implies that $db_f/dT < 0$ if and only if

$$-\frac{(1-\sigma)\left[(1-d_f) + (1-d_f)^3\right]}{2} < D < 0.$$

Expanding the polynomials on D and incorporating the condition for existence of steady-state ($\alpha = \sigma$), this inequality can be rewritten as

$$-\frac{(1-\sigma)\left[(1-d_f) + (1-d_f)^3\right]}{2} < -(1-\sigma)2 + (2-\sigma)2d_f - (1+\sigma)d_f^2 + \sigma d_f^3 < 0.$$

The effect of the change in longevity on the quality of children, therefore, depends on the equilibrium fraction of time allocated to the household (d_f). In addition, the set of possible values of d_f compatible with Equilibrium A being optimum depends on the other parameters of the model. From the extensive margin choice between the two technologies, the maximum level of longevity compatible with Equilibrium A is determined implicitly from

$$e_f = \frac{T(1-d_f)}{2} = p\frac{a}{1-a}\frac{B}{A}.$$

Since the left hand side is constant and $T(1-d_f)$ increases more than linearly with T, we know that there is a T high enough so that this expression is satisfied (even though it may be already with $d_f = 0$). This is the level of longevity at which the family stops using time to invest in the children and starts using market goods. Call this level of longevity and the associated fraction of time spent in the household $\hat{T}$ and $\hat{d}_f$, respectively. Notice that the expression above implies that any value of d_f between 0 and 1 is always compatible with Equilibrium A being optimum for some set of parameters. Particularly, changes in p do not affect the intensive margin choice of d_f in Equilibrium A, but do affect the extensive margin choice between Equilibria A and B. So, for any given values of the other parameters, there is always a p such that virtually all values of d_f (corresponding to changing T's) are compatible with the optimality of Equilibrium A.

In principle, this means that d_f can really take any value between 0 and 1 for any value of σ, depending on the set of parameters. So, in order to check when the inequalities above are satisfied, we have to check whether they hold for all the values of d_f and σ between 0 and 1. Based on the inequalities above, the table below shows whether db_f/dT is negative or positive for a grid of values of d_f and σ between 0 and 1.

It is clear from the table that, in terms of this grid, db_f/dT is positive in the vast majority of cases (or, generally, it is positive in the vast majority of the area defined when d_f and σ vary between 0 and 1). Nevertheless, this result does not mean that db_f/dT is positive for all the empirically relevant cases.

But is does mean that when female labor force participation starts rising from low levels (typically well below 40%), child quality increases together with the increased attachment of women to the labor market. After this process takes place for some time, it is possible that further increases in female labor force participation are accompanied, in a certain interval, by reductions in the quality of children. Yet, even in this case, this would be no more than a temporary phenomenon. Further increases in longevity eventually bring back the positive association between increased female labor force participation and child quality. This happens either because, as d_f is reduced within Equilibrium A, the range of negative values of db_f/dT is surpassed, or because the household eventually finds it optimum to move from Equilibrium A into Equilibrium B.

Table A.1: Signal of db_f/dT for Different Values of d and σ

$d\backslash\sigma$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
0.05	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.10	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+
0.15	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	+	+	+
0.20	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	+	+	+	+
0.25	+	+	+	+	+	+	+	+	+	+	-	-	-	-	+	+	+	+	+
0.30	+	+	+	+	+	+	+	+	+	-	-	-	-	+	+	+	+	+	+
0.35	+	+	+	+	+	+	-	-	-	-	-	+	+	+	+	+	+	+	+
0.40	+	+	+	+	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+
0.45	+	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+	+
0.50	-	-	-	-	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.55	-	-	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.60	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.65	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.70	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.75	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.80	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.85	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.90	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
0.95	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Therefore, the model generates either a positive or a non-monotonic relationship between female labor force participation and quality of children. In the case of the non-monotonic relation, increases in female labor force participation are initially accompanied by increases in child quality, then by reduced investments in children (for intermediary values of female labor force participation), and then again by increased quality of children. Section 4 discusses the intuition for these results.

A.5 The Equilibrium with Market Investments in Children

When investments in children's basic human capital is done through the market, increases in longevity reduce the opportunity cost of investments in children, because of a lowered fertility rate and an expanded budget constraint (higher educational attainment on the part of both members of the family)

From the first order conditions in this situation, we can obtain $c^{\sigma-1}pn = \rho h_c^{\alpha-1}(1-a)$. Working with this expression and incorporating the condition for a steady-state ($\alpha = \sigma$), we obtain the following relation between the changes in c, n, and h_c.

$$\frac{dh_c}{h_c} = \frac{dc}{c} + \frac{dn}{n}\frac{1}{(\sigma-1)}.$$

In addition, previous results related to educational attainment and fertility hold, so that we have $e_i = l_i = \frac{T}{2}$ for $i = m, f$, and $\frac{dn}{dT} < 0$.

When T increases, n falls and y increases more then proportionately ($y = l_m H_m + l_f H_f = Ah_p T^2/2$), so either c or x_y must increase (from the budget constraint, $Tc + fn + px_y = y$). If c increases, from the equation above, h_c must also increase, so x_y grows. If c falls, from the budget constraint, x_y increases, and so does h_c. In any case, increases in longevity lead to increases in investments in children.

A.6 Child Mortality and the Allocation of Time

Proposition 3 *A reduction in the child mortality rate increases the amount of time that the woman allocates to household activities.*

Proof. A reduction in the child mortality rate reduces the fertility rate (see derivation in the text). In Figure A.1 from the Appendix, it can be seen that changes in child mortality affect the function g. The reduction in fertility generated by the child mortality reduction shifts the the curve g' upwards, shifting the optimal point III to the left (increasing the amount of time that the woman allocates to the household). ■

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