NASH BARGAINING, ON-THE-JOB SEARCH AND LABOR MARKET EQUILIBRIUM

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The objective of this study is to investigate market equilibrium in a labor market where employed workers can search while employed but only at a cost. The wage paid in any match between a previously unemployed worker and a firm is established by a bilateral Nash bargain. If an employed worker does search while employed, and another firm with a vacancy is contacted, the two firms are assumed to bid for the worker's services. Using such a framework it is shown that even in a labor market where both workers and firms are homogeneous, if search costs are small but strictly positive, then there exists a unique equilibrium where a positive fraction of employees search on-the-job. In such a situation there is a dispersed wage equilibrium.

Apart from Monks and Nuns, and possibly lighthouse keepers, all, at sometime or other, look for a job while employed. Surprisingly, until recently there were few studies on the topic even though a significant percentage of job changes by workers in the US involves no interim unemployment. The small number of studies on this topic possibly reflects the fact that the standard competitive labor market model implies there is no reason for a worker to look for another job while employed. In recent years, however, models of labor markets have been developed where frictions are suitably taken into account. These imply a worker at any moment in time faces only a limited number of job opportunities and these opportunities change through time. In such a framework it is not unreasonable for a worker to accept a job but continue to look for another one while employed. The objective here is to analyze equilibrium in a labor market with frictions where employed workers can, at a cost, search for new job opportunities.

The work on labor market models with frictions can be usefully partitioned into two - those where firms post wages and those that assume a worker and firm bargain over the wage to be paid if employment is accepted. Within the context of wage posting models (see, for example, Burdett and Mortensen (1998), Van den Berg (1999), and Postel-Vinay and Robin (2002)), on-the-job search by employees plays a central role. In these models, however, workers by assumption receive new job offers when employed as well as when unemployed - there is no decision to search, or not, by employees.[1]

The vast majority of studies that assume workers and firms bargain to establish wages do not consider on-the-job search. Recently, however, Shimer (2005) and Cahuc, Postel-Vinay and Robin (2005) have both analyzed an equilibrium model where employees search for new job opportunities within a strategic bargaining environment. In these two studies there is again no on-the-job search choice, - both employed and unemployed workers receive new offers from time to time, by assumption.

The paper perhaps closest to one presented here is by Pissarides (1994). He investigates equilibrium in the context of a labor market with heterogeneous firms. Employed workers at a cost can choose to search. By assumption, workers and firms cannot negotiate long-term contracts in the sense that what is negotiated is not binding when the outside options faced by the parties change. This restriction motivates Pissarides to assume that a firm and worker will utilize a split the surplus bargain. The present study differs from Pissarides study in three ways. First. a simpler labor market is considered where both workers and firms are homogeneous. There is no need to consider heterogeneous firms as it will be shown that search while employed can be an equilibrium outcome when all firms are homogeneous.[2] Second, we assume here that what a firm and worker negotiate is binding in the long-run. To illustrate, assume an employee searches on-the-job and receives an offer from another firm. In response the worker's current employer increases the wage paid. Assume this response is successful and the other firm withdraws forever. In the present study the firm's increased wage offer, unlike Pissarides' study, is binding on the firm. Finally, we assume that when a firm and unemployed worker bargain they select a wage that maximizes the Nash product, i.e., they use what is termed the Nash bargain. As will be shown this does not always imply they split the surplus in equal shares. These differences lead to very different results than those presented by Pissarides. In

[1] Burdett (1978) presents a simple model of the worker's decision to search, or not while employed.

[2] Of course, the generalization to heterogeneous firms is readonably straightforward.

particular, they imply that if search costs are small, then on-the-job search exists in equilibrium even when workers and firms are homogeneous.[3]

There are other studies that have considered costly on-the-job search. First, Burdett, Imai and Wright (2004) have analyzed the search decision within the context of a marriage model where utility is not transferable.[4] . In this setting the decision to look for a new partner while married depends, among other things, on whether the individual's partner is also looking around, or not. Note, in the marriage market setting either party can choose to search while matched. In the labor market context, by assumption, only workers can select to search while matched. Second, Nagypal (2005) has developed and analyzed an on-the-job search model where the where a worker's utility from employment at a particular firm depends on a idiosyncratic element as well as the wage offered. This idiosyncractic element is private information to the worker Hence, even if all firms offer the same wage, those employed workers in a bad match (i.e., a low idiosyncratic term) may elect to search. Thirdly, Moscarini (2005) considers the decision to search, or not, within a model of learning about a worker's ability. In this case a worker who turns out not to be a good match with his/her employer, can elect to search while employed

The market used here works as follows. As stated before, both workers and firms are homogeneous. Suppose that when an unemployed worker and a firm with a vacancy contact each other they bargain over the wage paid if employment is accepted. By assumption, the firm cannot observe employee search behavior and therefore they cannot condition on whether the worker searches or not. It is assumed that the unemployed worker and firm with a vacancy select the wage that maximizes the Nash product. Suppose for a moment that a worker searches while employed. If this worker contacts a firm with a vacancy, the two firms are assumed to bid for the workers services (see Postel-Vinay and Robin (2002b) for a similar approach). As firms are homogeneous, it is clear that either firm is willing to bid up to a wage that

[3] It should not be that difficult to extend the analysis to one where there are heterogeneous workers and/or firms. Such complexities are not attempted, however, so we can focus on explaining the logic genearting the results.

[4] As utility is non-transferable, there is no bargaining - what you see is what you get.

makes the firm indifferent between hiring the worker and posting a vacancy. Let z denote this wage. This wage can be seen as the payoff to search while employed. Given z, the worker's cost of search, and the arrival rate of job offers while employed, it is shown that if a worker hired from unemployment is paid wage w, this worker will search while employed if and only if w is less than a search wage, Q. This search, or not, decision generates a non-convexity in the feasible set of outcomes over which the unemployed worker and firm bargain.

The convexity of the feasible set of alternatives, of course, causes a problem as the Nash axioms only apply to a convex feasible set. The standard way to convexifying the feasible set in bargaining situations is by the use of lotteries. This is the approach used here. Luckily, the lottery is a simple one.

Within this context, it is shown that depending on the cost of search faced by an employee, three types of equilibria can be generated. First, if search costs are large, in equilibrium the Nash bargain implies the worker and firm split the surplus in equal shares and no employee searches while employed. Second, if the cost of search is in a "mid-range", the Nash bargain does not split the surplus in equal shares, but the worker is paid a wage that implies he/she does not search while employed. Finally, if search costs a small but strictly positive, an unemployed worker and firm with a vacancy use a lottery to maximize the Nash product. This lottery implies the wage paid is either a "low" wage and the worker searches while employed, or a "high" wage and the worker does not search while employed.

1 The Model

Assume there is a large fixed number (a continuum) of both workers and firms. We normalize the number of both to one. Time is continuous. When any worker is employed by a firm, the worker generates revenue p per unit of time. Independent of employment status, if a worker pays flow cost c, a firm with a vacancy is contacted at Poisson rate α. Vacancies are not contacted if the worker does not pay c. An unemployed worker obtains b per unit of time.

Each firm employs at most one worker. If a worker is employed at a firm, the partnership breaks up at an exogenous rate δ. If such an event occurs, the firm costlessly posts a vacancy, whereas the worker becomes unemployed. A firm with a vacancy contacts a searching worker at Poisson rate α_f. The probability a worker contacted is unemployed is denoted by π. Throughout we assume the market is in a steady-state and therefore the above aggregates stay constant through time.

If a firm with a vacancy contacts an unemployed worker the wage paid is determined by a Nash bargain. An employee may, or may not, search but this is not observable to the firm. This implies the worker and firm cannot condition on the worker's search behavior. Suppose a firm's employee does search on-the-job. Further, assume this worker contacts another firm with a vacancy. Here, the newly contacted firm and the worker's current employer are assumed to enter a wage bidding competition for the worker services.

All discount the future at rate r. Each worker maximizes expected discounted income, whereas each firm maximizes its expected discounted profit. To reduce descriptions, in what follows we shall say workers and firms maximize their expected payoffs. We now describe the behavior of firms and workers.

1.1 Firms

Given the model described above let V denote the firm's expected payoff when it posts a vacancy. The object here is to specify a firm's expected return when it hires a worker, taking V as given. Let $J(w, 0)$ denote a firm's expected payoff when currently employing a worker at wage w given the worker does not search while employed. It follows

$$rJ(w, 0) = p - w + \delta[V - J(w, 0)]$$

Now let $J(w, 1)$ denote the expected return to a firm that employs a worker at wage w and the worker searches on-the-job. In this case

$$rJ(w, 1) = p - w + \delta[V - J(w, 1)] + \alpha[J_H - J(w, 1)]$$

where J_H denotes the firm's expected payoff after its employee contacts another firm and they bid for the worker's services.

Suppose for a moment that a firm's employee contacts another firm with a vacancy. In this case by assumption the two firms bid for the worker's services. As firms are homogeneous each firm is willing to bid a maximum wage, z, such that if the worker accepts the firm's expected return is the same as if the firm posts a vacancy, i.e., $J_H = V$. Without loss of generality, assume the worker stays at his current employer. Further, as no firm is willing to pay a worker more than this, at wage z, the employee does not search, i.e., $J_H = V = J(z, 0)$.This implies

$$J(w,0) = \frac{p - w + \delta V}{r + \delta} \tag{1}$$

and

$$J(w,1) = \frac{p - w + (\delta + \alpha)V}{r + \delta + \alpha} \tag{2}$$

It follows immediately that $J(z, 1) = J(z, 0) = V$ and

$$z = p - rV \tag{3}$$

1.2 Workers

Let U_0 denote a unemployed worker's expected lifetime discounted income. Suppose for the moment a worker is employed at wage w and does not search on-the-job. This worker's expected return in this case, $U(w, 0)$, can be written as

$$rU(w,0) = w + \delta[U_0 - U(w,0)] \tag{4}$$

Let $U(w, 1)$ denote a worker's expected payoff when employed at wage w and searching. It follows

$$rU(w,1) = w + \delta[U_0 - U(w,1)] + \alpha[U(z,0) - U(w,1)] - c \tag{5}$$

where $U(z, 0)$ denotes the worker's expected return after the worker has contacted another firm and the two firms have bid for his/her services. Manipulation establishes that

$$U(w,0) = \frac{w + \delta U_0}{r + \delta} \tag{6}$$

and

$$U(w,1) = \frac{w(r+\delta)+(r+\delta+r)\delta U_0+\alpha z-c(r+\delta)}{(r+\delta+\alpha)(\delta+r)} \tag{7}$$

We are now in a position to define two reservation wages; R_0 and R_1. In particular,

$$U(w,0) \gtrless U_0 \text{ as } w \gtrless R_0$$

and

$$U(w,1) \gtrless U_0 \text{ as } w \gtrless R_1$$

Hence, at any wage greater than R_0 (R_1), a worker strictly prefers to work and not search (search) than remain unemployed. Without any real loss of generality we assume at any wage that makes a worker indifferent between employment and unemployment, the worker accepts employment. From (5) and (6) it follows that

$$R_0 = rU_0$$

and

$$R_1 = \frac{(\alpha+\delta+r)rU_0-\alpha[z-c(r+\delta)/\alpha]}{(r+\delta)} \tag{8}$$

We are now in a position to specify Q - the search wage of a worker. This is the wage that makes the worker indifferent between searching while employed and not searching while employed. For any fixed U_0, it follows from (6) and (7) that $U(w,1)$ $\gtrless U(w,0)$ if and only if $w \lesseqgtr Q$, where

$$Q = p - rV - \frac{c(r+\delta)}{\alpha}$$

What is the relationship between R_0, R_1, and Q? The first Claim establishes the relevant results.

Claim 1

(a) If $c < c_0$, then $R_1 < R_0 < Q$.

(b) If $c > c_0$, then $Q < R_0 < R_1$.

(c) If $c = c_0$, then $R_0 = R_1 = Q$.

where

$$c_0 = \frac{\alpha[p-r(V+U_0)]}{(r+\delta)} \tag{9}$$

Proof

A little math establishes that $\partial U(w,0)/\partial w > \partial U(w,1)/\partial w > 0$. The results now follow and are illustrated in Figures 1 and 2. This completes the proof.

Inspection of Figure 1 establishes that if $c < c_0$, then R_0 is irrelevant as a worker strictly prefers to search while employed if offered wage R_0. The above Claim leads to a complete description of an unemployed worker's strategy.

If $c < c_0$ and wage w is offered to an unemployed worker, then

(a) $w < R_1$ implies the worker prefers to remain unemployed,

(b) $Q > w \geq R_1$ implies the worker prefers employment and search on-the-job, and

(c) $w > Q$ implies the worker prefers employment and no search on-the-job.

Note, at wage $w = Q$ the worker is indifferent between employment and not searching and employment and searching on-the-job. To simplify the analysis, and without any real loss of generality, we assume a worker in this situation does as told by the firm. As shown below, the firm will always prefer to tell the worker not to search at $w = Q$.

If $c > c_0$, then inspection of Figure 2 establishes that R_1 and Q are irrelevant in this case. In particular, if $c > c_0$ and wage w is offered to an unemployed worker, then

(a$'$) $w < R_0$ implies the worker prefers unemployment, and

(b$'$) $w \geq R_0$ implies the worker prefers employment and not to search on-the-job.

2 Feasible Sharing Arrangements

In this section we study the feasible sharing arrangements by varying the wage paid by the firm. As is standard, we call the frontier of the set of feasible sharing arrangements in the positive orthant the Nash Frontier. Suppose wage w is paid and the worker does not search on-the-job. In this case the surplus going to the worker, $S_e(w,0)$, and surplus to the firm, $S_f(w,0)$, can be written as

$$S_e(w,0) = U(w,0) - U_0 = \frac{w - rU_0}{r+\delta}$$

and

$$S_f(w,0) = J(w,0) - V = \frac{p - w - rV}{r+\delta}$$

The total surplus, S_0, is

$$S_0 = S_f(w,0) + S_e(w,0) = \frac{p - r(V + U_0)}{r+\delta} \tag{10}$$

for any w where the worker does not search on-the-job. Each point on the frontier shows how much of this surplus goes to the worker and how much to the firm. For example, at wage $w = z$, where z is defined in (3), $S_f(z,0) = 0$ and the worker receives all the surplus created by the match.

In what follows we assume that V and U_0 are such that $S_0 > 0$. It was shown above that if $c > c_0$,then no worker searches while employed and therefore S_0 describes the surplus generated by the match. The situation is not so straightforward when $c \leq c_{0.}$.

Given $c \leq c_0$, for the worker not to search while employed we require $w \geq Q$. Suppose the wage offered is w where $R_1 \leq w < Q$. In this case an employee will search on-the-job. Let $S_e(w,1)$ and $S_f(w,1)$ denote the surplus going to the employee and firm respectively. After some substitution it follows

$$S_e(w,1) = U(w,1) - U_0 = \frac{w(r+\delta) - rU_0 + \alpha Q}{(r+\delta+\alpha)(\delta+r)} \tag{11}$$

and

$$S_f(w,1) = J(w,1) - V = \frac{p - w - rV}{r+\delta+\alpha}$$

The total surplus generated when the worker searches on-the-job, $S_1(c)$, can be written as

$$S_1(c) = S_0 - \frac{c}{(r+\delta+\alpha)} \tag{12}$$

It is straightforward to check that that $S_1(c) > 0$ if $c \leq c_0$.

Suppose the cost of search is such that $c < c_0$. In this case $R_1 < Q < z$. Figure 3 illustrates the situation where S_e and S_f is the surplus going to each party. As all the surplus goes to the worker when wage z is paid $S_0 = S_e(z,0)$. Similarly, as all the surplus goes to the firm when wage R_1 is paid, $S_1(c) = S_f(R_1,1)$. For wage, w, such that $Q < w \leq z$, the worker does not search on-the-job, and therefore surplus

S_0 is generated by the match. If $R_1 \leq w < Q$, however, the worker selects to search and hence total surplus $S_1(c)$ is generated. This implies the feasible set, given only the search, or not search options are considered, is given by the area contained in ABCDE. in Figure 3. Clearly, this set is not convex.

As stated before, to make the feasible set convex we use a lottery. In particular, suppose with probability ρ wage $w = R_1$ is paid, and with probability $(1-\rho)$ wage $w = Q$ is paid. If wage R_1 is paid the worker will search on-the-job, whereas the worker is told not to search if wage Q is used. Note, the firm prefers the bargain where the worker is paid R_1 and searches, whereas the worker prefers the bargain where wage Q is paid. Given the lottery is used with mixing probability ρ, the surplus going to the worker can be written as

$$S_e(\rho) = (1-\rho)S_e(Q, 0)$$

and the firm's surplus can be written as

$$S_f(\rho) = \rho S_f(R_1, 1) + (1-\rho)S_f(Q, 0)$$

The total surplus, $S(\rho)$ can be expressed as

$$\begin{aligned} S(\rho) &= (1-\rho)S_1 + \rho S_0 \\ &= S_0 - \rho c \end{aligned}$$

Using this construction it is possible to construct the desired convex feasible set. In particular, in Figure 3 when the lottery option is added, the feasible set become the convex area ACDE.

3 Bargaining

Suppose for the moment that the firm and worker bargain on the assumption the worker will not search on-the-job. The Nash bargain in this case satisfies the following program

$$\arg\max_{w} S_e(w, 0) S_f(w, 0)$$

on the feasible set whose frontier was described above. As $\partial S_e(w,0)/\partial w = -\partial S_e(w,0)/\partial w$,it follows that w_s solves the above program where this wage splits the surplus (STS) in that $S_e(w_s,0) = S_f(w_s,0)$. This implies

$$w_{ns} = \frac{1}{2}[p + r(U_0 - V)] \tag{13}$$

Indeed, if $\partial S_e(w,0)/\partial w = -\partial S_e(w,0)/\partial w$, and no other constraints are taken into account, then the Nash bargain is always a STS bargain. If $c > c_0$, we have shown employees do not search on-the-job and therefore the above bargain is the one used. This is illustrated Figure 4. Indeed, this is the standard bargaining case much considered in the literature.

Assume now that $c < c_0$. In this case an employee will search if the wage paid is less than the search wage, Q. Hence, the STS bargain is feasible if $w_{ns} > Q$. Manipulation, establishes $w_s > Q$ if and only if $c > c_1$ where

$$c_1 = \frac{\alpha S_0}{2} < c_0 = \alpha S_0 \tag{14}$$

This implies the Nash bargain is a STS bargain if $c_0 > c \geq c_1$.

To understand this result in a slightly different way, note $S_e(Q,1)$ can be written as

$$S_e(Q,1) = S_0 - \frac{c}{\alpha}.$$

It follows that $S_e(Q,1) = S_e(R_1,1)$,when $c = c_0$. Further, $S_e(Q,1)$ increases as c decreases when $0 < c < c_0$. It now follows from inspection of Figure 5 that as long as $S_e(Q,1) \leq S_0/2$, the unique Nash bargain is a STS bargain illustrated by (S_e^*, S_f^*). It is simple to show $S_e(Q,1) \leq S_0/2$ if and only if $c \geq c_1$.

When $c < c_1$(i.e., when $S_e(Q,1) < S_0/2$) the STS bargain is not feasible. In this case the Nash bargain is either a constrained Nash (CN) bargain (where wage Q is paid and the worker is told not to search), or a lottery, where the worker is paid Q with probability $(1-\rho)$ (and the worker told not to search, or and R_1 with probability ρ. Without loss of generality define the Nash surplus in this case as:

$$N(\rho) = S_e(\rho)S_f(\rho) = (1-\rho)[S_w(Q,0)][\rho S_1 + (1-\rho)S_f(Q,0)]$$

as $S_f(R_1,1) = S_1(c)$. Note, if maximization implies ρ is chosen, where $0 < \rho < 1$, then the lottery is best.. If maximization implies $\rho = 0$, then a CN bargain is best.[5]. The next result summarizes the results.

Claim 3

Given U_0 and V fixed such that $p - r(U_0 + V) > 0$:

(a) If c is such that $c_2 \leq c < c_1$, a CN bargain maximizes the Nash product, .where c_2 is defined by

$$c_2 = \frac{\alpha S_0}{2}\frac{(r+\delta+\alpha)}{(\delta+r+(3/2)\alpha)} < c_1 \tag{15}$$

(b) If $0 < c < c_2$, then the lottery with mixing probability ρ^* maximizes the Nash product where $0 < \rho^* < 1/2$,where

$$\rho^* = \frac{S_1(c) - 2S_f(Q,0)}{2[S_1(c) - S_f(Q,0)]}$$

Proof

See Appendix.

Note, when $c = c_2$, then $S_e(Q,1) = S_k$,where

$$S_k = \frac{S_0}{2}\frac{(\delta+r+2\alpha)}{\alpha(\delta+r+(3/2)\alpha)} > \frac{S_0}{2}$$

We know already that if $c < c_1$, then $S_e(Q,1) > S_0/2$ and the STS bargain is not feasible. If $c_2 < c < c_1$, then $S_e(Q,1) < S_k$, and the bargain that maximizes the Nash product is a CN bargain where the worker is paid Q and told not to search. This is illustrated in Figure 6. Suppose a worker and firm use a STS bargain when $c_2 < c < c_1$. I the firm and worker use a STS bargain, then the worker will receive a wage less then the worker will search while employed and the surplus from the match is $S_1(c)$ which is significantly less than S_0. A greater Nash product can be obtained by paying Q (and telling the worker not to search). This bargain implies the firm gets less than half the surplus S_0 but this preferred to obtaining the surplus $S_1(c)$

When the cost of search, is in the range $0 < c < c_2$, then $S_e(Q,1) > S_k$, and therefore the lottery bargain maximizes the Nash product. Such a bargain is illustrated in

[5] As will be shown later, $\rho = 1$ is never optimal.

Figure 7. In this case a worker is paid either R_1,or Q (and told not to search). Those paid R_1 search, those paid Q don't. Paying the worker Q and telling the worker not to search becomes more costly to the firm as the cost of search decreases. For $c < c_2$ it is too costly and the lottery bargain yields a higher Nash product.

4 Market Equilibrium

In the previous Section it was established that given U_0 and V, there are three possible bargaining outcomes depending on the cost of search:

A: If $c > c_1$, workers and firm use a STS bargain

B: If $c_2 < c < c_1$, workers and firms use a CN bargain.

C: If $c < c_2$,workers and firms use the lottery bargain.

The objective here is to demonstrate when each of the above bargains is an element of a market equilibrium. Intuitively, a market equilibrium can be described as follows. Suppose the market is in steady-state where the market aggregates remain constant through time. Suppose, for example, given U_0 and V, assume $c > c_1$,and therefore a STS bargain is used. This implies the wages paid can be calculated and therefore (given an encounter function is specified) it is possible to calculate the expected return to an unemployed worker, $\widetilde{U}_0$ and the expected return to a firm posting a vacancy, $\widetilde{V}$, in a steady-state A market equilibrium exists if $U_0 = \widetilde{U}_0 > 0$ and $V = \widetilde{V} > 0$, i.e., expectations are fulfilled and both prefer to participate. A market equilibrium can be defined in the same way for the other two types of bargains.

To make progress we first need to specify a steady-state in the market described above. To achieve this goal we first define an encounter function. This specifies the number of encounters (e) per unit of time between number of searching workers (s) and the firms with a vacancy (v), i.e., $e = e(s, v)$. To simplify the math, assume $e = sv$. The simplicity generated by using a quadratic encounter function can now be stated.[6] The number of searchers who make contact with a firm per unit of time,

[6] Essentially the same results follow from assuming the encounter function has constant returns to scale. The math, however, is more complicated.

αs, equals the number of encounters between workers and firms, sv, which, in turn, equals the number of firms that contact a worker, $\alpha_f v$. Hence, under all circumstances $\alpha = v$ and $\alpha_f = s$. By construction, however, the number of vacancies must equal the number unemployed and therefore $\alpha = u = v$. Firms with vacancies always hire unemployed on contact and therefore any steady-state $u\alpha = (1-u)\delta$. This implies the steady-state number unemployed is always $u = \delta/(\alpha+\delta)$. As $\alpha = u$, the steady-state number of unemployed (and therefore vacancies) can be written as

$$\alpha = v = u = \frac{\sqrt{\delta}\sqrt{(\delta+4)} - \delta}{2} \tag{16}$$

Hence, given the market is in a steady-state, the number of unemployed, the number of vacancies, and the Poisson arrival rate of offers faced by workers who search are the same and can be written as a function of the job destruction rate.

All that needs to be specified now is the steady-state arrival of workers faced by firms. Assume a fraction γ ($\gamma \geq$) of employees search on-the-job. Then $s = u+\gamma(1-u)$ is the number of workers who search. Hence, $\pi = u/[u+\gamma(1-u)]$ denotes the fraction of searchers who are unemployed. This implies $\alpha_f \pi = u = \alpha$. Hence, independent of the bargain obtained, the arrival rate of offers faced by searching workers equals the arrival rate of unemployed workers faced by firms.

Assuming that one of three bargaining outcomes occurs and the market is in a steady-sate, an unemployed worker's expected return can be written as

$$rU_0 = \begin{cases} b - c + \alpha[U(w_{ns},0) - U_0], \text{ if all use } STS \text{ bargain} \\ b - c + \alpha[U(Q,0) - U_0], \text{ if all use a } CN \text{ bargain} \\ b - c + \alpha\rho[U(R_1,1) - U_0] + \alpha(1-\rho)[U(Q,0) - U_0], \text{ if all use a lottery} \\ \text{bargain.} \end{cases}$$

Further, the expected return to a firm with a vacancy can be written as

$$rV = \begin{cases} \alpha_f[J(w,0) - V], \text{ if all use } STS \text{ bargain} \\ \alpha_f[J(Q,0) - V], \text{ if all use a } CN \text{ bargain} \\ \alpha_f\pi\rho[J(R_1,0) - V] + \alpha_f(1-\rho)\pi[J(Q,0) - V], \text{ if all use the lottery} \\ \text{bargain} \end{cases}$$

The conditions required for all three types of equilibria are specified in the final Claim. The proof of the Claim follows from straightforward algebra and is relegated to an Appendix.

Claim 4

(a) If $k_1 \leq c < k_0$, where

$$k_0 = \frac{\alpha(b+p) + 2b(r+\delta)}{\alpha + 2(r+\delta)} \text{ and } k_1 = \frac{(p-b)\alpha}{2(r+\delta)+\alpha},$$

then there is a unique equilibrium where workers and firms reach a STS bargain. No employed workers searches and the wage paid is $w = (p+b-c)/2$

(b) If $k_2 \leq c < k_1$, where

$$k_2 = \frac{\alpha(p-b)}{2(r+\alpha+\delta)},$$

then there exists a unique equilibrium where workers and firms reach a CN bargain. The wage paid is $w = Q = (p\alpha - c(r+\delta+\alpha))/\alpha$

(c) If $0 < c < k_2$, then there exists a unique equilibrium where workers and firms use a lottery. In this case those employees who are paid R_1 search, whereas employees paid Q or z do not search.

Proof.

See Appendix.

Note when $c < k_2$the market equilibrium involves three wages R_1, Q,and z. If an unemployed worker contacts a firm, then the lottery bargain implies the worker will be hired either at wage R_1,or Q. If the worker is paid Q, then the worker is told not to search and this becomes the wage received by this worker until the job is destroyed. Those paid R_1, however, search. If another firm is contacted, this worker's wage is increased to z.

5 Discussion

Above we have shown that if search costs are small enough then the unique equilibrium is where some workers search while employed and there are three wages in the market even though firms and workers are homogeneous. Note, unlike the model analyzed by Pissarides (1994), we assumed that bargains are binding. Thus, when a firm offers to pay an employee the high wage z, in response to the worker receiving an offer from another firm, the firm cannot renege later. In the Pissarides model wages are not binding in this sense, and hence there is little purpose in the two firms bidding for a worker as when one of the firms leaves the threat has gone and they are assumed to revert to a STS bargain. It should also be noted that STS bargains yield a smaller Nash product than the CN bargain if the costs of search are such that $k_3 < c < k_2$.[7]

For ease of exposition we have used a market model slightly different than the one used in the Mortensen and Pissarides model (1994). It is, however, simple to apply the bargaining model described here into such a framework. This generates an interesting alternative model to the one they present.

To illustrate the results established above we assign values to the parameters. In particular, assume $r = 0.1$, $\delta = 0.1$, $p = 50$, and $b = 15$. Hence, from (16) it follows $\alpha = 0.2701562119$. Further, making the relevant substitutions it follows that

$$k_1 = 35.15,\ k_2 = 14.10,\ \text{and } k_3 = 10.05$$

The resulting bargains reached as the cost of search is varied is illustrated in Figure 8. Notice that $R_1 < b$. Unemployed workers accept a wage less than b if they will search on-the-job, as working and searching is the only way to obtain the highest wage $z.-$ its a "foot in the door" effect.

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Appendix

Proof of Claim 3

Taking the derivative of the Nash product implies

$$N'(\rho) = S_e(Q,0)[(1-2\rho)S_1 - 2(1-\rho)S_f(Q,0)]$$

Further,

$$N"(\rho) = -2S_e(Q,0)[S_1 - S_f(Q,0)]$$

As $S_1 = S_f(R_1,1)$, and $N(.)$ is a concave function if $S_f(R_1,1) - S_f(Q,0) > 0$. It is straightforward to show $c < c_1$ implies $S_f(R_1,1) > S_f(Q,0)$ and therefore $N(.)$is concave. Further,

$$\begin{aligned} lim_{\rho\to 0}N'(\rho) &= S_e(Q,0)[S_f(R_1,1) - 2S_f(Q,0)], \text{ and} \\ lim_{\rho\to 1/2}N'(\rho) &= -S_e(Q,0)S_f(Q,1) \end{aligned}$$

These results imply the Nash product $N(.)$ reaches an interior maximum at ρ^* ($0 < \rho^* < 1/2$), if $S_1 > 2S_f(Q,0)$. It is now simple to establish $S_1 > 2S_f(Q,0)$ if and only if $c < c_2$, where c_2 is defined in the Claim. Given ρ^* is interior, i.e., $0 < \rho^* < 1$, it follows and therefore

$$\frac{d\rho^*}{dc} = -\frac{[\frac{(1-2\rho^*)}{(r+\delta+\alpha)} + 2(1-\rho^*)/\alpha]}{[S_1 - S_f(Q,0)]2} < 0$$

This completes the proof.

Proof of Claim 4

Suppose all workers and firms reach a STS bargain. From (**??**) and (**??**) it follows that the steady-state expected return to an unemployed worker, U_0, and the expected return to a firm posting a vacancy, V, are given

$$U_0 = \frac{b + \alpha U(w,0) - c}{r+\alpha}$$

and

$$V = \frac{\alpha J(w,0)}{r+\alpha}$$

However, $U(w,0)$ and $J(w,0)$, when the STS wage is paid can be written as

$$U(w_{ns},0) = \frac{p + U_0(2\delta + r) - rV}{2(r+\delta)}$$

$$J(w_{ns},0) = \frac{p + V(2\delta + r) - rU_0}{2(r+\delta)}$$

At a market equilibrium V and U_0 must satisfy the above four equations. i.e.,

$$V = \frac{(p+c-b)}{2r(\alpha+\delta+r)}$$

$$U_0 = \frac{\alpha p + (b-c)(\alpha + 2(\delta + r))}{2r(\alpha+\delta+r)}$$

Using these equations it follows $U_0 \geq 0$ if and only $c \leq k_0$, where k_0 is defined in the Claim. Further, it can be established that $c \geq c_1$ if and only if $c \geq k_1$, where $k_1$is also defined in the Claim. This establishes (a).

Proceeding in the same way as above it follows that if all use a CN bargain, then at a market equilibrium

$$U_0 = \frac{p\alpha - c(2r + \alpha + 2\delta) + b(r+\delta)}{r(r+\alpha+\delta)}$$

$$V = \frac{c}{r}$$

and the wage is given by (**??**). For a CN bargain to be the chosen we use the above to check that $c_1 > c \geq c_2$ It follows that that in a market equilibrium $c_3 = k_3$, where k_3 is defined in the Claim.

Given all use the lottery bargain, it is possible to show

$$V = \frac{c(\phi^2(1-\rho) + \alpha^2\rho^2) + \alpha\rho\phi(p-b)}{\phi^2 r}$$

$$U_0 = \frac{\phi(c(r+\delta) + \alpha(b-p)) + c\rho\alpha^2(1-\rho) - \phi(c(2r+2\delta+\alpha) - b(r+\delta) - \alpha p)}{\phi^2 r}$$

where $\phi = (\delta + \alpha + r)$ and $\rho = \rho^*$. It is now straightforward to establish (c). For completeness, the three wages used are presented below:

$$z = \frac{c[\phi^2(\rho-1) - \alpha^2\rho^2] + \phi[p((1-\rho)\alpha + r + \delta) + \alpha b\rho]}{\phi^2}$$

$$Q = \frac{c[\phi^2(\alpha(\rho-1)-r-\delta)-\alpha^3\rho^2]-\alpha\phi[p(\alpha(\rho-1)-r-\delta)+\alpha\rho b]}{\alpha\phi^2}$$

$$R_1 = \frac{\phi[b(\alpha(1+\rho)+r+\delta)-\alpha p\rho]-c[\phi(\alpha+(1-\rho)(r+\delta))+\alpha^2\rho^2]}{\phi^2}$$

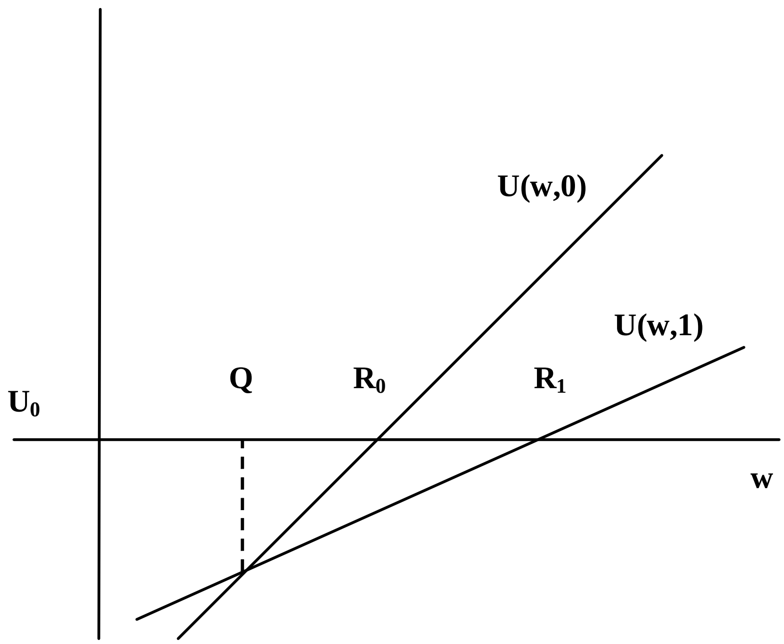

Figure 1: $c > c_0$

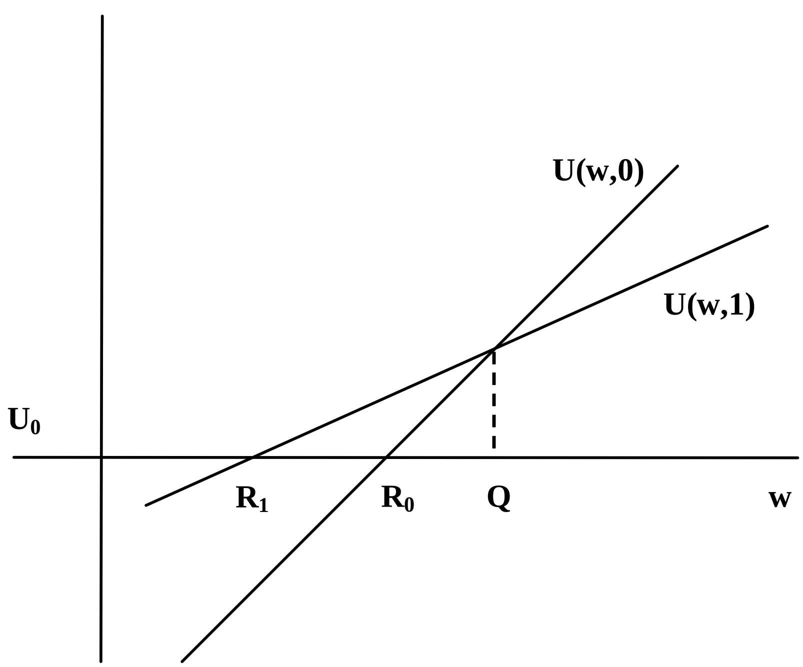

Figure 2: $c \leq c_0$

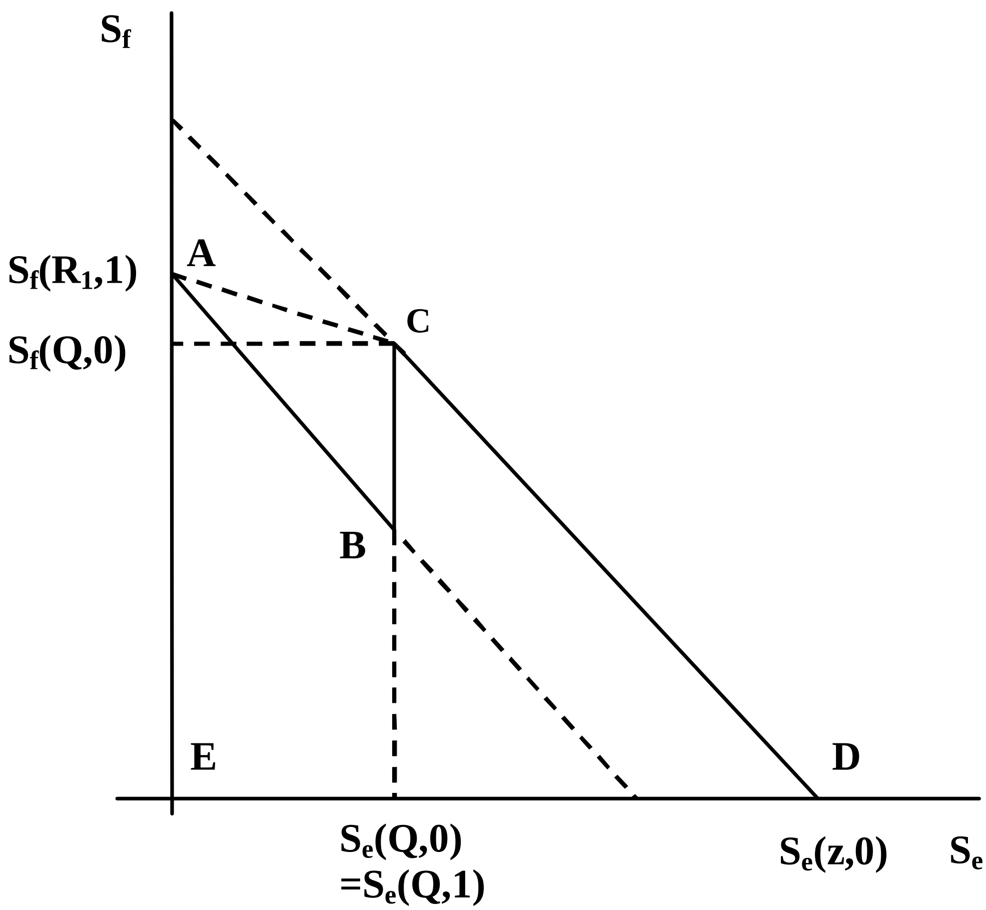

Figure 3: The Feasible Bargaining Set

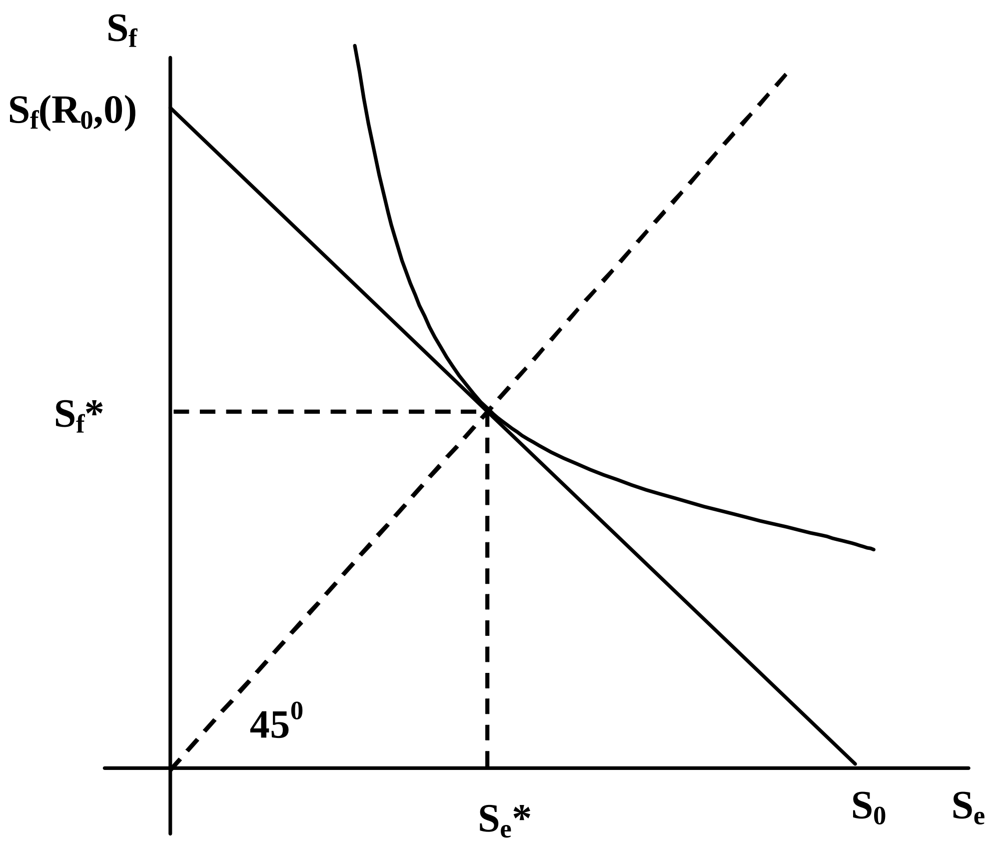

Figure 4: Nash Bargaining with $c > c_0$

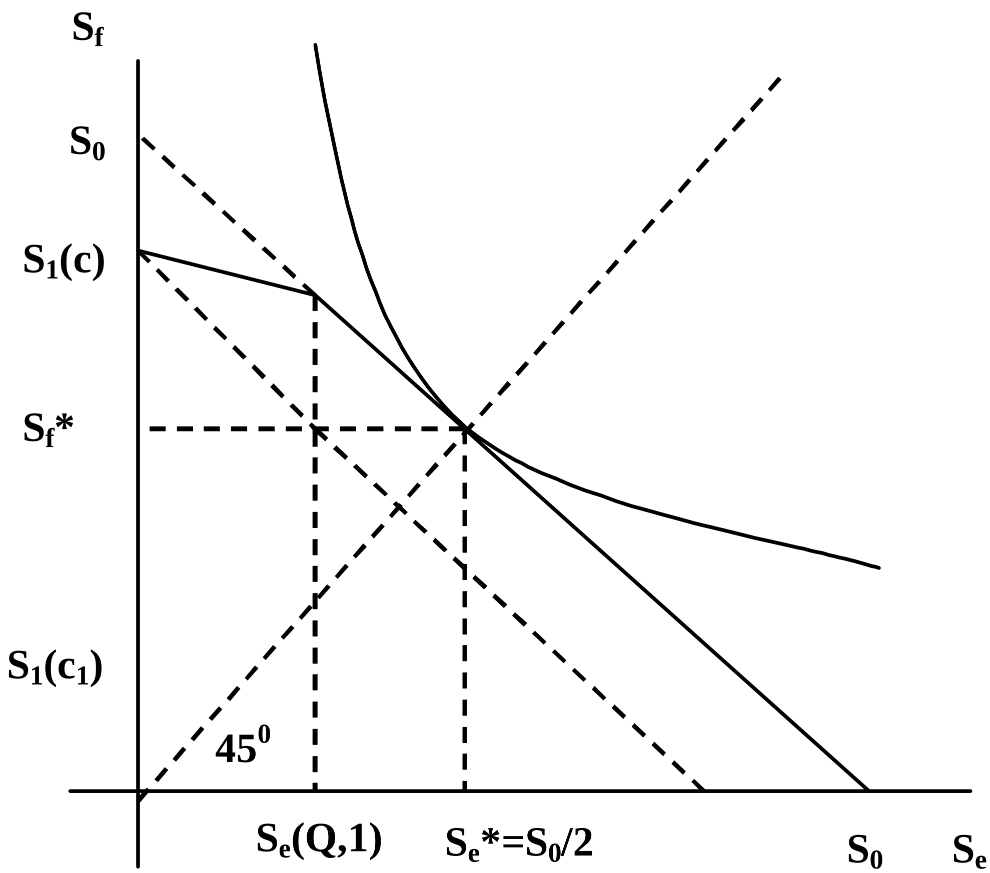

Figure 5: Nash Bargaing when $c_1 < c < c_0$

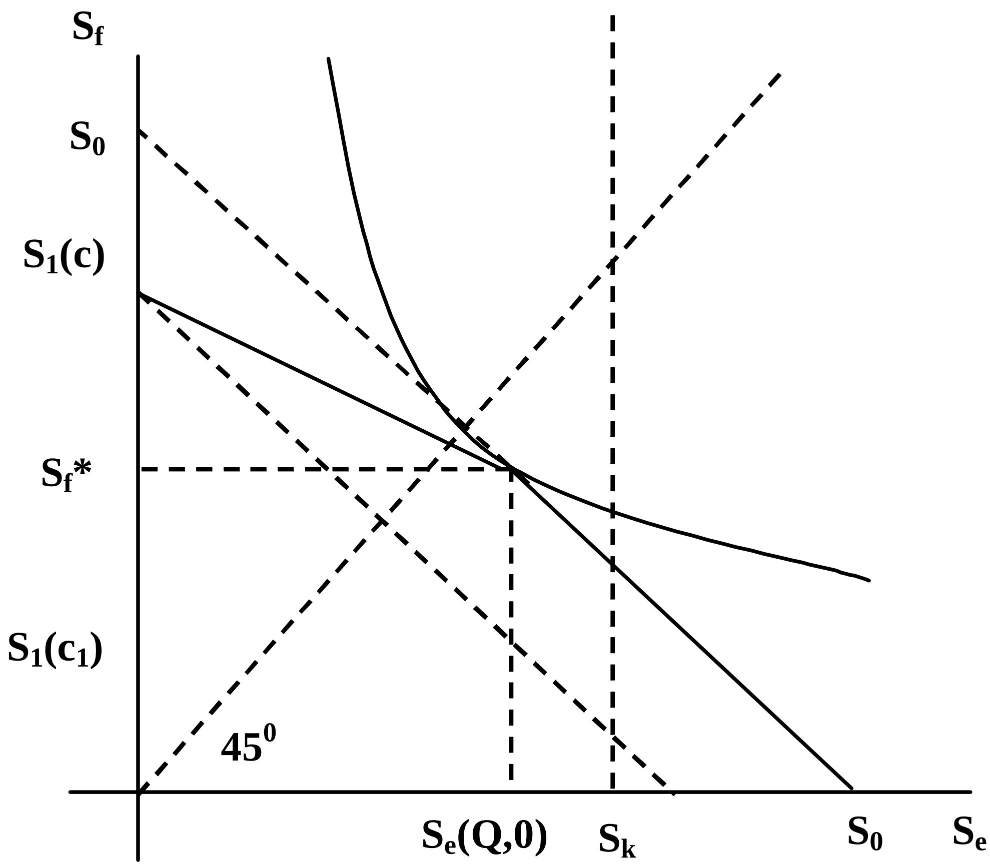

Figure 6: The CN Bargain

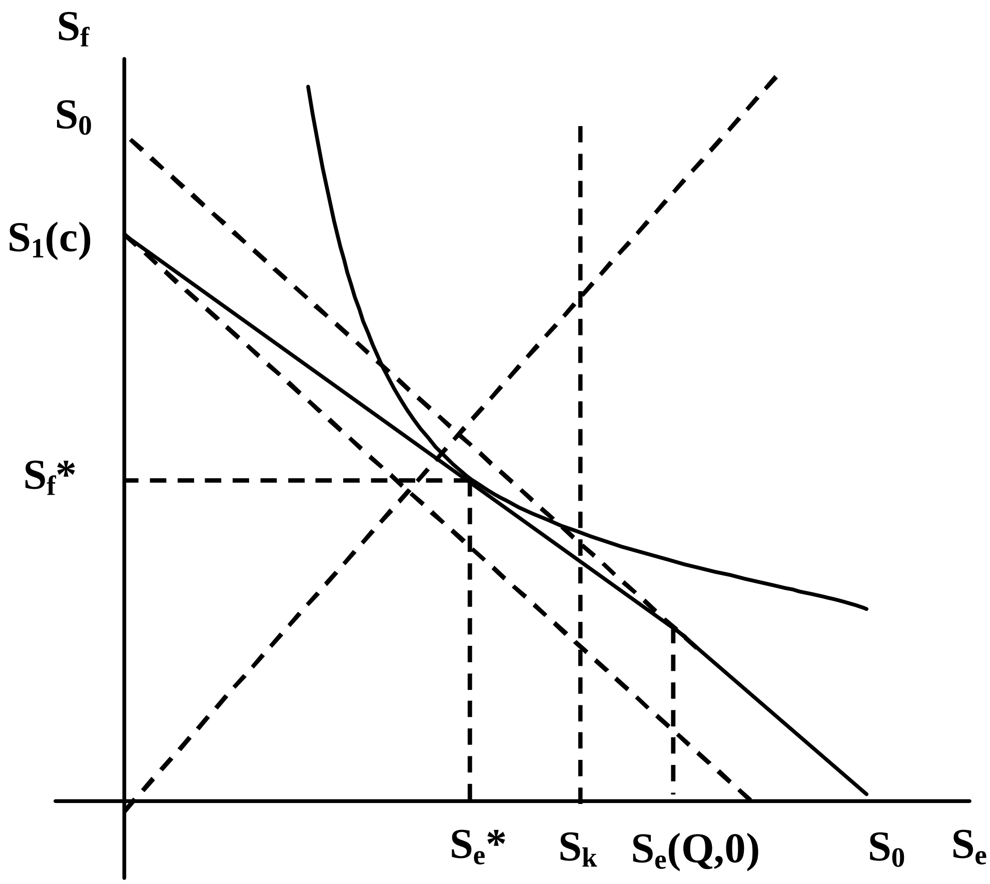

Figure 7: The Lottery Bargain

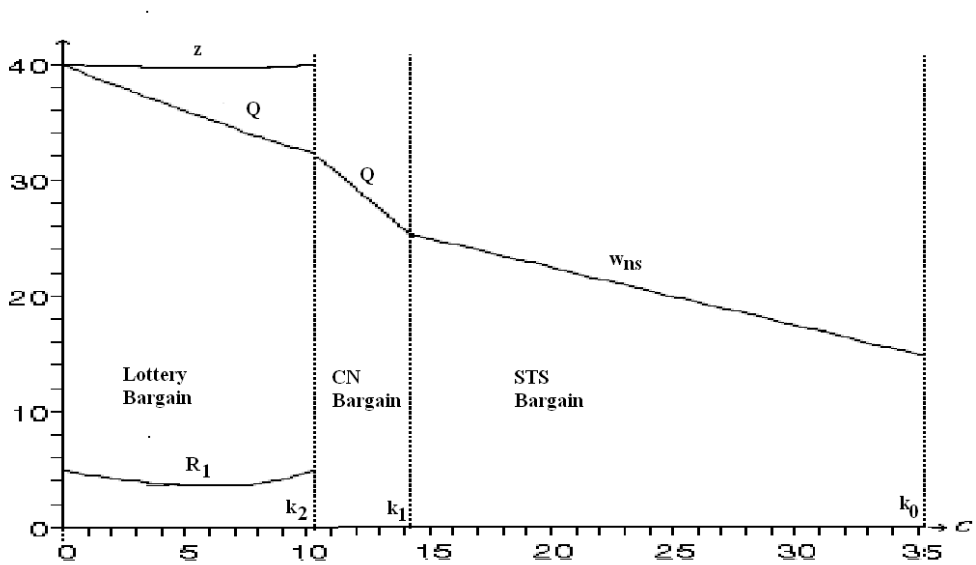

Figure 8: Market Equilibrium and Wages Paid