Do I have what it takes?
Equilibrium Search With Uncertainty About the Self

Armin Falk
IZA, Bonn
University of Bonn
CEPR

David Huffman
IZA, Bonn

Uwe Sunde*
IZA, Bonn
University of Bonn
CEPR

October 23, 2006

Abstract

This paper presents a general equilibrium model of the labor market in which an unemployed worker's decision to stop search, and enter a state of nonparticipation, is made in a non-stationary environment. Nonstationarity arises because each time a worker decides to search, and is unsuccessful, this decreases the worker's self-confidence, and subjective belief about the probability of future success. This nonstationarity gives rise to structural flows from unemployment to nonparticipation in equilibrium. In contrast, existing search models typically appeal to stochastic shocks to generate transitions from unemployment into nonparticipation. The behavioral assumptions in our model regarding uncertainty and updating of beliefs are microfounded in experimental evidence from a companion paper. Other important implications of the model include a novel channel through which unemployment duration leads to decreasing starting wages: unemployment duration reduces self-confidence, and thus affects workers' threat points in wage bargaining. The role of self-confidence in job search also points to a different way in which variables such as gender, race, and age may affect search behavior, and a way in which unemployment may damage self-esteem and mental health. Importantly, because search outcomes are only a noisy signal about ability in our model, some individuals can become overly discouraged and stop search too early due to wrong beliefs. We discuss how the model provides a new, unifying explanation for empirical evidence on search behavior and unemployment.

JEL-Classification:

Keywords: Learning, Discouraged Workers, Subjective Job Finding Probabilities

*Corresponding Author. Contact: sunde@iza.org

1 Introduction

This paper presents a general equilibrium model of the labor market in which an unemployed worker's decision to stop search, and enter a state of nonparticipation, is made in a non-stationary environment. Nonstationarity arises because each time a worker decides to search, and is unsuccessful, this decreases the worker's self-confidence, and subjective belief about the probability of future success. In contrast, most existing models maintain stationarity, and instead use exogenous stochastic shocks to generate transitions into nonparticipation, for instance shocks to the value of household production, or shocks to the job-finding rate. The particular mechanism highlighted in our model, changing self-confidence, has not been discussed in the search literature, and in fact is ruled out in standard search theory by the assumption that workers know, with certainty, their objective abilities. If uncertainty and learning are modelled at all in search models, it is usually uncertainty regarding market conditions, e.g., the shape of the wage offer distribution. In contrast, our model assumes that individuals are uncertain about *themselves* when they enter the search process, lacking knowledge about their abilities relative to those of competing applicants. We also assume that workers rationally update beliefs about themselves based on search outcomes. These behavioral assumptions are consistent with findings from a companion experiment on search and learning, which we survey below when we introduce our model (Falk, Huffman and Sunde, 2006).

The model makes various novel predictions, in addition to the key prediction that unemployment duration erodes self-confidence and willingness to continue search. For example, because search outcomes are only a noisy signal of ability, beliefs about own ability need not converge to the truth in the short run. Thus, in contrast to standard search models, workers may search too long, or may give up on search too early, due to wrong beliefs about the self. Importantly, once a worker stops search, she receives no new information, and thus never realizes her mistake. The fact that search decisions depend on self-confidence also leads to a new channel through which variables such as gender, or race, may affect search behavior. If self-confidence is lower for women, or racial minorities, this leads to faster discouragement, and greater rates of nonparticipation, consistent with empirical evidence on gender and racial differences in labor market outcomes. Adding firms and wage bargaining contributes additional insights, which are hard to anticipate without a general equilibrium model. This includes a novel way in which unemployment duration affects starting wages: falling self-confidence lowers workers' threat points in wage bargaining, and thus leads to a wage profile that is decreasing with unemployment duration. This also shows how a gender difference in confidence could translate into a gender difference in wages. A possible extension to the model would be to allow for beliefs about own ability to directly affect utility, such that workers lose utility when they receive negative information about the self. This could help explain empirical evidence that unemployment duration affects self-esteem, and mental health, even controlling for the drop in income associated with losing a job.

In the model there are two types of individuals, high and low. High types have a sufficiently high probability of receiving a job offer such that search is always worthwhile if an individual is certain of being a high type. Low types have a sufficiently low probability of success such that it is never worthwhile to search. The model builds on a single intuitive generalization of the standard model: instead of having perfect self-knowledge, individuals do not know their own type with certainty. As a result, search decisions are based on a subjective probability of success, which arises from the searcher's subjective beliefs about her type. Individuals update their beliefs about their type rationally based on successful or unsuccessful search outcomes. We show that for a given prior there is a unique cumulative unemployment duration after which it is rational for an individual to stop search, even if she is a high type, because subjective beliefs have converged sufficiently towards certainty of being a low type. This gives rise to endogenous, rational inactivity or discouragement after sufficient duration of unemployment.[1]

On the demand side, firms post vacancies optimally given their expectations about wages. Wages are determined once a match occurs based on Nash bargaining. However, individuals' subjective beliefs about their type affect their threat point in the wage negotiations. This implies that in equilibrium the wage an individual can earn decreases endogenously with her unemployment duration. In equilibrium, wages, the level of unemployment, the number of vacancies, and the unemployment duration threshold for discouragement are all determined endogenously. Everything else equal, both the equilibrium level of unemployment and the share of inactive unemployed are increasing in unemploy-

[1] In a less extreme form, this result can be interpreted as individuals deciding to stop search for a particular kind of job, rather than giving up on job search altogether, and redirecting efforts towards labor market segments that are less demanding in terms of ability but offer higher job finding chances.

ment benefits and decreasing in productivity. Moreover, in terms of responsiveness to variations in productivity, such as cyclical shocks, the model predicts a higher variability of unemployment than a standard model without uncertainty. This arises due to an additional feedback effect through the discouragement margin.

The model provides a new perspective on the empirical literature on search and nonparticipation. For example, it can explain the frequently documented pattern of falling exit rates from employment to employment as unemployment duration increases, and the tendency for wages to fall with unemployment duration (*e.g.*, Machin and Manning, 1999; Frijters and van der Klaauw, 2006). Leading explanations in the literature include nonstationarities arising from human capital depreciation, or employer stigma. Although these are plausible explanations, they have not been modelled explicitly, and there is little direct empirical evidence for either (*e.g.* the discussion in Machin and Manning, 1999). On the other hand, falling self-confidence is an alternative explanation that has not been discussed, and is supported empirically by our companion paper, where individuals exhibit substantial uncertainty about ability even in a very simple experimental setting. Notably, human capital depreciation and stigma are ruled out in the experiment by design.

To our knowledge Frijters and van der Klaauw (2006) is the only empirical paper to estimate a structural model with nonparticipation and nonstationarity. They distinguish between nonstationarity in the job-offer arrival rate and the wage offer distribution. Using German data they find that negative duration dependence is mainly evident for wages, with only weak evidence of negative duration dependence in job-offer probabilities. These findings are consistent with our model, which predicts no change in the objective probability of receiving a job offer, but falling wages due to changing subjective beliefs. It is less clear to what extent human capital depreciation or stigma can rationalize the lack of a decline in the job-finding probability. Interestingly, Frijters and van der Klaauw (2006) also find some evidence of a gender difference, although samples are too small for this to be robustly significant: women exhibit stronger negative duration dependence than men. This gender difference is difficult to explain with stigma or human capital depreciation; our model offers one potential explanation, which is lower self-confidence among women.

Our model also leads to a different perspective on what it means for a worker to be in the state of nonparticipation rather than unemployment.[2] It highlights the possibility

[2] For some early studies showing that these states are behaviorally distinct, for example because exit

that some nonparticipants may in fact be high ability types who have wrong beliefs. These individuals are objectively suited for work and are appropriately considered as part of the problem of involuntary unemployment, in contrast to definitions of unemployment based on the criteria of active search.[3] Notably, this perspective on nonparticipation matches the notion of discouraged workers identified in US labor force surveys. People are labelled as discouraged workers in these surveys if they state that they would like a job, but are not looking for one because they believe that their job finding chances are too low. Our model emphasizes that individuals reach this state after a personal experience of unsuccessful search depresses their subjective job-finding probability. One long-standing puzzle has been why discouraged workers exhibit only a weak response to improvements in aggregate conditions (McElhattan, 1980; Jones and Riddell, 1999). One explanation has been that these workers do not actually want a job, despite what they say in the survey. However, this discussion has assumed that people infer their personal job-finding chances mainly from aggregate conditions, whereas our model emphasizes that people pay attention to their own personal search histories. In our framework, even if someone does respond to a shock to aggregate conditions by resuming search, it may take only a few failures to push them back below the nonparticipation threshold again. Taken together, the view of job search provided by our model has consequences for policy interventions to reduce (long-term) unemployment. Rather than training measures designed to address a lack of objective ability, our model suggests early interventions that assist the unemployed in their search efforts and foster their subjective perception to be able to find new employment.

The model also suggests one explanation for why unemployment may be a psychologically painful process. Although we do not explicitly model an impact of positive beliefs about the self on utility, a plausible extension would be to allow for this type of "ego utility"(Köszegi, 2001 and 2006). In this case unsuccessful search would have an additional, psychological cost because it leads to a less positive self-perception and thus lower utility. This approach would move in the direction of linking search theory with evidence on the psychological costs of unemployment, which include declines in subjective well-being, mental health, and self-esteem, which are present even after controlling for loss of income

rates into employment are significantly lower for nonparticipants, see Flinn and Heckman, 1983; Blau and Robins, 1986; or Coleman, 1989)

[3] Low types who keep on searching because of uncertainty could also be seen as involuntarily unemployed in this context, since under perfect information they would immediately opt for nonparticipation.

(see, *e.g.*, Gerdtham and Johannesson, 2003; Bjorklund and Eriksson, 1998; Mathers and Schofield, 1998; and Winkelmann and Winkelmann, 1998). Standard search theory has little to say about these important social costs of unemployment.

Our paper contributes to a relatively small theoretical literature that models the nonparticipation decision in an environment with search frictions and unemployment. For the most part these models are based on individual heterogeneity with respect to the value of leisure or home production. Individuals decide about their labor market participation, and flows between unemployment and non-participation are driven by exogenous changes in the labor market environment, such as cyclical variations.[4] Only two recent general equilibrium search models, by Garibaldi and Wasmer (2005) and Pries and Rogerson (2004), are able to generate permanent structural flows between activity and inactivity, without changes in aggregate labor market conditions. These structural flows are caused by exogenous shocks to the individual productivity in home production, or exogenous shocks to the individual costs of labor market participation, respectively, rather than exogenous changes in the aggregate environment.[5] However, to our knowledge, there is no theoretical equilibrium search model that models transitions from unemployment to inactivity as the outcome of a nonstationary environment on the individual level, rather than of an exogenous stochastic variation. Nor is there a theoretical framework that can rationalize the empirical findings of negative duration dependence in job contacts and wages.

Our paper also relates to several recent theoretical contributions that consider uncertainty in a search framework. Andolfatto *et al.* (2004) consider an environment where individuals are uncertain about their abilities, but their focus is on modelling ego utility, or preference for positive beliefs about the self, and for this purpose they focus on a simple partial equilibrium framework with only two-periods. Our aim is different, namely to nest uncertainty about the self in a general equilibrium search framework. This allows us to investigate whether firms' vacancy posting or the wage determination have effects

[4] Examples for models in this vein are Bowden (1980), McKenna (1987), Pissarides (2000, chapter 7) and Sattinger (1995).

[5] In another recent paper, Rosholm and Toomet (2005) investigate the role of unemployment benefits for transitions from unemployment to non-participation. In their partial equilibrium model, discouragement arises as consequence of shocks to the job offer arrival rate. In a different context, focused on the role of active labor market programs in a union wage-setting framework, Calmfors and Lang (1995) consider exogenous shocks to psychological well-being, which generate discouragement and negative duration dependence.

that overturn the results under partial equilibrium, and to provide a behaviorally founded equilibrium search model in the spirit of Frijters and van der Klaauw (2006). A different group of papers studies uncertainty and learning in search, but focuses on learning about market conditions, such as the shape of the wage distribution (Morgan, 1985; Burdett and Vishwanath, 1988; Flam and Risa, 2003; Dubra, 2004). Our paper is different in that it highlights the role of uncertainty about the self, rather than market conditions. Moreover, we view individual unemployment history as natural candidate for explaining the observed heterogeneity in individuals' behavior. In addition, and in contrast to the other literature, the learning process embedded in our model is micro-founded on the basis of experimental evidence.[6]

The paper is organized as follows. In section 2 we highlight findings from our companion paper that motivate new behavioral assumptions regarding search behavior. In section 3 we develop a general equilibrium search model that incoporates these new assumptions. Section 4 concludes.

2 Some Evidence on Uncertainty and Search Behavior

In order to motivate our equilibrium search model, we briefly survey some of the main findings from a companion paper, in which we conducted a laboratory experiment on learning and search (Falk, Huffman, and Sunde, 2006). In the experiment we tested whether people are in fact uncertain about relative abilities, and studied how this uncertainty affects decisions in a search environment.

In the experiment, subjects first took part in a math test. Subjects were compensated for each correct answer according to a piece rate. After the test, subjects were informed about their score, but not the scores of the other subjects in the room. Subjects then participated in eight rounds of a search process, which involved real stakes. In each round, they were given 80 points, and could decide whether to keep the 80 points or invest them. If they invested the points, they could win 200 or they could win nothing. Importantly, the probability of winning 200 depended on their relative performance on

[6] There are previous job search experiments that study search behavior when the wage or price distribution is unknown (*E.g.*, Hey, 1982 and 1987). This uncertainty is imposed rather than related to individual ability, however, and these experiments do not elicit beliefs directly, limiting the possibilities for verifying whether and how individuals update beliefs about wage distributions.

the initial math test. If they had scored above the median they were a high type, with a 60 percent chance of winning 200. If they had scored below the median they were a low type, with only a 30 percent chance of winning. These parameters were chosen so that a risk neutral individual would always search given certainty of being a high type, and never search if certain of being a low type.

Figure 1 shows that people are uncertain about their relative abilities.[7] Between the math test and the first period of search, we asked subjects how likely they thought it was, in percentage terms, that they had scored better than half of the other subjects in the room. The top panel of Figure 1 shows the distribution of subjective beliefs of all participants. If subjects knew with certainty whether they were a high or a low type, the distribution should be bi-modal, with half of the mass at 0 at half at 1. Instead, the distribution is nearly uniform. Subjects did have some idea of their type, as shown by the bottom two panels: the beliefs of low types and high types are clearly different, in the right direction. There is still substantial uncertainty, however. For instance, some individuals who are absolutely certain that they are of one type are absolutely wrong. Given that the environment in our experiment is relatively simple, involving a known number of competing applicants, and only a single skill dimension, these findings provide a strong empirical basis for assuming that job searchers in the real world are also uncertain of their relative abilities.

The second main finding of the experiment is that subjects change their beliefs about themselves based on search outcomes, in the direction predicted by rational updating. We asked subjects about the likelihood of being the high type after each subsequent round of search. Regressions (not shown) reveal that the most important determinants of an individual's beliefs in period t are the belief in $t-1$ and an indicator for whether search was successful or unsuccessful in $t-1$. Following an unsuccessful round of search, beliefs about the probability of being the high type drop significantly.

The third main finding of the experiment is that belief updating has an impact on subsequent search decisions. We find that beliefs in t are the most important determinant of whether an individual decides to search in period $t+1$, dominating demographic factors and risk preferences.

[7] In the experiment, subjects were informed about own absolute performance on the test, and so uncertainty about relative ability could come only from uncertainty about the abilities of others. Thus the degree of uncertainty about relative ability observed in the experiment is a lower bound.

In summary, the experiment provides an empirical basis for assuming that job searchers are uncertain of their own relative abilities. Furthermore, the data show how this uncertainty plays out in a search environment: people respond to search outcomes by updating beliefs about themselves, and this updating has an impact on subsequent search decisions.

The experiment also provides two additional results, which suggest possible extensions to our model. One is a gender difference in initial priors: women are significantly less confident about being the high type before search begins, even though they perform better on the test than men on average. A second finding suggests that the process of search may be psychologically painful. At the end of the search process, after learning whether they were in the top or bottom half, subjects were asked whether they wanted to know their exact rank on the math test. Although this information was free, roughly 30 percent of individuals declined to learn this information. In contrast to the null hypothesis that information aversion is random with respect to performance on the test, those who turned down information were overwhelmingly those who had scored in the bottom half. This finding suggests that people have a strong aversion to learning that they are one of the worst in terms of ability.

3 An Equilibrium Search Model with Type Uncertainty

In this section we construct a general equilibrium model of the labor market. We develop the model incrementally, beginning by describing the behavior of workers and then characterizing the steady state, without adding the complexity of firm behavior, or wage determination. This portion of the model makes it possible to analyze the implications of our alternative assumptions in a partial equilibrium. We then proceed to add the firm side of the market and wage bargaining, and allow for endogenous wages and vacancy posting to close the model. We conclude the section by characterizing equilibrium in the full model, and performing comparative statics exercises.

3.1 Workers

In our model, workers are either unemployed or employed. Unemployed workers decide in each instant whether to search actively for a job or not. Active search entails a flow cost of c, and with some probability leads to a job offer. Passive, or inactive search is costless but results in a job-finding probability of zero.[8] Employed individuals exhibit a flow productivity y and are assumed not to search while they have a job. Regardless of whether they are unemployed or employed, workers leave the labor force at a rate ρ, due to death or retirement. New workers are born, and enter the pool of unemployed at the same rate ρ.[9] Workers discount the future at rate r.

We depart from a standard model by assuming that individual workers do not know their objective job-finding probabilities with certainty. This assumption is motivated by our finding that people are uncertain about their relative ability even in the simple environment implemented in our laboratory experiment. This type of uncertainty is likely to be even more pronounced in real labor market decisions, given that these involve a much less transparent pool of competing applicants, and multiple skill dimensions rather than only one. Under the reasonable assumption that job-finding probabilities in the labor market are related to relative attributes, this uncertainty about the self leads to uncertainty about one's job-finding probability. For simplicity, we assume that there are only two types of workers in the market, high and low, with different job finding rates λ^h and λ^l, respectively. The precise job-finding rates are determined in equilibrium, and are increasing in labor market tightness, θ, as discussed below. However, we impose the restriction $\lambda^h > \lambda^l$, *i.e.*, the high type has a relatively good chance of finding a job while the low type has a relatively poor chance. In fact, we will assume below that search is always worthwhile for a worker who knows with certainty that she is a high type, and never worthwhile for a worker who knows she is a low type. We normalize the labor force to one and assume that a fraction $q^l = q \in (0,1)$ are low types, with the remaining fraction $q^h = (1-q)$ consisting of high types. The objective job-finding probabilities for high and low types, and the fraction of high and low types in the population, are both common knowledge to workers and firms.

The key assumption of the model is that workers do not know their true type with certainty. They are assumed to have an initial prior abut the likelihood of being a high

[8] Alternatively we could allow for a non-zero job-finding probability for passive searchers, without changing the main insights of the model as is discussed below. Without costs for search, however, individuals would never stop searching.

[9] An alternative interpretation would be that employed workers lose their jobs and re-enter the pool of unemployed with rate ρ. This could reflect exogenous job destruction due to bankruptcies or plant closings, putting individuals in the situation of having to start searching for new jobs in a new environment.

type, equal to the fraction of high types in the population, $p^h(0) = q^h$. We assume that firms are also uninformed about a worker's true type, because this simplifies the analysis considerably. In particular, it precludes the possibility that firms exploit an information asymmetry in wage bargaining with workers. We also assume that productivity on the job, and therefore the wage, does not depend on whether a worker is a high or low type. In this case types can still be interpreted as reflecting differences in relative ability, but ability in the search process itself, *e.g.*, in writing resumes or interviewing, rather than in the workplace.[10] As will become clear below, this simplifying assumption does not imply that there is only a single wage in the market. Rather, a worker's wage will depend on the worker's level of confidence about being the high type, through wage bargaining. Thus there will be a distribution of wages across individuals, but due to different confidence levels rather than due to differences in randomly drawn productivity as is typically the case in search models.

A worker's type plays a crucial role in her choice of whether to search actively or not, because it affects the expected value of search. In the model, the only information that a worker can use to infer something about her true type is the cumulative time, t, she has spent unemployed so far. We assume that workers use Bayes' rule to update beliefs, so that their subjective belief about the likelihood of being the high type is decreasing in the length of unsuccessful search t. This assumption is in line with the results of our experiment, in the sense that people were observed to update beliefs in a way that is qualitatively consistent with Bayes' rule.[11] Below, we will also impose the restriction that the initial prior is high enough that all individuals find it worthwhile to search in the initial period, regardless of their type. This ensures that some portion of the population finds it optimal to search.

Formally, the worker's subjective belief about the probability of being the high type, conditional on t, is given by the continuous time version of Bayes' rule

$$p^h(t,\theta) = \frac{e^{-\lambda^h(\theta)\cdot t}\cdot(1-q)}{e^{-\lambda^h(\theta)\cdot t}\cdot(1-q)+e^{-\lambda^l(\theta)\cdot t}\cdot q}\,, \tag{1}$$

[10] This assumption eliminates additional complications of complicated reservation wage strategies of workers. For a model considering this type of non-stationarity, see Burdett and Vishwanath (1988).

[11] In fact, people were somewhat too conservative in their updating in our experiment. For simplicity we maintain the assumption of fully-rational updating, but in principle it is possible to modify the model, replacing Bayes' rule with a more conservative updating rule, in which case the main results are unchanged except that updating occurs more slowly.

where $p^h(0) = 1 - q$ is the prior belief about the probability of being a high type. A longer cumulative unemployment duration t unambiguously implies a lower subjective belief of being a high type.[12] Hence, it takes some time for a low type with a high initial prior to become certain that she is the low type, and similarly, a run of bad luck can convince a high type that she is a low type. Given her subjective beliefs about the probability of being a high type, the worker's subjective job-finding probability is simply the weighted average of the job-finding probabilities of high and low types

$$\widetilde{p}(t,\theta) = \lambda^l(\theta) + p^h(t,\theta) \cdot \left(\lambda^h(\theta) - \lambda^l(\theta)\right) . \tag{2}$$

This subjective job-finding probability is clearly also decreasing in t.[13]

We can now characterize the unemployed worker's decision problem formally. Let $U(t)$ denote the expected discounted value of being unemployed for a worker with unemployment duration t, and let $W(t)$ denote the expected discounted value of a newly found job, which also depends on t through subsequent search behavior in the case that the job is destroyed. When unemployed, the worker receives an unemployment benefit, b, which is independent of time and unemployment duration. The benefit level is assumed to be sufficient to cover the worker's basic needs, but to be sufficiently low for her to accept any job offer she might receive, i.e. $W(t) - U(t) > 0$. The value of unemployment for a worker with history t can then be characterized as

$$(r+\rho)U(t) = b + \max\left\{\widetilde{p}(t,\theta) \cdot (W(t) - U(t)) - c + \dot{U}(t), 0\right\} , \tag{3}$$

where the expressions in curly brackets give the value of searching actively and searching

[12] By taking partial derivatives, we have

$$\frac{\partial p^h(t,\theta)}{\partial t} = \frac{\left(\lambda^l(\theta) - \lambda^h(\theta)\right) q(1-q)e^{-(\lambda^h(\theta)+\lambda^l(\theta))t}}{\left(e^{-\lambda^h(\theta)\cdot t} \cdot (1-q) + e^{-\lambda^l(\theta)\cdot t} \cdot q\right)^2} < 0$$

since $\lambda^l(\theta) < \lambda^h(\theta)$ for any θ.

[13] The model could be extended by relaxing the assumption that employment is an absorbing state that can only be left by retirement or death, and assuming instead that employment relationships split up with some probability δ. Such a model would complicate the analysis as one would also have to consider the informational content of the number of previous employment spells n of an individual with cumulative unemployment duration t. The equilibrium would have to be characterized by optimal policies for all combinations t and n and the respective consistent distributions, without altering the main insights of the model. Alternatively, to avoid a two-dimensional state space, one could assume perfect recall, i.e. that a worker, when being displaced, recalls the duration of unsuccessful search t that she had right before finding her job, or that the individual starts over with $t = 0$ each time she loses her job. Neither extension is essential for the basic insights of the model, namely that individuals with longer unemployment history have lower beliefs about themselves. We therefore stick to the simplest case in which individuals remain employed until retirement.

passively, respectively. The last expression, $\dot{U}(t)$, is the derivative with respect to unemployment duration t and reflects the decline in utility associated with another instant of unsuccessful search, as discussed in detail below.

Clearly, the unemployed worker will choose to search actively as long as the expected benefit from doing so exceeds the utility of passive search. This implies that there exists a critical subjective job-finding probability such that the individual is just indifferent between searching and not searching. We assume that high types always find it worthwhile to search, given full information about their type, and that low types never find it worthwhile to search, which is equivalent to $\lambda^h(\theta) > \widetilde{p}(T,\theta) > \lambda^l(\theta)$, where the critical probability $\widetilde{p}(T,\theta)$ is given by

$$\widetilde{p}(T,\theta) = \frac{c}{W(T) - U(T)}. \tag{4}$$

For each individual this probability is associated with a threshold duration of unemployment, $T < \infty$, sufficient to reduce their confidence about being the high type to the point that their subjective job-finding probability is equal to $\widetilde{p}(T,\theta)$. We will refer to unemployed workers with $t > T$ as discouraged workers, *i.e.*, workers whose personal search history has lead them to revise their subjective beliefs downwards, to the point where they no longer see a point in searching.[14] Note that high types should always search, by assumption, but will become discouraged and stop search if they are sufficiently unlucky to remain unemployed for more than T. Low types should never search, but will search until they reach T, assuming that they were sufficiently uncertain about their type to justify search initially.

To show that T exists and is unique for each individual, we use the fact that workers process no new information once they become discouraged and stop search. This implies that they have stationary values for $U(t) = \underline{U}$ and $W(t) = \underline{W}$ for all $t \geq T$, where $\underline{U}$ satisfies

$$(r+\rho)\,\underline{U} = b\,, \tag{5}$$

and $\underline{W}$ is discussed below. Active searchers with durations $t \leq T$, on the other hand, face a value of being unemployed of

$$(r+\rho)\,U(t) = b - c + \widetilde{p}(t,\theta)\,(W(t) - U(t)) + \dot{U}(t)\ . \tag{6}$$

[14] Note that once individuals become discouraged they stop search and thus receive no new information. This implies $\dot{U}(t) = 0$ for all $t \geq T$.

Optimality of T implies $U(T) = \underline{U}$ and $W(T) = \underline{W}$, which can be used to rewrite condition (4) as

$$\widetilde{p}\,(T,\theta) = \frac{c}{\underline{W} - \underline{U}}\;. \tag{7}$$

In a later section, we discuss the process of wage formation in detail, which is based on Nash bargaining between workers and firms. Bargaining occurs after a firm and a worker have met, and all characteristics of the match have been revealed, in particular t for the worker. For the moment, let $w(t,\theta)$ denote the wage of a worker that has an unemployment history of cumulative duration t, given an aggregate labor market characterized by θ. The value of being employed for a worker with cumulative unemployment duration $t \leq T$ can then be characterized as

$$(r+\rho)\,W\,(t) = w(t,\theta) + \dot{W}(t)\;, \tag{8}$$

while the value for discouraged workers with $t > T$ is given by

$$(r+\rho)\,\underline{W} = w(T,\theta)\;. \tag{9}$$

Using the value of being unemployed for discouraged workers given by (5), one obtains

$$\underline{W} - \underline{U} = \frac{w(T,\theta) - b}{r+\rho}\;, \tag{10}$$

which then makes it possible to express the critical condition for the subjective job offer arrival rate as

$$\widetilde{p}\,(T,\theta) = \frac{c}{w(T,\theta) - b}(r+\rho)\;. \tag{11}$$

This condition is crucial in the model, implicitly determining the maximum cumulative unemployment duration T after which individuals stop searching actively for employment. In partial equilibrium, i.e. taking the wages as exogenously given, existence follows directly from condition (11) and the fact that $\widetilde{p}\,(t,\theta)$ is strictly falling in t. In general equilibrium, the threshold also exists and is unique, since, as shown below, equilibrium wages depend negatively on unemployment duration and therefore on T. This implies that the right hand side is strictly increasing in T, while the left hand side falls in T.

In summary, we assume that the job-finding probability depends on the worker's true type, but that workers are uncertain of their type. Workers use search outcomes to update their beliefs, where the outcome is the cumulative duration of unsuccessful search. This updating in turn determines search decisions. Once the duration of unemployment reaches a critical threshold T, the likelihood of being a hight type, and thus the subjective job-finding probability, is perceived as being too low to justify further search. Importantly, uncertainty causes low types to search too long in the model, and can cause high types to stop search too early.

3.2 Steady State

Having defined the behavior of workers, we next characterize the conditions required to hold in a steady state. This yields a sufficiently developed framework for a partial equilibrium analysis focusing on worker behavior, without worrying about firms or how job-finding probabilities and wages are determined.

Note that there are, in principle, four groups of workers in the model. For characterizing the steady state it is important to keep track of all of these. For each type $j \in \{l, h\}$ we denote the proportion of unemployed that search actively by a_u^j, and let a_e^j be the proportion of employed workers who have not crossed the discouragement threshold. Let d_u^j be the proportion of unemployed of each type that are discouraged, that is, have experienced a cumulative unemployment duration exceeding T. By definition, these proportions satisfy

$$a_u^h + d_u^h + a_u^l + d_u^l = 1 \, . \tag{12}$$

$$a_e^h + a_e^l = 1 \, . \tag{13}$$

We denote the proportion of the labor force that is unemployed by u. Given that the fraction of low (high) types in the labor force is q $(1-q)$ at any point in time, the following must also hold

$$\left(a_u^l + d_u^l\right) u + a_e^l(1-u) = q \, , \quad \text{and} \tag{14}$$

$$\left(a_u^h + d_u^h\right) u + a_e^h(1-u) = 1 - q \, . \tag{15}$$

Let $f_u^j(T)$ be the density of actively searching unemployed, of type $j \in \{l, h\}$, who become discouraged at any point in time, because their cumulative unemployment duration reaches T. This density differs across types because of the difference in job finding rates. Note that workers cannot become discouraged while being employed.

With this notation in hand we can now characterize the steady state, which is reached when all of the following are stationary: $a_u^h, d_u^h, a_e^h, a_u^l, d_u^l, a_e^l$, the unemployment rate, u, and the density of unemployed individuals becoming discouraged in each moment of time, $f_u^j(T)$. In other words, the steady state is characterized by a collection of balancing conditions, such that the flows in and out of each of these states are equal, for both types of workers, $j \in \{l, h\}$. Using these conditions, one can derive an expression for the discouragement density $f_u^j(T) = \left(\lambda^j + \rho\right) \frac{e^{-(\lambda^j+\rho)T}}{1-e^{-(\lambda^j+\rho)T}}$, with $j = l, h$.[15] This density is strictly decreasing in T. Solving the steady state conditions for the fractions of active and discouraged workers in and out of employment, and using the expression for $f_u^j(T)$ one obtains

$$a_u^j = \frac{\rho(1-u)}{u}\frac{q^j}{\lambda^j}, \tag{16}$$

$$ua_u^j = \frac{\rho}{\lambda^j+\rho}\left(1-e^{-(\lambda^j+\rho)T}\right)q^j, \quad \text{and} \tag{17}$$

$$d_u^j = \frac{1-u}{u}\frac{e^{-(\lambda^j+\rho)T}}{1-e^{-(\lambda^j+\rho)T}}\frac{q^j}{\lambda^j}. \tag{18}$$

Finally, using conditions (14) and (15), one can recover a_e^l and a_e^h. These conditions express the masses of different groups of employed and unemployed workers in terms of exogenous parameters. One can easily verify that restrictions (14) and (15) are satisfied. Note also that the fraction of discouraged workers is strictly increasing in T, as one would expect. This completes the characterization of the steady state, treating the job finding probabilities and the wage schedule as given.

3.3 Labor Market Tightness and Job Arrival Rates

In this section we move beyond a partial equilibrium framework, defining the process that matches workers and firms. This matching process is assumed to be a function of labor market tightness, θ, which is given by the effective number of unemployed searchers in the market divided by the number of job vacancies available. Through the matching process, θ determines the equilibrium job-arrival rates for workers and the applicant arrival-rate for firms.

One component of labor market tightness is the effective number of unemployed workers searching for jobs. A formal investigation of the equilibrium requires an explicit treatment of the effective number of unemployed that determines the effective labor market tightness faced by all agents on the labor market. This effective mass of searching

[15] The full derivation of the steady state conditions can be found in Appendix A.1.

unemployed workers can be characterized in terms of efficiency units as

$$\tilde{u} = \left(a_u^h + \frac{\lambda^l}{\lambda^h} a_u^l \right) u \; , \tag{19}$$

where the search efficiency of actively searching high types is normalized to one. Low types have a strictly lower relative search efficiency since $\lambda^l < \lambda^h$, and the search efficiency of inactive unemployed workers is zero. The second component of labor market tightness is the number of vacancies, v, available in the market. Firms looking for workers are assumed to post vacancies according to their hiring policy, which is described in the next section.

The labor market exhibits search frictions, so productive matches do not form instantaneously, but require a costly search process by both firms and unemployed workers. Matches between searching firms and searching unemployed workers are assumed to arise randomly, search is undirected. The flow of successful job matches arising per unit of time is generated by a matching function m such that

$$m = m\left(\tilde{u}, v\right) \; , \tag{20}$$

where m is increasing and concave in both arguments, exhibits constant returns to scale and satisfies $m(0, v) = m(\tilde{u}, 0) = 0$.

Given this matching function, the rate of job-arrivals per effective searcher is $m\left(\tilde{u}, v\right)/\tilde{u} = m\left(\theta\right)$, where $\theta = \frac{v}{\tilde{u}}$ is market tightness in terms of search efficiency units.[16] At some points the discussion below will be facilitated by making specific assumptions about the relationship between the job finding probabilities of high and low types. Since we assume that $\lambda^l < \lambda^h$, a natural assumption to ensure this is that low types have a discretely lower job-arrival rate. The job finding probabilities for high and low types are then given by

$$\lambda^h = m\left(\theta\right), \text{ and } \lambda^l = \max\{m\left(\theta\right) - \phi, 0\} \; , \tag{21}$$

respectively, with $\phi > 0$, $\partial\lambda^l/\partial\theta > 0$ and $\partial\lambda^h/\partial\theta > 0$.[17] We will discuss the role of this assumption and the effects of alternative assumptions where needed. For firms, the analogue is the applicant arrival rate, or number of matches per vacancy, given by $m\left(\tilde{u}, v\right)/v = m(\theta)/\theta$, which is decreasing in θ.

[16] To ensure the existence of a unique effective mass of searchers $\tilde{u}$ it is sufficient to assume that λ^l/λ^h is non-decreasing in θ. Existence and uniqueness of $\tilde{u}$ given u and v can be shown by rewriting condition (19) in terms of $\tilde{u}/u$, which is strictly monotonically increasing in $\tilde{u}$, and noting that the corresponding right hand side is continuous and non-decreasing in θ, i.e. non-increasing in $\tilde{u}$; since $a_u^h + a_u^l < 1$ it also follows that $\tilde{u} < u$.

[17] The maximum operator is needed to rule out negative job finding probabilities.

3.4 Vacancy Posting

We now specify the hiring policy of firms in the model, which determines the number of vacancies in the market. We assume that jobs are created by small firms, offering only one job at a time, where v denotes the total number of vacancies posted in the economy. The productivity of a newly hired worker does not to depend on the worker's unemployment history t. Nevertheless, as mentioned earlier, t will affect the value of the match to the firm, through its impact on the wage that the firm has to pay. We assume that firms have to post a vacancy before it is possible to encounter a match. As long as a job is vacant, the firm incurs a flow cost κ for maintaining the vacancy. Therefore, firms decide whether to post a vacancy or not by comparing the expected cost of the vacancy to the expected wage that will have to be paid to the worker. Free entry of firms implies that the expected vacancy cost has to be equal to the expected return from the job for the firm, $EJ > 0$, where E denotes the expectations operator over worker types. Importantly, this must be true *ex ante*, that is before a particular match has been formed and the unemployment duration t of the respective worker is known.[18] Free entry (with positive job creation) implies

$$\kappa = \frac{m\left(\theta\right)}{\theta} EJ \; . \tag{22}$$

Given the assumptions on $m\left(\tilde{u}, v\right)$, this condition determines the equilibrium value of market tightness θ, as will be shown below.

With free entry of firms, the value of a vacancy is zero in equilibrium. As soon as a match between a vacancy and an unemployed applicant is realized, however, information about the worker's cumulated unemployment duration t is revealed. This t determines the worker's threat point and thus has an impact on wage bargaining. What is important from the firm's point of view is that a filled job generates an expected payoff that depends on the particular wage that has to be paid to the worker with state variable t. The value of a filled job to the firm is then given by

$$(r + \rho)J = y - w(t, \theta) \; . \tag{23}$$

[18] Note that we assume that y, q, T etc. are common knowledge and therefore known to firms. We explicitly rule out heterogeneity and uncertainty concerning the productivity of workers on the job. In contrast to the uncertainty of workers about their self studied here, uncertainty about productivity would give rise to statistical discrimination of workers by part of firms, and thus stigma effects, and would require a more involved information structure. While such an extension would be possible at the cost of considerable complication, in this paper we confine ourselves to highlight the consequences of uncertainty about the self on the side of the workers in the clearest possible way.

3.5 Wage Determination and Search Decisions

Wages are a key determinant of vacancy posting and job creation behavior on the side of firms. At the same time, wages determine the search decisions of workers. The set-up of the model implies that wages are determined once a match has been formed and all match characteristics, such as the worker's cumulative unemployment duration at the time of hiring, have materialized. We assume that the surplus S that a new match generates is divided according to the generalized Nash solution to the bargaining problem. Let β denote the workers' bargaining power. Then, the solution to the bargaining problem $\max_{W(t)-U(t),J(t)}(W(t)-U(t))^{\beta}(J(t))^{1-\beta}$ s.t. $S(t)=W(t)-U(t)+J(t)$ can be characterized as

$$\frac{W(t)-U(t)}{\beta}=S(t)=\frac{J(t)}{(1-\beta)} \tag{24}$$

as long as the surplus $S(t)$ is non-negative. The key issue to note is that a worker's threat point in the wage negotiations depends on his subjectively perceived job finding probability, $\tilde{p}(t,\theta)$, and, in particular, his subjective probability of being a high type, $p^h(t,\theta)$, which determines the value of being unemployed. Once a match is realized and firm and worker begin bargaining, the unemployment 'clock' is stopped, and the workers threat point does not decrease during the negotiations. This rules out that any party can change the surplus by delaying.[19] Using (3), (8) and (23), and eliminating $(r+\rho)$, the wage can be expressed as

$$w(t,\theta)=\beta y+(1-\beta)(b-c)+(1-\beta)\tilde{p}(t,\theta)\left(W(t)-U(t)\right)+(1-\beta)\left(\dot{U}(t)-\dot{W}(t)\right). \tag{25}$$

The last term reflects the fact that another instant of unsuccessful search alters (more precisely, decreases) the worker's self-evaluation, and therefore the values of being unemployed and employed, respectively.[20] In the current context, these terms can not simply be dismissed since they affect the worker's position in the bargaining, and therefore imply a discount on the wage, as we now show. From the sharing rule in equation (24), one can solve for $U(t)-W(t)=\frac{\beta}{1-\beta}\frac{1}{r+\rho}\left(w(t)-y\right)$. Taking derivatives with respect to t, one obtains an expression for $\left(\dot{U}(t)-\dot{W}(t)\right)$ that can be substituted into the wage expression.

[19] In this sense, the problem differs from a non-stationary bargaining problem where the possible payoffs change over time even during the negotiations, as in the environment studied by Coles and Muthoo (2003).

[20] It is important to note, however, that the time derivatives are derivatives with respect to unemployment duration, i.e. $\dot{U}(t)=\partial U(t)/\partial t$, not with respect to actual (calendar) time.

In this case, the wage function is characterized by a non-homogeneous, non-autonomous ordinary differential equation that can be written as

$$w(t,\theta) = \frac{r+\rho+\tilde{p}(t,\theta)}{r+\rho+\beta\tilde{p}(t,\theta)}\beta y + \frac{(1-\beta)(r+\rho)}{r+\rho+\beta\tilde{p}(t,\theta)}(b-c) + \frac{\beta}{r+\rho+\beta\tilde{p}(t,\theta)}\dot{w}(t,\theta)\,. \quad (26)$$

A general solution to this ordinary differential equation exists and can be obtained by applying an integrating factor method.[21] To see how the logic of discouragement shows up in the wage distribution, consider the wage of an individual that is just about to give up search at $t = T$. Using the fact that no new information arrives in the next instant when search is abandoned, $\dot{w}(T,\theta) = 0$, it turns out that the wage in the last instant of search is given by the respective share of the surplus without taking into account any option value of future job offers.[22] Using condition (7) and simplifying, the wage expression (26) can be simplified to yield

$$w(T,\theta) = \beta y + (1-\beta)(b-c) + (1-\beta)\tilde{p}(T,\theta)\left(\underline{W} - \underline{U}\right) = w(T,\theta) = b + \beta(y-b)\,. \quad (27)$$

Solving the differential equation yields a unique solution for the wage function conditional on T, θ, and u.

Given the solution to the wage function, we can now also explicitly state the assumptions mentioned in the discussion of worker behavior before, which ensure that search is optimal for a subset of the unemployed population. Without loss of generality and for the sake of illustration, in the following we assume first that active search is always preferred to inactivity when individuals first enter the labor market with their initial (flat) prior. Secondly, we assume that inactivity is better than active search if an individual is certain to be a low type. Formally, these assumptions imply

$$\tilde{p}(0,\theta) > \frac{(r+\rho)c}{\beta(y-b)}\,,$$
$$\text{and} \quad \tilde{p}(p^h = 0,\theta) < \frac{(r+\rho)c}{\beta(y-b)}\,.$$

Taken together, these conditions imply that search tends to be better than inactivity for small t, while for large t the opposite is the case because individuals become certain that they are the low type.

We now turn to the determination of the labor market equilibrium.

[21] See Appendix A.2 for details.

[22] Put differently, this condition implies a terminal condition that the differential equation (26) has to solve, see Appendix A.2.

3.6 Labor Market Equilibrium

Intuitively, a labor market equilibrium is given by a search threshold T^*, a labor market tightness θ^* and a level of unemployment u^*, such that individuals stop search optimally given their beliefs about themselves, firms post vacancies optimally with free entry ensuring the value of a vacancy being zero, and the level of unemployment as well as the composition of active and discouraged unemployed is stationary. In this section we specify the equilibrium condition for each of these three endogenous variables. As we go along, we derive the properties of each condition that are needed to show that the equilibrium exists and is unique.

For determining the equilibrium, note that the wage function is strictly decreasing in unemployment duration over the entire support. That is, for a given labor market tightness θ, the wage decreases as the cumulative unemployment duration t of an individual increases, $\dot{w}(t,\theta) < 0$ for all searchers with $t \in [0, T)$.[23] Intuitively, individuals with low cumulated unemployment history t have a high subjective probability of being a high type, and thus can credibly extract higher wages in equilibrium than individuals with a long unemployment record, because the threat point is better with a low unemployment duration. Recalling the implicit condition (11), and the fact that $\tilde{p}(t,\theta)$ is strictly decreasing in t, this implies that, for each θ, a search threshold T exists and is unique. By feeding the wage $w(T,\theta)$ back into the threshold condition (11), the condition that implicitly determines T in equilibrium is given by

$$G \equiv \tilde{p}(T,\theta) = \phi m(\theta) + (1-\phi) m(\theta) p^h(T,\theta) = \frac{c(r+\rho)}{\beta(y-b)} \,. \tag{28}$$

This implies that, for any state of aggregate labor market conditions, θ and u, there exists a unique search threshold, T, such that condition (28) holds. Given the assumptions on the job finding rates made in (21), one can rewrite $p^h(t,\theta) = \frac{1}{1+\frac{q}{1-q}e^{\phi t}} = p^h(t)$ and $\tilde{p}(t,\theta) = -\phi + m(\theta)\left(1 + p^h(t)\right)$. With this, and using the threshold conditions (11) and (27), the threshold can be explicitly written as function of equilibrium labor market tightness θ,

$$T(\theta) = \frac{1}{\phi} \ln \left[\left(\frac{m(\theta)\beta(y-b)}{c(r+\rho) + \phi\beta(y-b)} \right) \frac{1-q}{q} \right] \,. \tag{29}$$

Hence, in equilibrium, the search threshold T is strictly increasing in labor market tightness θ. This implies that the higher θ in equilibrium, the larger is the unemployment duration

[23] The result is shown in Appendix A.3.

T before people get discouraged in equilibrium.[24]

Note that, although there is a unique discouragement threshold T^*, there is a distribution of wages along the dimension of cumulative unemployment duration. More precisely, in equilibrium wages are distributed according to the density $\frac{a_e^l f_e^l(t)+a_e^h f_e^h(t)}{a_e^l+a_e^h}$ on a support that is bounded by the range of subjective probabilities $p^h(t)$ that sustain active search behavior (i.e. in equilibrium, wages only exist for individuals of unemployment duration $t \in [0, T^*]$).

The second equilibrium condition characterizes the optimal vacancy posting behavior of firms, and thus the equilibrium value of θ. In equilibrium the costs of posting a vacancy must be exactly outweighed by the expected value of a job, which depends crucially on the expected wage. Note that the firm has no way to influence the type of worker in terms of t that it encounters in the frictional labor market. From the characterization of the wage function before, wages are strictly decreasing in the t of the worker the firm is eventually matched with.[25] Note also that the subjective probability of being a high type at T defines the lower bound for the wage distribution, while the upper bound is given by the prior probability of being a high type that one presumes (e.g. $(1-q)$ with a flat prior). The expected wage therefore depends on the expected unemployment duration of the available workers that the firm can be matched with. Hence, in equilibrium it must hold that

$$\kappa = \frac{m(\theta)}{\theta} EJ = \frac{m(\theta)}{\theta} \frac{y - Ew(t, \theta)}{r+\rho} = \frac{m(\theta)}{\theta} \frac{y - w\left(E(t|T^*, \theta), \theta\right)}{r+\rho} \equiv H \,. \tag{30}$$

Because the probability to be matched with an applicant decreases with θ, $\frac{\partial (m(\theta)/\theta)}{\partial \theta} < 0$,

---

[24] This positive relationship follows from the assumption in (21) concerning the different job finding probabilities for high and low types. Allowing for a positive job finding probability $\underline{\lambda}$ even under inactivity would leave the results unaltered if $\underline{\lambda}$ is unrelated to labor market conditions θ. With a $\underline{\lambda}$ that increases in θ just as the job finding probabilities of active searchers of either type, however, the relationship between θ and T would be negative. In this case inactivity is relatively more attractive if labor market conditions are better. Finally, assuming that low types have a proportionally lower job finding probability than high types, i.e. $\lambda^l = \varphi \lambda^h$, $0 < \varphi < 1$, the sign of $\frac{\partial T}{\partial \theta}$ would be *a priori* ambiguous. As long as the direct, positive effect on the labor market outweighs the indirect, negative effect through self perception in that case, the subjective job finding probability $\tilde{p}(t, \theta)$ would be higher for a labor market with more vacancies per searcher, and thus $\frac{\partial T}{\partial \theta} > 0$. This would lead to the same results as in the presented scenario. In this context it is worth noting that the model can also be solved for a negative relationship between T and θ. In this case better labor market conditions imply a faster discouragement of unemployed. While changing the comparative statics results below, this would provide different, but potentially interesting implications. Which assumption is more appropriate is ultimately an empirical question.

[25] This is shown formally in Appendix A.3.

we have that the right hand side of the condition is strictly falling in θ, $\frac{\partial H}{\partial \theta} < 0$.[26]

To evaluate the behavior of the function H with respect to T^*, we again use the fact that the wage is decreasing in the unemployment duration of a worker that is hired, which implies that the wage is also decreasing in the expected duration. But the expected duration is a strictly monotonic increasing function of the search threshold T^*, $\frac{\partial E(t|T^*,\theta)}{\partial T^*} > 0$, and so the right hand side of condition (30) increases in T^*, $\frac{\partial H}{\partial T^*} > 0$.[27] This implies a one-to-one relationship between the optimal search threshold T^* and firms' vacancy posting, which determines θ. In particular, the higher the search threshold T^*, the more it pays off for firms to post vacancies, and therefore the higher is θ.

The third equilibrium condition concerns the level of unemployment. Note that for each level of unemployment u, there is a unique level of effectively searching unemployed $\tilde{u}$, so one can restrict attention to either of these variables in the determination of the equilibrium.[28] In the following, we consider the equilibrium level of aggregate unemployment u.

The level of unemployment only affects the level of vacancy posting through its effect on labor market tightness, and does not affect the structure of wages, which is entirely determined by the pattern of unemployment duration t. For consistency between expected wages and the wages that firms actually have to pay, we assumed before that firms cannot strategically target unemployed with certain unemployment histories. Using conditions (16) and (17) and the steady state condition for unemployment (35), the equilibrium unemployment rate can be expressed as

$$u = 1 - \frac{\lambda^h (1-q)\rho}{\rho + \lambda^h}\left(1 - e^{-(\lambda^h+\rho)T}\right) - \frac{\lambda^l q\rho}{\rho + \lambda^l}\left(1 - e^{-(\lambda^l+\rho)T}\right) , \quad (31)$$

[26] Under mild assumptions on ϕ, one can also show that the wage is strictly increasing in θ for any t, see Appendix A.3.

[27] This result follows from applying the Leibniz rule to both densities f_u^j, $j = h, l$, since the expected unemployment duration $E(t|T,\theta)$ is an average of the expected unemployment durations of both types weighted by their respective share of the total pool of unemployed. Because $E(t^j|T,\theta) = \int_0^T f_u^j(t)tdt = \int_0^T (\lambda^j + \rho)\frac{e^{-(\lambda^j+\rho)t}}{1-e^{-(\lambda^j+\rho)T}}tdt$, we have $\frac{\partial E(t^j|T,\theta)}{\partial T} = \int_0^T (\lambda^j+\rho)^2 \frac{e^{-(\lambda^j+\rho)t}e^{-(\lambda^j+\rho)T}}{\left(1-e^{-(\lambda^j+\rho)T}\right)^2}tdt + \int_0^T (\lambda^j+\rho)\frac{e^{-(\lambda^j+\rho)t}}{\left(1-e^{-(\lambda^j+\rho)T}\right)}dt + (\lambda^j+\rho)\frac{e^{-(\lambda^j+\rho)T}}{1-e^{-(\lambda^j+\rho)T}} > 0$.

[28] The proof of the injective relationship between u and $\tilde{u}$ depends on the assumptions concerning the relative search effectiveness of the two types in terms of λ^l and λ^h. Given the assumptions in (21), $\frac{\tilde{u}}{u} = a_u^h + a_u^l \frac{\max\{m(\theta)-\phi\}}{m(\theta)}$. The claim then follows, because the left-hand side is strictly increasing in $\tilde{u}$, while the right hand side is strictly decreasing in $\tilde{u}$. This can be seen since the right hand side increases in $m(\theta)$ while $\partial m/\partial\theta > 0$ and $\partial\theta/\partial\tilde{u} < 0$. For alternative assumptions like a proportional mark-off in search efficiency of low types φ mentioned in footnote 24, an analogous proof applies.

using the implicit notation of before.[29] Taking derivatives shows that unemployment is strictly falling in the horizon until unemployed become discouraged, *i.e.*, $\frac{\partial u}{\partial T} < 0$.[30] Likewise, unemployment is lower if labor market tightness is higher, *i.e.*, $\frac{\partial u}{\partial \theta} < 0$.[31]

The steady state equilibrium is fully characterized by a vector $\{T^*, \theta^*, u^*\}$ for which conditions (28), (30) and (31) hold simultaneously, and by a composition of the population a_u^j, a_e^j and d_u^j such that the steady state conditions (16), (17) and (18) hold as well. Hence, together with the steady state conditions, the equilibrium is characterized by the unique solution of a system of six equations in six unknowns. Apart from the unemployment rate u^*, all these variables are jump variables. The properties of the equilibrium conditions, identified above, imply that conditional on the prior the equilibrium exists and is unique.[32] The results so far are sufficient to illustrate the working of the model.[33]

To summarize, in equilibrium the outflows from unemployment into employment decline with cumulative unemployment duration, and become zero for durations above T^*. On the other hand, outflows into the inactive state of discouragement increase with unemployment duration, in the sense of a discrete jump at T^*. The extent of this negative duration dependence of the hazard into employment, and the positive duration dependence of hazards into discouragement is endogenously determined in equilibrium and character-

[29] Given the assumptions on the job finding probabilities λ^j, this condition can be explicitly written as

$$u = 1 - \frac{m(\theta)(1-q)\rho}{\rho + m(\theta)}\left(1 - e^{-(m(\theta)+\rho)T}\right) - \frac{(m(\theta)-\phi)q\rho}{\rho + (m(\theta)-\phi)}\left(1 - e^{-(m(\theta)-\phi+\rho)T}\right) .$$

[30] This result can be seen by the fact that $\partial(1 - e^{-(\lambda^j+\rho)T})/\partial T > 0$.

[31] The result follows since $\partial(1 - e^{-(\lambda^j+\rho)T})/\partial\theta > 0$ and $\partial(\frac{m(\theta)(1-q)}{\rho+m(\theta)})/\partial\theta > 0$.

[32] See also appendix B. In case of type specific priors, there are two type specific search thresholds in equilibrium. More generally, if priors are dispersed, the equilibrium is characterized by a distribution of threshold durations that correspond to the priors. A detailed analysis of these cases is beyond the scope of this paper.

[33] The main results of this section do not depend on the assumption of wage determination through Nash bargaining. If firms were to post wages upon having matched with a worker, a qualitatively similar wage distribution would arise. The problem faced by firms in this case would be very similar to the model by Albrecht and Vroman (2005), where individuals' unemployment benefits b vary, and can fall to a lower level $s < b$ according to a random process. With type uncertainty, it is not the value of unemployment that is changing along an unemployment spell, but the value of employment, due to the decreasing expected wage. In equilibrium, firms would post wages corresponding to the respective worker's reservation wage, and vacancies would depend on the reservation wage of the expected worker, i.e. the expected unemployment duration t. Firms would pay the reservation wage ex post, since offers that are below a worker's reservation wage are rejected. The main complication in such a model is the continuum of different reservation wages determined by the unemployed's beliefs about their type and characterized by their t. The determination of a continuous wage distribution with unknown types under wage posting is beyond the scope of this paper.

ized by T^* and the associated distributions f_u^i. Thus, in line with the evidence provided by Jones and Riddell (1999), our model provides a rationale that is consistent with permanent, structural flows between the states of unemployment and (marginally attached) non-participation because of discouragement after reaching the threshold T^*.[34] Finally, wages decline with unemployment history in terms of t, reflecting the lower self-confidence in terms of subjective job finding probabilities that eventually determines the bargaining position of the unemployed. Overall, fewer individuals with high unemployment duration are employed.

3.7 Some Comparative Static Results

Having characterized equilibrium in the model, we can now perform comparative statics. We illustrate by discussing the impact of a change in the income stream from unemployment, b, and a change in productivity, y. Comparative statics with respect to other parameters follow directly, based on analogous arguments.

An unexpected, exogenous increase in the income stream from unemployment, $b' > b$, leads to workers becoming discouraged earlier, and increases the level of unemployment. To see this, note that an increase in b improves all workers' payoff from inactivity. From condition (28) this leads to an increase in the subjective probability of being a high type that makes an unemployed worker indifferent between active search and discouragement. In other words, there is a new equilibrium unemployment duration T', such that the unemployed leave the active search pool earlier, i.e. $T' < T^*$. This, in turn, leads to an increase in the wages that firms expect to pay to newly hired workers, as implied by the discussion of condition (30), and therefore reduces the job creation activity of firms. A lower labor market tightness $\theta' < \theta^*$ and an earlier discouragement threshold $T' < T^*$, lead, in turn, to a higher equilibrium unemployment level, $u' > u^*$, as can be seen from condition (29). We obtain analogous comparative static results for an exogenous increase in the search cost, c, by similar arguments. Higher search costs imply faster discouragement,

[34] An extension of the model taking into account cyclical productivity shocks or likewise other shocks affecting individuals' incentives to search for employment, e.g. arising from marriage or child birth, can generate inflows from non-participation into (active) unemployment. An extension of the model with positive job finding probabilities even in the state of no active search would allow to generate flows from the state of discouragement back to employment. Likewise, the model could be extended to allow for stochastic shocks that affect the value of being unemployed along the lines of Garibaldi and Wasmer (2005) and Pries and Rogerson (2004), and therefore induce discouraged workers to return to the labor market and search actively.

and a higher level of unemployment.

Following an unexpected, exogenous increase in productivity to $y' > y$, the time it takes for workers to become discouraged increases, and unemployment decreases. This follows from the fact that, on the one hand, an increase in y implies that search is more profitable. This induces the unemployed to search longer, i.e. it must be that $T' > T^*$ for (28) to be satisfied because $\partial\tilde{p}(T^*, \theta)/\partial T^* < 0$. On the other hand, higher productivity makes vacancies more attractive, and thus for (30) to be satisfied, labor market tightness must increase. Consequently, the steady state is characterized by lower unemployment $u' < u^*$ from condition (29). Similar effects emerge for an unexpected exogenous decrease in vacancy costs κ. On the other hand, higher rates of r or ρ have precisely the opposite effect of an increase in productivity, since they make vacancy posting and search more expensive. They lead to a lower θ and faster discouragement $T' < T^*$ in equilibrium.[35]

As an aside, note that a policy enforcing active search, that is, increasing T^* exogenously regardless of whether this is optimal or not, unambiguously decreases unemployment since $u(T^* = \infty) < u(T^* = 0)$. Hence, if unemployed could be forced to search, not surprisingly, unemployment would be lower in steady state.

Beyond these comparative statics results, the presence of type uncertainty has important implications for the dynamics of the labor market. After an unexpected increase in productivity y, individuals' subjective beliefs about their type, and hence about their job finding probability, are unchanged. This is because the change in labor market conditions does not affect the information value of their previous search history in the old environment, and therefore does not affect their current subjective self-evaluation.[36] However, the threshold probability needed to justify search decreases with an increase in y. This induces individuals at the discouragement threshold to resume active search. In the model presented so far, there is a mass point of discouraged workers with unemployment duration T^*, so the response to an increase in y in terms of renewed search is substantial. In fact, immediately after the increase, all workers in the economy will be searching actively, and will continue to do so until they reach the new unemployment threshold T'.

[35] A similar result would hold in an extended model for an exogenous displacement rate δ.

[36] Recall that under the assumption (21) it follows that $p^h(t, \theta) = p^h(t) = \frac{1}{1+\frac{q}{1-q}e^{\phi t}}$. This claim would be also true for any change in labor market conditions under any other parametric specification of the difference in job finding probabilities, however, since there would be no reason to adjust the subjective belief retroactively.

This implies that type uncertainty leads to an amplification of the effects of changes in productivity on unemployment and vacancies, compared to a standard search model. The increase in y raises θ^* as jobs become more valuable for firms, but the attendant increase in T^* induces more individuals to search, which makes it even easier for firms to meet an applicant as labor market tightness is moderated by more searchers. This facilitates job creation, amplifying the firms' increase in vacancy posting.[37] In a model with a non-degenerate distribution of unemployment durations above T^*, the increase in the pool of active searchers would be less pronounced but still existent. Thus, in comparison to a standard model without uncertainty, unemployment decreases more in response to an increase in y.

4 Concluding remarks

This paper explores a simple idea: what happens when people are uncertain about their abilities, and thus search without knowing their objective chances of success? We develop a general equilibrium model of the labor market in which workers are uncertain of their type and update beliefs about their subjective job-finding chances based on the duration of unsuccessful search. The model offers a new perspective on several aspects of job-search behavior and unemployment, including falling hazards out of unemployment, lower wages for individuals with long unemployment history, the phenomenon of discouraged workers, the volatility of unemployment, gender differences in search behavior, and the damaging effects of unemployment on mental health. In terms of labor market policy, the results of this paper suggests that job search assistance should not be focused solely on the long-term unemployed. In fact, early interventions that credibly increases worker's subjective beliefs about the likelihood of finding a job may be as important for avoiding long-term unemployment as retraining schemes designed to address skill depreciation among long-term unemployed.

Our model need not be interpreted narrowly in terms of describing a choice between participation or non-participation in the labor force. Instead, the insights could apply

[37] In this respect, the model adds to the recent debate about the standard search model's ability to generate the empirically observed high unemployment volatility in response to cyclical variations. (see Shimer, 2005 and Mortensen and Nagypal, 2005). The effect generated by our model is based on inflows of discouraged workers, analogous to the added-worker and discouraged-worker effects discussed by Pissarides (2000, chapter 7).

equally well to explaining the behavior of someone searching a particular segment of the labor market, and then lowering aspirations and switching to search in a less remunerative but also less demanding sector. The mechanism highlighted in the model could also apply to other types of search besides job search. In many search settings, individuals are probably uncertain of their own abilities, and uncertain of their abilities relative to others. For example, someone searching for a mate may be uncertain about own attractiveness, compared to competitors in the marriage market. A prolonged period of unsuccessful search could lower self-confidence, and cause an individual to switch to a different type of mate, or give up on search altogether.

We draw on laboratory evidence to provide empirical support for our assumptions, but in the future it should also be possible to design surveys that elicit individuals' beliefs about their relative abilities and job-finding chances, and their certainty about these beliefs. Analogous to our experiment, respondents could be asked how certain they are that they job finding chance in the next month is higher than a certain percentage. Questions like this would allow an investigation of how duration of unsuccessful search affects confidence and future search decisions. It would also provide an indication of how confidence and updating varies according to different personal characteristics in the field. We believe that this is a fruitful direction for future research on explaining individual search behavior.

References

Albrecht, J., and S. Vroman (2005): "Equilibrium Search with Time-Varying Unemployment Benefits," *Economic Journal*, 115(505), 631–648.

Andolfatto, D., S. Mongrain, and G. Myers (2005): "Self-Esteem and Labour Market Choices," *mimeo, Simon Fraser University.*

Bjorklund, A., and T. Eriksson (1998): "Unemployment and Mental Health: Evidence from research in the Nordic Countries," *Scandinavian Journal of Social Welfare*, 7, 219–235.

Blau, D. M., and P. K. Robins (1986): "Labor Supply Response to Welfare Programs: A Dynamic Analysis," *Journal of Labor Economics*, 4(1), 82–104.

Bowden, R. J. (1980): "On the Existence and Secular Stability of u-v-Loci," *Economica*, 75, 35–50.

Burdett, K., and T. Vishwanath (1988): "Declining Reservation Wages and Learning," *Review of Economic Studies*, 55(4), 655–665.

Calmfors, L., and H. Lang (1995): "Macroeconomic Effects of Active Labour Market Programmes in a Union Wage Setting Model," *Economic Journal*, 105, 601–619.

Coleman, T. S. (1989): "Unemployment Behavior: Evidence from the CPS Work Experience Survey," *Journal of Human Resources*, 24(1), 1–38.

Coles, M. G., and A. Muthoo (2003): "Bargaining in a Non-Stationary Environment," *Journal of Economic Theory*, 109, 70–89.

Dubra, J. (2004): "Optimism and Overconfidence in Search," *Review of Economic Dynamics*, 7, 198–218.

Falk, A., D. Huffman, and U. Sunde (2006): "The Relevance and Implications of Imperfect Self-Knowledge for Search," *mimeo, IZA, Bonn.*

Flam, S. D., and A. E. Risa (2003): "Ability, Self-Confidence, and Search," *Journal of Institutional and Theoretical Economics*, 159, 439–456.

Flinn, C. J., and J. J. Heckman (1983): "Are Unemployment and Out of the Labor Force Behaviorally Distnict Labor Force States?," *Journal of Labor Economics*, 1(1), 28–42.

Frijters, P., and B. van der Klaauw (2006): "Job Search with Non-Participation," *Economic Journal*, 116(508), 45–83.

Garibaldi, P., and E. Wasmer (2005): "Equilibrium Search Unemployment, Endogenous Participation, and Labor Market Flows," *Journal of the European Economic Association*, 3(4), 851–882.

Gerdtham, U.-G., and M. Johannesson (2003): "A Note on the Effect of Unemployment on Mortality," *Journal of Health Economics*, 22(3), 505–518.

Hey, J. D. (1982): "Search for Rules for Search," *Journal of Economic Behavior and Organization*, 3(1), 65–81.

——— (1987): "Still Searching: Search for Rules for Search," *Journal of Economic Behavior and Organization*, 8(1), 137–144.

Jones, S. R., and C. W. Riddell (1999): "The Measurement of Unemployment: An Empirical Approach," *Econometrica*, 67(1), 147–162.

Köszegi, B. (2001): *Essays in Behavioral Economics*. Ph.D. Thesis, UC Berkeley.

——— (2006): "Ego Utility, Overconfidence, and Task Choice," *Journal of the European Economic Assciation*, 4(4), 673–707.

Machin, S., and A. Manning (1999): "The Causes and Consequences of Longterm Unemployment in Europe," in *Handbook of Labor Economics*, ed. by O. C. Ashenfelter, and R. Layard, vol. 3.

Mathers, C., and D. Schofield (1998): "The Health Consequences of Unemployment: The Evidence," *Medical Journal of Australia*, 168, 178–182.

McElhattan, R. (1980): "Should Discouraged Workers Be Counted in the Labor Force? A Job Search Approach," *Federal Reserve Bank of San Francisco Economic Review*, pp. 7–24.

McKenna, C. J. (1987): "Labor Market Participation in Matching Equilibrium," *Economica*, 54, 325–333.

Morgan, P. B. (1985): "Distributions of the Duration and the Value of Job Search with Learning," *Econometrica*, 53(5).

Mortensen, D. T., and E. Nagypal (2005): "More on Unemployment and Vacancy Fluctuations," *IZA Discussion Paper*, 1765.

Pissarides, C. A. (2000): *Equilibrium Unemployment Theory*. MIT Press, Cambridge, Massachusetts, 2nd edn.

Pries, M., and R. Rogerson (2004): "Search Frictions and Labor Market Participation," *mimeo, University of Maryland*.

Rosholm, M., and O. Toomet (2005): "A Search Model of Discouragement," *IZA Discussion Paper*, 1633.

Sattinger, M. (1995): "General Equilibrium Effects of Unemployment Compensation with Labor Force Participation," *Journal of Labor Economics*, 13, 623–652.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.

Simon, C. P., and L. Blume (1994): *Mathematics for Economists*. W.W.Norton & Company, New York.

Winkelmann, L., and R. Winkelmann (1998): "Why are the Unemployed so Unhappy? Evidence from Panel Data," *Economica*, 65(257), 1–15.

A Appendix: Proof of Steady State Conditions and Claims about the Wage Function

A.1 Derivation of Steady State Conditions

The steady state conditions stated in the text can be derived from equating inflows and outflows into the four respective states. Among the discouraged unemployed, $d_u^j u$, inflows due to newly discouraged unemployed must equal the outflows into employment or retirement.[38] Thus the following must hold

$$u a_u^j f_u^j (T) = \rho u d_u^j \,. \tag{32}$$

For actively searching unemployed, $a_u^j u$, inflows from new births must equal outflows due to discouragement, new jobs, and retirement

$$\rho q^j = u a_u^j f_u^j (T) + \left(\rho + \lambda^j\right) u a_u^j \,, \tag{33}$$

where $q^l = q$ and $q^h = (1-q)$. Inflows into the pool of employed workers that result from new hires must equal outflows due to retirement or death

$$\lambda^j u a_u^j = \rho (1-u) e_e^j \,. \tag{34}$$

If the unemployment rate is also to be stationary, inflows into unemployment due to new births must be exactly offset by outflows, which arise either due to active searchers finding new employment or due to death

$$\rho = \lambda^h u a_u^h + \lambda^l u a_u^l + \rho u \,. \tag{35}$$

To complete the characterization of the steady state we need an expression for $f_u^j (T)$, the density of unemployed individuals of type j who have just reached T. In order to derive this expression we first characterize the cumulative distribution functions $F_u^j (t)$ that denotes the c.d.f. of actively searching unemployed over all unemployment durations t and $F_e^j (t)$ that denotes the corresponding cumulative distribution for workers who are employed. To derive an expression for $F_u^j (t)$ and $F_e^j (t)$, consider a discrete time version of the model for small time increments Δ, and without loss of generality, consider just one type of worker.[39] The mass of 'duration type t' unemployed workers in time period s, denoted $x_u (t, s)$, evolves according to

$$x_u (t, s + \Delta) = x_u (t, s) - \Delta\lambda x_u (t, s) - \Delta\rho x_u (t, s) \,.$$

The first term on the right hand side reflects the fact that, for some unemployed, duration simply increases without a change in their status. The second term reflects outflows into employment, the third captures exit due to death. Likewise, the mass of 'duration type t' employed workers in time period s, denoted $x_e (t, s)$, evolves according to

$$x_e (t, s + \Delta) = x_e (t, s) + \Delta\lambda x_u (t, s) - \Delta\rho x_e (t, s) \,.$$

[38] The model could also be extended to allow for some low probability that inactive unemployed receive a job offer such that $(1-u)d_u^j > 0$.

[39] For simplicity, we suppress indices for types in the following derivation.

The next step is to impose the steady state conditions $x_k(t, s+\Delta) - x_k(t,s) = 0$, where $k = u, e$, and let $\Delta \to 0$. Dropping the time index, rearranging, and using $x_u(t) = a_u u f_u(t)$ and $x_e(t) = a_e(1-u) f_e(t)$ one arrives at

$$\begin{aligned} 0 &= -a_u u \frac{d}{dt} f_u(t) - \lambda a_u u f_u(t) - \rho a_u u f_u(t) \ , \quad \text{and} \\ 0 &= \lambda a_u u f_u(t) - \rho a_e (1-u) f_e(t) \ . \end{aligned}$$

The solution for $j \in \{l, h\}$ is then

$$f_e^j(t) = \tfrac{\lambda^j a_u}{\rho} \tfrac{u}{1-u} f_u^j(t) \ , \qquad \text{and} \qquad \frac{d}{dt} f_u^j(t) = -\left(\lambda^j + \rho\right) f_u^j(t) \ .$$

The solution of this system of differential equations can be shown to be

$$F_u^j(t) = \frac{1 - e^{-(\lambda^j+\rho)t}}{1 - e^{-(\lambda^j+\rho)T}} \quad \text{and} \quad f_u^j(t) = (\lambda^j + \rho) \frac{e^{-(\lambda^j+\rho)t}}{1 - e^{-(\lambda^j+\rho)T}} \ .^{40} \tag{36}$$

Substituting T into (36) we arrive at the necessary expression for completing the steady state: $f_u^j(T) = (\lambda^j + \rho) \frac{e^{-(\lambda^j+\rho)T}}{1-e^{-(\lambda^j+\rho)T}}$, which is strictly decreasing in T. The conditions (16) and (17) in the text can be derived by solving (33) and (34) and using the expression for $f_u^j(T)$. Finally, combining conditions (16) and (17) with (32), one obtains (18).

A.2 General Solution of the Wage Function

After some substitution, the wage function given by equation (26) can be expressed as

$$\begin{aligned} \dot{w}(t,\theta) \quad &- \quad \left[\frac{r+\rho}{\beta} + \tilde{p}(t,\theta)\right] w(t,\theta) = \\ &- \quad \left[(r+\rho+\tilde{p}(t,\theta))\, y + (r+\rho)\frac{1-\beta}{\beta}(b-c)\right] \ , \end{aligned}$$

which is equivalent to

$$\dot{w}(t,\theta) + P(t,\theta) \cdot w(t,\theta) = Q(t,\theta) \ , \tag{37}$$

where

$$\begin{aligned} P(t,\theta) &= -\left[\frac{r+\rho}{\beta} + \tilde{p}(t,\theta)\right] \\ Q(t,\theta) &= -\left[(r+\rho+\tilde{p}(t,\theta))\, y + (r+\rho)\frac{1-\beta}{\beta}(b-c)\right] \ . \end{aligned}$$

Using $e^{\int P(t,\theta)dt}$ as integrating factor (see e.g. Simon and Blume, 1994, pp. 637ff) the solution can be written as a wage function in terms of unemployment duration,

$$w(t,\theta) = \left[C + \int Q(t,\theta) e^{\int P(t,\theta)dt} dt\right] e^{-\int P(t,\theta)dt} \ , \tag{38}$$

[40] It is straightforward to show that $\frac{\partial}{\partial t} f_u^j(t) = -(\lambda^j + \rho) f_u^j(t)$, that $\frac{\partial}{\partial t} F_u^j(t) = f_u^j(t)$ and that $F_u^j(0) = 0$ and $F_u^j(T) = 1$.

where C is a constant still to be determined. Condition (27) provides a terminal condition that the differential equation (37) has to solve because wages are only defined for $t \in [0, T]$. Using this, one can determine a unique particular solution for the wage function by setting

$$C = \frac{Q(T,\theta)}{P(T,\theta)} e^{\int P(T,\theta)dt} - \int Q(T,\theta) e^{\int P(T,\theta)dt} dt\,. \tag{39}$$

Substituting this expression into (38) yields a unique solution for the wage function conditional on T, θ, and u.

A.3 Characterization of the Wage Function

Claim 1. *The wage is strictly decreasing in unemployment duration, $\dot{w}(t,\theta) < 0 \;\forall\; t \in [0, T)$.*

Proof. To show the claim, first use conditions (6) and (8) to obtain

$$(r + \rho + \tilde{p}(t,\theta))\,(W(t) - U(t)) = w(t,\theta) - (b - c) + \left(\dot{W}(t) - \dot{U}(t)\right).$$

Taking time derivatives we have that

$$(r + \rho + \tilde{p}(t,\theta))\left(\dot{W}(t) - \dot{U}(t)\right) + (W(t) - U(t))\,\dot{\tilde{p}}(t,\theta) = \dot{w}(t,\theta)$$

applying an approximation with $\left(\ddot{W}(t) - \ddot{U}(t)\right) \approx 0$. From the solution of the bargaining problem, $(W(t) - U(t)) = \frac{\beta}{(1-\beta)} J(t) = \frac{\beta}{(1-\beta)} \frac{y - w(t,\theta)}{r+\rho}$, and $\left(\dot{W}(t) - \dot{U}(t)\right) = \frac{\beta}{(1-\beta)} \dot{J}(t) = -\frac{\beta}{(1-\beta)} \frac{\dot{w}(t,\theta)}{r+\rho}$. Substituting and collecting terms, one can express the wage increment as

$$\dot{w}(t,\theta) = \frac{\beta(y - w(t,\theta))\dot{\tilde{p}}(t,\theta)}{r + \rho + \beta\tilde{p}(t,\theta)}\,. \tag{40}$$

The claim follows since $(W(t) - U(t)) > 0$, or alternatively $y - w(t,\theta) > 0$, and $\dot{\tilde{p}}(t,\theta) < 0$ for any admissible t on the support, excluding the last instant of active search when $\dot{\tilde{p}}(T,\theta) = 0$. Hence, $\dot{w}(t,\theta) < 0 \;\forall\; t \in [0, T)$ and $\dot{w}(T,\theta) = 0$, which proves the claim. ☐

Claim 2. *By making appropriate assumptions on parameters, in particular making the difference in search effectiveness between high and low types, ϕ sufficiently large, and the size q of the group of low types sufficiently small, one can always ensure that the wage is strictly increasing in labor market tightness, $\frac{\partial w(t,\theta)}{\partial\theta} > 0 \;\forall\; t \in [0, T^*]$.*

Proof. First note that, by using the sharing rule given by (24) to eliminate $W(t) - U(t)$ and substituting, the wage function (25) can be rewritten as in (26),

$$w(t,\theta) = \frac{(r + \rho + \tilde{p}(t,\theta))\,\beta y + (1-\beta)(r+\rho)(b-c) + \beta\dot{w}(t,\theta)}{r + \rho + \beta\tilde{p}(t,\theta)}\,.$$

Taking derivatives with respect to θ, one obtains

$$\frac{\partial w(t,\theta)}{\partial\theta} = \frac{\beta\left[(r+\rho)(1-\beta)[y - (b-c)] - \beta\dot{w}(t,\theta)\right]\frac{\partial\tilde{p}(t,\theta)}{\partial\theta}}{(r + \rho + \beta\tilde{p}(t,\theta))^2} + \frac{\beta\frac{\partial\dot{w}(t,\theta)}{\partial\theta}}{(r + \rho + \beta\tilde{p}(t,\theta))}\,. \tag{41}$$

Now remember that $\tilde{p}(t,\theta) = -\phi + m(\theta)\left(1 + \frac{1}{1+\frac{q}{1-q}e^{\phi t}}\right)$, and hence $\frac{\partial \tilde{p}(t,\theta)}{\partial \theta} = \frac{\partial m(\theta)}{\partial \theta}\left(1 + \frac{1}{1+\frac{q}{1-q}e^{\phi t}}\right) > 0$. Since $\dot{w}(t,\theta) < 0$, it follows that the first term on the right hand side of equation (41) is unambiguously positive. This condition implies that the wage increases in labor market tightness θ as long as the direct effect of better outside labor market conditions (the first, positive, term) outweighs the indirect effect of a worse self perception due to faster negative updating under better labor market conditions (the second, negative, term). To see this, take $\dot{w}(t,\theta)$ from condition (40) and derive with respect to unemployment duration t. For notational simplicity, in the following we denote derivatives with respect to unemployment duration t by $\dot{x} = \frac{\partial x}{\partial t}$, and derivatives with respect to labor market tightness θ by $x' = \frac{\partial x}{\partial \theta}$. Then,

$$\dot{w}'(t,\theta) = \frac{(y - w(t,\theta))\,\beta}{(r+\rho+\beta\tilde{p}(t,\theta))^2}\left[\dot{\tilde{p}}'(t,\theta)(r+\rho+\tilde{p}(t,\theta)) - \beta\dot{\tilde{p}}'(t,\theta)\tilde{p}'(t,\theta)\right] - \frac{\beta\dot{\tilde{p}}w'(t,\theta)}{(r+\rho+\beta\tilde{p}(t,\theta))}.$$

Using this to eliminate $\dot{w}'(t,\theta) = \frac{\partial \dot{w}(t,\theta)}{\partial \theta}$ from condition (41), and isolating $\frac{\partial w(t,\theta)}{\partial \theta}$, we finally get

$$\begin{aligned}\frac{\partial w(t,\theta)}{\partial \theta} &= \frac{\beta\left[(r+\rho)(1-\beta)[y-(b-c)] - \beta\dot{w}(t,\theta)\right]\frac{\partial \tilde{p}(t,\theta)}{\partial \theta}}{(r+\rho+\beta\tilde{p}(t,\theta))^2 + \beta^2\dot{\tilde{p}}} \\ &\quad + \frac{\beta\,(y-w(t,\theta))\left[\dot{\tilde{p}}'(t,\theta)(r+\rho+\tilde{p}(t,\theta)) - \beta\dot{\tilde{p}}'(t,\theta)\tilde{p}'(t,\theta)\right]}{(r+\rho+\beta\tilde{p}(t,\theta))\left((r+\rho+\beta\tilde{p}(t,\theta))^2 + \beta^2\dot{\tilde{p}}(t,\theta)\right)}. \end{aligned} \tag{42}$$

Given that the denominator of the first term is positive, a sufficient condition for the term in the second line being positive is that the term in brackets is positive. Using the properties of $\tilde{p}(t,\theta)$, in particular $\dot{\tilde{p}}(t,\theta) = \frac{\partial \tilde{p}(t,\theta)}{\partial t} = m(\theta)\left(\frac{-\phi\frac{q}{1-q}e^{-\phi t}}{\left(1+\frac{q}{1-q}e^{\phi t}\right)^2}\right) < 0$ and consequently, $\dot{\tilde{p}}'(t,\theta) = \frac{\partial \dot{\tilde{p}}(t,\theta)}{\partial \theta} = \frac{m'(\theta)}{m(\theta)}\dot{\tilde{p}}(t,\theta)$, the term in brackets can be simplified to

$$\dot{\tilde{p}}(t,\theta)\left[(r+\rho+\beta\tilde{p}(t,\theta))\frac{m'(\theta)}{m(\theta)} - \beta\tilde{p}'(t,\theta)\right] = \dot{\tilde{p}}(t,\theta)\frac{m'(\theta)}{m(\theta)}\left[r+\rho-\beta\phi\right],$$

where the last step involves using the definition of $\tilde{p}(t,\theta)$. Since $\dot{\tilde{p}}(t,\theta) < 0$ and $\frac{m'(\theta)}{m(\theta)} > 0$, one can always find a $\phi \geq (r+\rho)/\beta$ such that the entire term is non-negative.

Finally, to see that the denominator can indeed be made positive, first note that the condition that ensures search is optimal, $\tilde{p}(0,\theta) = -\phi + m(\theta)(2-q) > \frac{r+\rho}{\beta(y-b)}$, implies an upper bound on the size of the group of low types, q. Developing the denominator for a given admissible ϕ, it is straightforward to show that a sufficiently small q ensures positivity of the denominator. But then we have that, for a sufficiently large ϕ and a sufficiently low q that satisfies both constraints, $\frac{\partial w(t,\theta)}{\partial \theta} > 0$ for any admissible t. □

Note, however, that $\frac{\partial w(t,\theta)}{\partial \theta} > 0$ requires much weaker conditions because of the positive first term in condition (41). The parametric conditions merely ensure positiveness of the second term, which is a second-order effect.

B Existence and Uniqueness of Equilibrium

Existence of an interior equilibrium can be inferred from an analysis of the loci implied by G as given by condition (28) and H as given by condition (30) in the $\theta - T$-space. In fact, given the results in the text and that $\frac{\partial \tilde{p}(T,\theta)}{\partial \theta} > 0$, both loci are upward sloping.[41] For a stable equilibrium to exist, it suffices to show that

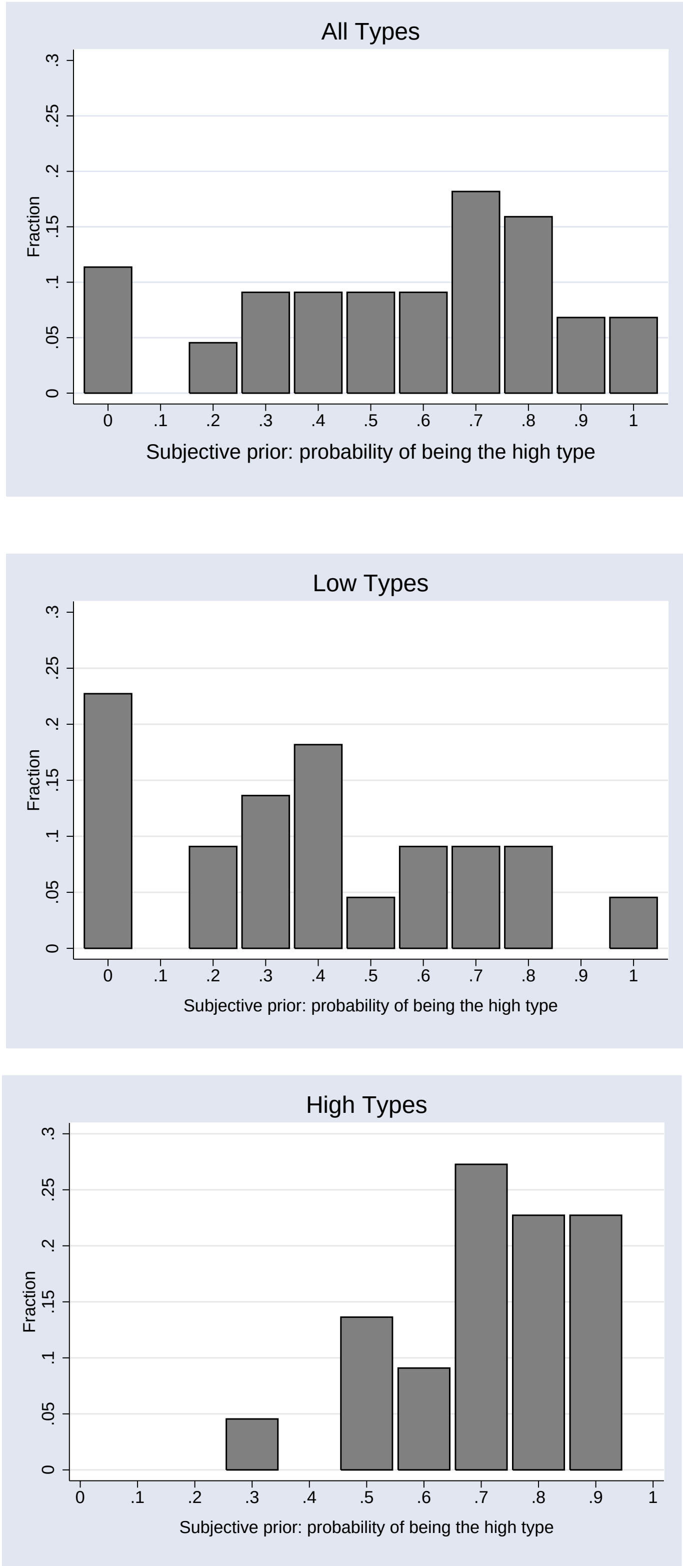
$$\left.\frac{d\theta^*}{dT^*}\right|_G > \left.\frac{d\theta^*}{dT^*}\right|_H \Leftrightarrow -\frac{\partial G/\partial T}{\partial G/\partial \theta} > -\frac{\partial H/\partial T}{\partial H/\partial \theta} .$$

But note that as a consequence of rent sharing under Nash bargaining and by the fact that $E(t|T \to \infty, \theta)$ is finite implying that the wage is bounded from below, we have that $\left|\frac{\partial G}{\partial T}\right| > \left|\frac{\partial H}{\partial T}\right|$. Thus, the direct (negative) effect of a longer unemployment spell on the subjective job finding probability $\tilde{p}(T,\theta)$ is larger in absolute terms than the indirect effect of a lower expected wage as consequence of a delayed inactivity threshold. Note also that, $\left|\frac{\partial G}{\partial \theta}\right| < \left|\frac{\partial H}{\partial \theta}\right|$: the direct negative effects of labor market tightness on the value of vacancies through congestion and wages are larger in absolute terms than the effect on subjective job finding probabilities which might be mitigated by (negative) self-perception effects. With the assumption about the difference in job finding rates of low and high types in 21 the latter are zero, however. The existence of a unique stable equilibrium can be verified by analyzing the remaining conditions studying the loci implied by G as given by condition (28) and u given by condition (16) in the $u - T$-space and the loci of H and u implied by conditions (30) and (16) in the $\theta - u$-space.

[41] With a downward sloping locus G as implied by the alternative assumptions discussed in footnote 24, the existence of a unique interior equilibrium follows immediately.

C Figures

Figure 1: Uncertainty About the Self



Notes: Subjects were assigned a high job finding probability in the search experiment (high type) if they scored higher than the median on an initial math test. After being informed of their own test score, but not the scores of others, subjects were asked: how likely do you think it is, in percentage terms, that you answered more questions correctly than half of the other subjects in the room today?