Optimal Unemployment Insurance with Hidden Search Effort and Endogenous Savings*

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Abstract

After first reviewing recent results on optimal unemployment insurance (UI) with unobserved search effort and hidden savings, this paper identifies that lump sum layoff payments play an important role. Simulations find that co-ordinating constant UI paid to the unemployed, with a severance payment that fully compensates for the drop in permanent income by being laid-off yields payoffs which are surprisingly close to the full information benchmark. By improving the value of employment, severance payments improve search incentives through re-entitlement effects. Thus severance payments are employment enhancing, though for reasons which are quite distinct from those given in the firing cost literature.

Keywords: Unemployment Insurance, Hidden Search Effort, Hidden Savings, Severance payments.

JEL Classification: J3, J6.

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1 Introduction.

Following seminal work by Shavell and Weiss (1979) on optimal unemployment insurance (UI) with hidden job search effort, a large literature has asked whether UI payments should stop after 6 months unemployment (as in the U.S.), one year (as in the U.K.) or be paid indefinitely (e.g. Davidson and Woodbury (1997), Millard and Mortensen (1997), Fredriksson and Holmlund (2001), Cahuc and Lehmann (2002), Coles (2006)). Recent work, however, extends the analysis to allow (hidden) savings (e.g. Werning (2002), Kocherlakota (2004), Lentz and Tranaes (2005)). Introducing savings into the analysis is important not only because employed workers can use a savings strategy to self-insure against layoff risk, but also because ruling out savings by assumption leads to distorted policy prescriptions. Indeed Werning (2002) argues that when workers can save:

- "optimal unemployment benefits are not necessarily decreasing, and, in fact, are typically increasing with unemployment durations."

This suggests UI payments should be backloaded with duration, rather than frontloaded. In contrast, I show UI payments should instead be frontloaded **even more** than is suggested by Shavell and Weiss (1979): the laid-off worker is given a lump sum severance payment and continuation payments $b(.)$ are set low thereafter.

The literature on optimal UI with hidden search effort and hidden savings is complex. With no savings, Hopenhayn and Nicolini (1997) show that setting income tax premia (on re-employment) which depend on the length of the completed unemployment spell can generate large welfare gains. Their proposed program is not unlike a loans program, where benefits received while unemployed are repaid though higher taxes when re-employed. But in the more realistic case that workers can save, and if in addition there are perfect capital markets (i.e. no liquidity constraints) then one can normalise the Hopenhayn/Nicolini tax premia to zero in the optimal policy (e.g. Fudenberg et al (1990), Werning (2002)). In this no liquidity constraints case, Werning (2002) further argues that UI payments should increase with duration.[1] Kocherlakota (2004) instead assumes liquidity constraints and for a special

[1] Werning (2002) allows liquidity constraints in the description of the model but for the most part (from page 10 onwards) assumes this constraint never binds on the optimal program

case - linear search costs - argues that the Planner pays constant UI during the unemployment spell, and a re-employment bonus which does not depend on the length of the unemployment spell. Unfortunately to derive this result Kocherlakota (2004) makes a possibly counterfactual assumption which I discuss fully in the text. The underlying difficulty with this literature is that the principal's programming problem is not concave and so the first order approach as developed in Werning (2002) is not valid (see Kocherlakota (2004) for a full critique).

This paper instead builds on the analytic insights obtained in Coles (2006), using numerical examples to demonstrate the argument. Coles (2006) characterises optimal unemployment policy in a standard Pissarides (2000) matching equilibrium with hidden search effort but where workers cannot save. An important finding there is that the first UI payment received when laid-off, denoted $b(0)$, equals the wage w; i.e. optimal unemployment insurance implies consumption is smooth across the job destruction shock.[2] Shavell and Weiss (1979) do not identify this result as the UI budget there is exogenous. Indeed this is the case with much of the insurance literature - the budget level is not determined optimally, the focus instead is on how UI payments vary within the spell. But optimal unemployment insurance not only smooths consumption within the unemployment spell, it also smooths consumption across job destruction shocks and re-employment shocks.

A useful perspective then is that unemployment risk has two separate components. First being laid-off implies a drop in permanent income and the employed worker would like to buy insurance against layoff risk. Second there is re-employment risk - finding a job implies an increase in permanent income and re-employment is also a stochastic process. Thus the risk averse worker would like to purchase insurance against both types of risk. The central insight here is that benefits received while unemployed, $b(.)$, insure unemployed job seekers against re-employment risk, while a severance layoff payment B_0 insures employed worker's against the drop in permanent income through being laid-off. If the layoff payment B_0 fully compensates for the drop in permanent income by being laid-off, then an optimal dissavings strategy while unemployed not only smooths consumption across the unemployment spell, it also smooths consumption across the job destruction shock.[3] Of course the optimal UI

[2] this result requires additively separable preferences

[3] also see Abdulkadiroglu et al (2002)

program co-ordinates B_0 and $b(.)$ to maximise worker welfare given worker search and savings strategies.

Augmenting the UI program with a lump sum severance payment which fully compensates for the drop in permanent income is welfare improving for three reasons:

(i) employed workers are fully insured against the drop in permanent income when laid-off through a job destruction shock;

(ii) re-entitlement effects imply unemployed workers have improved search incentives and;

(iii) the employed have weaker incentives to over-accumulate assets and so more likely to search actively for work when laid-off.

Point (iii) is related to Kocherlakota's explanation for why the Planner's programming problem is not concave: unemployed workers have the incentive to underconsume and, by saving some of their early UI payments for later consumption, choose lower search effort. An important insight here is that employed workers have the same incentive: the precautionary savings motive while employed implies they over-accumulate savings and choose too little search effort when laid-off. Somewhat surprisingly, the lump sum severance payment reduces this incentive so much that the asset over-accumulation problem largely disappears. The simulations find that co-ordinating policy choices $b(.)$ and B_0 optimally yields welfare payoffs which are surprisingly close to the full information benchmark

There are several related literatures. A different optimal UI approach assumes instead that search effort is exogenously fixed but job offers are not observed. Thus a worker might reject a low wage offer and continue search. But with no UI, workers might have too low reservation wages and it is then efficient to subsidise search. Papers in this literature include Mortensen (1977), van den Berg (1990), Mortensen and Pissarides (1999), Marimon and Zilibotti (1999), Shimer and Werning (2005).[4] Some papers instead have considered how, with unobserved job offers, a duration dependent UI program distorts wages either with wage posting by firms (Albrecht and Vroman (2005)) or when wages are determined by strategic bargaining and UI payments raise the option value of remaining unemployed (e.g. Coles and Masters (2006a), (2006b)).

[4] Also see Acemoglu and Shimer (1999) who consider efficient UI within a directed search framework.

This paper also provides an important link with the firing cost literature. Essentially the paper shows that the Shavell and Weiss (1979) optimal UI problem, extended to allow hidden savings, implies lump sum severance payments are optimal. The firing cost literature typically considers how firing costs protect employment levels (e.g. Bentolila and Bertola (1990), Hopenhayn and Rogerson (1993)). Lazear (1990) argues that if workers are risk neutral then a legislated firing cost, which is paid to the worker on layoff, has no real effects - wage bargaining at the point of hire implies the negotiated wage falls one-for-one with the firing cost, while the worker is only laid-off when a separation is jointly efficient. Fella (2006) extends Lazear's insight to the case when workers are risk averse and shows that legislated firing costs have real effects, but those effects have little welfare significance (and are not welfare improving).[5] But an important insight, both here and in Pissarides (2004) and Fella (2006) is that severance payments paid to laid-off workers have valuable insurance properties. I discuss further the parallels between these approaches in the conclusion.

Finally the results identified here are closely linked to Stevens (2004) and Burdett and Coles (2003)). Those papers consider optimal wage tenure contracts where the principal offers a contract whose wage paid $w(\tau)$ depends on tenure τ. That problem is isomorphic to the optimal UI problem. Rather than minimise the cost of the UI program given the search incentives of unemployed workers, the wage-tenure contract problem instead maximises firm profits given the quit propensities of employees. In that framework, Stevens (2004) establishes an optimal contract either charges lump sum entry fees when the new hire first starts work, or charges a lump sum exit fee should the worker quit. Her results are strongly resonant of the findings here: that in the optimal UI program workers should either be given a lump sum severance payment when laid-off, or given a re-employment bonus when next finding work.

The paper has two main parts. The first part describes a simplified model which allows me to critique the recent literature on optimal UI with hidden search effort and savings. The second part considers optimal layoff insurance in an economy where workers face multiple unemployment spells during a working lifetime. Using simulations, the second part establishes that a constant UI program with low b and a lump sum layoff payment which fully compensates for the drop in permanent income

[5] A different way to generate real effects is to assume some type of wage rigidity; e.g. Alvarez and Veracierto (2001), Garibaldi and Violante (2005).

by being laid off yields welfare payoffs which are close to those obtained in the full information benchmark.

2 Model and Overview of the Optimal UI Problem.

The model in this section is a simplified version of that considered in Section 3. In this section workers are initally unemployed with an exogenous level of assets A_0, and becoming re-employed is an absorbing state; i.e. there is only a single spell of unemployment. Shavell and Weiss (1979), Hopenhayn and Nicolini (1997), Werning (2002), Kocherlakota (2004) all adopt this approach. The aim of this first section is to understand and reconcile the results in this literature. The second section then considers a steady state framework where workers face job destruction shocks while employed and so experience multiple unemployment spells over a lifetime. The second section shows how the UI program, in addition to distorting search effort while unemployed, distorts savings behaviour while employed.

Throughout the paper time is continuous and has an infinite horizon. Workers are ex-ante identical, strictly risk averse and have subjective rate of time preference ρ which is also the market interest rate. Workers are finitely lived and die according to a Poisson process with parameter $\lambda > 0$. λ also describes the inflow of new entrants into the labour market and so implies a unit mass of workers in a steady state. To avoid bequest behaviour I assume workers save in a competitive annuity market which offers rate of return $\rho + \lambda$ where, on death, the worker's assets pass to the annuity seller. The positive death rate implies all will discount the future at gross rate $r = \rho + \lambda$.

The model uses a principal/agent framework where the Planner insures risk averse workers against unemployment risk. There is a moral hazard problem - the job search effort of an individual job seeker is not observed by the Planner. This hidden action problem implies UI payments $b(.)$ cannot be conditioned on search effort. There are also hidden savings. At any unemployment duration τ, an unemployed worker has financial assets A which are unobserved by the Planner. The unemployed worker also faces liquidity constraints: an unemployed worker with no assets, and hence no colateral, is unable to borrow against future earnings. Thus liquidity constraints imply assets A cannot become negative.

At each unemployment duration a job seeker chooses search effort k and consumption $x \geq 0$. The job seeker chooses $k \in \{0,1\}$ and so either does not search or searches for work. Typically it is instead assumed that k is a continuous choice variable with search cost $c(k)$. It is important to note, however, that the model is also consistent with k being a continous choice variable $k \in [0,1]$ with linear search costs.[6] Thus the results obtained in Kocherlakota (2004) are pertinent.

Given effort $k \in \{0,1\}$, the job seeker becomes employed according to a Poisson process with parameter γk. γ describes how easy it is to find work and, in a matching equilibrium, it depends on labour market tightness. Coles (2006) provides a complete description of optimal unemployment policy for the case when γ is endogenously determined but workers cannot save. To abstract from those policy issues I assume here that $\gamma > 0$ is exogenous. The flow cost of search is zero if $k = 0$ and is $c > 0$ if $k = 1$. If the worker is indifferent between choosing $k = 0$ or 1 assume the worker chooses $k = 1$. This tiebreaking assumption plays no important role. In this section, once re-employed the worker earns wage $w > 0$ forever (until death).

Preferences are additively and time separable. If the job seeker consumes $x\Delta \geq 0$ and searches with effort $k \in \{0,1\}$ over arbitrarily small time period $[t, t+\Delta)$, the worker obtains utility payoff $[u(x) - ck]\Delta$ over that period. $u(.)$ describes the utility from consumption, is strictly increasing, strictly concave, satisfies $\lim_{x\to 0^+} u'(x) = \infty$ and is twice differentiable for all $x > 0$.

The UI program is denoted $B = \{B_0, b(.), D(.)\}$ and has three components:

(i) $B_0 \geq 0$ is a lump sum layoff payment which an unemployed worker receives at duration $\tau = 0$;

(ii) $b(\tau) \geq 0$ describes the flow UI payment to job seekers at unemployment durations $\tau > 0$ and

(iii) $D(\tau)$ is a lump sum tax deduction on re-employment which depends on the length τ of the completed unemployment spell. The tax is implemented by setting an income tax premium $\widehat{t}(\tau)$ on future wages such that $\widehat{t}w = rD(\tau)$; e.g. Hopenhayn and Nicolini (1997).

Thus the worker begins the unemployment spell with initial assets $A(0) = A_0 + B_0$ where $A_0 \geq 0$ are the assets carried over from the previous employment spell. For now

[6] The worker can always convexify search effort by choosing $k = 1$ for fraction θ of any time period Δ.

A_0 is exogenously given. While unemployed the worker's assets evolve with duration according to

$$\frac{dA(\tau)}{d\tau} = rA(\tau) - x + b(\tau) \tag{1}$$

where x is contemporaneous consumption. If the job seeker becomes re-employed at duration τ with assets $A \geq 0$, the optimal savings strategy implies consumption equals permanent income $w + r(A - D(\tau))$ from then onwards. Thus given B, the expected lifetime value of becoming re-employed at duration τ with assets $A \geq 0$ is

$$W^E(A, \tau|B) = \frac{u(w + r(A - D(\tau))) - d}{r},$$

where $d \geq 0$ is the disutility of labour. Given B and initial assets $A_0 \geq 0$, each unemployed worker chooses a consumption and job search strategy to maximise expected lifetime utility.

2.1 Optimal Job Search and Consumption.

Conditional on being unemployed and the UI program B, let $W^U(A, \tau \mid B)$ denote the worker's expected lifetime utility using an optimal savings and job search strategy given current assets $A \geq 0$ and unemployment duration $\tau \geq 0$. Over arbitrarily small time period $\Delta > 0$, the Bellman equation describing W^U is

$$W^U(A, \tau \mid B) = \max_{\substack{x \geq 0 \\ k \in \{0,1\}}} \left\{ \begin{array}{c} [u(x) - ck]\Delta \\ +e^{-r\Delta} \left[(1 - e^{-\gamma k \Delta}) W^E(A', \tau + \Delta|B) + e^{-\gamma k \Delta} W^U(A', \tau + \Delta|B)\right] \end{array} \right\} \tag{2}$$

subject to

$$A' = e^{r\Delta}[A - x\Delta + b(\tau)\Delta] \geq 0.$$

The first term in (2) describes the flow payoff while unemployed. The second describes the expected continuation payoff where, conditional on survival, the worker has continuation assets A' and with probability $(1 - e^{-\gamma k \Delta})$ finds employment over the next instant $[\tau, \tau + \Delta)$ and so enjoys $W^E(.)$, otherwise he remains unemployed and continues search. Note that $W^E(.)$ has already been determined. Thus in the limit as $\Delta \to 0$, the Bellman equation implies a pair of policy rules $x = x^*(A, \tau|B), k = k^*(A, \tau|B)$ which describe optimal consumption and optimal search while unemployed.

Given B and initial assets A_0 at the start of the unemployment spell, let $A(\tau|A_0, B)$ describe how assets evolve with duration $\tau \geq 0$ when the worker uses the optimal strategy. Thus I can define

$$\begin{aligned} x &= x(\tau|A_0, B), \text{ the optimal consumption path} \\ k &= k(\tau|A_0, B), \text{ the optimal search path} \end{aligned}$$

for durations $\tau \geq 0$, where these paths are obtained by substituting $A = A(\tau|A_0, B)$ into the above policy rules x^*, k^*. Also with $A = A(\tau|A_0, B)$ define

$$\begin{aligned} \mu(\tau|A_0, B) &= \frac{\partial W^U(A, \tau \mid B)}{\partial A} \\ V(\tau|A_0, B) &= W^U(A, \tau \mid B) \end{aligned}$$

so that μ describes the marginal value of savings and V describes the value of being unemployed along the optimal path. This notation makes explicit that job seekers with different initial assets A_0 make different consumption and search decisions while unemployed.

In the limit as $\Delta \to 0$, the Bellman equation implies $k = 1$ is privately optimal if and only if

$$c/\gamma \leq \left[W^E(A, \tau|B) - W^U(A, \tau|B)\right]$$

where c/γ is referred to as the effective cost of search. Define **the no-holiday constraint** as the condition

$$W^E(A(.), \tau|B) - W^U(A(.), \tau|B) \geq c/\gamma \text{ for all } \tau \geq 0, \tag{3}$$

with $A(.) = A(\tau|A_0, B)$. Thus the no-holiday constraint implies the worker chooses $k = 1$ along the optimal path. Conversely the worker chooses $k = 0$ whenever $W^E(.) - W^U(.) < c/\gamma$.

I first describe the worker's savings strategy along the opitmal path, given the optimal search path $k = k(.)$. The Inada condition $u'(0) = \infty$ implies consumption $x > 0$ at all durations. There are two cases depending on whether the liquidity constraint is binding or not.

(i) Unconstrained consumption ($A' > 0$).

If the liquidity constraint $A' \geq 0$ is not binding at duration τ, standard arguments imply optimal consumption x is

$$u'(x) = \frac{\partial W^U(A, \tau|B)}{\partial A}; \tag{4}$$

i.e., the marginal utility of consumption equals the marginal value of savings. The Envelope Theorem implies over (arbitrarily small) time period $\Delta > 0$, the marginal value of savings evolves according to

$$\frac{\partial W^U(A, \tau \mid B)}{\partial A} = (1 - e^{-\gamma k \Delta}) \frac{\partial W^E(A', \tau + \Delta \mid B)}{\partial A'} + e^{-\gamma k \Delta} \frac{\partial W^U(A', \tau + \Delta \mid B)}{\partial A'} \tag{5}$$

where $k = k(.) \in \{0, 1\}$ is the optimal search effort choice and $A' = e^{r\Delta}[A - x\Delta + b(\tau)\Delta]$ is the continuation asset level given the optimal consumption choice $x(.)$. Thus an optimal savings strategy implies today's marginal value of savings equals tomorrow's expected marginal value of savings. Rearranging appropriately and letting $\Delta \to 0$, recalling that $\mu(.)$ is defined as $\partial W^U / \partial A$ along the optimal path yields the following differential equation for $\mu(.)$:

$$\gamma k \mu(\tau|.) - \frac{d\mu(\tau|.)}{d\tau} = \gamma k \frac{\partial W^E(A(.), \tau|B)}{\partial A} \tag{6}$$

where $k = k(\tau|A_0, B)$ along the optimal path. Note for what follows that whenever $k(.) = 0$, (6) implies $\mu(.)$ is (locally) constant and so optimal consumption $x(.)$ is constant during such phases.

(ii) Liquidity constrained consumption ($A' = 0$). If the liquidity constraint binds at τ, then optimal consumption $x = b(\tau)$ and the Kuhn-Tucker condition for optimality is

$$u'(b(\tau)) \geq \left[\frac{\partial W(0, \tau|B)}{\partial A} \right] \tag{7}$$

so that the marginal utility of today's consumption exceeds the marginal value of savings. The Envelope theorem then implies

$$\frac{\partial W(0, \tau|B)}{\partial A} = u'(b(\tau)).$$

Thus $\mu(\tau|.) = u'(b(\tau))$ when the liquidity constraint binds. The above establishes the following Claim.

Claim 1. Optimal Consumption.
Given the optimal search effort path $k = k(.)$, the optimal consumption path $x(.|A_0, B)$ satisfies

$$u'(x) = \mu \tag{8}$$

where $\mu = \mu(.|A_0, B)$ evolves according to

$$\mu = u'(b(\tau)) \text{ while } A(.) \geq 0 \text{ is binding}$$

$$\gamma k\mu - \frac{d\mu}{d\tau} = \gamma k u'(w + r(A - D(\tau)) \text{ while } A(.) \geq 0 \text{ is non-binding} \tag{9}$$

and $A(.) \geq 0$ evolves according to the differential equation:

$$rA - \frac{dA}{d\tau} = x - b(\tau) \tag{10}$$

subject to the initial condition $A(0) = A_0 + B_0$.

The optimal search effort choice k is not determined in claim 1. If A_0, B satisfy the no-holiday constraint then V evolves according to

$$rV - \frac{dV}{d\tau} = u(x) - c + \gamma\left[\frac{u(w + r(A - D)) - d}{r} - V\right],$$

where the flow value of being unemployed equals the flow payoff while unemployed (using the optimal strategy) plus the expected capital gain by finding work. Of course the no holiday constraint must then hold with W^U replaced by V.

2.2 An Illustrative Example

Anticipating the arguments below, it is worth illustrating the optimal strategy for a scheme with $b(.) = \underline{b} > 0$ and $B_0 > 0, D = 0$; i.e. flow UI payments $\underline{b}$ are duration independent and there are no income tax premia when re-employed. As UI payments are not duration dependent, the worker's optimal job search and consumption problem is stationary. We can simplify the above notation by defining $W^U(A|\underline{b})$ as the value of being unemployed with assets A (noting that $A(0) = A_0 + B_0$).

Unfortunately stationarity is not sufficient to establish that programming problems of this type are globally concave; e.g. Lentz and Tranaes (2005). Nevertheless the discrete choice structure for search effort allows a relatively straightforward characterisation of the policy optimum. The proof of Theorem 1 below essentially considers two possibly optimal strategies. One is a "retirement strategy" where the unemployed worker never seeks employment. The second is a "holiday" strategy where the unemployed worker seeks employment at some future (finite) duration, including the case that he/she searches immediately.

The characterisation of the optimal retirement strategy is simple: the unemployed worker always consumes permanent income $\underline{b}+rA$. Part III of Theorem 1 establishes that the retirement strategy is optimal for sufficiently high assets $A > A^R$.

The proof of Theorem 1 fully characterises the optimal "holiday strategy". The proof establishes there is a holiday level of assets, denoted A^H, where a worker with inital assets $A_0 + B_0 \leq A^H$ subsequently chooses $k = 1$ until a job is found; i.e. the no holiday constraint is satisfied for low $A_0 + B_0$. For intermediate assets $A_0 + B_0 \in (A^H, A^R)$ the worker takes a "holiday": the worker initially chooses zero search effort but, as consumption exceeds income during this phase, assets $A(.)$ eventually decline to A^H whereupon the worker switches to active search. For high assets $A_0+B_0 \geq A^R$, the unemployed worker never searches for work and instead consumes permanent income $\underline{b} + r[A_0 + B_0]$.

Theorem 1. Optimal Job Search and Consumption
Given UI scheme $D = 0$ and $b(.) = \underline{b} > 0$ where $\underline{b}$ also satisfies $u(\underline{b}) < u(w)-d-rc/\gamma$, then optimal job search and consumption is characterised by a pair of asset thresholds $A^H(\underline{b}), A^R(\underline{b}) \geq 0$ such that:

(I) for $A \leq A^H$, the optimal policy rules are $k^* = 1$ and $x^* = x^U(A;\underline{b})$ where x^U is a continuous and strictly increasing function of A with $x^U = \underline{b}$ at $A = 0$ and $x^U \in (\underline{b}+rA, w+rA)$ for all $A > 0$;

(II) for $A \in (A^H, A^R)$, the optimal policy rules are $k^* = 0$ and $x^* = x^H$, where $x^H = x^U(A^H;.)$ and satisfies $x^H > \underline{b}+rA$;

(III) for $A \geq A^R$, the optimal policy rules are $k^* = 0$ and $x^* = \underline{b}+rA$.
The proof of Theorem 1 is in the Appendix.

For low asset levels satisfying (I), the no-holiday constraint is satisfied: the unemployed worker chooses $k^* = 1$ and the optimal consumption rule $x^U(.)$ implies

$x^U > \underline{b} + rA$ and so savings strictly fall with duration. As $x^U(.)$ is a continuous and strictly increasing function, consumption gradually falls as the worker's assets fall, converging to $x^U = \underline{b}$ when the liquidity constraint binds.

For intermediate asset levels satisfying (II), a worker chooses $k^* = 0$ and so takes an unemployment holiday. During this phase, optimal consumption smoothing implies (i) a constant level of consumption, x^H, and (ii) "holiday consumption" x^H exceeds flow income $\underline{b} + rA$ and so savings strictly fall with duration. Once savings reach the critical level A^H, the worker switches to phase (I) and then actively searches for work. Optimal consumption smoothing, however, implies consumption is continuous across $A = A^H$ and so $x^H = x^U(A^H; .)$. The holiday level of assets, A^H, is identified where the no-holiday constraint fails; i.e.

$$\frac{u(w + rA^H) - d}{r} - W^U(A^H|\underline{b}) = c/\gamma.$$

A^R describes the retirement level of assets: for $A \geq A^R$ the worker chooses zero search effort and consumes permanent income $\underline{b} + rA$. As assets do not change over time, this is an absorbing state (until death). The retirement level of assets, A^R, occurs where holiday consumption x^H equals flow income; i.e. $A = A^R$ where $\underline{b} + rA = x^H$. For $A < A^R$ the holiday strategy is optimal but as $A \rightarrow A^R$, phase II becomes arbitrarily long in duration and the payoff to the holiday strategy converges to the payoff to the retirement strategy; the worker consumes x^H indefinitely.

As the Envelope Theorem implies $\partial W^U/\partial A = u'(x^*(.))$, and Theorem 1 implies x^* is increasing in A, it follows that $W^U(.)$ is concave. But a curious result is that that $W^U(.)$ is linear for $A \in [A^H, A^R]$; thus workers taking unemployment holidays are indifferent to (small) gambles, even though $u(.)$ is strictly concave.

Theorem 1 establishes that the no-holiday constraint fails at high asset levels. To understand why this occurs, it is useful to define re-employment surplus:

$$S(A|\underline{b}) = W^E(A|\underline{b}) - W^U(A|\underline{b}) \equiv \frac{u(w + rA) - d}{r} - W^U(A|\underline{b})$$

and note $k = 1$ is privately optimal if and only if $S(A|\underline{b}) \geq c/\gamma$. Differentiating with respect to A yields:

$$\begin{aligned} \frac{\partial S(A|\underline{b})}{\partial A} &= u'(w + rA) - \frac{\partial W^U(A|\underline{b})}{\partial A} \\ &= u'(w + rA) - u'(x^*(A; \underline{b})) \end{aligned}$$

where $x^*(.)$ is the optimal consumption rule as described in Theorem 1. But Theorem 1 implies optimal consumption $x^*(.) < w + rA$ and so re-employment surplus S is strictly decreasing in A: the incentive to search is weaker as assets A increase. The intuition is that the optimal dissaving strategy while unemployed cushions the cost of being unemployed. Although an increase in A increases both the value of being employed and the value of being unemployed, it closes the value gap as the marginal value of savings is greater for the unemployed.[7]

As the notation makes clear, the asset thresholds $A^H(\underline{b}), A^R(\underline{b})$ depend on the generosity of the UI program $\underline{b}$. For example a too generous program, one with $u(\underline{b}) \geq u(w) - d - rc/\gamma$, implies $A^R(\underline{b}) = 0$; all unemployed choose $k = 0$ and consume $x = \underline{b} + rA$. Conversely $\underline{b}$ small implies workers with sufficiently low assets search for work, but richer workers do not. In the extended model considered in section 3, the worker's initial assets A_0 are accumulated endogenously in a prior employment spell. With less than full insurance, workers have a precautionary savings motive to self-insure against layoff risk. The Planner's optimal UI program not only has to consider optimal job search behaviour when unemployed, but also how this affects savings incentives while employed. The obvious concern is that a generous UI program not only encourages unemployment holidays, it might sustain 'early retirement' should accumulated assets exceed A^R.

2.3 An Overview of The Optimal UI Literature.

Given an unemployed worker with exogenous assets $A_0 \geq 0$, optimal layoff insurance chooses B to maximise the value of being laid-off, which is $W^U(A_0 + B_0, 0|B)$, subject to a budget constraint that the cost of the UI program is no greater than some (exogenous) cost $C_0 > 0$. Recall that $k = k(\tau|A_0, B)$ describes the worker's optimal search effort at duration τ along the optimal path. Now define

$$\Psi(\tau|A_0, B) = e^{-\int_0^\tau \gamma k(t|.)dt}$$

which, conditional on survival, is the probability the laid-off worker remains unemployed at duration τ. The budget constraint can then be written as:

$$B_0 + \int_0^\infty \Psi(\tau|.)e^{-r\tau}b(\tau)d\tau - \int_0^\infty \Psi(\tau|.)e^{-r\tau}\gamma k(\tau|.)D(\tau)d\tau \leq C_0. \tag{11}$$

[7]Lentz and Tranaes (2005) establish this result in a more general framework but assume lotteries.

The first term in (11) is the lump sum layoff payment, the second is the expected discounted cost of further UI payments should the job seeker remain unemployed at duration $\tau > 0$, and the third is the reclaimed tax should the worker become re-employed at duration τ. The total expected budget cannot exceed cost C_0. There are two main strands to the existing literature.

2.4 Optimal UI with no savings.

Shavell and Weiss (1979) introduced the optimal layoff insurance problem with unobserved search effort, assuming workers can neither save nor dissave. This implies $A_0 = 0$ and also consumption must equal the UI payment $b(\tau)$ at every duration τ. By also setting $D = 0$ they show that UI payments optimally decrease with duration. Coles (2006) extends that model to a matching equilibrium with endogenous wage formation and job creation rates. It identifies an optimal insurance condition, given by (12) below, which is particularly insightful.

In (12) below, $Z(\tau|B)$ denotes the expected continuation cost of further UI payments given a currently unemployed worker with duration τ. Thus $Z(.)$ is given by

$$Z(\tau|.) = \int_{\tau}^{\infty} e^{-\int_{\tau}^{t} \gamma k(x|.)dx} e^{-r(t-\tau)} b(t)dt.$$

$S(\tau|.)$ in (12) is the re-employment surplus at duration τ, that is $S(\tau|.) = W^E(.) - W^U(.)$. $k^*(S)$ is the optimal search effort choice given re-employment surplus $S = S(\tau|.)$. In a more general framework, k^* is given by the first order condition $c'(k) = \gamma S$, where the marginal cost of search effort equals its expected marginal gain. In contrast, the discrete choice case considered above implies $k^* = 1$ whenever $c/\gamma \leq S$.

With no discounting, $r = 0$, Coles (2006) establishes that optimal insurance in the Shavell/Weiss framework reduces to

$$\frac{u'(b(t))}{u'(b(0))} = 1 + \int_0^t \gamma \frac{\partial k^*(S(\tau|.))}{\partial S} Z(\tau|.)d\tau. \tag{12}$$

To understand this insurance condition, suppose the Planner marginally reduces UI benefit $b(t)$ at duration $t > 0$. This marginally reduces the value of remaining unemployed at all durations $\tau < t$ and so worker re-employment surplus $S(\tau|.)$ marginally increases at those durations. By inducing greater search effort at each duration $\tau < t$, worker exit rates increase by $\gamma \partial k^* / \partial S$. An increase in the exit rate of benefit claimants then saves the Planner the continuation cost of further UI, $Z(\tau|.)$. The integral in (12) computes this total return which then distorts optimal UI away from a flat UI profile. As the integral is increasing in t, it follows that UI payments fall with duration.

The insurance condition (12) identifies the essential distortion: **insured job seekers ignore that by finding employment they save the Planner the continuation cost of further UI.** In the above case with no savings, the Planner partially internalises this externality by reducing UI payments with duration. But more generally, the objective of an optimal UI program is to internalise this externality on the job seeker's job search problem at minimum distortion.

Consider then Hopenhayn and Nicolini (1997). That paper assumes workers cannot save/dissave and that unemployed workers are subject to an income tax premium when re-employed, where that tax depends on the length of the completed spell of unemployment. Their simulation is particularly insightful as it shows that the optimal income tax premium levied at each re-employment duration rises one for one with previous UI receipts. Hopenhayn and Nicolini (1997) show such a scheme generates large welfare gains. Formally such gains arise as their scheme fully internalises the externality identified in (12): as the worker must repay any further UI receipts through future taxes, remaining unemployed does not extract further rents from the Planner. Thus workers have efficient job search incentives. Given workers cannot save/dissave by assumption, the tax program also allows the Planner to smooth consumption over the unemployment spell and across the re-employment shock.

Unfortunately the large welfare improvements suggested by the simulations of Hopenhayn and Nicolini (1997) need not carry over to the case when workers use savings strategies. With perfect capital markets, the optimal UI contract reduces to a payment function $\Pi^*(\tau)$ which is the amount paid to the worker conditional on obtaining re-employment at duration τ (e.g. Fudenberg et al (1990)). Here this payment can be implemented as

$$\Pi^*(\tau) = B_0 e^{r\tau} + \int_0^\tau e^{r(\tau-t)} b(t)dt - D(\tau).$$

Thus with no liquidity constraints one can normalise $D = 0$ and the optimal compensation sequence $\Pi^*(\tau)$ is implemented with $B_0 = \Pi^*(0)$ and $b(\tau) = d\Pi^*/dt - r\Pi^*$.[8] Of

[8]Kocherlakota (2004) chooses a different normalisation: instead he sets $B_0 = 0$ and b equal to

course setting $D = 0$ is not a normalisation if there are binding liquidity constraints. The Hopenhayn and Nicolini (1997) approach suggests that when the liquidity constraint binds, the Planner might offer loans which are repaid in the next employment spell. There are two criticisms of such a policy proposal. First the presence of a liquidity constraint implicitly presumes some market failure in the financial sector so that banks are not willing to loan funds to unemployed workers with no collateral. Without modelling an explicit market failure in the financial sector, it seems ad-hoc to assume the Planner will offer loans that banks are not willing to make. But perhaps more importantly, Kocherlakota (2004) argues that it is optimal not to offer such loans: the Planner improves job search incentives by leaving the worker liquidity constrained.

2.5 Optimal UI with hidden savings but $\mathbf{A}_0 = 0$.

Before reviewing the results in Werning (2002) and Kocherlakota (2004), it is useful to consider first the full information benchmark when search effort is perfectly contractible. Suppose for the moment there is job turnover where all jobs are destroyed at rate $\delta > 0$. If employed workers receive gross wage w^G, then full (and fair) layoff insurance implies workers pay an insurance premium $\pi = \delta w^G/(\ r + \gamma + \delta)$ while employed and so earn net wage $w = (r + \gamma)w^G/(r + \gamma + \delta)$. When laid-off, a worker receives a lump-sum layoff payment $C_0 = \pi/\delta$ which fully compensates for his/her drop in permanent income by becoming unemployed. Given the unemployed worker contracts to search with effort $k = 1$, the Planner then sells unemployment insurance annuities. Such annuities pay \$1 per period unemployed and, given the worker contracts to search with effort $k = 1$, the fair price for this annuity is $1/(r + \gamma)$.[9] The risk-averse laid-off worker fully insures against re-employment risk by purchasing w unemployment annuities at cost $w/(r + \gamma) \equiv C_0$. By design there is perfect consumption smoothing where the worker consumes $x = w$ at all dates. As there is no precautionary savings motive, $A_0 = 0$ is also privately optimal.

optimal consumption so that assets $A(.)$ are zero along the optimal path.

[9] For example if the time reference is a year with discount rate $r = 5\%$ and $\gamma = 4$ (implying expected duration of unemployment equal to 13 weeks), then an annuity which pays flow payoff \$1 per year while unemployed has price 24.7 cents. Purchasing 365 unemployment annuities in order to receive \$1 per day while unemployed costs 90.1 dollars (where 91 days is the expected duration of unemployment).

Thus a useful perspective is that when laid-off, the worker receives a cash layoff payment C_0 (to compensate for the drop in permanent income) and then purchases an unemployment annuity plan to insure against re-employment risk. If search effort $k = 1$ is incentive compatible, each unemployment annuity has fair price $1/(r + \gamma)$. If instead search effort $k = 0$ is incentive compatible, each unemployment annuity has fair price $1/r$ which simply reflects the market savings rate. Given the worker can already save/dissave at market rate r, the Planner's main financial role is to sell unemployment annuities at price $1/(r + \gamma)$ but rations sales so that $k = 1$ remains incentive compatible. An optimal annuity plan suggests $b^*(.)$ is rationed to the point where the worker is indifferent to $k^* = 1$ or 0 along the optimal path, but chooses $k^* = 1$ by convention. But this outcome suggests the results in Kocherlakota (2004) should apply.

Kocherlakota (2004) also considers the case where a laid-off worker has initial assets $A_0 = 0$, faces liquidity constraint $A \geq 0$, and chooses search effort $p \in [0, 1]$ where search costs are linear. He also conjectures that in the optimal program *"the principal wants to (weakly) implement a sequence of effort choices* $p_t \in (0, 1)$ *for all t"*. Linear search costs and incentive compatability then imply the worker must be indifferent across any choice $p \in [0, 1]$. Kocherlakota (2004) then establishes that the UI program which maximises the value of being laid-off, while also satisfying the above conjecture, implies $B_0 = 0$ (no severance payments), $b(.) = b_0$ at every duration and $D(.) = D_0$ is duration independent. The optimal choice of (b_0, D_0) satisfies the no holiday constraint with equality. In my notation this implies

$$\frac{u(w - rD_0) - d}{r} - \frac{u(b_0) - c + \gamma \frac{u(w - rD_0) - d}{r}}{r + \gamma} = \frac{c}{\gamma}$$

where the first term is the value of being re-employed, the second term is the value of being unemployed and searching with effort $k = 1$. As this condition implies $b_0 < w - rD_0$, the worker always consumes $x = b_0$ while unemployed and so never accumulates savings. Note in particular the Planner chooses to leave the worker liquidity constrained (i.e. there are no loans; $D(.)$ is independent of previous UI receipts). Instead $-D_0$ describes a re-employment bonus which is paid when the job seeker gets a job. Furthermore, by increasing the re-employment bonus, $-D_0$, the Planner can increase b_0 and still ensure $k = 1$ remains incentive compatible. Given the budget constraint

$$\frac{b_0 - \gamma D_0}{r+\gamma} = C_0,$$

the optimal policy implies the Planner increases b_0 and the re-employment bonus $-D_0$ together, to ensure $k = 1$ remains incentive compatible, until the budget constraint binds.[10] I shall refer to this policy as the re-employment bonus program.

Such a policy seems plausibly optimal. The re-employment bonus partially internalises the externality described above - that job seekers ignore that finding work saves the Planner the continuation cost of further UI. But a simple numerical example, described below, illustrates a better policy can exist.

In the absence of any UI, being laid-off implies a drop in permanent income equal to $w/(r+\gamma)$. Thus consider a UI program which offers compensation $C_0 = w/(r+\gamma)$. One approach might simply give the worker a lump sum severance payment $B_0 = C_0$ and the worker smooths consumption by dissaving during the spell of unemployment. As the worker receives no further UI, the worker also has efficient search incentives; i.e. this approach also internalises the externality described above. But as there is re-employment risk, a better policy exists. Suppose instead the worker is given constant UI payments $\underline{b} > 0$ while unemployed, no re-employment bonus ($D = 0$), and a lump sum layoff payment which ensures the no holiday constraint is satisfied with equality. In the notation of Theorem 1, severance payment $B_0 = A^H(\underline{b})$ and so

$$\frac{u(w + rB_0) - d}{r} - W^U(B_0|\underline{b}) = c/\gamma.$$

Note that an optimal dissaving strategy implies $A(.) < B_0 = A^H$ at strictly positive durations and the no holiday constraint is satisfied. The severance payment program is defined as a pair $(\underline{b}, B_0)$ where $B_0 = A^H(\underline{b})$ and which satisfies budget balance

$$B_0 + \frac{\underline{b}}{r+\gamma} = C_0.$$

For the high turnover economy described in the next section and budget allocation $C_0 = w/(r+\gamma)$ [i.e. the UI program fully compensates for the drop in permanent income through being laid-off] it can be shown numerically that the severance payment

[10] As the no holiday constraint is satisfied with equality and search costs are linear, this program is consistent with Kocherlakota's conjecture.

program achieves a higher welfare payoff than the re-employment bonus program.[11] Relative to the severance payment program, the re-employment bonus program allows a more generous b_0 consistent with inducing search effort $k = 1$. But laid-off workers in the re-employment bonus program are liquidity constrained (no loans) and $b_0 \ll w$ implies a large fall in consumption relative to that while employed. In contrast, the severance payment program and an optimal dissavings strategy ensures consumption is smooth across the job destruction shock. Although the job seeker is not initially liquidity constrained, the threat of becoming liquidity constrained in the near future (no loans) ensures the worker chooses high job search effort. No re-employment bonus also implies consumption does not increase so sharply across the re-employment shock. The smoother consumption profile induced by the severance payment program yields the higher welfare payoff.

The reason why Kocherlakota's conjecture is not consistent with optimality is that the optimization problem is not concave. Although the Planner might ration unemployment annuities so that the worker is indifferent to choosing $p = 0$ or 1 along the optimal path, the second order effects described in Kocherlakota (2004) imply the worker is worse off choosing an interior $p \in (0, 1)$ (strictly in a discrete time framework). Instead an optimal UI program implies the usual bang-bang property, where the worker prefers either to look for work with $p = 1$ and choose high consumption, or to take an unemployment holiday with $p = 0$ and choose low consumption.[12]

It is now useful to reconsider Werning (2002) with $D = 0$. As Kocherlakota (2004) points out, the non-concavity problem implies the first order approach may

[11] For $C_0 = w/(r + \gamma)$ and parameters for the high turnover economy specified in the next section (with $\pi = 0$), and also assuming re-employment is an absorbing state, I obtain the following policy outcomes:

Re-employment bonus program: $b_0 = 74.9$ and $D_0 = -0.0627$ (i.e. replacement rate of 74.9% and a re-employment bonus equal to 3.4 weeks wages).

Severance payment program: $\underline{b} = 69.1$ and $B_0 = 0.0761$ (i.e. replacement rate of 69.1% and layoff payment equal 4.0 weeks wages).

The latter program yields the higher payoff though the gain is small (occuring at the 4th significant figure). The smallness reflects that the expected spell of unemployment is only 13 weeks which is short relative to consuming $w + r(A - D)$ indefinitely once re-employed..

[12] Kocherlakota (2004) justifies his conjecture in a footnote by appealling to congestion externalities. Coles (2006) explicitly considers a matching equilibrium and shows congestion externalities play no role in the optimal UI problem. Indeed linear search costs with unbounded domain $k \in [0, \infty)$ implies there is no value to an unemployment insurance program: optimality implies zero UI and equilibrium unemployment spells are arbitrarily short as job seekers choose arbitrarily high search effort.

not identify the optimal policy. Indeed it is this non-concavity which explains why a lump sum layoff payment is optimal. But reconsider the above interpretation: that the laid-off worker receives layoff payment C_0 and purchases an unemployment annuity plan $b(.)$ from the Planner. The Planner rations $b(.)$ so that search effort $k = 1$ is incentive compatible. The critical insight from Theorem 1 is that unemployed workers with lower assets $A(.)$ have a greater incentive to search (also see Lentz and Tranaes (2005)). Anticipating that optimal dissaving implies the asset path $A(.)$ falls over the unemployment spell, the Planner can then sell more unemployment annuities with duration and still ensure $k = 1$ remains incentive compatible. This suggests $b(.)$ increases with duration in the optimal annuity plan. Of course as incentive compatability implies $b < w$, then the cost of this annuity plan is less than C_0 assuming C_0 compensates for the drop in permanent income through being laid-off. The difference implies a positive lump sum layoff payment $B_0 > 0$. The simulations in Werning (2002) all have this structure - consumption falls over the unemployment spell as $A(.)$ declines and UI benefits $b(.)$ increase slowly with duration.[13]

Unfortunately the above is necessarily heuristic as the optimal UI problem is analytically intractable. But even if one could characterise analytically the optimal rationing scheme, the solution would be of limited value. The extended case assumes $A_0 \geq 0$ is endogenously determined in a previous employment spell. Suppose I identified the optimal rationing scheme $b^*(.)$ for $A_0 = 0$. A laid-off worker with assets $A_0 > 0$, where A_0 is hidden, would then deviate by taking an unemployment holiday.

Kocherlakota (2004) focusses on the unemployed worker's incentive to underconsume and take an unemployment holiday. By doing this, the worker extracts greater rents from the Planner who is then mis-selling unemployment annuities at price $1/(r + \gamma)$ on the presumption the worker is actively seeking employment. But in anticipation of job destruction shocks, employed workers have the same incentive to underconsume and so overaccumulate assets. When the job destruction shock occurs, the laid-off worker with greater (hidden) assets then enjoys an unemployment holiday, extracting rents from the Planner who is selling unemployment annuities too cheaply.

[13] Werning (2002) never specifies an initial asset level A_0 for the laid-off worker; e.g. the worker's problem in his section 2.3. Essentially the paper uses Euler-type arguments to describe how consumption and search varies optimally with duration. The actual financing of the optimal consumption plan is never clearly discussed - there is no mention of lump sum layoff payments.

3 Unemployment Insurance with Endogenous Savings

Using the insights identified above, this section considers optimal unemployment insurance with endogenous savings. It extends the model by now assuming employed workers face job destruction shocks and self-insure using savings strategies. For ease of exposition I set $D = 0$. The market outcome with optimal layoff payments generates value which is already very close to the full information benchmark. Thus the additional welfare benefits of allowing $D \neq 0$ is necessarily small. Neverthless I shall return to the potential value of re-employment bonusses in the Conclusion.

The model is the same as before except now employed workers face idiosynchratic job destruction shocks which occur according to a Poisson process with parameter $\delta > 0$. A job destruction shock implies an employed worker becomes unemployed with duration $\tau = 0$. Employed workers earn the same gross wage which is denoted w^G and w^G remains exogenous. The net wage is $w = w^G - \pi$ where π is the insurance premium set by the Planner.

Labour market entrants are initially unemployed, have no assets and are not entitled to receive UI payments. They search at rate $k = 1$.[14] I distinguish between insured unemployed workers (those who were previously employed and so receive unemployment benefits) and uninsured new labour market entrants. Let U^I, U^{NI} denote the respective numbers of 'insured' and 'not insured' unemployed workers, where $U = U^{NI} + U^I$ denotes total unemployment.

Each worker can save/dissave at the market rate $r > 0$. Market failures in the capital market imply banks do not offer loans to unemployed workers with no assets. While unemployed, a worker enjoys flow payoff $u(x) - ck$ where $k \in \{0, 1\}$ describes search effort and $c > 0$ describes the flow cost of search. An employed worker instead enjoys flow payoff $u(x) - d$ where $d \geq 0$ is the opportunity cost of leisure.

The Planner offers UI program $B = \{\pi, B_0, b(.)\}$ where

(i) π is the insurance premium paid by an employed worker;

(ii) B_0 is a lump sum payment when laid-off;

(iii) $b(.)$ is the flow UI payment while unemployed.

[14] The focus here is designing optimal unemployment insurance, rather than a social security system for new labour markets. In this world one might presume new labour market entrants live with their parents until first employed.

Note that unemployment annuities $b(.)$ insure unemployed workers against re-employment risk, while severance layoff payments B_0 insure employed worker's against the drop in permanent income through being laid-off. In what follows, co-ordinating policy choices $b(.)$ and B_0 yields welfare payoffs which are surprisingly close to the full information benchmark.

At least in principle these policy instruments at date t might be conditioned on the worker's entire employment history up to that date. Practicalities suggest, however, that governments are unlikely to implement very complicated policies. Furthermore policies which design punishment phases which are optimal for particular preferences c, d and $u(.)$ may lead to highly inefficient outcomes for other preferences. This issue is important when preferences are unobserved. For example, the simulations in Hopenhayn and Nicolini (1997) suggest a non-collateralised loan scheme for unemployed workers, where loans are repaid when re-employed. But the simulations there report an optimal loan sequence for particular worker preferences. A practical policy issue is that preferences are unobserved by the Planner. Faced with that specific loan sequence, an unemployed worker with higher work disutility d or discount rate ρ instead may enjoy those loans and never search for work, and thus avoid repaying the debt. An important policy issue, therefore, is that the proposed program must be efficient for a large swathe of (unobserved) worker preferences. In the simulations that follow, I show that relatively simple UI programs which ignore information on previous unemployment spells can still achieve payoffs which are close to the full information benchmark.

The simulations of Werning (2002) suggest that in the optimal scheme UI payments might gently increase with duration as worker assets decline over the unemployment spell. But here there is ex-post worker heterogeneity and, as will be made clear below (see Table 1), workers with longer unemployment durations may have higher, rather than lower, assets. If assets are positively correlated with duration, then raising UI payments with duration need not be welfare enhancing. For tractability I assume the Planner instead offers constant UI while unemployed, $b(.) = \underline{b}$.

Thus policy B is constrained to a constant premium π paid while employed, a severance payment B_0 when laid-off, and constant UI $b(.) = \underline{b}$ at strictly positive unemployment durations $\tau > 0$. Given these policy restrictions, the worker's optimisation problem is then stationary. For given $B = \{\pi, \underline{b}, B_0\}$, let $W^E(A|B)$, $W^U(A|B)$

denote the value of being employed and unemployed respectively, with current assets A. For notational convenience, I shall subsume reference to B in these functions. The Bellman equation describing these functions over arbitrarily short time period $\Delta > 0$ are:

$$W^E(A) = \max_{x \geq 0} \left\{ \begin{array}{c} [u(x) - d]\Delta \\ +e^{-r\Delta} \left[(1 - e^{-\delta\Delta}) W^U(A' + B_0) + e^{-\delta\Delta} W^E(A') \right] \end{array} \right\} \tag{13}$$

where continuation assets

$$A' = e^{r\Delta}[A - x\Delta + w\Delta] \geq 0.$$

Note that when laid-off through a job destruction shock, he/she receives severance payment B_0. $W^U(A)$ is given by

$$W^U(A) = \max_{\substack{x \geq 0 \\ k \in \{0,1\}}} \left\{ \begin{array}{c} [u(x) - ck]\Delta \\ +e^{-r\Delta} \left[(1 - e^{-\gamma k\Delta}) W^E(A') + e^{-\gamma k\Delta} W^U(A') \right] \end{array} \right\} \tag{14}$$

with continuation assets

$$A' = e^{r\Delta}[A - x\Delta + \underline{b}\Delta] \geq 0.$$

Solving this pair of Bellman equations numerically is straightforward. The solution yields a pair of optimal consumption rules $x^U(A)$, $x^E(A)$ while unemployed and employed respectively, and a pair of asset thresholds $A^H, A^R \geq 0$ where

(i) the unemployed worker chooses $k = 1$ if $A \leq A^H$;

(ii) the unemployed worker chooses $k = 0$ and strictly dissaves if $A \in (A^H, A^R)$;

(iii) the unemployed worker never searches if $A \geq A^R$.

There is ex-post worker heterogeneity as individual assets A evolve stochastically depending on the realised employment history . The optimal consumption and search rules in a steady state generates a distribution of assets across insured unemployed workers, denoted $G^U(A)$, and across employed workers $G^E(A)$. In a steady state those distributions are jointly determined by a pair of first order differential equations. Those equations are fully described in Appendix B.

The simulations that follow have two sections. The next section first shows how worker savings and search behaviour adjust to different levels of b. Section 4 then considers optimal layoff insurance.

3.1 Model Specification.

Following Lentz (2005) who estimates a structural job search model with savings, I assume CRRA utility function $u(x) = x^{1-\sigma}/(1-\sigma)$ with risk aversion parameter σ =2.2. The average wage rate in Lentz (2005) is 144 units but to facilitate the discussion I round this wage to $w^G = 100$. Lentz (2005) does not estimate a disutility of labour. I set d so that $u(100) - d = u(75)$ which implies a worker is indifferent to a 25% wage cut in return for full leisure. Note that a constant UI program with $b \geq 75$ implies no insured worker searches for employment. I set $c = d$ which implies active job search is as unpleasant as working. Using one year as the reference unit of time, I set $\lambda = 0.02$, thus implying an expected working lifetime of 50 years, and $\rho = 0.04$ which implies gross discount rate $r = 6\%$ per annum.

As made clear in Hassler and Rodriguez (1999), the value of the unemployment insurance program depends on job turnover rates. I consider three different economies. In the high turnover economy (HT), I set $\gamma = \gamma_{HT} = 4$ which implies the expected duration of unemployment with $k = 1$ is 13 weeks (which is reasonable for the U.S.). If all choose k=1, the baseline steady state unemployment rate is

$$U = \frac{\lambda + \delta}{\lambda + \delta + \gamma}.$$

Assume the high turnover economy has job destruction rate $\delta = \delta_{HT} = 20\%$ per annum. This implies average employment spells of 5 years and baseline steady state unemployment equal to 5.2%. Of course "unemployment holidays" and "early retirement" raise unemployment above this baseline level.

Average employment spells and unemployment spells are longer in the low turnover economy (LT). Specifically I set $\gamma_{LT} = 2$ which implies the average unemployment spell of an active job seeker is 6 months (as in the U.K.). The interpretation is that equilibrium market tightness is lower in the low turnover economy and so it takes longer to find work. Holding baseline unemployment at 5.2% requires job destruction rate $\delta_{LT} = 9\%$. Thus the low turnover economy has long average employment spells of around 11 years, average unemployment spells of 6 months but the baseline unemployment level is the same as in the high turnover economy.

The third case is a high unemployment (HU) economy. In this case $\gamma_{HU} = 1.5$ which implies average unemployment spells of 8 months. An intermediate job destruction rate $\delta_{HU} = 12.5\%$ implies average employment spells of 8 years and the baseline unemployment rate is then a relatively high 8.8%.

To provide a feel for how UI payments distort these economies, Table 1 describes the steady state market outcome with a pure constant UI program with $\pi = 0$, $B_0 = 0$. As the results are qualitatively identical across the economies, Table 1 describes the results for the high turnover economy.

Table 1: Market Outcomes by UI Generosity (High Turnover)

Replacement Rate	A^H	A^R	$\overline{A}_E$	$\overline{A}_U$	$p_U^H(\%)$	$p_U^R(\%)$	$U(\%)$	$m_u(\%)$
0.4	5.8	14	0.87	0.74	0^+	0^+	5.2	0.9
0.5	3.8	11	0.70	0.60	0.1	0^+	5.2	1.2
0.6	1.7	7.3	0.62	0.90	18.2	0.6	6.2	1.1
0.7	0.24	3.5	0.73	1.59	76.7	16.6	41.3	0.1

Anticipating the results that follow, it is useful to focus on the third row with $\underline{b} = 60$ and thus replacement rate 0.6. Table 1 reports the asset thresholds A^H, A^R which are measured in units of one years salary. Thus for replacement rate 0.6, unemployed workers take a holiday if assets exceed 1.7 years salary and take "early retirement" if assets exceed 7.3 years salary. $\overline{A}_E$ denotes the average assets held by employed workers also measured in units of one years salary. Thus replacement rate 0.6 implies employed workers hold average assets equal to 0.62 years salary. $\overline{A}_U$ denotes the average assets held by insured unemployed workers, which is 0.9 years salary. At first sight it is surprising that the insured unemployed, on average, are wealthier than the employed. This is not always true - see the rows with lower replacement rates. But high replacement rates imply the holiday asset threshold A^H is relatively low. As those unemployed with $A < A^H$ search for work and quickly exit unemployment, the pool of unemployed workers is over-represented by relatively wealthy types, those with $A > A^H$, who take a holiday and remain claimant unemployed. This composition effect becomes large at higher replacement rates.

m_U describes the percentage of insured unemployed workers who are liquidity constrained. Here and in all the simulations that follow, this number is always very small. Note that if the Planner offers low UI, employed workers compensate through increased savings. The optimal dissaving strategy while unemployed implies most laid-off workers find a job before exhausting their savings. In Kocherlakota's policy

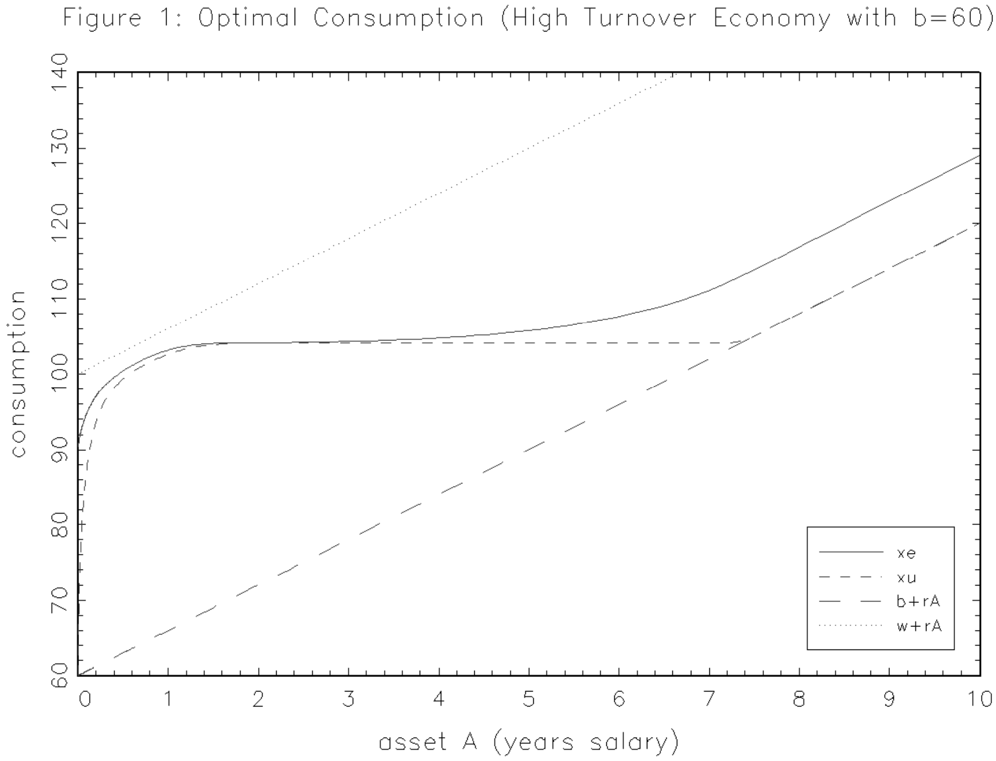

Figure 1:

exercise, the Planner leaves unemployed workers liquidity constrained to improve job search incentives. But workers when employed accumulate assets to avoid this outcome.

p_U^H denotes the proportion of insured unemployed workers who are "on holiday", which is 18% in this case. This might seem surprisingly large given average assets across the employed are only 0.62 years salary. The distribution of assets, however, is highly skewed reflecting the Kocherlakota underconsumption problem: workers overaccumulate assets to enjoy unemployment holidays. Figure 1 describes the optimal consumption rules $x^E(A)$, $x^U(A)$ with replacement rate 0.6 in the high turnover economy.

Not surprisingly at low A, the optimal consumption levels $x^E(A)$, $x^U(A)$ are low - an employed worker builds up a savings buffer to self-insure against job destruction shocks, while an unemployed worker chooses low consumption in anticipation of the binding liquidity constraint. At intermediate asset levels however, and for $A < A^R =$

7.3 years salary, consumption x^U increases with A but quickly flattens out so that $x^U(.) = x^H$, the holiday level of consumption. Optimal consumption smoothing while employed then implies $x^E(.)$ also flattens out around x^H. The upshot is that the savings rate while employed is a minimum at asset level $A = 0.55$, but is strictly positive. For higher asset levels, the savings rate while employed rapidly increases as A further increases. This yields a fat right tail in the distribution of assets across employed workers - steady state implies a surprisingly large number of employed workers accumulate assets exceeding A^H.[15]

Of course in an ideal world the Planner would raise the replacement rate to $\rho = 1$ and so offer full insurance against job destruction risk. But at $\rho = 0.7$ the moral hazard problems become extreme. The next section now considers optimal UI.

4 Optimal Layoff Insurance.

The problem for an optimal insurance scheme, then, is to improve consumption smoothing between spells of employment and unemployment without inducing holiday-taking or early retirement. There are two standard approaches to the optimal UI problem. The principal agent literature maximises the value of being laid-off (where the employed worker is laid-off at exogenous rate $\delta > 0$) while the macro approach instead maximises a Utilitarian welfare function in a steady state (e.g. Davidson and Woodbury (1997), Frederiksson and Holmlund (2001), Coles and Masters (2006a)). With savings the latter approach is not appropriate. For example a poor choice of replacement rate implies employed workers, in a steady state, have high average savings $\overline{A}_E$. As workers consume the interest on these savings, this implies high average consumption which overstates the value of the UI program. Instead I adopt the layoff insurance problem: the Planner's objective is to maximise $W^E(0|B)$, which is the expected payoff of each new labour market entrant when first employed. Thus the Planner designs the UI program to insure all recently hired new labour market entrants against unemployment risk.

The most natural budget constraint to consider is fair insurance - that expected discounted benefits received while unemployed must equal expected discounted premia paid while employed. This case is considered in section 4.2. An interesting result,

[15] Lise (2006) uses this mechanism in a model of on-the-job search to explain why equilibrium wage dispersion leads to even larger wealth dispersion.

however, is that this program yields a steady state flow budget deficit. Given the political process yields turnover in governments, this suggests a dynamic consistency problem where future governments might renege on the original UI program. It is illuminating, therefore, to first consider optimal layoff insurance when the UI program instead satisfies steady state budget balance.

4.1 Optimal Layoff Insurance with Steady State Budget Balance.

In this section the Planner's objective is to choose $B = \{\pi, B_0, \underline{b})$ to maximise $W^E(0|B)$ subject to steady state budget balance

$$U^I b + [1-U]\delta B_0 = [1-U]\pi.$$

To evaluate the impact of lump sum layoff payments on market outcomes, I first constrain $B_0 = 0$ and suppose the Planner chooses (b,π) to maximise $W^E(0|B)$ subject to steady state budget balance. Table 2 reports the policy optimum and corresponding market outcome for each of the three economies.

Table 2: Optimal constant UI programs with $B_0 = 0$.

	high turnover	low turnover	high unemployment
optimal b	55	52	49
optimal π	2.76	2.37	4.37
A^H	2.70	3.00	3.73
A^R	8.62	9.65	9.98
$\overline{A}_E$	0.59	0.85	1.20
$\overline{A}_U$	0.53	0.79	1.42
$p_U^H(\%)$	0.9	1.9	6.4
$p_U^R(\%)$	0	0.4	1.5
$m_u(\%)$	1.4	2.6	2.3

Consider the high turnover economy as described in column 1. Table 1 established that holiday-taking by the unemployed becomes large at around $b = 60$. Optimal b is 0.55 and budget balance then requires π =2.76% on gross wages. At this replacement rate, A^H is sufficiently high that very few workers take unemployment holidays (p_U^H), and the number reaching early retirement (p_U^R) is negligible. The same insights apply to the LT and HU economies. In those economies the lower re-employment rate implies the effective cost of search, c/γ, is higher. As this increases the magnitude of the moral hazard problem, the Planner reduces the offered b. The stronger precautionary savings motive leads to an increase in savings and greater holiday taking and early retirement.

I now augment the above UI programs with a lump sum severance payment. An obvious benchmark is that the layoff payment fully compensates the worker for his/her drop in permanent income by being laid-off. Given UI payment $\underline{b}$ while unemployed, a fully compensating layoff payment (assuming an unemployed worker always chooses $k = 1$) requires $B_0 = (w - \underline{b})/(r + \gamma)$ where $w = w^G - \pi$. Note that a higher $\underline{b}$ implies a lower compensating payment B_0. Given the level of UI $\underline{b}$ described in Table 2, Table 3 describes the market outcomes when the UI program is augmented with a fully compensating layoff payment and there is budget balance.

Table 3: Fully compensating layoff payment schemes

	high turnover	low turnover	high unemployment
flow UI ($\underline{b}$)	55	52	49
layoff payment B_0	0.10	0.21	0.27
premium π	4.86	4.50	8.21
A^H	2.74	3.10	3.92
A^R	8.59	9.62	9.91
$\overline{A}_E$	0.47	0.63	0.91
$\overline{A}_U$	0.49	0.69	1.1
$p_U^H(\%)$	0.3	0.6	3.6
$p_U^R(\%)$	0	0	0.3

In each case the augmented UI scheme increases welfare. I defer discussion of the magnitude of those welfare effects to the next section. The focus here is on how fully compensating layoff payments change market behaviour.

Column 1 again describes the high turnover economy. Comparing with column 1 in Table 2, note $\underline{b}$ is unchanged at 55 but the fully compensating lump sum layoff payment is $B_0 = 0.10$. As this lump sum is measured in units of one years salary, it equals 5.2 weeks wages (gross). The more generous UI program requires increasing the insurance premium π to 4.86%. But a surprising result is that for each economy, the more generous UI program leads to a fall in unemployment; compared with Table 2, the proportion of unemployed workers taking holidays (p_U^H) and early retirement (p_U^R) are smaller in Table 3.

The lump sum layoff payment not only provides better insurance against layoff

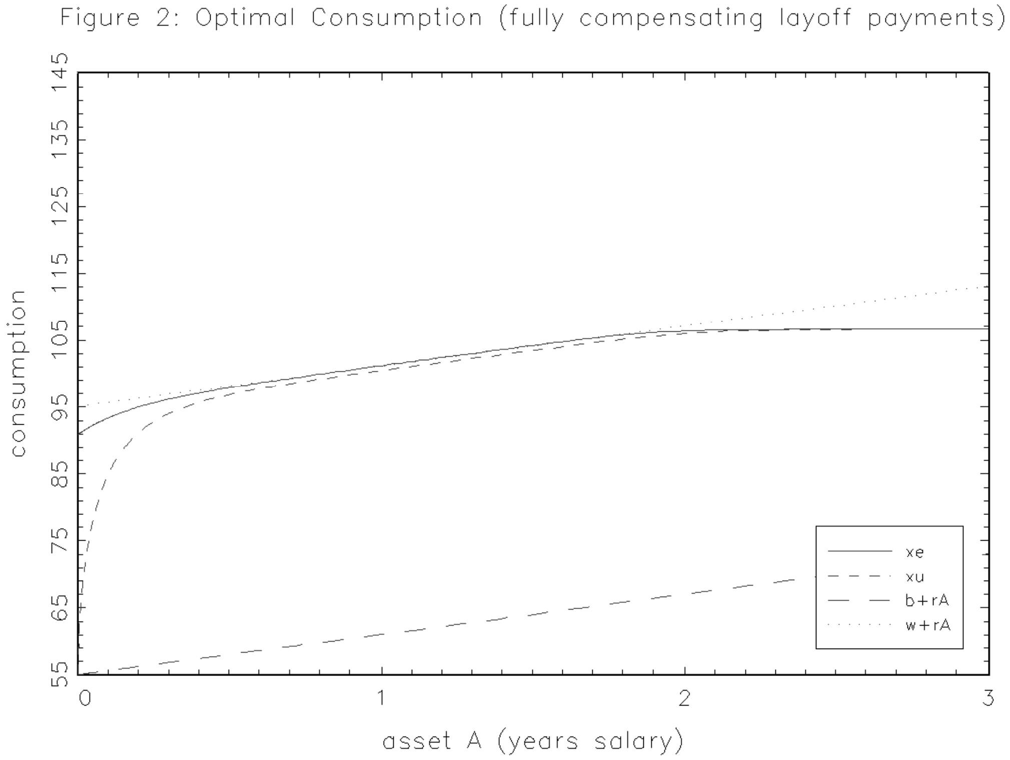

Figure 2:

risk, it also improves search incentives. One reason is re-entitlement effects. Re-entitlement effects arise when UI payments are duration dependent (e.g. Mortensen (1977)) and duration dependence here is extreme: laid-off workers enjoy a lump sum severance payment. Re-entitlement to full insurance through becoming re-employed increases the value of becoming re-employed and so improves search incentives.

Lump sum layoff payments also reduce the incentive of employed workers to over-accumulate assets. Figure 2 describes the optimal consumption rules x^E, x^U given this augmented UI program for the high turnover economy.

In contrast to Figure 1, where savings rates by the employed are always strictly positive and assets potentially grow without bound, a fully compensating layoff payment implies the savings rate is **negative** for assets in the range $A \in [0.8, 1.7]$. Thus individual level assets while employed (if less than 1.7 years salary) revert to $A = 0.8$ years salary.

For a large range of asset values, there is almost perfect consumption smoothing across the job destruction shock; i.e. $x^U(A + B_0) \simeq x^E(A)$. Indeed for assets $A \in [0.8, 1.7]$, $x^U(A+B_0)$ is slightly higher than $x^E(A)$ which is why an employed worker slowly dissaves in this range.

When laid off in this example, the worker receives a lump sum severance payment equal to 5.2 weeks gross income. Roughly speaking, if the unemployed worker gets a job before 13 weeks, then assets A will exceed those held when first laid-off. A longer employment spell instead implies lower assets when re-employed. If the unemployment spell is so long that assets fall below $A = 0.8$, the worker when re-employed chooses relatively low consumption to rebuild his savings buffer back to $A = 0.8$. If instead the unemployment spell is so short that assets exceed $A = 0.8$ (but not 1.7) the worker when re-employed chooses relatively high consumption and allows his savings buffer to decline to $A = 0.8$. Optimal savings behaviour implies savings tend to cycle around $A = 0.8$ years salary. Achieving the holiday level of assets, $A^H = 2.7$ years salary requires several times being laid-off and quickly becoming re-employed so that assets escape the $A \leq 1.7$ years salary barrier. Of course such a sequence is an unlikely outcome and, in a steady state, only 0.3% of the currently unemployed take unemployment holidays.

The above interpretation applies in all three economies. The fully compensating severance payment scheme has three efficiency advantages:

(i) employed workers are fully insured against the drop in permanent income when laid-off;

(ii) re-entitlement effects imply unemployed workers have improved search incentives and;

(iii) the employed have weaker incentives to over-accumulate assets - assets instead tend to cycle about some intermediate value $A^c \ll A^H$.

It turns out that the programs described in Table 3 yield a $W^E(0|B)$ which is remarkably close to the full information benchmark. Numerical findings show it is marginally welfare improving to increase $\underline{b}$ further: the unemployment holiday distortion in Table 3 is relatively small and raising $\underline{b}$ further improves the quality of insurance against re-employment risk. It is also marginally welfare improving to increase further the lump sum layoff payment and so strengthen the mean reversion in asset accumulation. But for these numerical values, the additional welfare improvement is very small.

4.2 Welfare and Fair Insurance.

To identify welfare measures, this section considers the alternative welfare problem: what is the cost of the UI program which yields welfare $W^E(0|B) = W^*$ where W^* is the worker's payoff in the full information benchmark.[16] To be consistent with that benchmark, it is important this cost is measured in terms of fair insurance rather than steady state budget balance.

Let $p(t|B)$ denote the probability the newly hired labour market entrant is unemployed at future date t given program B. The funding shortfall is defined as

$$\text{Shortfall}= \int_0^\infty e^{-rt}\left[p(t|B)\underline{b} + [1-p(t|B)]\delta B_0 - [1-p(t|B)]\pi\right] dt.$$

Note that a fair insurance premium implies the shortfall is zero. But given welfare is set at W^*, the moral hazard problems imply the shortfall is necessarily positive. The difference in shortfalls across different programs then measures their efficiency differences in terms of cost per new labour market entrant. Those costs are reported below in units of annual salary.

$p(t|B)$ depends on worker search effort choice and thus on how the worker's assets evolve over time. Computing shortfall directly requires not only computing the probability distribution of assets $G(A|t, B)$ at each date $t > 0$,but also doing this over the infinite horizon. This is not computationally feasible. An alternative approach is to estimate the shortfall statistically by considering a large number of independent labour market histories. I do this using the following procedure. First I fix a UI program $(\underline{b}, B_0)$. The insurance premium π is then determined by the welfare constraint $W^E(0|B) = W^*$. Given this policy $(\pi, \underline{b}, B_0)$, solving the Bellman equations yields the optimal search and consumption rules. I then generate 100,000 independent labour market histories over a 50 year time horizon. Assuming $p(.)$ at 50 years is close to its ergodic distribution, I can then compute the mean shortfall per worker and its variance. A sample of 100,000 histories yields sufficiently small standard errors that the shortfall is tightly estimated.

I repeat the methodology described in the previous section. For each economy I first restrict $B_0 = 0$ and find the constant UI program $(\pi, \underline{b})$ which generates least shortfall. I then augment that UI program with fully compensating layoff payments.

[16]The full information benchmark implies $\pi = \delta w^G/(r+\gamma+\delta)$, $b = w = (r+\gamma)w^G/(r+\gamma+\delta)$, and $B_0 = 0$. The parameter restriction $c = d$ implies $W^* = (u(w) - d)/r$.

Table 4 reports those policies and their corresponding shortfalls (standard errors are reported in brackets).

Table 4: Optimal UI and Welfare with Fair Insurance

Economy	b	B_0	$premium$	$shortfall$ $\begin{pmatrix} \text{standard} \\ \text{error} \end{pmatrix}$
High Turnover				
	60	0	2.77	0.044 (0.0009)
	60	0.09	4.61	0.030 (0.0010)
Low Turnover				
	59	0	2.29	0.052 (0.0011)
	59	0.18	4.05	0.028 (0.0015)
High Unemployment				
	57	0	4.05	0.094 (0.0016)
	57	0.23	7.19	0.046 (0.0022)

A surprising result is that for each economy, the optimal b in the constant UI case (with no severance payments) is signficantly higher with the fair insurance budget constraint than with steady state budget balance. In the high turnover economy, $b = 60$ is optimal and Table 1 shows that around 18% of the currently unemployed are then on an unemployment holiday. In contrast in Table 2 where $b = 55$ is optimal, less than 1% are on an unemployment holiday. The reason for the difference is that workers taking unemployment holiday are relatively old: a worker must first accumulate assets $A > A^H$ which takes time. The fair insurance constraint discounts those payments heavily - the new labour market entrant is, say, 20 years old and might take an unemployment holiday when 50. In contrast, the steady state budget balance condition does not discount those payments and the Planner keeps the number of workers on unemployment holidays relatively low. In fact the optimal UI program with fair insurance yields a steady state budget deficit. On startup, the fair insurance program generates an initial budget surplus as all workers admitted onto the program are employed and pay premium $\pi > 0$. But turnover of governments through the political process then finds a future government faces a flow budget deficit as many older workers enjoy unemployment holidays. As such governments have the incentive to redesign the UI program to at least yield flow budget balance, the fair insurance case may not be dynamically consistent.

A second important feature of Table 4 is that the shortfall is relatively small. Assuming workers all have these same preferences, then a constant UI program with optimally chosen $\underline{b}$ achieves value which is already close to the full information benchmark. The shortfall is highest in the high unemployment economy and is around 5 weeks income per worker. In each case, augmenting the constant UI program with a fully compensating layoff payment is welfare improving and roughly halves the shortfall. The welfare gain is highest in the high unemployment economy and has value around 2.5 weeks income (per worker). The welfare gain in the high turnover economy is around 0.7 weeks income, reflecting that the market outcome is already close in value to the full information benchmark.

Table 4 describes the optimal UI program for a given worker with particular preferences $u(.), c, d$. But such preferences vary widely across workers, and such preferences are not observed by the government. Suppose instead the Planner chooses b for some other representative worker, and so b is not optimal for our particular worker.

For example suppose the policy-maker believes work disutility d for most workers is appreciably higher than for my representative worker. Suppose then the Planner sets a very low b to reduce average holiday taking; e.g. suppose $b = 20$. Given my worker above, Table 5 reports the resulting shortfall and the welfare value of fully compensating layoff payments.

Table 5. Efficiency Gains with Suboptimal b;b=20

Economy	b	B_0	*premium*	*shortfall*
High Turnover				
	20	0	-0.4	0.22 (0.0003)
	20	0.19	4.1	0.10 (0.001)
Low Turnover				
	20	0	-1.2	0.34 (0.0003)
	20	0.37	3.3	0.14 (0.0013)
High Unemployment				
	20	0	-2.0	0.56 (0.005)
	20	0.47	6.0	0.24 (0.002)

Setting $b = 20$ offers too little insurance against re-employment risk and the efficiency loss, as measured by shortfall, is correspondingly larger.[17] Reflecting that b is very

[17] The premium is sometimes negative in Table 5 as it has to ensure $W^E(0|B) = W^*$.

low, the fully compensating layoff payment in Table 5 is correspondingly high. Note in each case, the fully compensating layoff payment more than halves the shortfall. The welfare gain is highest in the high unemployment economy and is worth around 4 months income per labour market entrant. The welfare gain remains relatively modest in the high turnover economy, worth around 5 weeks income. But comparing results with Table 4, it is particularly noteworthy that choosing b much too low does not yield an immoderate increase in shortfall.

Suppose in contrast the Planner sets b just a little too high, say $b = 65$. An important point here is that the welfare losses are highly asymmetric to the choice in b.

Table 6. Efficiency Gains with Suboptimal b;b=65

Economy	b	B_0	*premium*	*shortfall*
High Turnover				
	65	0	3.16	0.75 (0.0009)
	65	0.07	4.70	0.66 (0.0013)
Low Turnover				
	65	0	2.76	0.72 (0.0012)
	65	0.15	4.18	0.67 (0.0017)
High Unemployment				
	65	0	5.26	1.45 (0.0015)
	65	0.18	7.55	1.41 (0.0024)

In contrast to the previous Table, a small increase in b above the optimum leads to a steep increase in shortfall. Augmenting the UI program with a lump sum severance payment remains welfare enhancing but the gains are not large. This is not surprising as the Planner is already offering 'too much' insurance. But it is important to note that such layoff payments remain welfare improving.

For too low b, increasing b is welfare improving as it provides better insurance against re-employment risk. But the gain through smoother consumption is relatively small. Thus setting a too low b does not yield a large welfare decrease. In contrast for too high b, too many workers take unemployment holidays. At this margin the number taking unemployment holidays increases steeply with b and welfare quickly drops as b is further increased. Given disperse worker preferences which are unobserved by the Planner, this asymmetry suggests it is important to be relatively conservative on the choice of b. Nevertheless regardless of the choice of b, augmenting the UI program with a fully compensating lump sum severance payment is always welfare enhancing

5 Conclusion

By reviewing results on optimal UI with unobserved search effort and hidden savings, the paper identifies that lump sum layoff payments are an important policy tool. Simulations find that co-ordinating constant UI paid, $b(.) = \underline{b}$, with a fully compensating layoff payment $B_0 = (w - \underline{b})/(r + \gamma)$ can yield payoffs which are surprisingly close to the full information benchmark. It has also been shown that welfare losses are highly asymmetric with respect to the choice of $\underline{b}$. For given worker preferences, a $\underline{b}$ which is much too low generates a relatively small welfare loss, while a $\underline{b}$ which is slightly too high generates large welfare losses. In an extended world with heterogeneous worker preferences which are unobserved by the Planner, and where the Planner designs a "one size fits all" UI program, this asymmetric loss structure suggests it is important to keep $\underline{b}$ small. Fully compensating layoff payments will then generate relatively large welfare gains. Establishing this formally, assuming ex-ante heterogeneous agents, is an important issue which is left for future research.

The results identified here are strongly complementary to those found in Pissarides (2004), Fella (2006). Those papers assume the UI program pays constant $\underline{\text{b}}$ and consider optimal contracting between a firm and risk averse employee. In those papers, severance layoff payments are privately optimal. Here instead I have considered optimal UI with hidden search effort and hidden savings but ignored privately optimal contracting between firms and employees. The finding here is that UI payments should be duration dependent, but the dependency is extreme and results in lump sum layoff payments. As public insurance crowds out private insurance, the underlying question is who should provide the lump sum layoff payment? Given publicly provided UI may generate strategic layoff behaviour (e.g. temporary layoffs as seen in the U.S., see Feldstein (1976)), the most likely answer is that severance layoff payments should be made by the firing firm. This suggests the decentralised policy optimum might simply offer constant, but low $\underline{\text{b}}$, anticipating that firms and workers privately negotiate fully compensating severance payments B_0. Of course if firms are risk averse or there is a risk of default (e.g. firing firms might declare bankruptcy), the Planner might pool those risks by insuring firms against such dismissal costs.

A third research direction is that workers enjoying unemployment holidays are (locally) risk neutral even though $u(.)$ is strictly concave. Thus workers when laid off with assets $A \in (A^H, A^R)$ are more willing to gamble than others. In an extended framework, such workers might invest in relatively risky self-employment projects noting that if the project fails, they return to unemployment with reduced assets. Should this happen, they again decide whether to search for work or maybe take a short holiday before looking for work or perhaps, if losses were not too great, invest in another self-employment project. Such an approach seems a promising way forward in explaining the self-employment decision.

Identifying a coherent unemployment policy also requires embedding this insurance problem into an equilibrium matching framework. With equilibrium wage bargaining, more generous flow UI payments $b(.)$ raise the option value of remaining unemployed and so tend to increase negotiated hiring wages. If hiring wages rise too high, job creation rates fall and γ (which describes how easy it is to find work) falls. This makes unemployed workers worse off through the thick market externality. But re-employment bonusses, which are financed through taxes while employed, act as a hiring subsidy. Coles (2006) shows that hiring subsidies target directly the underlying hold-up distortion and so re-employment bonusses potentially play an important policy role in a matching equilibrium.

6 Appendix A

Proof of Theorem 1.

For any given $A > 0$, there are two possibly optimal strategies; (i) a retirement strategy where along the optimal path the worker never searches for a job, and (ii) a holiday strategy where along the optimal path the worker begins search for a job at a finite duration. The text establishes that the value of the retirement strategy, denoted here as $W^R(A; \underline{b})$, is $W^R = u(\underline{b} + rA)/r$. The following describes the value of the optimal holiday strategy, which I denote $W^H(A; \underline{b})$, and compares that payoff against $W^R(.)$. As $\underline{b}$ is fixed, I simplify notation here by subsuming reference to $\underline{b}$ in these functions.

The proof identifies unimprovable strategies and a critical asset thresholds $A^R > 0$.

For $A < A^R$, the holiday strategy is optimal and implies assets strictly decline with duration. Using backward induction from $A = 0$ the following first identifies an unimprovable holiday strategy for assets $A < A^R$. This backward induction process identifies $W^H(A)$ and the iteration stops when the retirement strategy dominates; i.e. where $W^R \geq W^H$. This identifies A^R. I then establish a single crossing property, that $W^H > W^R$ if and only if $A < A^R$. The Principle of Unimprovability [Proposition 4, page 813 in Kreps (1993)] then establishes the Theorem.

First consider $A = 0$. Optimal consumption smoothing implies $x^* = \underline{b}$ (as future income is never lower) and the restriction $u(\underline{b}) < u(w) - d - rc/\gamma$ implies $k^* = 1$ is optimal. Thus policy rules $x^* = \underline{b}$ and $k^* = 1$ are strictly optimal when $A = 0$.

Consider now $A > 0$ but sufficiently small that $k^* = 1$ remains optimal. The optimal consumption path is identified by solving (8),(9),(10) in Claim 1 for $\{x, \mu, A\}$ with $k = 1$. As (8) implies $u''(x)\dot{x} = \dot{\mu}$, use this to solve out μ in (9), and note that optimal (x, A) must then evolve according to the autonomous pair of equations:

$$[-u''(x)]\dot{x} = \gamma[u'(w + rA) - u'(x)] \tag{15}$$

$$\dot{A} = rA - x + \underline{b} \tag{16}$$

while $x, A > 0$. Figure A describes the corresponding phase diagram.

$\underline{b} < w$ implies the $\dot{x} = 0$ locus lies above the $\dot{A} = 0$ locus. Given $k = 1$, Claim 1 implies the optimal consumption path corresponds to the flow line which limits to $(A, x) = (0, \underline{b})$. Let $x = x^U(A; \underline{b})$ denote that path. As $u(.)$ is twice differentiable for all $x \geq \underline{b} > 0$, it follows that x^U is continuous and it is also strictly increasing in A. As x^U cannot cross the $\dot{x} = 0$ locus nor the $\dot{A} = 0$ locus, it also follows that $x^U \in (\underline{b} + rA, w + rA)$ and thus assets A are strictly decreasing with duration. As $(A, x) = (0, \underline{b})$ is not a stationary point in this dynamic system, (A, x) reaches $(0, \underline{b})$ in finite time. This consumption path with $k^* = 1$ determines $W^H(A)$ for small A.

Now define re-employment surplus

$$S(A) = \frac{u(w + rA) - d}{r} - W^H(A)$$

and note that $k = 1$ is optimal only if $S(A) \geq c/\gamma$. The restriction $u(\underline{b}) < u(w) - d - rc/\gamma$ implies $S(0) > c/\gamma$. Differentiating with respect to A implies:

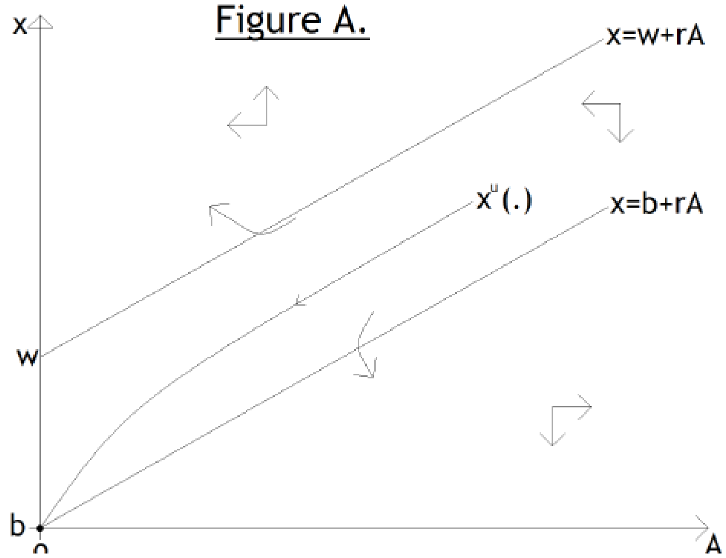

Figure 3: Figure A

$$
\begin{aligned}
\frac{dS}{dA} &= u'(w+rA) - \frac{\partial W^H}{\partial A} \\
&= u'(w+rA) - u'(x^U(A;\underline{b})) < 0
\end{aligned}
$$

along the optimal consumption path $x^U(.)$ and so is strictly decreasing with A. Thus an $A^H > 0$ exists where $S(A^H) = c/\gamma$.

For $A > A^H$ the no holiday constraint fails and optimal job search $k^* = 0$. (9) in Claim 1 then implies μ, and hence consumption, is constant with duration. Let x^H denote that consumption choice. An optimal holiday strategy exists only for A satisfying $x^H > \underline{b} + rA$ so that assets strictly decrease with duration and reach A^H at a finite duration. But optimal consumption smoothing then implies $x^H = x^U(A^H)$ at $A = A^H$. Thus an optimal holiday strategy implies

$$
\begin{aligned}
k &= 1,\ x = x^U(.) \text{ for } A \leq A^H \\
k &= 0,\ x = x^H \text{ for } A > A^H
\end{aligned}
$$

with $x^H = x^U(A^H)$. Strictly declining assets during the holiday phase ($k = 0$) requires assets $A < A^R$ where A^R is defined by $x^H = \underline{b} + rA^R$. As $x^H = x^U(A^H) > \underline{b} + rA^H$ this implies $A^R > A^H$. Further note that as $A \to (A^R)^-$, the holiday phase becomes arbitrarily long in duration and $W^H(A)$ converges in value to the retirement strategy (i.e. consume permanent income $x^H = \underline{b} + rA^H$ and never search). I now compare $W^H(A)$ with the value of the retirement strategy $W^R(A) = u(\underline{b} + rA)/r$.

The above has established $W^H(A^R) = W^R(A^R)$. The Envelope Theorem implies dW^H/dA equals the marginal utility of consumption given the optimal holiday strategy. Furthermore, the definition of A^R (that $A^R = A$ satisfying $x^H = \underline{b} + rA$) implies optimal consumption in the holiday strategy exceeds $\underline{b} + rA$ for all $A \in (0, A^R)$. As $u(.)$ is strictly concave, the slope of W^H is therefore strictly less than the slope of W^R for all $A \in (0, A^R)$. As $W^H = W^R$ at $A = A^R$ then $W^H > W^R$ for $A < A^R$. Thus the holiday strategy dominates the retirement strategy for all such A. Further for $A > A^R$, consuming holiday consumption x^H indefinitely is strictly dominated by the retirement strategy where consumption instead equals permanent income $\underline{b}+rA > x^H$.

By construction, these strategies (as described in the Theorem) are unimprovable strategies. To establish that they describe an optimal strategy, I now use the Principle of Unimprovability. Note that in an optimal savings strategy, it is never optimal to consume $x < \underline{b}$ as $\underline{b}$ is a lower bound on future income. Hence there is no loss in generality by imposing the additional restriction $x \geq \underline{b}$. In this extended case, the restrictions $x \geq \underline{b}$ and $k \in \{0, 1\}$ imply the "one period" payoff function $u(x) - ck$ is bounded below by $u(\underline{b}) - c$. Thus for $\underline{b} > 0$, the Principle of Unimprovability implies Theorem 1.

7 Appendix B: steady state asset distributions.

Claim B. Given a constant UI program $b(.) = b$ with lump sum layoff payment $B_0 \in [0, A^H]$, steady state turnover implies:
(i) the number of unemployed workers $U = U^{NI} + U^I$ where

$$\begin{aligned} U^I &= \frac{\gamma}{\lambda + \gamma} \frac{\delta}{\delta + \lambda + \gamma G^U(A^H)} \\ U^{NI} &= \frac{\lambda}{\lambda + \gamma} \end{aligned}$$

(ii) $G^E(.)$ satisfies the differential equations

$$(\lambda + \delta)G^E(A|.) + (rA - x^E(A|.) + w)\frac{dG^E}{dA} = \frac{\gamma U^{NI} + \gamma G^U(A|.)U^I}{1 - U} \quad \text{for } A < A^H$$

$$(\lambda+\delta)G^E(A|.) + (rA - x^E(A|.) + w)\frac{dG^E}{dA} = \frac{\gamma U^{NI} + \gamma G^U(A^H|.)U^I}{1-U} \text{ for } A > A^H$$

subject to the initial value $G^E(0|.) = 0$;

(iii) $G^U(.)$ satisfies the differential equations

$$(\lambda+\gamma)G^U(A|.) + (rA - x^U(A|.) + b)\frac{dG^U}{dA} = \frac{1-U}{U^I}\delta G^E(A-B_0|.) \text{ for } A < A^H$$

$$\lambda G^U(A|.) + \gamma G^U(A^H|.) + (rA - x^U(A|.) + b)\frac{dG^U}{dA} = \frac{1-U}{U^I}\delta G^E(A-B_0|.) \text{ for } A \in (A^H, A^R)$$

$$\lambda G^U(A|.) + \gamma G^U(A^H|.) = \frac{1-U}{U^I}\delta G^E(A-B_0|.) \text{ for } A \geq A^R.$$

Proof:

(i) Steady state turnover implies the number of uninsured unemployed workers satisfies

$$\lambda = (\gamma+\lambda)U^{NI},$$

while the number of insured unemployed workers satisfies

$$\delta[1-U] = U^I[\lambda + \gamma G^U(A^H)].$$

Using $U = U^I + U^{NI}$ then implies Claim B(i).

(ii) Consider the pool of employed workers with assets no greater than $A > 0$. Over arbitrarily small time period $\Delta > 0$, the number who exit this pool is:

$$\text{outflow} = [1-U]\left[G^E(A|.)(\lambda+\delta)\Delta + [G^E(A|.) - G^E(A'|.)]\right]$$

where $A' < A$ satisfies $A = e^{r\Delta}[A' - x^E(A'|.)\Delta + w\Delta]$. The first term describes the outflow through layoff or death, the second describes exit through asset accumulation, where employed workers with assets in $(A', A]$ have assets strictly greater than A after further time period Δ. The inflow for $A > 0$ is

$$\begin{aligned} \text{inflow} &= [\gamma U^{NI} + \gamma G^U(A|.)U^I]\Delta \text{ for } A \leq A^H \\ &= [\gamma U^{NI} + \gamma G^U(A^H|.)U^I]\Delta \text{ for } A > A^H \end{aligned}$$

Setting inflow equal to outflow and letting $\Delta \to 0$ implies Claim B(ii). There can be no mass point in G^E at $A = 0$ as optimal consumption implies $x^E(0) < w$ and assets $A' > 0$ in the next instant. Thus $G^E(0) = 0$.

(iii) Consider the pool of insured unemployed workers with assets no greater than A. Over arbitrarily small time period $\Delta > 0$, steady state turnover for $A < A^H$ implies

$$[1-U]G^E(A-B_0|.)\delta\Delta + U^I\left[G^U(A'|.) - G^U(A|.)\right] = U^I G^U(A|.)(\lambda+\gamma)\Delta$$

where $A' > A$ satisfies $e^{r\Delta}[A' - x^U(A'|.)\Delta + b\Delta] = A$. The left hand side describes the inflow through job destruction and through insured workers dissaving over time where insured unemployed workers with assets in $[A, A')$ have assets strictly less than A after further time period Δ. The right hand side describes the outflow through finding work or death.

For $A \in (A^H, A^R)$ steady state turnover implies:

$$\begin{aligned} & [1-U]G^E(A-B_0|.)\delta\Delta + U^I\left[G^U(A'|.) - G^U(A|.)\right] \\ = \ & U^I G^U(A^H|.)(\lambda+\gamma)\Delta + U^I[G^U(A|.) - G^U(A^H|.)]\lambda\Delta \end{aligned}$$

where insured unemployed workers with $A > A^H$ only exit through death. Finally for $A \geq A^R$ the steady state turnover equations are

$$[1-U]G^E(A-B_0|.)\delta\Delta = U^I G^U(A^H|.)(\lambda+\gamma)\Delta + U^I[G^U(A|.) - G^U(A^H|.)]\lambda\Delta$$

as workers do not dissave over time in the early retirement phase. Letting $\Delta \to 0$ implies the conditions in Claim B(iii).

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