The Effects of Labor Market Policies in an Economy with an Informal Sector*

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Abstract

In many economies, there is substantial economic activity in the informal labor market, beyond the reach of government policy. Labor market policies, which by definition apply only to the formal sector can have important spillover effects on the informal sector. The relative sizes of the informal and formal sectors adjust, the skill composition of the workforce in the two sectors changes, etc.

In this paper, we build an equilibrium search and matching model to analyze the effects of labor market policies in an economy with an informal sector. Our model extends Mortensen and Pissarides (1994) by allowing for ex ante worker heterogeneity with respect to formal-sector productivity. We analyze the effects of labor market policy on informal- and formal-sector output, on the division of the workforce into unemployment, informal-sector employment and formal-sector employment, and on wages. Finally, our model allows us to examine the distributional implications of labor market policy; specifically, we analyze how labor market policy affects the distributions of wages and productivities across formal-sector matches.

1 Introduction

In this paper we construct a search and matching model that allows us to analyze the effects of labor market policies in an economy with a significant informal sector. What we mean by an informal sector is a sector that is unregulated and hence not directly affected by labor market policies such

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as severance or payroll taxes. Nonetheless we find that labor market policies in effect only in the formal sector affect the size and the composition of employment in the informal sector. This is important since in many economies, there is substantial economic activity in the informal sector. This is particularly true in developing countries. Estimates for some Latin American countries put the informal sector at more than 50 percent of the urban work force.[1] There is also evidence that the informal sector is important in many transition countries as well as in some developed economies.[2]

Although much of the literature treats the informal sector as a disadvantaged sector in a segmented labor market framework, this interpretation is not consistent with the empirical evidence. Under a segmented or dual labor market interpretation, one would expect jobs to be rationed in the primary sector and workers to be involuntarily in the secondary or informal sector. Maloney (2004) presents evidence for Latin American countries that challenges this view and instead interprets the informal sector as an unregulated micro-entrepreneurial sector. Gong and van Soest (2002) analyze the Mexican urban labor markets and their findings also challenge the dual labor market view. Both studies find that employment in the informal sector is a worker's decision determined by his or her level of human capital and potential productivity in the formal sector, which explains the negative association between informal sector employment and education level within countries.[3] This does not mean that workers in the informal sector are as well off as those in the formal sector. As Maloney notes (2004, p.12) "to say that workers are voluntarily informally employed does not imply that they are either happy or well off. It only implies that they would not necessarily be better off in the other sector." To summarize, the evidence suggests that (i) the informal sector is important in many countries, (ii) self employment represents the bulk of informality in many economies, and (iii) workers' potential productivity, determined by the level of human capital, is a major determinant of participation in the informal sector. In addition, there is evidence of significant mobility between the formal and the informal sector, and of large informal sectors in economies with very flexible labor markets.[4] This is the view of the informal sector taken in this paper.

[1] According to Maloney (2004), the informal sector includes 30 to 70 percent of urban workers in Latin American countries.

[2] Schneider and Enste (2000) give estimates for a wide range of countries. They calculate that the informal sector accounts for 10 to 20 percent of GDP in most OECD countries, 20 to 30 percent in Southern European OECD countries and in Central European transition economies. They calculate that the informal sector accounts for 20 to 40 percent for the former Soviet Union countries and as much as 70 percent for some developing countries in Africa and Asia.

[3] There is also a negative relationship between average years of education of the population and the size of the informal sector as a percentage of GDP across countries. See Masatlioglu and Rigolini (2005).

[4] Maloney (1999) presents interesting evidence for Mexico, where there is a large informal sector, even though the usual sources of wage rigidity are absent. He notes that in Mexico minimum wages are not binding, unions are more

There are other recent papers that analyze the effects of labor market and fiscal policies in models with search unemployment and an informal sector. Most of these adopt the view that the informal sector is illegal, with tax evasion and noncompliance with legislation as its identifying characteristics. These papers focus on the disutility of participation in the underground economy and analyze the effect of monitoring and punishment on informality.[5] A distinctive aspect of our paper is that, consistent with the evidence in developing countries, we consider an unregulated informal sector, which is not necessarily illegal, but rather the sector in which low-productivity workers decide to work.

Our model is also related to the macro literature that introduces home (nonmarket) production into growth models (Parente, Rogerson and Wright, 2000) and RBC models (Benhabib, Rogerson and Wright, 1991). The main idea in these papers is that introducing a home production sector improves the fit of RBC models and helps explain cross-country differences in income. By introducing nonmarket production in otherwise standard models, policies or productivity shocks not only have effects on market production but also on the substitution between market and nonmarket activities. Compared with standard models, these extensions generate lower fluctuations in output and larger cross-country differences in income, which is consistent with the data. The introduction of an informal sector in these models could produce similar effects.

Our model is a substantial extension of Mortensen and Pissarides (1994), hereafter MP, a standard model for labor market policy analysis in a search and matching framework.[6] This model is particularly attractive because it includes endogenous job separations. Specifically, we extend MP by (i) adding an informal sector and (ii) allowing for worker heterogeneity. The second extension is what makes the first one interesting. We allow workers to differ in terms of what they are capable of producing in the formal (regulated) sector. All workers have the option to take up informal sector opportunities as these come along, and all workers are equally productive in that sector, but some workers – those who are most productive in formal-sector employment – will reject informal-sector work in order to wait for a formal-sector job. Similarly, the least productive workers are shut out of the formal sector. Labor market policy, in addition to its direct effects on the formal sector, changes the composition of worker types in the two sectors. A policy change can disqualify some workers from formal-sector employment; similarly, some workers accept informal-sector work who would not have done so earlier. Labor market policy thus affects the mix of worker types in the two sectors. These compositional effects, along with the associated distributional implications, are

worried about employment preservation than about wage negotiations, and wages have shown downward flexibility.

[5] See, for example, Fugazza and Jacques (2003), Kolm and Larsen (2003), and Boeri and Garibaldi (2005).

[6] There is a substantial literature that analyzes the equilibrium effects of labor market policies in developed economies using a search and matching framework, e.g., Mortensen and Pissarides (2003).

what our heterogeneous-worker extension of MP buys.[7]

The basic MP model can be summarized as follows. First, there are frictions in the process of matching unemployed workers and vacant jobs. These frictions are modeled using a matching function $m(\theta)$, where $m(\theta)$ is the rate at which the unemployed find work, and θ, which is interpreted as labor market tightness equals v/u (the ratio of vacancies to unemployment). Second, when an unemployed worker and a vacancy meet, they match if and only if the joint surplus from the match exceeds the sum of the values they would get were they to continue unmatched. This joint surplus is then split via Nash bargaining. Third, there is free entry of vacancies, so θ is determined by the condition that the value of maintaining a vacancy equals zero. Fourth – and this is the defining innovation of MP – the rate of job destruction is endogenous. Specifically, when a worker and a firm start their relationship, match productivity is at its maximum level. Shocks then arrive at an exogenous Poisson rate, and with each arrival of a shock, a new productivity value is drawn from an exogenous distribution (and the wage is renegotiated accordingly). The productivity of a match can go up or down over time, but it can never exceed its initial level. When productivity falls below an endogenous reservation value, R, the match ends. R is determined by the condition that the value of continuing the match equals the sum of values to the two parties of remaining unmatched. Labor market tightness, θ, and the reservation productivity, R, are the key endogenous variables in MP. Firms create more vacancies (θ is higher), the longer matches last on average (the lower is R); and matches break up more quickly (the higher is R), the better are workers' outside options (the higher is θ). Equilibrium, a (θ, R) pair, is determined by the intersection of a job-creation schedule (θ as a decreasing function of R) and a job-destruction schedule (R as an increasing function of θ).

Our innovation is to assume that workers differ in their maximum productivities in formal-sector jobs. In particular, we assume that maximum productivity ("potential") is distributed across a continuum of workers of measure one according to a continuous distribution function $F(y)$, $0 \leq y \leq 1$. Workers with a high value of y start their formal-sector jobs at a high level of match productivity; workers with lower values of y start at lower levels of match productivity. As in MP, job destruction is endogenous in our model. Productivity in each match varies stochastically over time, and eventually the match is no longer worth maintaining. The twist in our model is that different worker types have different reservation productivities; that is, instead of a single reservation productivity, R, to be determined in equilibrium, there is an equilibrium reservation

[7] Dolado, Jansen and Jimeno (2005) is a related paper. They construct an MP-style model to analyze the effect of targeted severance taxes. Firms are homogeneous and can fill their vacancies with either high or low-skill workers (the fraction of workers of each type is given). When a worker and a vacancy meet, they realize the productivity of the match. They assume that the distribution of match productivity for high-skill workers stochastically dominates the one for low-skill workers.

productivity schedule, $R(y)$.[8]

The connection between our assumption about worker heterogeneity and our interest in the informal sector is as follows. We assume that the unemployed encounter informal-sector opportunities at an exogenous Poisson rate α; correspondingly, informal-sector jobs end at exogenous Poisson rate δ. Any informal-sector opportunity, if taken up, produces output at flow rate y_0, all of which goes to the worker. There is, however, a cost to taking one of these jobs; namely, we assume that informal-sector employment precludes search for a formal-sector job. This opportunity cost is increasing in y. The decision about whether or not to accept an informal-sector job thus depends on a worker's type. Workers with particularly low values of y take informal-sector jobs but do not find it worthwhile to take formal-sector jobs. For these workers, the value of waiting for an informal-sector opportunity exceeds the expected surplus that would be generated by taking a formal-sector job. These "low-productivity" workers are indexed by $0 \leq y < y^*$. Workers with intermediate values of y, "medium-productivity" workers, find it worthwhile to take both informal-sector and formal-sector jobs. These workers are indexed by $y^* \leq y < y^{**}$. Finally, workers with high values of y, "high-productivity" workers, reject informal-sector opportunities in order to continue searching for formal-sector jobs. These workers are indexed by $y^{**} \leq y < 1$. The cutoff values, y^* and y^{**}, are endogenous and are influenced by labor market policy.

We use our model to analyze the effects of two policies that are particularly important in developing countries, namely, severance taxes and payroll taxes.[9] We do this by solving our model numerically and performing policy experiments. We find that a severance tax reduces the rate at which workers find formal-sector jobs but at the same time increases average employment duration in the formal sector. There are also compositional effects, namely, fewer workers take formal-sector jobs and fewer workers reject informal-sector jobs. The net effect is that unemployment among medium- and high-productivity workers falls, as does aggregate unemployment. A payroll tax has somewhat different effects. It also reduces the rate at which workers find formal-sector jobs, but, unlike a severance tax, it decreases average employment duration in the formal sector. Again, there are compositional effects. As with the severance tax, fewer workers reject the informal sector, and more workers also reject the formal sector. Unemployment among medium- and high-productivity workers increases, as does aggregate unemployment. Even though severance taxation decreases unemployment while payroll taxation increases unemployment in our policy experiments, payroll taxation seems to be the less distorting policy. A severance tax has strong negative effects on productivity because firms keep jobs intact even when productivity is low to avoid paying the

[8] In Dolado et al. (2005), there are two reservation productivities, one for high-skill workers and one for low-skill workers.

[9] For Latin America, see Heckman and Pagés (2004); for the specific case of Colombia, see Kugler (1999) and Kugler and Kugler (2003).

tax. On the other hand, payroll taxation has a positive effect on formal-sector productivity. Only high-productivity matches are worth sustaining in the presence of a payroll tax. Both policies lead to a fall in net output, but the severance tax has the much stronger effect. Finally, our model generates distributions of productivities and wages. A severance tax leads to greater dispersion of formal-sector wages and productivity than a payroll tax does.

In the next section, we describe our model and prove the existence of a unique equilibrium. To simplify the exposition, we present our model without labor market regulations. We present the details of the model with payroll and severance taxation in the Appendix. In Section 3, we work out the implications of the model for the distributions of productivity and wages across workers in formal-sector jobs. Section 4 is devoted to our policy experiments. Our simulations give a qualitative sense for the properties of our model as well as a quantitative sense for the impact of the policies. Finally, Section 5 concludes.

2 Basic Model without Taxes

We consider a model in which workers can be in one of three states: (i) unemployed, (ii) employed in the informal sector, or (iii) employed in the formal sector. Unemployment is the residual state in the sense that workers whose employment in either an informal- or a formal-sector job ends flow back into unemployment. Unemployed workers receive b, which is interpreted as the flow income equivalent to the value of leisure. The unemployed look for job opportunities. Formal-sector opportunities arrive at endogenous rate $m(\theta)$, and informal-sector opportunities arrive at exogenous rate α.

In the informal sector, a worker receives flow income y_0, where $y_0 > b$. As mentioned above, opportunities to work in the informal sector arrive to the unemployed at Poisson rate α. Employment in this sector ends at Poisson rate δ. We assume that employment in the informal sector precludes search for a formal-sector job; i.e., there are no direct transitions from the informal to the formal sector.[10]

A worker's output in a formal-sector job depends on his or her type. Formal-sector matches initially produce at the worker's maximum potential productivity level y. Thereafter, as in MP, productivity shocks arrive at Poisson rate λ, which change the match productivity. These shocks are *iid* draws from a continuous distribution $G(x)$, where $0 \leq x \leq 1$. There are three possibilities to consider. First, if the realized value of a draw x is sufficiently low, it is in the mutual interest of the worker and the firm to end the match. Here "sufficiently low" is defined in terms of an endoge-

[10] Alternatively, one could assume that workers in informal-sector jobs can search for formal-sector opportunities but less effectively than if they were unemployed.

nous reservation productivity, $R(y)$, which depends on the worker's type. Thus, with probability $G(R(y))$, a shock ends the match. Second, if $R(y) \leq x \leq y$, the productivity of the match changes to x. That is, with probability $G(y) - G(R(y))$, the match continues after a shock, but at the new level of productivity. Finally, if the draw is such that $x > y$, we assume that the productivity of the match reverts to y. That is, with probability $1 - G(y)$, the match continues after a shock, but the productivity of the match is reset to its maximum value.[11]

The surplus from a formal-sector match is split between worker and firm using a Nash bargaining rule with an exogenous worker share, β. This surplus depends both on the current productivity of the match, y', and on the worker's type, y. As in MP, we assume the wage is renegotiated whenever match productivity changes.

2.1 Value functions

We can summarize the worker side of the model by the value functions

$$\begin{aligned}
rU(y) &= b + \alpha \max\left[N_0(y) - U(y), 0\right] + m(\theta) \max\left[N_1(y,y) - U(y), 0\right] \\
rN_0(y) &= y_0 + \delta\left(U(y) - N_0(y)\right) \\
rN_1(y',y) &= w(y',y) + \lambda G(R(y))\left(U(y) - N_1(y',y)\right) \\
&\quad + \lambda \int_{R(y)}^{y} \left(N_1(x,y) - N_1(y',y)\right) g(x)dx + \lambda\left(1 - G(y)\right)\left(N_1(y,y) - N_1(y',y)\right),
\end{aligned}$$

where $U(y)$ is the value of unemployment, $N_0(y)$ is the value of informal-sector employment, and $N_1(y',y)$ is the value of formal-sector employment in a job with current productivity level y', all of the above for a worker of type y. The final three terms in the expression for $N_1(y',y)$ reflect our assumptions about the shock process.

Next, consider the vacancy-creation problem faced by a formal-sector firm. Let V be the value of creating a formal-sector vacancy, and let $J(y',y)$ be the value of employing a worker of type y in a match with current productivity y'. The latter value can be written as

$$\begin{aligned}
rJ(y',y) &= y' - w(y',y) + \lambda G(R(y))\left(V - J(y',y)\right) \\
&\quad + \lambda \int_{R(y)}^{y} \left(J(x,y) - J(y',y)\right) g(x)dx + \lambda\left(1 - G(y)\right)\left(J(y,y) - J(y',y)\right).
\end{aligned}$$

[11] Two alternative assumptions are: (i) if a shock is drawn such that $x > y$, the productivity of the match remains where it is rather than reverting to y and (ii) shocks are drawn randomly from $[0, y]$ rather than $[0, 1]$. We prefer to assume that y is an upper bound and that shocks do not turn a low productivity (y) workers into high productivity workers. In short, we assume that sows' ears cannot be turned into silk purses.

A firm that employs a worker of type y in a match of productivity y' receives flow output y' and pays a wage of $w(y', y)$. The final three terms in this expression again reflect our assumptions about the shock process. At rate λ, a productivity shock arrives. With probability $G(R(y))$, the job ends, in which case the firm suffers a capital loss of $V - J(y', y)$. If the realized shock x falls in the interval $[R(y), y]$, the value changes from $J(y', y)$ to $J(x, y)$. Finally, with probability $1 - G(y)$, the shock resets the value of employing a worker of type y to its maximum level, $J(y, y)$.

The value of a vacancy is defined by

$$rV = -c + \frac{m(\theta)}{\theta} E \max[J(y, y) - V, 0]. \tag{1}$$

This expression reflects the assumption that match productivity initially equals the worker's type. A vacancy, however, does not know in advance what type of worker it will meet. It may, for example, meet a worker of type $y < y^*$, in which case it is not worth forming the match. If the worker is of type $y \geq y^*$, the match forms, but the job's value depends, of course, on the worker's type. Finally, note that in computing the expectation, we need to account for contamination in the unemployment pool. That is, the distribution of y among the unemployed will, in general, differ from the corresponding population distribution. We deal with this complication below in the subsection on steady-state conditions.

As usual in this type of model, the fundamental equilibrium condition is the one given by free entry of vacancies, i.e., $V = 0$. Equation (1), with $V = 0$, determines the equilibrium value of labor market tightness. The other endogenous objects of the model, namely, the wage schedule, $w(y', y)$, the reservation productivity schedule, $R(y)$, and the cutoff values, y^* and y^{**}, can all be expressed in terms of θ.

2.2 Wage Determination

We use the Nash bargaining assumption with an exogenous share parameter β to derive the wage function. Given $V = 0$, the wage for a worker of type y on a job producing at level y' solves

$$\max_{w(y',y)} [N_1(y', y) - U(y)]^{\beta} J(y', y)^{1-\beta}.$$

It is straightforward to verify that

$$w\left(y', y\right) = \beta y' + (1 - \beta) rU(y).$$

That is, the wage is a weighted sum of the current output and the worker's continuation value.

2.3 Reservation Productivity

Filled jobs are destroyed when a sufficiently unfavorable productivity shock is realized. The reservation productivity $R(y)$ is defined by

$$N_1(R(y), y) - U(y) + J(R(y), y) = 0.$$

Given the surplus sharing rule, this is equivalent to

$$J(R(y), y) = 0.$$

Substitution gives

$$R(y) = rU(y) - \frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1 - G(x)]dx. \tag{2}$$

For any fixed value of y, this is analogous to the reservation productivity in MP. Since $U(y)$ is increasing in θ for all workers who take formal-sector jobs, equation (2) defines an upward-sloping "job-destruction" locus in the $(\theta, R(y))$ plane.

An interesting question is how the reservation productivity varies with y. On the one hand, the higher is a worker's maximum potential productivity, the better are her outside options. That is, $U(y)$ is increasing in y. This suggests that $R(y)$ should be increasing in its argument. On the other hand, a "good match gone bad" retains its upside potential. The final term in equation (2), which can be interpreted as a labor-hoarding effect, is decreasing in y. This suggests that $R(y)$ should be decreasing in y. As will be seen below once we solve for $U(y)$, which of these terms dominates depends on parameters.

2.4 Unemployment values and cutoff productivities

Workers with $y < y^*$ only work in the informal sector, workers with $y^* \leq y \leq y^{**}$ accept both informal-sector and formal-sector jobs, and workers with $y > y^{**}$ accept only formal-sector jobs. Thus y^* is defined by the condition that a worker with productivity $y = y^*$ be indifferent between unemployment and a formal-sector offer, and y^{**} is defined by the condition that a worker with productivity $y = y^{**}$ be indifferent between unemployment and an informal-sector offer.

Consider a worker with $y^* \leq y \leq y^{**}$. The value of unemployment for this worker is given by

$$rU(y) = b + \alpha[N_0(y) - U(y)] + m(\theta)[N_1(y, y) - U(y)].$$

The condition that $N_1(y^*, y^*) = U(y^*)$ then implies

$$rU(y^*) = b + \alpha[N_0(y^*) - U(y^*)]$$

and substitution gives

$$rU\left(y^{*}\right)=\frac{b\left(r+\delta\right)+\alpha y_{0}}{r+\alpha+\delta}.$$

(Note that $rU\left(y\right)$ takes this value for all $y\leq y^{*}$.) Setting this equal to $N_{1}\left(y^{*},y^{*}\right)$ gives

$$y^{*}=\frac{b\left(r+\delta\right)+\alpha y_{0}}{r+\alpha+\delta}-\frac{\lambda}{r+\lambda}\int_{R(y^{*})}^{y^{*}}\left(1-G\left(x\right)\right)dx. \tag{3}$$

Since $R(y)$ is increasing in θ, so too is y^{*}.

Similarly, the condition that $N_{0}\left(y^{**}\right)=U\left(y^{**}\right)$ implies that at $y=y^{**}$

$$rU\left(y^{**}\right)=b+m\left(\theta\right)\left[N_{1}\left(y^{**},y^{**}\right)-U\left(y^{**}\right)\right].$$

Setting $U\left(y^{**}\right)=N_{0}\left(y^{**}\right)$ implies that $rU\left(y^{**}\right)=y_{0}$ and substitution gives

$$N_{1}\left(y^{**},y^{**}\right)=\frac{(r+m(\theta))y_{0}-rb}{rm(\theta)}.$$

Substituting for $N_{1}\left(y^{**},y^{**}\right)$ and solving gives

$$y^{**}=\frac{(y_{0}-b)(r+\lambda)+m\left(\theta\right)\beta y_{0}}{m\left(\theta\right)\beta}-\frac{\lambda}{r+\lambda}\int_{R(y^{**})}^{y^{**}}[1-G(x)]dx. \tag{4}$$

While $rU(y)$ has a simple form for $y\leq y^{*}$ and at y^{**}, it is more complicated at other values of y. For $y^{*}\leq y<y^{**}$, we have

$$rU\left(y\right)=\frac{\left[b\left(r+\delta\right)+\alpha y_{0}\right]\left(r+\lambda\right)+\left(r+\delta\right)m\left(\theta\right)\beta\left\{y+\frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1-G(x)]dx\right\}}{\left(r+\alpha+\delta\right)\left(r+\lambda\right)+\left(r+\delta\right)m\left(\theta\right)\beta}$$

and for $y\geq y^{**}$, we have

$$rU\left(y\right)=\frac{b\left(r+\lambda\right)+m\left(\theta\right)\beta\left\{y+\frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1-G(x)]dx\right\}}{r+\lambda+m\left(\theta\right)\beta}.$$

Note that the two expressions would be identical if the informal sector did not exist, that is, were $\alpha=\delta=0$.

Given the expression for $R(y)$, equation (2), the differing forms for $rU(y)$ mean that the form of $R\left(y\right)$ differs for high and medium productivity workers. For any fixed value of θ, equation (2) has a unique solution for $R(y)$. One can also check, given a unique schedule $R(y)$, that equations (3) and (4) imply unique solutions for the cutoff values, y^{*} and y^{**}, respectively.

2.5 Steady-State Conditions

The model's steady-state conditions allow us to solve for the unemployment rates, $u(y)$, for the various worker types. Let $u(y)$ be the fraction of time a worker of type y spends in unemployment, let $n_0(y)$ be the fraction of time that this worker spends in informal-sector employment, and let $n_1(y)$ be the fraction of time that this worker spends in formal-sector employment. Of course, $u(y) + n_0(y) + n_1(y) = 1$.

Workers of type $y < y^*$ flow back and forth between unemployment and informal-sector employment. There is thus only one steady-state condition for these workers, namely, the flows out of and into unemployment must be equal,

$$\alpha u(y) = \delta(1 - u(y)).$$

For $y < y^*$ we thus have

$$\begin{aligned} u(y) &= \frac{\delta}{\delta + \alpha} \\ n_0(y) &= \frac{\alpha}{\delta + \alpha} \\ n_1(y) &= 0. \end{aligned} \tag{5}$$

There are two steady-state conditions for workers with $y^* \leq y \leq y^{**}$, (i) the flow out of unemployment to the informal sector equals the reverse flow and (ii) the flow out of unemployment into the formal sector equals the reverse flow,

$$\begin{aligned} \alpha u(y) &= \delta n_0(y) \\ m(\theta) u(y) &= \lambda G(R(y)) (1 - u(y) - n_0(y)). \end{aligned}$$

Combining these conditions gives

$$\begin{aligned} u(y) &= \frac{\delta \lambda G(R(y))}{\lambda (\delta + \alpha) G(R(y)) + \delta m(\theta)} \\ n_0(y) &= \frac{\alpha \lambda G(R(y))}{\lambda (\delta + \alpha) G(R(y)) + \delta m(\theta)} \\ n_1(y) &= \frac{\delta m(\theta)}{\lambda (\delta + \alpha) G(R(y)) + \delta m(\theta)} \end{aligned} \tag{6}$$

for $y^* \leq y \leq y^{**}$.

Finally, for workers with $y > y^{**}$ there is again only one steady-state condition, namely, that the flow from unemployment to the formal sector equals the flow back into unemployment,

$$m(\theta) u(y) = (1 - u(y)) \lambda G(R(y)).$$

This implies

$$
\begin{aligned}
u(y) &= \frac{\lambda G(R(y))}{\lambda G(R(y)) + m(\theta)} \\
n_0(y) &= 0 \\
n_1(y) &= \frac{m(\theta)}{\lambda G(R(y)) + m(\theta)}
\end{aligned}
\tag{7}
$$

for $y > y^{**}$.

Total unemployment is obtained by aggregating across the population,

$$u = \int_0^{y^*} u(y) f(y)dy + \int_{y^*}^{y^{**}} u(y) f(y)dy + \int_{y^{**}}^1 u(y) f(y)dy.$$

2.6 Equilibrium

We use the free-entry condition to close the model and determine equilibrium labor market tightness. Setting $V = 0$ we have

$$c = \frac{m(\theta)}{\theta} E \max[J(y, y), 0].$$

To determine the expected value of meeting a worker, we need to account for the fact that the density of types among unemployed workers is contaminated. Let $f_u(y)$ denote the density of types among the unemployed. Using Bayes Law,

$$f_u(y) = \frac{u(y)f(y)}{u}.$$

The free-entry condition can thus be rewritten as

$$c = \frac{m(\theta)}{\theta} \int_{y^*}^1 J(y, y) \frac{u(y)}{u} f(y)dy.$$

After substitution, this becomes

$$c = \frac{m(\theta)}{\theta} (1 - \beta) \int_{y^*}^1 \left(\frac{y - R(y)}{r + \lambda} \right) \frac{u(y)}{u} f(y)dy. \tag{8}$$

This expression takes into account that $J(y, y) < 0$ for $y < y^*$, i.e., some contacts do not lead to a match. Note also that the forms of $R(y)$ and of $u(y)$ differ for medium-productivity and high-productivity workers.

A steady-state equilibrium is a labor market tightness θ, together with a reservation productivity function $R(y)$, unemployment rates $u(y)$, and cutoff values y^* and y^{**} such that

(i) the value of maintaining a vacancy is zero

(ii) matches are consummated and dissolved if and only if it is in the mutual interest of the worker and firm to do so

(iii) the steady-state conditions hold

(iv) formal-sector matches are not worthwhile for workers of type $y < y^*$

(v) informal-sector matches are not worthwhile for workers of type $y > y^{**}$.

A unique equilibrium exists if there is a unique value of θ that solves equation (8), taking into account that $R(y)$, $u(y)$, y^*, and y^{**} are all uniquely determined by θ. Note that

1. $R(y)$ is increasing in θ for each y;

2. $\int_{y^*}^{1} \frac{u(y)}{u} f(y)dy$ is decreasing in θ;

3. y^* is increasing in θ.

These three facts imply that the right-hand side of the free-entry condition is decreasing in θ. This result, together with the facts that the limit of the right-hand side of (8) as $\theta \to 0$ equals ∞ and equals 0 as $\theta \to \infty$, implies the existence of a unique θ satisfying the free-entry condition.

3 Distributional Characteristics of Equilibrium

Given assumed functional forms for the distribution functions, $F(y)$ and $G(x)$, and for the matching function, $m(\theta)$, and given assumed values for the exogenous parameters of the model, equation (8) can be solved numerically for θ. Given θ, we can then recover the other equilibrium objects of the model, namely, the cutoff values y^* and y^{**}, the reservation productivity schedule, $R(y)$, the wage schedule, $w(y', y)$, the type-specific unemployment rates, $u(y)$, etc. In fact, we can do more than this. Once we solve for equilibrium, we can compute the distributions of productivity and wages in formal-sector employment. We can then use these distributions to evaluate both the aggregate and distributional effects of labor market policy.

To begin, we discuss the computation of the joint distribution of (y', y) across workers employed in the formal sector. Once we compute this joint distribution (and the corresponding marginals), we can find the distribution of wages in the formal sector. To find the distribution of (y', y) across workers employed in the formal sector, we use

$$h(y', y) = h(y'|y)h(y).$$

Here $h(y', y)$ is the joint density, $h(y'|y)$ is the conditional density, and $h(y)$ is the marginal density across workers employed in the formal sector. It is relatively easy to compute $h(y)$. Let E denote "employed in the formal sector." Then by Bayes Law,

$$h(y) = \frac{P[E|y]f(y)}{P[E]} = \frac{n_1(y)f(y)}{\int n_1(y)f(y)dy},$$

where from equations (5) to (7),

$$\begin{aligned} n_1(y) &= 0 && \text{for } y \leq y^* \\ &= \frac{\delta m(\theta)}{\delta m(\theta) + (\alpha+\delta)\lambda G(R(y))} && \text{for } y^* \leq y < y^{**} \\ &= \frac{m(\theta)}{m(\theta) + \lambda G[R(y)]} && \text{for } y \geq y^{**}. \end{aligned}$$

Next, we need to find $h(y'|y)$. Consider a worker of type y who is employed in the formal sector. Her match starts at productivity y; later, a shock (or shocks) may change her match productivity. Let N denote the number of shocks this worker has experienced to date (in her current spell of employment in a formal-sector job). Since we are considering a worker who is employed in the formal sector, we know that none of these shocks has resulted in a productivity realization less than $R(y)$.

If $N = 0$, then $y' = y$ with probability 1. If $N > 0$, then $y' = y$ with probability $\dfrac{1 - G(y)}{1 - G(R(y))}$, i.e., the probability that the productivity shock is greater than or equal to y (conditional on the worker being employed) in which case the productivity reverts to y. Combining these terms, we have that conditional on y, $y' = y$ with probability $P[N = 0] + \dfrac{1 - G(y)}{1 - G(R(y))}P[N > 0]$. Similarly, the conditional density of y' for $R(y) \leq y' < y$ is

$$h(y'|y) = \frac{g(y')}{1 - G(R(y))}P[N > 0] \text{ for } R(y) \leq y' < y.$$

Thus, for a worker of type y, we need to find $P[N = 0]$.

To do this, we first condition on elapsed duration. Consider a worker of type y whose elapsed duration of employment in her current formal sector job is t. This worker type exits formal sector employment at Poisson rate $\lambda G(R(y))$; equivalently, the distribution of completed durations for a worker of type y is exponential with parameter $\lambda G(R(y))$. The exponential has the convenient property that the distributions of completed and elapsed durations are the same.

Let N_t be the number of shocks this worker has realized given elapsed duration t. Shocks arrive at rate λ. However, as the worker is still employed, we know that none of the realizations of these shocks was below $R(y)$. Thus, N_t is Poisson with parameter $\lambda(1 - G(R(y))t$, and $P[N_t = 0] = \exp\{-\lambda(1 - G(R(y))t\}$. Integrating $P[N_t = 0]$ against the distribution of elapsed duration gives

$$P[N = 0] = \int_0^\infty \exp\{-\lambda(1 - G(R(y))t\}\lambda G(R(y)) \exp\{-\lambda G(R(y))t\}dt = G(R(y))$$

for a worker of type y.

Thus, the probability that a worker of type y is working to her potential when she is employed in a formal sector job (i.e., that $y' = y$) is

$$P[y' = y|y] = G(R(y)) + \frac{1 - G(y)}{1 - G(R(y))}(1 - G(R(y))) = 1 - (G(y) - G(R(y))).$$

The density of y' across all other values that are consistent with continued formal-sector employment for a type y worker is

$$h(y'|y) = \frac{g(y')}{1 - G(R(y))}(1 - G(R(y))) = g(y') \text{ for } R(y) \leq y' < y.$$

Given the marginal density for y and the conditional density, $h(y'|y)$, we thus have the joint density, $h(y', y)$, which is defined for for $y^* \leq y \leq 1$ and $R(y) \leq y' \leq y$, i.e., for the (y', y) combinations that are consistent with formal-sector employment.

The final steps are to compute the densities of productivity in formal-sector employment (i.e., the marginal density of y') and of formal-sector wages. The density of y' is computed as $h(y') = \int h(y'|y)h(y)dy = \int h(y', y)dy$.

To derive the distribution of wages across formal-sector employment, we use $m(w) = \int m(w|y)h(y)dy$. We thus need the conditional distribution of wages given worker type, i.e., $m(w|y)$. Worker y receives $w(y, y)$ up to the time that the first shock to match productivity is realized, i.e., while $N = 0$. After the realization of the first shock (when $N > 0$), $y' = y$ with probability $\frac{1 - G(y)}{1 - G(R(y))}$. The probability that worker y receives $w(y; y)$ is thus $P[y' = y|y] = 1 - (G(y) - G(R(y)))$. In addition, a worker of type y can earn wages in the range $w \in [w(R(y); y), w(y; y))$ once the first productivity shock is realized. Using $h(y'|y) = g(y')$ for $R(y) \leq y' < y$ and $w\,(y', y) = \beta y' + (1-\beta)rU(y)$ implies that the density of $w\,(y', y)$ conditional on y is $\frac{1}{\beta}g(\frac{w - (1 - \beta)rU(y)}{\beta})$ for $w \in [w(R(y); y), w(y; y))$.

Summarizing, given y we have

$$\begin{aligned} P[w &= w(y; y)] = 1 - (G(y) - G(R(y))) \\ m(w|y) &= \frac{1}{\beta}g(\frac{w - (1 - \beta)rU(y)}{\beta}) \text{ for } w \in [w(R(y); y), w(y; y)). \end{aligned}$$

Since there is a mass point in the distribution of current productivity given worker type, there is likewise a mass point in the conditional distribution of wages given worker type. The final step is then to carry out the integration, $m(w) = \int m(w|y)h(y)dy$.

4 Policy Experiments

We now present our numerical analysis of the model and examine the effects of labor market policy. Specifically, we look at a severance tax and a payroll tax.[12] In the Appendix, we present the model augmented to incorporate these two taxes. Incorporating a payroll tax is straightforward, but incorporating a severance tax is not. The presence of a severance tax means that the initial Nash bargain and subsequent Nash bargains when productivity shocks occur differ because, following Mortensen and Pissarides (1999), "termination costs are not incurred if no match is formed initially but must be paid if an existing match is destroyed". In other words, if the bargaining breaks down in the initial negotiation, the firm does not have to pay a severance tax, but in subsequent negotiations, the firm's outside option must include the severance tax.

For all our simulations, we assume the following functional forms and parameter values. First, we assume that the distribution of worker types, i.e., y, is uniform over $[0, 1]$ and that the productivity shock, i.e., x, is likewise drawn from a standard uniform distribution. We assume the standard uniform for computational convenience, but it is not appreciably more difficult to solve the model using flexible parametric distributions, e.g., betas, for $F(y)$ and/or $G(x)$. Second, we assume a Cobb-Douglas matching function, namely, $m(\theta) = 4\theta^{1/2}$. Third, we chose our parameter values with a year as the implicit unit of time. We set $r = 0.05$ as the discount rate. We normalize the flow income equivalent of leisure to $b = 0$. The parameters for the informal sector are $y_0 = 0.35$, $\alpha = 5$ and $\delta = 0.5$, and the formal-sector parameters are $c = 0.3$, $\beta = 0.5$, and $\lambda = 1$. Note that the share parameter, β, equals the elasticity of the matching function with respect to labor market tightness. Our parameter values were chosen to produce plausible results for our baseline case in which there is no severance tax or payroll tax.

Consider first the baseline case given in row 1 of Table 1. With no severance or payroll taxation, our baseline generates a labor market tightness of 1.21. More than 30 percent of the labor force is "low productivity" and works only in the informal sector, while about 60 percent of the labor force works only in the formal sector. The remaining 10 percent would work in either sector. The reservation productivity for the worker who is just on the margin of working in the formal sector ($y = y^*$) is the same as that worker's type. With no severance tax, it is worthwhile employing this worker even though the match would end were its productivity to go even a bit below its maximum level. Next, note that $R(y^{**}) < R(y^*)$. As noted earlier, there are two effects of y on the reservation productivity. First, more productive workers have greater "upside potential"; on the other hand, more productive workers have better outside options. The first effect dominates among medium-productivity workers for this parameterization, however, the first panel of Figure 1

[12] For simplicity, we ignore any benefits from the use of the tax revenues that arise from these policies.

shows that $R(y)$ is rising above y^{**}, i.e., for all high-productivity workers. The next four columns in Table 1 give average unemployment rates. The average unemployment rate for the baseline case is 8.6 percent. Among low-productivity workers, the unemployment rate is $\delta/(\alpha+\delta) = 9.1$ percent. The average unemployment rate for medium-productivity workers is much lower, reflecting the fact that these workers take both informal- and formal-sector jobs. Finally, the average unemployment rate for high-productivity workers is 9.1 percent. This reflects the fact that these workers do not take up informal-sector opportunities. Next, we present average productivity for workers in the formal sector, which is 0.639, and the average wage paid to formal-sector workers, which is 0.579. The final column gives net total output, Y, i.e., the sum of outputs from the informal and formal sectors net of vacancy creation costs, $c\theta u$.

The next four rows of Table 1 show the effect of raising the severance tax, s. Since the severance tax makes vacancy creation less attractive, we find that θ, labor market tightness, decreases. The severance tax shifts the reservation productivity schedule down. This is a consequence of the fact that the severance tax makes it more costly to end matches. In addition, the severance tax affects the composition of the sectors. The two cutoff values y^* and y^{**} increase with s; that is, formal-sector employment is less attractive to the previously marginal workers. The reason is that although jobs last longer when the severance tax is higher, the expected formal-sector wage decreases with s. There is a slight decrease in the difference between y^{**} and y^*; that is, the number of workers who accept a job in either sector decreases slightly. The unemployment rate for low-productivity workers is unaffected by the severance tax, but the unemployment rates for medium- and high-productivity workers fall significantly. The effect of increasing job duration outweighs the reduction in the job arrival rate. Since the reduction in unemployment associated with formal-sector jobs outweighs the effect of the increase in the number of workers in the high-unemployment informal sector, the overall unemployment rate falls. While the unemployment effects of the severance tax make it seem attractive, this policy has strong negative effects on productivity. The large downward shift in the $R(y)$ schedule (see Figure 1) implies that jobs last longer, leading to a reduction in average productivity ($\overline{y}'$) in the formal sector. Wages in the formal sector fall, as does net output.

Table 2 presents the effects of varying the payroll tax, τ, holding the severance tax at zero. We consider payroll taxes ranging from zero to 20 percent. A payroll tax of $\tau = 0.2$ imposes approximately a 50% greater cost than does a severance tax of $s = 0.2$.[13]. As shown in Table 2, increasing the payroll tax reduces θ since it makes formal-sector vacancy creation less attractive. In contrast to the effect of the severance tax, a payroll tax decreases job duration by shifting up the reservation productivity schedule, as can be seen in Figure 1. The payroll tax also has compositional effects.

[13] When $\tau = 0.2$, the average formal-sector wage is 0.509, giving an approximate annual cost of 0.1, while $s = 0.2$ leads to an expected employment duration of approximately 3 years and a corresponding annual cost of about 0.067

The fraction of workers who never take formal-sector jobs ($y < y^*$) increases substantially with τ, and the fraction who only take formal-sector jobs ($y > y^{**}$) decreases substantially with τ. Given that a payroll tax has a stronger effect on the unemployment value of high-productivity workers than on that of medium-productivity workers, y^{**} increases by more than y^* with τ. This means that the fraction of workers who would take any job increases with τ. The fact that both labor market tightness and expected formal sector job duration decrease implies that unemployment increases among high-productivity workers. The effect on overall unemployment, however, is mitigated to some extent by the compositional changes. Consistent with the compositional change and the shift in the reservation productivity schedule, average formal-sector productivity rises. Formal sector wages fall, as does net output.

It is interesting to compare the effects of the two taxes. Although the severance tax reduces aggregate unemployment (by increasing average job duration), it is inferior to the payroll tax on almost all other dimensions. The main negative effect of the severance tax is on formal-sector productivity. This is why a severance tax leads to a substantially greater decrease in aggregate net output than does a revenue-equivalent payroll tax. The severance tax encourages firms to maintain low-productivity matches and has a strong negative effect on vacancy creation. While both taxes increase the size of the informal sector, they have opposite effects on formal-sector employment. Both taxes increase y^{**} but the job-duration effects of the severance tax increase employment in the formal sector.

The final table examines the effects of increasing s and τ simultaneously to $s = \tau = 0.1$. Since both of these taxes make vacancy creation less attractive, labor market tightness falls. There is a slight decrease in the average unemployment rate among medium- and high-productivity workers, which leads to a corresponding fall in the aggregate rate. Employment duration in the formal sector increases, i.e., the severance tax effect dominates, as can be seen in Figure 1. This results in a decrease in average productivity in the formal sector reflecting the downward shift in the reservation productivity schedule. Net output decreases.

Figures 2 and 3 illustrate the effects of s and τ on the distribution of types (y), current productivities (y') and wages in the formal sector. The density of y is the contaminated one; i.e., it incorporates the different job-finding and job-losing experiences of the various worker types. Since no worker's current productivity can exceed his or her type, the distribution of y necessarily first-order stochastically dominates that of y'. Figure 2 shows how the severance tax and the payroll tax compress the distribution of types in the formal sector. The severance tax shifts the density of current productivity to the left, reflecting the decrease in reservation productivities. The payroll tax shifts the density of y' to the right, reflecting the upward shift in the reservation productivity schedule. Regarding wages (Figure 3), both taxes compress the wage distribution, although the effect of the payroll tax is greater. Finally, while Figures 2 and 3 suggest that the distributional effects of the payroll tax are more pronounced, when analyzing the two policies together the severance tax effects dominate.

5 Conclusions

In this paper, we build a search and matching model to analyze the effects of labor market policies in an economy with a significant informal sector. In light of the empirical evidence for many developing countries, we model an economy where workers operate as self employed in the informal sector. Additionally, depending on their productivity levels, some workers only work in the formal sector, others only work in the informal sector, and an intermediate group of workers goes back and forth between the formal and the informal sectors.

We examine the effects of two particularly important labor market policies, a severance tax and a payroll tax. Despite the fact that both policies reduce the rate at which workers find formal-sector jobs, their effects on unemployment duration, unemployment rates, and the distribution of workers across the sectors are different. A severance tax greatly increases average employment duration in the formal sector, reduces overall unemployment, reduces the number of formal-sector workers, and reduces the number of workers who accept any type of offer (formal or informal). In contrast, a payroll tax reduces average employment duration in the formal sector, greatly reduces the number of formal-sector workers, and significantly increases the size of the informal sector and the number of workers accepting any type of offer. Total unemployment rises under the payroll tax. The two policies also have different effects on the distributions of productivity and wages in the formal sector (Figures 2 and 3). The severance tax decreases average productivity, while the payroll tax increases it, but under both policies, net output falls.

6 Appendix - The Model with Taxes

In this appendix, we augment the model presented in Section 2 to include a severance tax and a payroll tax. The introduction of a payroll tax is straightforward, but the introduction of a severance tax requires us to distinguish between the initial negotiation between a worker and a vacant job and subsequent negotiations. In the initial negotiation, if the bargaining breaks down, the firm does not have to pay a severance tax; but in subsequent negotiations, the firm's outside option must include the severance tax. Thus, the worker has two values of employment, $N_1(y)$, the initial value of employment for a worker of productivity y, and $N_1(y', y)$, the value of employment for a worker of productivity y in a match that has experienced one or more shocks and has current productivity y'.

The worker value functions become

$$
\begin{aligned}
rU(y) &= b + \alpha \max\left[N_0(y) - U(y), 0\right] + m(\theta) \max\left[N_1(y) - U(y), 0\right] \\
rN_0(y) &= y_0 + \delta\left(U(y) - N_0(y)\right) \\
rN_1(y) &= w(y) + \lambda G(R(y))\left(U(y) - N_1(y)\right) + \lambda \int_{R(y)}^{y} \left(N_1(x, y) - N_1(y)\right) g(x)dx \\
&\quad + \lambda\left(1 - G(y)\right)\left(N_1(y, y) - N_1(y)\right) \\
rN_1(y', y) &= w(y', y) + \lambda G(R(y))\left(U(y) - N_1(y', y)\right) + \lambda \int_{R(y)}^{y} \left(N_1(x, y) - N_1(y', y)\right) g(x)dx \\
&\quad + \lambda\left(1 - G(y)\right)\left(N_1(y, y) - N_1(y', y)\right),
\end{aligned}
$$

where $w(y)$ is the initial wage paid to a worker of type y and $w(y', y)$ is the wage paid to this worker once one or more shocks to the match have been realized. As we show below, $w(y) \neq w(y, y)$.

We assume the two taxes are nominally paid by employers so the value functions for the job must reflect this. We denote the initial value of filling a job by $J(y)$ and the current value of a filled job (after a shock has been realized and the current productivity is y') by $J(y', y)$. We then have

$$
\begin{aligned}
rJ(y) &= y - w(y)(1 + \tau) + \lambda G(R(y))\left(V - J(y) - s\right) \\
&\quad + \lambda \int_{R(y)}^{y} \left(J(x, y) - J(y)\right) g(x)dx + \lambda\left(1 - G(y)\right)\left(J(y, y) - J(y)\right) \\
rJ(y', y) &= y' - w(y', y)(1 + \tau) + \lambda G(R(y))\left(V - J(y', y) - s\right) \\
&\quad + \lambda \int_{R(y)}^{y} \left(J(x, y) - J(y', y)\right) g(x)dx + \lambda\left(1 - G(y)\right)\left(J(y, y) - J(y', y)\right),
\end{aligned}
$$

where τ is the payroll tax rate and s is the severance tax. The value of a vacancy is

$$rV = -c + \frac{m(\theta)}{\theta} E \max[J(y) - V, 0].$$

The initial wage $w(y)$ for a worker of type y solves

$$\max_{w(y)} [N_1(y) - U(y)]^{\beta} J(y)^{1-\beta},$$

while the wage $w(y', y)$ for a type y worker producing at y' solves

$$\max_{w(y',y)} \left[N_1(y', y) - U(y)\right]^{\beta} \left[J(y', y) + s\right]^{1-\beta}.$$

The initial wage and the subsequent wages can then be written as

$$\begin{aligned} w(y) &= \frac{\beta(y - \lambda s) + (1-\beta)(1+\tau) rU(y)}{1+\tau} \\ w(y', y) &= \frac{\beta(y' + rs) + (1-\beta)(1+\tau) rU(y)}{1+\tau}. \end{aligned}$$

Clearly, $w(y, y) > w(y)$. Note that this introduces a complication into the distributional analysis; namely, conditional on y, the density of wages now has two mass points, one at $w(y)$ and one at $w(y, y)$.

The reservation productivity for a worker of productivity y is defined by

$$N_1(R(y), y) - U(y) + J(R(y), y) = -s.$$

Given the surplus sharing rule, this is equivalent to $J(R(y), y) = -s$. Substitution gives

$$R(y) = (1+\tau) rU(y) - sr - \frac{\lambda}{r+\lambda} \int_{R(y)}^{y} [1 - G(x)] dx. \tag{A1}$$

This makes it clear that s shifts the $R(y)$ schedule down, while τ shifts it up. Again, since $U(y)$ is increasing in θ for each fixed value of y, so too is $R(y)$.

The next step is to solve for y^* and y^{**}. The first cutoff value is given by

$$y^* = (1+\tau) \frac{b(r+\delta) + \alpha y_0}{r + \alpha + \delta} + \lambda s - \frac{\lambda}{r+\lambda} \int_{R(y^*)}^{y^*} (1 - G(x)) dx.$$

As in the basic model, since $R(y)$ is increasing in θ, y^* is also increasing in θ. The second cutoff value, y^{**}, is given by

$$y^{**} = (1+\tau) \frac{(y_0 - b)(r+\lambda) + m(\theta) \beta y_0}{m(\theta) \beta} + \lambda s - \frac{\lambda}{r+\lambda} \int_{R(y^{**})}^{y^{**}} [1 - G(x)] dx.$$

The flow value of unemployment, $rU(y)$, for $y \leq y^*$ depends on neither y nor taxes and is given by

$$rU(y) = \frac{b(r+\delta) + \alpha y_0}{r+\alpha+\delta}.$$

The expression for $U(y)$ for low-productivity workers is the same as the one given in the basic model since these workers are unaffected by what goes on in the formal sector. For $y^* \leq y < y^{**}$, we have

$$rU(y) = \frac{[b(r+\delta) + \alpha y_0](r+\lambda) + \frac{(r+\delta)m(\theta)\beta}{1+\tau}\left\{y - \lambda s + \frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1-G(x)]dx\right\}}{(r+\alpha+\delta)(r+\lambda) + (r+\delta)m(\theta)\beta}$$

and for $y \geq y^{**}$, we have

$$rU(y) = \frac{b(r+\lambda) + \frac{m(\theta)\beta}{1+\tau}\left\{y - \lambda s + \frac{\lambda}{r+\lambda}\int_{R(y)}^{y}[1-G(x)]dx\right\}}{r+\lambda+m(\theta)\beta}.$$

As in the model with no taxes, there are unique solutions for y^* and y^{**} given θ.

The model's steady-state conditions are the same as those given in the text and so are not repeated here. Finally, we use the free-entry condition to close the model and determine equilibrium labor market tightness. Setting $V = 0$ we have

$$c = \frac{m(\theta)}{\theta} E \max[J(y), 0].$$

After substitution, the free-entry condition is

$$c = \frac{m(\theta)(1-\beta)}{\theta}\int_{y^*}^{1}\left(\frac{y - R(y)}{r+\lambda} - s\right)\frac{u(y)}{u}f(y)dy. \tag{A2}$$

A unique equilibrium exists if there is a unique value of θ that solves equation $(A2)$, taking into account that $R(y)$, $u(y)$, and y^* all depend on θ. As in the basic model,

1. $R(y)$ is increasing in θ for each y;

2. $\int_{y^*}^{1} \frac{u(y)}{u} f(y)dy$ is decreasing in θ;

3. y^* is increasing in θ.

These three conditions imply that the right-hand side of the free-entry condition is decreasing in θ. This result, together with the facts that the limit of the right-hand side of $(A2)$ as $\theta \rightarrow 0$ equals ∞ and equals 0 as $\theta \rightarrow \infty$, implies the existence of a unique θ satisfying the free-entry condition.

Of course, this argument assumes that s and τ are low enough so that some worker-firm matches are profitable. There is a simple sufficient condition to ensure this. Since $J(y)$ is increasing in y, it suffices to check that $y^* < 1$. Using the expression for y^* given above, that condition is in turn ensured by

$$(1+\tau)\frac{b\,(r+\delta)+\alpha y_0}{r+\alpha+\delta}+\lambda s<1.$$

This condition is satisfied trivially in the model with no taxes.

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Table 1: Effects of Varying s with $\tau = 0$

						u rates by type				formal		output
s	θ	y^*	y^{**}	$R(y^*)$	$R(y^{**})$	low	med	high	total	$\overline{y}'$	$\overline{w}$	Y
0	1.21	0.315	0.410	0.315	0.243	0.091	0.037	0.091	0.086	0.639	0.579	0.459
.05	1.12	0.331	0.425	0.278	0.200	0.091	0.035	0.085	0.082	0.631	0.562	0.458
.10	1.03	0.345	0.439	0.240	0.153	0.091	0.031	0.079	0.078	0.617	0.544	0.453
.15	0.94	0.357	0.451	0.200	0.104	0.091	0.027	0.071	0.074	0.601	0.528	0.448
.20	0.85	0.368	0.458	0.158	0.049	0.091	0.021	0.061	0.068	0.578	0.510	0.440

Table 2: Effects of Varying τ with $s = 0$

						u rates by type				formal		output
τ	θ	y^*	y^{**}	$R(y^*)$	$R(y^{**})$	low	med	high	total	$\overline{y}'$	$\overline{w}$	Y
0	1.21	0.315	0.410	0.315	0.243	0.091	0.037	0.091	0.086	0.639	0.579	0.459
.05	1.19	0.331	0.434	0.331	0.257	0.091	0.039	0.093	0.087	0.647	0.560	0.459
.10	1.17	0.347	0.459	0.347	0.272	0.091	0.040	0.095	0.088	0.656	0.540	0.458
.15	1.15	0.363	0.484	0.363	0.287	0.091	0.041	0.098	0.088	0.665	0.524	0.456
.20	1.12	0.378	0.511	0.378	0.302	0.091	0.043	0.100	0.089	0.675	0.509	0.455

Table 3: Results with $s = \tau = 0$ and $s = \tau = 0.1$

					u rates by type				formal		output
θ	y^*	y^{**}	$R(y^*)$	$R(y^{**})$	low	med	high	total	$\overline{y}'$	$\overline{w}$	Y
1.21	0.315	0.410	0.315	0.243	0.091	0.037	0.091	0.086	0.639	0.579	0.459
0.98	0.380	0.495	0.275	0.186	0.091	0.035	0.084	0.081	0.636	0.509	0.451

Figure 1: $R(y)$ for Various (s, τ) Combinations

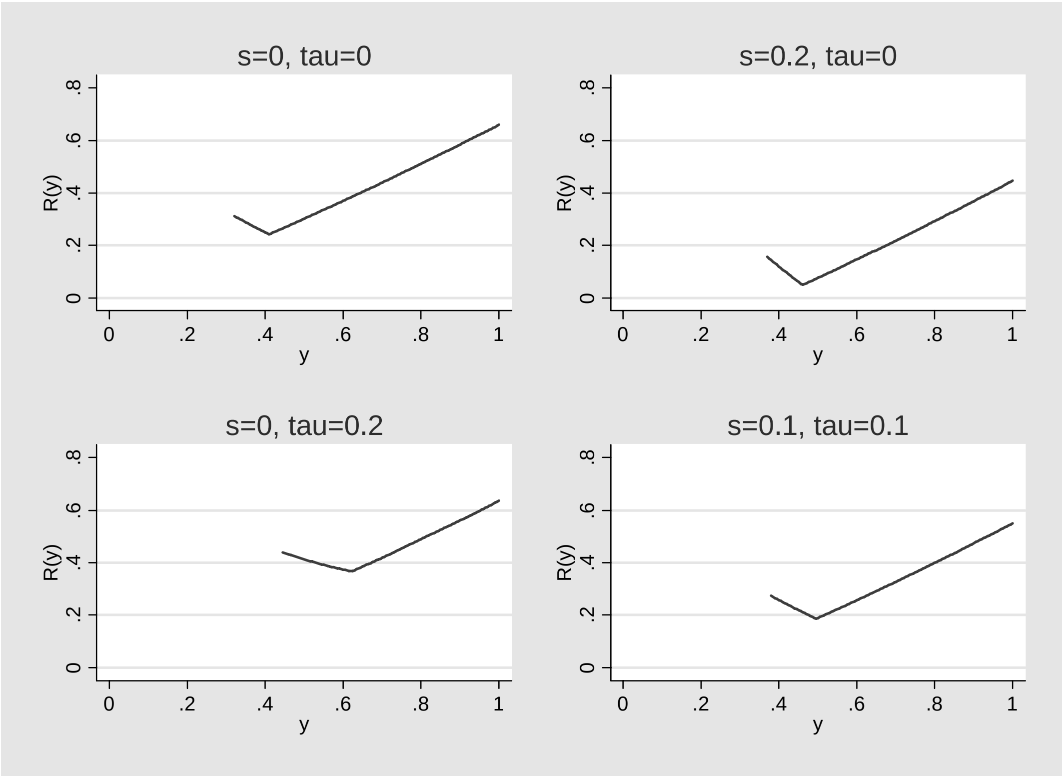

Figure 2: Densities of y and y' for Various (s, τ) Combinations

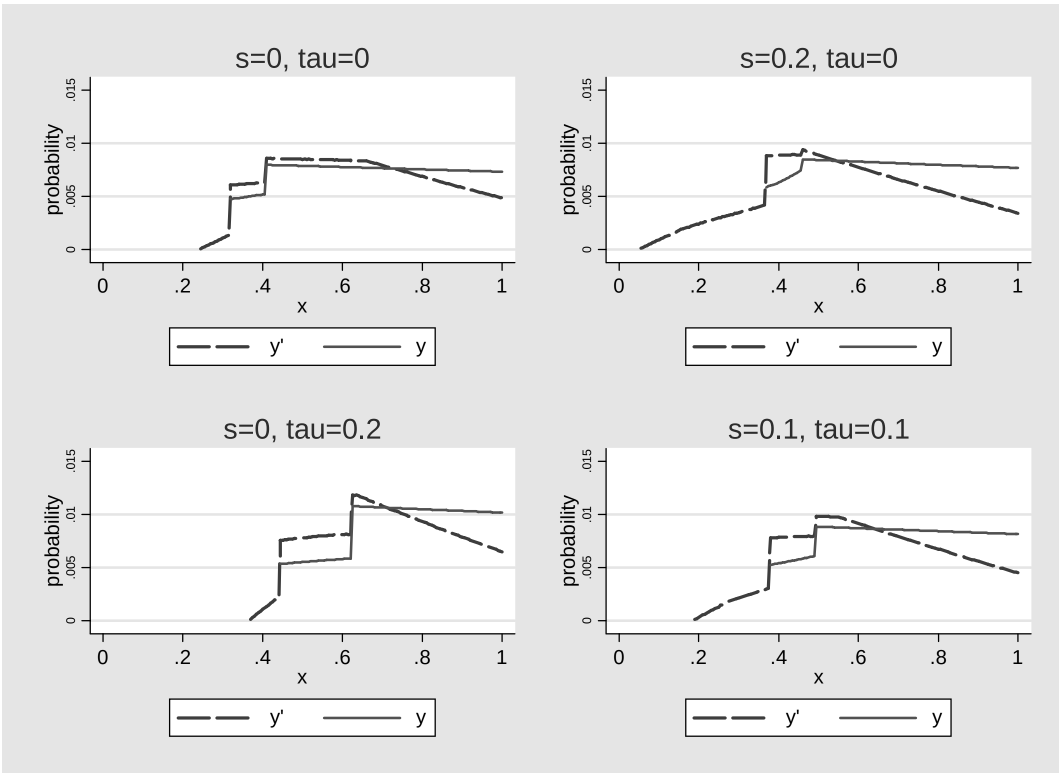

Figure 3: Densities of y' and w for Various (s, τ) Combinations