Public Sector Wage Policy, Informality and Labor Market Equilibrium in Developing Countries: The Egyptian and Jordanian Cases *

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Preliminary and Incomplete

Abstract

In this paper, we develop and estimate a structural model with two different employment sectors; namely public and private sectors, in a labor market with search frictions with heterogeneous productivities. This allows us to then extend the model to segment the private sector into formal and informal sectors. The model aims at exploring the labor market dynamics within an imperfect-information environment particularly in developing countries where sizable public and informal sectors exist. Following Bradley et al. (2012) we assume the wage distribution and employment rate in the public sector as exogenous policy parameters. The private sector wage distributions and employment rates are however determined endogenously. Profit-maximizing firms choose endogenously whether to locate in the informal or formal sector, all taking into consideration their optimal response to the public sector employment policies. Aiming to reflect reality the closest possible, job turnover is sector specific and transitions between sectors depend on the worker's decision trying to maximize his expected lifetime utility, i.e current utility as well as the expected gain or loss. The model is estimated using Egyptian and Jordanian labor market panel surveys (ELMPS2012, ELMPS2006 & JLMPS2010) by a method of matching empirical and theoretical moments. We use the model then to simulate the impact of various counter-factual labor market policies.

Keywords: labor market frictions, wage posting, on-the-job search, informality, public sector.

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1 Introduction

In any economy, the public sector occupies a sizeable share of the jobs' market.Moreover, The segmentation of labor markets and the existence of an informal sector is almost a fact in developing as well as developed countries. It is often argued indeed that they are the engine growth because their existence allows firms to operate in an environment where wage and regulatory costs are lower. Yet, and on the other hand, informality means less insurance and stability to the worker. In the Egyptian and Jordanian labor markets, this phenomenon is quite common where informality counts to approximately 60% of the labor market. Thus, if the theory omits one of these three sectors, the measures of the job turnovers, as transitions between sectors, can be biased. Indeed, the workers' occupation choices are based on the comparisons between their expected job values in the current or all prospective jobs.

Consequently, limiting the analysis, as previous literature, to an unsegmented Private sector labor market did not sufficient to reflect the different transitions. These partial analysis also underestimate the search frictions in the labor markets of developing countries. Hence, this can lead to a fail in explaining of the persistent unemployment and the wage dispersion among similar workers.

Aiming to adapt to the nature of the economy in developing countries, this paper extends the Burdett and Mortensen (1998)[4] model to allow for three sectors of employment in a labor market with search frictions: a sizeable public sector, a formal and an informal private sector. In this paper, we combine previous approaches such as those adopted by Meghir et al.(2011)[10], El Badaoui et al.(2010)[6] and Bradely et al. (2011)[3], to extend the Burdett Mortensen (1998) model to include more than one employment sector. This original framework allows us to simulate the labor market transitions between the different sectors as well as between the employment and unemployment states. Following Bradley et al.(2011)[3], moments of the wage distribution and the employment rate in the public sector are taken as exogenous policy parameters. The distinction between formal and informal firms is made by the introduction of heterogenous production functions, such that the less productive firms are in the informal sector.

Previous preliminary attempts, by Yassine C.(2011)[11] to estimate the Burdett Mortensen

(1998)[4] model showed that the Egyptian and Jordanian are relatively rigid labor markets when compared to other European and USA countries. However, this rigidity might have been biased due to the peculiar nature of these markets where the public and informal sectors employ a high proportion of the market's wage workers. By adding the public and the informal sector to the basic equilibrium job search model, it would therefore be interesting to check if the public sector has got a crowding-out effect on the Private Sector. Indeed, if workers are risk adverse, they tend to accept more easily the more stable jobs of the public sector even if this implies a lower salary or more time in the public jobs queues. Adding the informal sector, seeks to verify previous ideas about informality, being a remarkable characteristic of developing countries in general, Egypt and Jordan in specific and whether labor market entrants choose this sector as an intermediary till they manage to obtain a formal private or a public job, or otherwise.

The empirical analysis uses the Egyptian and Jordanian labor market panel surveys (ELMPS2006[8], ELMPS2012[7] & JLMPS2010 [9]) where the data contain sufficient information on wages, labor force states, durations and transitions to generate estimates of the model's structural transition parameters, enabling us to explore the nature of labor market dynamics in the region and hence possibly explain persistent high unemployment rates. Using the rich retrospective information available in these data sets, we are able to extract different employment panels for Egypt and Jordan from the available cross-sectional data sets, showing a detailed employment vector of each individual for every year. The overlap of the two panels created from the available Egyptian cross-sectional surveys in Egypt enabled us to even lower the error margin that might arise due to human's memory and attrition.

Following Bradley et al. (2011)[3], the model is estimated using the method of simulated moments. We use the resulting model estimates to simulate the impact of various counterfactual public sector employment policies and to determine how an effective policy on informality needs to be designed.

The rest of the paper is divided as follows. In the second section, we build up our theoretical model with public and private sectors. The third section provides a description of the data and sample used, the estimation methodology adopted and the empirical results obtained. The fourth section extends the model to segment the private sector into formal and informal sectors. The fifth section is devoted to perform simulations and counterfactual policy analysis. Finally, Section 6 concludes.

2 The model with public and private sectors

2.1 Workers' Behavior

Workers maximize the expected lifetime income discounted at a rate of ρ. At any instant, unemployed workers receive an income stream b, taken to be constant across individuals, regardless of their history. The b is traditionally viewed as unemployment benefits. In our context of developing countries, it is more convenient to consider it as self-employment or support of the non-employed by their family which is relatively a common feature of these economies. Let W_k, where $k \in f, i, g$ denote the values of a wage contract w in a specific sector and let U be the value of unemployment. Lastly, as in Burdett and Mortensen (1998) [5], F_k, defined on $[\underline{w_k}, \overline{w_k}]$, denotes the (equilibrium) distribution of (present values of) contracts from which the workers sample their offers. The offer distributions are endogenous for both the formal and informal private sectors and exogenous for the public sector. We explain in the rest of the paper how the endogenous distributions are determined.

As mentioned before, employed workers will have the following reservation wages R_{fg}, R_{gf}. Following Bradley et al. (2012) [3] A worker's reservation wage will be a function of their current wage. The reservation wage applying to a one sector's offers made to another sector worker earning w makes this worker indifferent between his current present value and the present value of the other sector employment at his reservation wage. This therefore implies that for a worker in the Private formal sector $W_f(R_{gf}(\omega)) = W_g(\omega)$, for a worker employed in the Public sector, $W_g(R_{fg}(\omega)) = W_f(\omega)$. It follows that the above reservation wages between any given two sectors are reciprocal of eachother:

$$R_{fg}(R_{gf}(\omega)) \quad = \quad \omega \tag{1}$$

The reservation wages of the unemployed R_{uf}, R_{ug} are simply the wages at which an individual is indifferent between unemployment and employment in l sector. We therefore obtain $U = W_f(R_{uf}) = W_g(R_{ug})$ in case the offer received by the unemployed individual is from the Private formal sector or the public sector. Applying the reciprocity property of the reservation wages explained above in (1), we derive the following;

$$R_{fg}(R_{uf}) \quad = \quad R_{ug} \tag{2}$$

To portray unemployed individuals' attitudes towards their insertion into the labor market in the different sectors, we assume that the reservation wages follow the inequality $R_{uf} < R_{ug}$ [1].

Since workers seek to maximize their expected lifetime income discounted at the rate ρ, the value functions are obtained from the following bellman equations where they combine both the immediate gains of being in s state, as well as the resulting option values, such as the possibility of moving to better jobs within or between the sectors, or the impact of exogenous shocks, such as the possibility of job destruction leading to unemployment. The value function for an unemployed worker is therefore defined by the following Bellman equation:

$$\rho U \quad = \quad b + \lambda_{ug} \int_{R_{ug}}^{+\infty} [W_g(x) - U] dF_g(x) + \lambda_{uf} \int_{R_{uf}}^{+\infty} [W_f(x) - U] dF_f(x) \tag{3}$$

The first term, b is the flow utility an individual gets from being in unemployment. Offers arrive from the public, formal private sectors at a rate of λ_{ug} and λ_{uf} respectively . Wage offers, x are drawn from the formal private sector from an endogenous distribution, $F_f(w)$, which will be derived from the firms' side later. An unemployed worker will accept the job offer if the wage is higher than the workerŠs reservation wage for that sector, the lower bound of the integral. Inside the integral is the gain the worker makes from switching from unemployment to public sector employment at wage w. The third term is the formal private sector analogue to the second. The theoretical difference between the private and the public sectors is that the distribution from which public-sector job offers are drawn is an exogenous policy parameter of the model.

Similar value functions define a worker employed in one of the employment public or

[1]This is empirically verified through the descriptive statistics obtained from the Egyptian Labor market panel Survey 2012

private sectors. The value of a formal private sector employee is, for instance, as follows:

$$\begin{aligned} \rho W_f(\omega) &= \omega_f + \delta_f\{U - W_f(\omega)\} + \lambda_{ff}\int_{\omega}^{+\infty}[W_f(x) - W_f(\omega)]dF_f(x) \\ &+ \lambda_{fg}\int_{R_{fg(\omega)}}^{+\infty}[W_g(x) - W_f(\omega)]dF_g(x). \end{aligned} \tag{4}$$

A worker employed in the formal private sector and earning a wage ω_f has a discounted value from employment given by the right hand side of (4). The first term ω_f is the instantaneous wage paid in the current formal private sector firm. The next term, is the loss of value an individual would get if he were to transit into unemployment $[U - W_f(\omega)]$ multiplied by the flow probability of such an event occurring, the private sector job destruction rate, δ_f. At rate $\lambda_f f$ the worker receives an offer from another formal private sector firm, where the offer is drawn from the distribution $F_f(x)$. If this offer is greater than his current wage ω_f he will accept, the lower bound of the integral. Given the offer is received and it meets his acceptance criteria, the individual will make an unambiguous gain in value given by $[W_f(x) - W_f(\omega)]$. The next term represents the equivalent, except for offers from the public sector. Thus the wage is drawn from a different distribution and the acceptance criteria, the lower bound of the integral is instead $R_f g(\omega)$.

Analogously, we obtain the value functions for employed workers in public sectors in (12) and (13) respectively.

$$\begin{aligned} \rho W_g(\omega) &= \omega_g + \delta_g[U - W_g(\omega)] + \lambda_{gg}\int_{\omega}^{+\infty}[W_g(x) - W_g(\omega)]dF_g(x) \\ &+ \lambda_{gf}\int_{R_{gf(\omega)}}^{+\infty}[W_f(x) - W_g(\omega)]dF_f(x). \end{aligned} \tag{5}$$

Given that $W_g(R_{fg}(\omega)) = W_f(\omega)$, we deduce that

$$R'_{fg}(\omega)W'_g(R_{fg}(\omega)) = W'_p(\omega)$$

Using (4) and (5) we have

$$\begin{aligned} W'_f(\omega) &= \frac{1}{\rho + \delta_f + \lambda_{ff}\overline{F}_f(\omega) + \lambda_{fg}\overline{F}_g(R_{fg}(\omega))} \\ W'_g(R_{fg}(\omega)) &= \frac{1}{\rho + \delta_g + \lambda_{gg}\overline{F}_g(R_{fg}(\omega)) + \lambda_{gf}\overline{F}_f(R_{gf}(R_{fg}(\omega)))} \end{aligned}$$

$$= \frac{1}{\rho + \delta_g + \lambda_{gg}\overline{F}_g(R_{fg}(\omega_p)) + \lambda_{gf}\overline{F}_f(\omega_p)}$$

which leads to

$$R'_{fg}(\omega) = \frac{\rho + \delta_g + \lambda_{gg}\overline{F}_g(R_{fg}(\omega)) + \lambda_{gf}\overline{F}_f(\omega)}{\rho + \delta_f + \lambda_{ff}\overline{F}_f(\omega) + \lambda_{fg}\overline{F}_g(R_{fg}(\omega))}$$

The expression of the reservation wages $R_{fg}(\omega)$ and $R_{gf}(\omega)$ are provided in the appendix A.

2.2 The stock-Flows equations

The economy being in steady-state, the flows in and out of any given sector, for each class of workers, are equal. Applying this to unemployment, one obtains:

$$(\lambda_{uf} + \lambda_{ug})N_u = \delta_f N_f + \delta_g N_g$$

given that we assume that $F_p(R_{up}) = 0$ and $F_g(R_{ug}) = 0$. A worker can only be in one of three states, u, f or g so: $N_u + N_p + N_g = N$, where N is the total population of workers, a given number. Thus, at this stage, we have three unknowns: N_p, N_g and $G_f(w)$, given $G_g(w)$ and $F_g(w)$. We then present the tree restrictions allowing us to determine these quantities.

The following equation is the flow-balance equation for private sector workers:

$$\begin{aligned} &N_f G_f(w)\left\{\delta_f + \lambda_{ff}\overline{F}_f(w)\right\} + N_f\lambda_{fg}\int_{R_{up}}^{w}\overline{F}_g(R_{fg}(x))dG_g(x) \\ &-N_g\lambda_{gf}\int_{R_{ug}}^{R_{fg}(w)}[F_f(w) - F_f(R_{gf}(x))]dG_g(x) = N_u\lambda_{uf}F_f(w) \end{aligned} \quad (6)$$

For the public sector, we have

$$\begin{aligned} &N_g G_g(w)\left\{\delta_g + \lambda_{gg}\overline{F}_g(w)\right\} + N_g\lambda_{gf}\int_{R_{ug}}^{w}\overline{F}_f(R_{gf}(x))dG_g(x) \\ &-N_f\lambda_{fg}\int_{R_{ug}}^{R_{gf}(w)}[F_g(w) - F_g(R_{fg}(x))]dG_g(x) = N_u\lambda_{ug}[F_g(w) - F_g(R_{ug})] \end{aligned} \quad (7)$$

By differentiating the equation (6) with respect to w, we obtain

$$\frac{d}{dw}\left\{N_f G_f(w)\left[\delta_f + \lambda_{ff}\overline{F}_f(w)\right]\right\} + N_f\lambda_{fg}\overline{F}_g(R_{fg}(w))g_g(w) \\ -N_g\lambda_{gf}G_g(R_{fg}(w))f_f(w) = N_u\lambda_{uf}f_f(w) \tag{8}$$

which an ODE in $g_p(w)$ (the probability distribution of observed wages in the private sector), if it was not for the term featuring $G_g(R_{fg}(w))$. Following Bradly, Posel-Vinay and Turon (2012), we derive this term using the equation (7) applying at $R_{fg}(w)$ instead of w:

$$N_g G_g(R_{fg}(w))\left\{\delta_g + \lambda_{gg}\overline{F}_g(R_{fg}(w))\right\} + N_g\lambda_{gf}\int_{R_{ug}}^{R_{fg}(w)}\overline{F}_f(R_{gf}(x))dG_g(x) \\ -N_f\lambda_{fg}\int_{R_{ug}}^{w}[F_g(R_{fg}(w)) - F_g(R_{fg}(x))]dG_g(x) = N_u\lambda_{ug}[F_g(R_{fg}(w)) - F_g(R_{ug})] \tag{9}$$

Now, adding (6) and (9), we obtain

$$N_f G_f(w)\left\{\delta_f + \lambda_{ff}\overline{F}_f(w) + \lambda_{fg}\overline{F}_g(w)\right\} + N_g G_g(w)(R_{fg}(w))\left\{\delta_g + \lambda_{gf}\overline{F}_f(w) + \lambda_{gg}F_g(R_{fg}(w))\right\} \\ = N_u\lambda_{uf}F_f(w) + N_u\lambda_{ug}F_g(w)$$

if we assume that $F(R_{uf}) = F(R_{ug}) = 0$. Thus, we have

$$G_g(w)(R_{fg}(w)) = \frac{N_u\lambda_{uf}F_f(w) + N_u\lambda_{ug}F_g(w) - N_f G_f(w)\left\{\delta_f + \lambda_{ff}\overline{F}_f(w) + \lambda_{fg}\overline{F}_g(w)\right\}}{N_g\left\{\delta_g + \lambda_{gf}\overline{F}_f(w) + \lambda_{gg}F_g(R_{fg}(w))\right\}} \tag{10}$$

Plugging this solution into (8), we obtain an ODE that defines $G_f(w)$.

Finally, applying the equation (6) for $w \to \infty$, we obtain:

$$N_f G_f(w)\delta_f + N_f\lambda_{fg}\int_{R_{up}}^{\infty}\overline{F}_g(R_{fg}(x))dG_g(x) \\ -N_g\lambda_{gf}\int_{R_{ug}}^{\infty}\overline{F}_f(R_{gf}(x))]dG_g(x) = N_u\lambda_{uf} \tag{11}$$

Using the system formed by the equations (8), (10) and (11), we determine simultaneously $\{G_f(\cdot), N_f, N_g\}$.

2.3 The private firm behaviors

As in Bradley, Postel-Vinay and Turon (2012), we first assume that the productivity distribution is exogenous, and that there is no matching externality. Thus, the economy is a continuum $[0,1]$ of private sector firms who are profit maximizers and heterogeneous in their level of productivity, y, where $y \sim \Gamma(\cdot)$ over the support $[\underline{y}, \overline{y}]$ in the population of firms. Firms set their wage w and their search effort in order to make a number of contacts m. The pair (w, m) is chosen so as to maximize steady-state profit flow.

A private sector firm choosing to pay w will experience a quit rate of $d(w)$ of its employees and an average acceptance rate $h(w)$ of the contacts it is making with prospective employees (bearing in mind that search is random), where:

$$\begin{aligned}
d(w) &= \delta_f + \lambda_{ff}\overline{F}_f(w) + \lambda_{fg}\overline{F}_g(R_{fg}(w)) \\
h(w) &= \frac{\lambda_{uf}N_u + \lambda_{ff}N_f G_f(w) + \lambda_{gf}N_g G_g(R_{fg}(w))}{\lambda_{uf}N_u + \lambda_{ff}N_f + \lambda_{gf}N_g}
\end{aligned}$$

The hiring policy of the public sector interacts directly with the quit rate via λ_{fg} and $F_g(\cdot)$. If the public sector provides low wages to its workforce ($G_g(\cdot)$ concentrated on the left), then it will be more easy for the private sector to attract workers, increasing its acceptance rate $h(w)$.

As a consequence, the steady-state size of this firm will be $l(w, m)$:

$$l(w,m) = m\frac{h(w)}{d(w)} = mL(w)$$

and its steady-state profit flow:

$$\Pi(w,m) = (y - w)l(w,m) - c(m) = (y.w)mL(w).c(m)$$

where $c(m)$ is the cost incurred by the firm to make m contacts. This function acts as a search cost without the externality linked to the usual matching process. Optimal wage and search policies $w(y)$ and $m(y)$ can thus be characterized using the following first-order conditions:

$$
\begin{aligned}
y &= w(y) + \frac{L(w(y))}{L'(w(y))} \\
c'(m(y)) &= \frac{L(w(y))^2}{L'(w(y))}
\end{aligned}
$$

It follows that the total number of contacts in the economy is:

$$M = \int_{\underline{y}}^{\overline{y}} m(y) d\Gamma(y)$$

and that the fraction of these contacts that is attached to wage lower than a given w, in other words the probability that a wage offer is less than w can be written in the two following manners:

$$F_f(w) = \frac{1}{M} \int_{\underline{y}}^{\widehat{y}} m(y) d\Gamma(y)$$

where $\widehat{y}$ is such that $w(\widehat{y}) = w$. Similarly, the fraction of employees earning a wage less than $w(y)$ do so because they are employed by firms with a productivity lower than y. Thus:

$$H(l(w(y), m(y))) = G_f(w(y))$$

where $H(\cdot)$ is the distribution of firm sizes among employed workers.

To close the model, we assume that the relative search intensities of workers in the three labor market states, i.e. unemployment, employment in the private sector and employment in the public sector are constant. These will be denoted s_{uf} (normalised ot 1 without loss of generality), s_{ff} and s_{gp} respectively. The arrival rates of private sector offers hence have the following expressions: $\lambda_{uf} = \lambda_f$, $\lambda_{ff} = s_{ff}\lambda_f$ and $\lambda_{gf} = s_{gf}\lambda_f$. The private sector job offer arrival rate λ_p is given by:

$$M = \lambda_p(N_u + s_{ff}N_f + s_{gf}N_g)$$

where the parameters governed by the policy of the public sector interact.

2.4 An alternative model with matching externality and endogenous productivity

In this section, we present an extension which allow us to account for the matching externality and for the endogeneity of the productivity distribution. This extension is based on the MORT/00 equilibrium search model with wage posting and training investment by firms. This wage posting has been estimated on French data by Chéron, Hairault and Langot [2008]. Statistical tests shown that this type of model is able to predict the impact of an observed reform, contrary to the traditional model à la Burdett and Mortensen [1998] with exogenous productivity distribution.

2.4.1 Matching Technology

For simplicity, we assume that the search is directed: in one market, private firms meet workers who apply for private jobs, and in the other market, public jobs meet workers who search for a public job. As it will be discussed below, this assumption doesn't imply that the private firm decisions are independent from the public sector policy. According to Pissarides [1990], the aggregate number of hirings in the private sector, H, is determined by a conventional constant returns to scale matching technology:

$$H = h(v, h_{ff}N_f + h_{gf}N_g + h_{uf}N_u)$$

where v is the number of vacancies in the private sector, $h_{ff} \geq 0, h_{gf} \geq 0, h_{uf} \geq 0$ are the exogenous search efficiencies (intensities) for employed workers in the private or public sectors and for the unemployed workers, represented (in number) by N_f, N_g and N_u, respectively. We normalize $N_f + N_g + N_u$ to 1 and we denote $\overline{h} = h_{ff}N_f + h_{gf}N_g + h_{uf}N_u$.

If we set $\theta = \frac{v}{\overline{h}}$ as labor market tightness of the private, the arrival rates of wage offers for workers are:

- for the employees in the private sector

$$h_{ff}\lambda(\theta) \equiv \frac{h_{ff}}{\overline{h}} \frac{H}{N_f + N_g + N_u} = h_{ff}\frac{H}{\overline{h}}$$

- for the employees in the public sector

$$h_{gf}\lambda(\theta) \equiv \frac{h_{gf}}{\overline{h}} \frac{H}{N_f + N_g + N_u} = h_{gf} \frac{H}{\overline{h}}$$

- for the unemployed

$$h_{uf}\lambda(\theta) \equiv \frac{h_{uf}}{\overline{h}} \frac{H}{N_f + N_g + N_u} = h_{uf} \frac{H}{\overline{h}}$$

The transition rate at which vacant jobs are filled is:

$$q(\theta) = \frac{H}{v} = h\left(1, \frac{\overline{h}}{v}\right)$$

The link between $q(\theta)$ and $\lambda(\theta)$ is given by $\theta q(\theta) = \lambda(\theta)$, or $\frac{\lambda(\theta)}{v} = q(\theta)$. Finally, we have

$$\lambda_{ff} = h_{ff}\lambda(\theta) \qquad \lambda_{gf} = h_{gf}\lambda(\theta) \qquad \lambda_{uf} = h_{uf}\lambda(\theta)$$

If we normalize $h_{uf} = 1$, then the observation of the worker transition rate between unemployment and the private sector gives the value of $\lambda(\theta)$. Thus, if we assume that the matching function is a usual Cobb Douglas function, we then deduce the value of θ from the estimation of the transition rate $U \to E_f = \widehat{\lambda}$:

$$h(v, h_{ff}N_f + h_{gf}N_g + h_{uf}N_u) = v^{\psi}(h_{ff}N_f + h_{gf}N_g + h_{uf}N_u)^{1-\psi} \Rightarrow \lambda(\theta) = \theta^{\psi} \Rightarrow \theta = \widehat{\lambda}^{\frac{1}{\psi}}$$

Concerning the public sector, we assume that the number of vacancies is exogenous. This implies that the transition rates $\{\lambda_{gg}, \lambda_{gf}, \lambda_{ug}\}$ are exogenous and thus can be changed by the policy maker.

2.4.2 The private firm behaviors

A private sector firm choosing to pay w will experience a quit rate of $d(w)$ of its employees and an average acceptance rate $h(w)$ of the contacts it is making with prospective employees (bearing in mind that search is random), where:

$$
\begin{aligned}
d(w) &= \delta_f + \lambda_{ff}\overline{F}_f(w) + \lambda_{fg}\overline{F}_g(R_{fg}(w)) \\
h(w) &= \frac{\lambda_{uf}N_u + \lambda_{ff}N_f G_f(w) + \lambda_{gf}N_g G_g(R_{fg}(w))}{\lambda_{uf}N_u + \lambda_{ff}N_f + \lambda_{gf}N_g}
\end{aligned}
$$

The hiring policy of the public sector interacts directly with the quit rate via λ_{fg} and $F_g(\cdot)$. If the public sector provides low wages to its workforce ($G_g(\cdot)$ concentrated on the left), then it will be more easy for the private sector to attract workers, increasing its acceptance rate $h(w)$.

Let k be the match specific investment per worker and $f(k)$ the value of worker productivity which is an increasing concave function of this investment. It is assumed that whenever an employed worker finds a job paying more than w (voluntary quit), then the employer seeks another worker. When an exogenous quit (destruction) occurs, the job receives no value. We assume that there is a free entry conditions at each wage level imply that the asset value of a vacant job is equal to zero: $V = 0$. Hence, the expected present value of the employer's future flow of quasi-rent once a worker is hired at wage w stated as $J(w, k)$, solves:

$$
rJ(w,k) = f(k) - w - d(w)J(w,k) \qquad \Rightarrow J(w,k) = \frac{f(k) - w}{r + d(w)}
$$

In turn, the asset value of a vacant job solves the continuous time Bellman equation:

$$
rV = \max_{w \geq R_{uf}, k \geq 0} \{\eta(w)\left[J(w,k) - p_k k - V\right] - \gamma\}
$$

where γ is the recruiting cost, p_k stands for the relative price of one unit of human capital, and $\eta(w) = \frac{\lambda(\theta)}{v} h(w)$ is the probability that a vacancy with posted wage w is filled.

Hence, labor market tightness θ, the wage distribution function $F_f(w)$ and firms' investment in human capital $k(w)$ can be derived from the system of equations defined by, $\forall w \geq \underline{w}$:

$$
\gamma = \eta(w)\left[\max_{k \geq 0}\{J(w,k) - p_k k\}\right]
$$

$$\Leftrightarrow \frac{\gamma\theta}{\lambda(\theta)} = \frac{h(w)}{r+d(w)}\left[\max_{k\geq 0}\{f(k) - w - p_k(r+d(w))k\}\right]$$

with $F_f(\underline{w}) = 0$. Employers have two reasons for offering a wage greater than $\underline{w}$. First, the firm's acceptance rate ($\eta(w)$) increases with the wage offer, since a higher wage is more attractive. Second, the firm's retention rate increases with the wage paid by limiting voluntary quits that lead to an increase in $J(w,k)$. The wage strategy implemented by firms is strongly interrelated with human capital investment decisions.

As each employer pre-commits to both the wage offered and the specific capital investment in the match, it is easy to show that the optimal investment solves:

$$\frac{f'(k)}{r+d(w)} = p_k \quad \Longrightarrow k = k(w) \quad \forall w \geq \underline{w}$$

Therefore, the level of specific human capital increases with the level of the wage offer. Indeed, a higher wage reduces the probability that an employee will accept job offers from other firms. The negative relationship between wage and labor turnover creates incentives to train employees. When the wage is high, the expected duration of the match is longer and the period during which the firm can recoup its investment increases. Therefore, firm-specific productivity increases with wages. If the public sector decides to hire more worker, this increases $d(w)$, $\forall w < \overline{w}_g$: thus, this public policy leads to a decrease of the productivity of the private sector. This productivity channel is an additional effect of the public sector impact on the labor market equilibrium. As it is shown in Chéron, Hairault and Langot [2008], this channel must not be ignored during an evaluation of a policy change. From this dimension, we then extend the theoretical framework of Bradly, Postel-Vinay and Turon [2013].

At the bottom of the wage distribution, $\underline{w} = R_{uf}$, we have:

$$\begin{aligned}
\frac{\gamma\theta}{\lambda(\theta)} &= \frac{h(\underline{w})}{r+d(\underline{w})}\left[f(k(\underline{w})) - \underline{w} - p_k(r+d(w))k(\underline{w})\right] \\
d(\underline{w}) &= \delta_f + \lambda_{ff} + \lambda_{fg}\overline{F}_g(R_{fg}(R_{uf})) \\
h(\underline{w}) &= \frac{\lambda_{uf}N_u + \lambda_{gf}N_g G_g(R_{fg}(R_{uf}))}{\lambda_{uf}N_u + \lambda_{ff}N_f + \lambda_{gf}N_g}
\end{aligned}$$

Given that (2) states that $R_{fg}(R_{uf}) = R_{ug}$, and assuming that the public sector doesn't offer wage lower than the reservation wage R_{ug}, we have $G_g(R_{ug}) = F_g(R_{ug}) = 0$. We deduce

$$\frac{\gamma\theta}{\lambda(\theta)} = \frac{h_{uf}}{\overline{h}}\frac{N_u}{r+\delta_f+\lambda(\theta)h_{ff}+\lambda(\theta)h_{fg}}\left[f(k(\underline{w}))-\underline{w}-p_k(r+\delta_f+\lambda(\theta)h_{ff}+\lambda(\theta)h_{fg})k(\underline{w})\right]$$

because $d(\underline{w}) = \delta_f+\lambda_{ff}+\lambda_{fg}$ and $h(\underline{w}) = \frac{\lambda_{uf}N_u}{\lambda_{uf}N_u+\lambda_{ff}N_f+\lambda_{gf}N_g}$, which leads, using the matching function to $h(\underline{w}) = \frac{\lambda(\theta)h_{uf}N_u}{\lambda(\theta)(h_{uf}N_u+h_{ff}N_f+h_{gf}N_g} = \frac{h_{uf}}{\overline{h}}N_u$. Given that $\frac{f'(k(\underline{w}))}{r+\delta_f+\lambda(\theta)h_{ff}+\lambda(\theta)h_{fg}} = p_k$ implying $k(\underline{w}) = f'^{-1}\left(p_k(r+\delta_f+\lambda(\theta)h_{ff}+\lambda(\theta)h_{fg})\right) \equiv \mathcal{K}(\theta,p_k)$, we deduce that

$$\frac{\gamma\theta}{\lambda(\theta)} = \frac{h_{uf}}{\overline{h}}\frac{N_u}{r+\delta_f+\lambda(\theta)h_{ff}+\lambda(\theta)h_{fg}}\begin{bmatrix} f(\mathcal{K}(\theta,p_k))-\underline{w} \\ -p_k(r+\delta_f+\lambda(\theta)h_{ff}+\lambda(\theta)h_{fg})\mathcal{K}(\theta,p_k)\end{bmatrix}$$

This equation imposes a restriction between $\{\theta, f(\cdot), \gamma, p_k\}$, with $\underline{w} = R_{uf}$.

The equilibrium of wage posting game is such that there exists a mixed-strategy equilibrium where different wage policies lead to the same profit for each firm. In this case, the equilibrium is stable, an we have

$$\begin{aligned}\frac{\gamma\theta}{\lambda(\theta)} &= \frac{h(w)}{r+d(w)}\left[f(k(w))-w-p_k(r+d(w))k(w)\right] \\ &= \frac{1}{\overline{h}}\frac{h_{uf}N_u+h_{ff}N_fG_f(w)+h_{gf}N_gG_g(R_{fg}(w))}{r+\delta_f+\lambda_{ff}\overline{F}_f(w)+\lambda_{fg}\overline{F}_g(R_{fg}(w))}\left\{\begin{array}{l} f(k(w))-w \\ -p_k\begin{bmatrix} r+\delta_f+\lambda_{ff}\overline{F}_f(w) \\ +\lambda_{fg}\overline{F}_g(R_{fg}(w))\end{bmatrix}k(w)\end{array}\right\}\end{aligned}$$

Given the link between $F_f(\cdot)$ and $\{G_f(\cdot); G_g(\cdot); F_g(\cdot)\}$ described in section 2.2, this equation provides the solution for $F_f(\cdot)$.

3 The empirical analysis

3.1 Data

In this paper, we use a new and original data set, the Egypt Labor Market Panel Survey of 2012 (ELMPS12)[7] whose aim was to revisit the households previously interviewed in the Egypt Labor Market Panel Survey 2006[8], which we also use in our estimation, to trace the evolution of their work and unemployment trajectories. The households selected in this longitudinal data set are national-represenatitive and are randomly selected. The final sample interviewed in 2012 consists of 12,060 households, which includes 6,752 original households (out of 8371 interviewed in 2006) from the 2006 sample,3,308 new households that emerged from those households as a result of splits (i.e. split households), and a refresher sample of 2,000 households. In terms of individuals, the 2012 sample includes 49,186 individuals, who are made up of 28,770 individuals initially interviewed in 2006 and successfully re-interviewed in 2012 and 20,416 new individuals. Of those new individuals, 5,009 joined the original 2006 households, 6,900 joined split households, and 8,507 were members of the refresher sample of households.[2] This longitudinal feature of the data along with the rich retrospective accounts included in the questionnaire, we construct two panel datasets of respondents at a yearly frequency (1998-2006) and (2000-2012) showing the employment trajectory of each indivual over time. We note our choice for these two panels depend on the fact that for the estimation of our model, it is necessary that the time period is short and has approximately constant shares in each of the four states $\{s = u, g, i, f\}$ across time.

The data used in our analysis for the Jordanian labor market is the JLMPS2010 [9]. Similar to the ELMPS, and even though it is only the first wave of what is to be a longitudinal survey, it contains a number of retrospective questions that allow us to reconstruct the entire employment trajectories of the respondents rather than simply get a snapshot of a single point in time. We therefore choose the panel 2000-2010 to carry out our estimations.

3.2 Sample and Descriptive Statistics

For both the Egyptian and Jordanian data, we include in our panels male workers between the age of 15-65 years old. We trim the income distributions in each sector, treating data as if it is missing below the 1st or above the 99th percentile in the distribution of wage in either employment sector. We also exclude individuals with holes in their employment history and once someone moves to non-wage employment,they are from then on excluded. Thus consistent with our model, an agent can be in in one of the four states, non-employement or employment in the public, formal private or informal private sectors. Due to the cumbersome calculations faced to derive the optimum behavior to endogenously demonstrate the choice of firms to offer employment contracts formally or informally, it suffices at a first step to estimate here our transition parameters by lumping both the formal and informal sectors into a private sector p. Throughout our estimation function, we therefore have an individual in one of three states; non-employment, public or private wage employment. We define private sector employment as one declaring him/herself as employed in a private sector firm, investment, joint-venture, foreign or non-profit organization, or co-operatives. The formal employed worker is a worker with either a contract and/or social insurance. The public sector is defined as a worker employed in the central or local government, as well as the public enterprises. The non-employed state in our analysis comprises anyone who is unemployed (i.e not having a job and available for work) or a non-participant (i.e out of the labor force) who transits into or out of the labor market [2].

To be able to derive a panel of employment trajectories over time, we use the retrospective chapters in the ELMPS and JLMPS questionnaires. We were lucky enough to have a question about the date of start of a job status precising the month and year of start of status. However, the month precision information is not available for all individuals and is only available for the ELMPS12 and the JLMPS10. We therefore develop a method to approximately determine the month of start of a job status if two of them occur in the same year. Since for a status to be recorded in the questionnaire, the individual should have spent at least 6 months in it, we therefore record one starting at the beginning of year 'x' and the second starting by mid year 'x'. For job statuses occurring in different years and unavailable month start, we will simply have to accept the assumption that they start at the beginning of the year 'x'.

Table 1 shows basic descriptive statistics including the composition of the labor market sample in Egypt (2006& 2012) and Jordan (2010), as well as the summary statistics of the monthly wages[3]. As mentioned earlier, we carry our estimation over a period of time

[2] In our analysis, we're not interested to distinguish between the unemployment and non-participation since in countries such as Egypt, both categories are according to our definition more or less "job searchers" even if not regualar and hence have got to be considered when analyzing the monopsony power exerted by firms when setting wages and making their offers to individuals.Moreover, one has to note that these shares do not represent the share of non-employment among the population but only among the group of individuals who ever worked and hence were tracked over time in the questionnaire. Our model aims at identifying what happens to people who were already inserted into the labor market, and the dynamics that follows throughout their work lives.

[3] Egyptian monthly wages are in Egyptian pounds whilst the Jordanian monthly wages are in Jordanian

where we notice that these shares remain fairly stable and constant. It's also worth noting that we are aware of the backward attrition problem affecting our data due to the recall and retrospective technique adopted to create our panels. This might be clear from the underestimation of the share of non-employment. This however shall not affect the results of our model since our model captures a picture of the labor market dynamics rather than the composition of the labor market population as a whole. The attrition can be easily corrected for using a weighting technique at a later step.

As for table 2, it conveys the transition rates both within and between sectors showing the extent of mobility in each country. Our data is a yearly panel of workers, bearing in mind the backward attrition forces influencing the raw data we obtain, we calculate our transition matrices 3 years only before the interview's date. Counting in each month the number of people making each type of transition and the number in each state, we construct a yearly cross-sector transition matrices. Table 2 shows that the private sector workers in general possess a better chance of changing their jobs than their counterparts in the public sector; they are therefore more mobile. This trend tends to be very obvious among the within-sectors transitions. When noting the between-sectors transitions, we note however that it tends to be dominated by the public sector employees moving to the private sector. Cross-mobility in the other direction i.e from the Private Sector to the Public Sector is surprisingly not that frequent in Egypt. Despite the wide spread traditional beliefs about stability and extra benefits a worker can have when employed in the public sector, it seems that there is a clear distinction between the two private and public sectors, in a way that people would prefer to queue in the non-employment state for a government job rather than entering a private job as a transit state till they get the public employment contract or appointment.

Dinars. Estimations are carried out with the log of the wages.

Table 1: Descriptive Statistics 1

	Private Sector	Public Sector	Non-employment
Size of each sector			
ELMPS06	52.75%	35.69%	11.56%
ELMPS12	61.61%	29.35%	9.04%
JLMPS10	%	%	%
Mean Monthly earnings			
ELMPS06	583.0088	865.571	–
ELMPS12	1072.874	1333.728	–
JLMPS10	586.68	414.85	–
Standard deviation of monthly wages			
ELMPS06	986.643	2523.879	–
ELMPS12	1181.316	1555.426	–
JLMPS10	2738.43	763.96	–

Table 2: Job Mobility within and between sectors

ELMPS06	Private Sector	Public Sector	Unemployment
Private Sector	0.0342	0.0073	0.0210
Public Sector	0.0029	0.0189	0.0150
Unemployment	0.1801	0.0391	–
ELMPS12	Private Sector	Public Sector	Unemployment
Private Sector	0.0395	0.0077	0.0177
Public Sector	0.0058	0.0091	0.0127
Unemployment	0.1289	0.0199	–
JLMPS10	Private Sector	Public Sector	Unemployment
Private Sector			
Public Sector			
Unemployment			–

We finally have data on the distribution of private firm sizes in the population of employed workers in Egypt , obtained from the questionnaire ELMPS 2012. The distribution of employers' sizes are reported in Table 3

Table 3: Employement in Egypt by firm size, 2012

Firm size	Employment (thousands)	Percent	Cumul
1-4	10500	60.88	60.88
5-9	2386.262	13.84	74.72
10-24	1076.108	6.24	80.96
25-49	519.515	3.01	83.97
50-99	481.973	2.79	86.77
100+	2281.91	13.23	100.00
Total	**17245.768**	**100.00**	

3.3 The econometric strategy

This section describes the econometric method we used to estimate the structural parameters of the model. Because structural econometric models are sensitive to misspecification, we choose an empirical strategy which ensures robust estimates of the unknown parameters. As the likelihood function cannot be derived analytically, it can be replaced the exact likelihood function of an approximated model (GOUR/MONF/94). Following e.g. Chéron, Hairualt and Langot [2008] or Bradley, Postel-Vinay and Turon (2012), we choose the latter strategy and, more specifically, the indirect inference method applied to equilibrium search model.[4] The benefit derived from estimating the model, however, comes at the cost of making parametric assumptions.

3.3.1 Estimation method

The indirect inference method consists of replacing the computation of analytic moments with simulations. The moments underlying the estimation are based on wage distributions and on the transition between labor market states. We focus on a sub-sample workers, who are known a period of employment during in the past. This excludes the young workers and the individuals who are permanently out-of-the-labor force. We also exclude the women. This enables us to detect the dimensions along which our simple structural model is capable of mimicking a set of moment restrictions.

Following Bradley, Postel-Vinay and Turon (2012), we make parametric assumptions about $F_f(\cdot)$ and $F_g(\cdot)$. First we assume that:

$$F_f(w) = \begin{cases} \frac{1-(w/R_{uf})^{\alpha_f}}{1-(\overline{w}_f/R_{uf})^{\alpha_f}} & \text{if } w \in [R_{uf}, \overline{w}_f] \\ 1 & \text{if } w > \overline{w}_f \\ 0 & \text{if } w < R_{uf} \end{cases}$$

where $\overline{w}_f$ is set equal to the top percentile in the observed wage distribution. For simplicity, we parameterize $F_g(\cdot)$ as:

$$F_g(R_{fg}[F_f^{-1}(x)]) = x^{\alpha_g} \quad \text{for } x \in [0, 1]$$

This parameterization carries the implicit assumption that the lower support of $F_g(\cdot)$ is R_{ug}.

The vector Φ ($\dim(\Phi) = 10$) contains all the parameters of the model:

$$\Phi = \{h_{ff}, h_{fg}, h_{uf}, h_{gg}, h_{gf}, h_{ug}, \delta_f, \delta_g, \alpha_f, \alpha_g\}$$

for a given value of the interest rate r, calibrated at 10%.

The estimation method is conducted as follows:

[4]See GOUR/MONF/RENA/93 or GOUR/MONF/94 for a general presentation of these methods. See COLL/FEVE/LANG/PERR/02 for an applied study.

Step 1: The vector of moments ψ is estimated using our data set. The choice of ψ moments is a critical step in the estimation method, but it is not driven by the model's specification. Rather, it should encompass as many data features as possible to avoid an arbitrary choice and reduce estimation biases. Therefore, we choose a set of moments that fully explain wage distribution and the labor market transitions: we take as moments to be matched 50 quantiles of the wage distributions, giving 100 moments in total: $\{w_{s,j}\}_{s=f,g,j=1..50}$, and the transition matrix 3×3, regrouping 9 moments $\pi_{ss'}$, for $s = f, g, u$.

Step 2: Given the vector of structural parameters Φ, the simulated wage density is computed from the set of equations defining the theoretical model.

Step 3: An estimate $\widehat{\Phi}$ for Φ minimizes the quadratic form $J(\Phi) = g' g$, where $g = \left(\widehat{\psi} - \widetilde{\psi}(\Phi)\right)$, $\widehat{\psi}$ is the vector of the estimated moments, and $\widetilde{\psi}(\Phi)$ denotes the set of moments implied by the model simulations.

Steps 2 and 3 are conducted until convergence *i.e.* until a value of Φ minimizing the objective function is obtained.

Contrary to Chéron, Hairault and Langot [2008], we do not use this method in order to perform a statistical test of the model, in the spirit of HANS/82. We just use this SMM algorithm to obtain the value of the structural parameters: this procedure is thus closest to a calibration exercise than an statistical test of the model.

3.4 The results

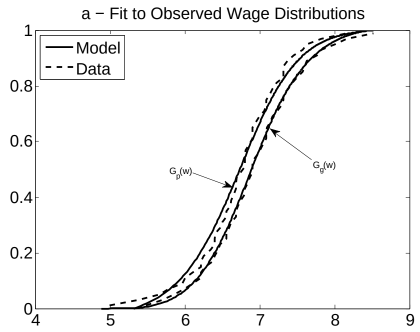
The following Table 4 shows the labor market transitions' parameters, the offer distributions' parameters and the sector-specific reservation wages estimated from the model. The unit of time associated with the transition/offer arrival rates is a year. Given our estimates of parameters, those transition rates produce a perfect fit to the transition probabilities obtained from the data as shown in Table 5. These estimations leave us with a private sector share in Egypt(2012) of 57.42% and a non-employment rate 8.67%. Bearing in mind that this non-employment rate is among those who are already inserted into the labor market, we note that it's an extremely high rate compared to other european countries where such a rate would not exceed the 3% for those who already got a first job in the labor market. Table 4 also reveals the flow value of unemployment b, at about EGP and 421.24EGP for Egypt in 2006 and 2012 respectively. We note that unemployed workers are prepared to give up a substantial amount in terms of income flow to benefit from the on-the-job search technology.

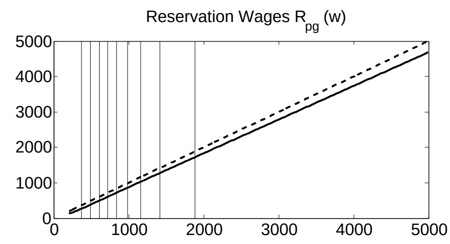
Table 4: Parameter Estimates

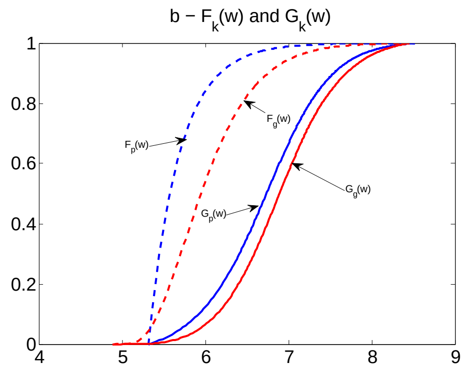
	ELMPS06	ELMPS12	JLMPS10
δ_p	0.0234	0.0169	
δ_g	0.0148	0.0113	
λ_{up}	0.2034	0.1396	
λ_{ug}	0.0433	0.0216	
λ_{pp}	0.1924	0.6045	
λ_{gg}	0.2044	0.0844	
λ_{pg}	0.0627	0.0277	
λ_{gp}	0.0302	0.3045	
α_p	2.27	2.67	
α_g	0.74	6.92	
b		421.24	
R_{up}	142.76	203.13	
R_{ug}	162.28	133.29	

Table 5: Comparison between Empirical and Model's π_{kl}

	ELMPS06		ELMPS12		JLMPS10	
	Model	Data	Model	Data	Model	Data
π_{up}	0.1803	0.1801	0.12895	0.1289		
π_{ug}	0.0384	0.0391	0.01995	0.0199		
π_{pu}	0.0227	0.021	0.01513	0.0177		
π_{gu}	0.0145	0.0189	0.01199	0.0127		
π_{pp}	0.0315	0.0342	0.0403	0.0395		
π_{gg}	0.0197	0.015	0.0095	0.0091		
π_{pg}	0.0082	0.0073	0.007436	0.0077		
π_{gp}	0.0037	0.0029	0.006500	0.0058		
N_p	51.27%		57.42			
N_u	7.34%		8.67			


a − Fit to Observed Wage Distributions
Model
Data
$G_p(w)$
$G_g(w)$


Reservation Wages R_{pg} (w)


b − F_k(w) and G_k(w)
$F_p(w)$
$F_g(w)$
$G_p(w)$
$G_g(w)$

4 An extended model with public, formal and informal sectors

In progress...

5 Simulations and Policy Implications

In progress...

6 Conclusion

References

[1] Albrecht J., Navarro L. and Vromann S. (2011) 'Public Sector Employment in an Equilibrium Search and Matching Model', *mimeo Georgetown University.*

[2] Assad R., Krafft C. and Yassine C. (2012) 'An Analysis Sample Attrition and Calculation of Sample Weights in Egypt Labor Market Panel Survey 2012' , *mimeo Economic Research Forum.*

[3] Bradley J., Postel-Vinay F. and Turon H. (2012)'Public Sector Wage Policy and Labor Market Equilibrium: A Structural Model', *mimeo University of Bristol.*

[4] Burdett K. (2011) 'Towards a Theory of the Labor Market with a Public Sector', *Journal of Labour Economics.*

[5] Burdett K. and Mortensen D.T. (1998) 'Wage Differentials, Employer Size, and Unemployment', *International Economic Review.*

[6] El Badaoui E., Strobl E. and Walsh F. (2010) 'The Formal Sector Wage premium and Firm Size' *Journal of Development Economics.*

[7] ELMPS 2012. Egypt Labor Market Panel Survey of 2012 dataset. Economic Research Forum, Cairo, Egypt.

[8] ELMPS 2006. Egypt Labor Market Panel Survey of 2006 dataset. Economic Research Forum, Cairo, Egypt.

[9] ELMPS 2010. Jordan Labor Market Panel Survey of 2010 dataset. Economic Research Forum, Cairo, Egypt.

[10] Meghir C., Narita R. and Robin J-M. (2010) 'Wages and Informality in Developing Countries', *mimeo UCL.*

[11] Yassine C. (2011) 'Structural Labor Market Transitions and Wage Dispersion in Egypt and Jordan', *ERF Working Papers Series.*

A The reservation wage

Using (4) and (5) we know that

$$W_f'(\omega) = \frac{1}{\rho + \delta_f + \lambda_{ff}\overline{F}_f(\omega) + \lambda_{fg}\overline{F}_g(R_{fg}(\omega))} \tag{12}$$

$$W_g'(\omega) = \frac{1}{\rho + \delta_g + \lambda_{gg}\overline{F}_g(\omega) + \lambda_{gf}\overline{F}_f(R_{gf}(\omega))} \tag{13}$$

Integrating by parts in the Worker Value Functions above:

$$(\rho + \delta_f)W_f(\omega) = \omega + \delta_f U + \lambda_{ff}\int_\omega^\infty W_f'(x)\overline{F}_f(x)dx + \lambda_{fg}\int_{R_{fg}(\omega)}^\infty W_g'(x)\overline{F}_g(x)dx$$

$$(\rho + \delta_g)W_g(\omega) = \omega + \delta_g U + \lambda_{gg}\int_\omega^\infty W_g'(x)\overline{F}_g(x)dx + \lambda_{gf}\int_{R_{gf}(\omega)}^\infty W_f'(x)\overline{F}_f(x)dx$$

The reservation wage for an employee in the private sector, with a current wage ω is defined by $W_g(R_{fg}(\omega)) = W_f(\omega)$, whereas the wage for an employee in the public sector, with a current wage ω is defined by $W_f(R_{gf}(\omega)) = W_g(\omega)$. We first want to compute $R_{gf}(\omega)$. The value functions in the future job is:

$$(\rho + \delta_f)W_f(R_{gf}(\omega)) = R_{gf}(\omega) + \delta_f U + \lambda_{ff}\int_{R_{gf}(\omega)}^\infty W_f'(x)\overline{F}_f(x)dx + \lambda_{fg}\int_\omega^\infty W_g'(x)\overline{F}_g(x)dx$$

The value function of the current job is given by

$$(\rho + \delta_g)W_g(\omega_g) = \omega_g + \delta_g U + \lambda_{gg}\int_{\omega_g}^\infty W_g'(x)\overline{F}_g(x)dx + \lambda_{gf}\int_{R_{gf}(\omega_g)}^\infty W_f'(x)\overline{F}_f(x)dx$$

Given that $W_f(R_{gf}(\omega)) = W_g(\omega)$, we deduce:

$$\begin{aligned} R_{gf}(\omega) = {} & \frac{\rho + \delta_f}{\rho + \delta_g}\omega + \left(\frac{\rho + \delta_f}{\rho + \delta_g}\delta_g - \delta_f\right)U \\ & + \left(\frac{\rho + \delta_f}{\rho + \delta_g}\lambda_{gg} - \lambda_{fg}\right)\int_\omega^\infty W_g'(x)\overline{F}_g(x)dx + \left(\frac{\rho + \delta_f}{\rho + \delta_g}\lambda_{gf} - \lambda_{ff}\right)\int_{R_{gf}(\omega)}^\infty W_f'(x)\overline{F}_f(x)dx \end{aligned}$$