# Panel cointegration rank testing with cross-section dependence*

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**Abstract**

In this paper we propose a test statistic to determine the cointegration rank of VAR processes in panel data allowing for cross-section dependence among the time series in the panel data. The cross-section dependence is accounted for through the specification of an approximate common factor model, which covers situations where there is cointegration among the cross-section dimension. The framework that is considered in the paper embedes as a particular case the situation where time series in the panel data set are cross-section independent. Finite sample performance is investigated via a Monte Carlo experiment. We show that not accounting for common factors when they are present can lead to over estimate the cointegrating rank.

**JEL Classification:** C12, C22

**Keywords:** Panel data cointegration, cointegrating rank, cross-section dependence, cross-cointegration

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# 1 Introduction

Since the pioneering work of Engel and Granger (1987) and Johansen (1988, 1991, 1995), the literature on cointegration grew at a rapid pace. Even though this topic has been covered extensively in time series, the analysis of cointegration in panel data is in early stages of development. The two main approaches to the analysis of cointegration in panels are the residual-based and the system-based approaches. Regarding the residual-based approximation, we can find in the econometric literature proposals that specify either the null hypothesis of no cointegration – see Pedroni (1999, 2004) and Kao (1999), among others – or the null hypothesis of cointegration – see McCoskey and Kao (1998). The system-based approach relies on the panel extension of Johansen's methodology. Thus, based on the vector autoregressive model (VAR) framework, Larsson, Lyhagen and Löthgren (2001) propose the panel Johansen's test analog to determine the rank of the cointegrating space.

The proposals mentioned above define the so-called first generation of panel cointegration tests, in which the time series (individuals) that define the panel data set are assumed to be cross-section independent. Unfortunately, this assumption is crucial for the limiting distributions that are obtained in these papers and, in most cases, it is not satisfied from an empirical point of view. Violation of the cross-section independence assumption implies that Central Limit Theorems (CLT) cannot be applied and, hence, the panel data based statistics do not converge to the standard normal distribution. Provided that in most cases the economic time series of different sectors, cities, regions or countries are closely related, the use of these panel data statistics to analyse the presence of cointegration can lead to misleading conclusions. The challenge to overcome this limitation has given rise to the so-called second generation of panel cointegration tests.

Proposals that consider the presence of cross-section dependence among the time series that define the panel data set include Bai and Carrion-i-Silvestre (2005), Banerjee and Carrion-i-Silvestre (2006) and Gengenbach, Palm and Urbain (2006) for the single equation framework, and Groen and Kleinberger (2003) and Breitung (2005) for the vector error correction (VECM) framework. For a more detailed literature review on panel cointegration see Breitung and Pesaran (2007).

The aim of this paper is to solve some limitations of the existing literature on panel cointegration analysis. To this end, we propose a test statistic to determine the cointegrating rank in a panel system of equations allowing for the presence of cross-section dependence across the systems of variables in the panel setup. We deal with cross-section dependence by means of approximate common factor models as proposed in Bai and Ng (2004), Bai and Carrion-i-Silvestre (2005), Banerjee and Carrion-i-Silvestre (2006), and Gengenbach, Palm and Urbain (2006), among others. The novelty of our approach is that it takes into account the possibility that there might be more than one cointegrating relationship among the variables that define the system for each individual, and at the time that it controls for the presence of cross-section dependence among the different systems in a parsimonious way through the use of common factors. This situation has not been addressed in the literature. Finally, simulations have revealed that the consideration of common factors is even required when we conduct the analysis for only one individual, i.e., the system for only one individual of the panel data set. The presence of unattended common factors can bias the analysis towards the over estimation of the cointegrating rank.

Our proposal relates to other test statistics that are available in the literature to determine the number of stochastic trends in individual systems. Thus, the statistic determines the number of stochastic trends using the principal component approximation as in Stock and Watson (1988) and Bai and Ng (2004). Other statistics that are based on the degeneration of the moment matrix that involves the time series in the system are the ones in Phillips and Ouliaris (1990), Shintani (2001), Harris and Poskitt (2004) and Cai and Shintani (2006). However, none of these approaches considers the case where the variables in the model are affected by global stochastic trends, which in our setup are captured by the common factors.

In order to investigate the small-sample properties of the panel cointegration rank test that we propose, we conduct a Monte Carlo simulation. We estimate two different models depending on the form of the deterministic component: one model where the deterministic term consists of only the constant and another where the deterministic term consists of the constant and the linear time trend. The results of the simulation study indicate that, in general, panel data statistic performs better than the univariate one.

The remainder of the paper is organized as follows. Section 2 presents the model, the assumptions and the test statistic that is used to determine the cointegrating rank. In Section 3, we discuss the way in which the individual statistics can be combined to specify a panel data cointegrating rank statistic. Section 4 analyses the finite sample of our approach, both in an individual-by-individual framework and in a panel setup, by means of Monte Carlo simulation. Finally, some concluding remarks are presented in Section 5. The Appendix collects all the proofs.

# 2 Model and assumptions

Let $Y_{i,t}$ be a $(k \times 1)$ vector of stochastic process, where $k$ is assumed to be finite throughout the paper, with the data generating process (DGP) defined as:

$$
\begin{aligned}
Y_{i,t} &= D_{i,t} + u_{i,t} & (1)\\
u_{i,t} &= \lambda_i F_t + e_{i,t} & (2)\\
(I - L)\, F_t &= C\,(L)\, w_t & (3)\\
(I - L)\, e_{i,t} &= G_i\,(L)\, \varepsilon_{i,t}, & (4)
\end{aligned}
$$

where $D_{i,t}$ denotes the deterministic component, which in this paper can be either $D_{i,t} = \mu_i$ – henceforth, this specification is denoted as the only constant case – or $D_{i,t} = \mu_i + \delta_i t$ – hereafter, the linear time trend case – $t = 1, \ldots, T$ and $i = 1, \ldots, N$. Note that the case of non-deterministic components $D_{i,t} = 0$ is also covered in our framework as a particular case of the only constant case. The component $F_t$ denotes a $(q \times 1)$ vector of common factors and $\lambda_i$ is a $(k \times q)$ matrix of factor loadings. Finally, $e_{i,t}$ is a $(k \times 1)$ vector that collects the idiosyncratic stochastic component. Despite the operator $(1 - L)$ in equations (3) and (4), neither $F_t$ or $e_{i,t}$ have to be I(1). In fact, $F_t$ and $e_{i,t}$ can be I(0), I(1), or a combination of both, depending on the rank of $C(1)$ and $G_i\,(1)$. For instance, if $C(1) = 0$, then $F_t$ is I(0). If $C(1)$ is of full rank, then each component of $F_t$ is I(1). If $C(1) \neq 0$, but not full rank, then some components of $F_t$ are I(1) and some are I(0). The same applies for $G_i\,(1)$.

It is worth mentioning that the presentation of the model is done in a general way, so that the case where there are no common factors at all, i.e., $\lambda_i = 0\ \forall i$, can be embedded in our framework – further specific comments on this concern are given below. Let $M < \infty$ be a generic positive number, not depending on $T$ and $N$. Throughout the paper, we use $\|A\|$ to denote the Euclidean norm $trace\,(A'A)^{1/2}$ of matrix $A$. The stochastic processes that participate on the definition of the DGP are assumed to satisfy the following assumptions:

*Assumption A:* (i) for non-random $\lambda_i$, $\|\lambda_i\| \leq M$; for random $\lambda_i$, $E\,\|\lambda_i\|^4 \leq M$, (ii) $\frac{1}{N}\sum_{i=1}^{N}\lambda_i'\lambda_i \xrightarrow{p} \Sigma_\Lambda$, a $(q \times q)$ positive definite matrix.

*Assumption B:* (i) $w_t \sim iid\,(0, \Sigma_w)$, $E\,\|w_t\|^4 \leq M$, and (ii) $Var\,(\Delta F_t) = \sum_{j=0}^{\infty} C_j \Sigma_w C_j' > 0$, (iii) $\sum_{j=0}^{\infty} j\,\|C_j\| < M$; and (iv) $C\,(1)$ has rank $q_1$, $0 \leq q_1 \leq q$.

*Assumption C:* (i) for each $i$, $\varepsilon_{i,t} \sim iid\,(0, \Sigma_{\varepsilon_i})$, $E\,\|\varepsilon_{i,t}\|^4 \leq M$, (ii) $Var\,(\Delta\varepsilon_{i,t}) = \sum_{j=0}^{\infty} G_{i,j}\Sigma_{\varepsilon_i}G_{i,j}' > 0$, (iii) $\sum_{j=0}^{\infty} j\,\|G_{i,j}\| < M$; and (iv) $G\,(1)$ has rank $r$.

*Assumption D:* The errors $\varepsilon_{i,t}$, $w_t$, and the loadings $\lambda_i$ are three mutually independent groups.

*Assumption E:* $E\,\|F_0\| \leq M$, and for every $i = 1, \ldots, N$, $E\,\|e_{i,0}\| \leq M$.

The definition of the $(k \times q)$ loading matrix $\lambda_i$ is given by

$$\lambda_i = \begin{bmatrix} \lambda_{i,1,1} & \lambda_{i,1,2} \\ \lambda_{i,2,1} & \lambda_{i,2,2} \end{bmatrix},$$

so that we can impose restrictions on how the factors affect the elements of $Y_{i,t}$ in (1). Thus, some of the factors can only affect one subset of the variables, say, the series that defines the cointegrating space, but not the other variables, and the other way round. Therefore, situations where $\lambda_{i,1,2} = 0$ and/or $\lambda_{i,2,1} = 0$ are covered in this setup. Note that it is possible that both $\lambda_{i,1,2} \neq 0$ and $\lambda_{i,2,1} \neq 0$, which is the general situation that is assumed henceforth.

The unobservable common factors are estimated using the principal component approach suggested in Bai and Ng (2002, 2004). Let us consider the general deterministic component given by $D_{i,t} = \mu_i + \delta_i t$. Taking the first difference of the model we have

$$\Delta Y_{i,t} = \delta_i + \lambda_i \Delta F_t + \Delta e_{i,t}. \tag{5}$$

We can define the idempotent matrix $M = I_{T-1} - \iota\,(\iota'\iota)^{-1}\,\iota'$, with $\iota$ a $(T-1) \times 1$ vector of ones. Then,

$$\begin{aligned} M\Delta Y_i &= M\Delta F \lambda_i' + M\Delta e_i. \\ y_i &= f\lambda_i' + z_i. \end{aligned}$$

Note that when the deterministic component is $D_{i,t} = \mu_i$, taking first differences removes the constant term, so that in this case we can define $M = I_{T-1}$ and the rest of our discussion applies without sole modification. The common factors are extracted as the $q$ eigenvectors corresponding to the $q$ largest eigenvalues of the $(T-1) \times (T-1)$ matrix $yy'$, where $y = [y_1, \ldots, y_N]$ is a $(T-1) \times kN$ matrix that is defined using the $(T-1) \times k$ matrices $y_i$, $i = 1, \ldots, N$. The matrix of estimated weights, $\hat{\Lambda} = \left(\hat{\lambda}_1, \ldots, \hat{\lambda}_N\right)$, is given by $\hat{\Lambda} = y'\hat{f}$. We can obtain an estimate of $z_i$ from $\hat{z}_i = y_i - \hat{f}\hat{\lambda}_i'$, as in Bai and Ng (2004). Note that we can recover the common factors as $\hat{F}_t = \sum_{j=2}^{t}\hat{f}_j$ and the idiosyncratic component as $\hat{e}_{i,t} = \sum_{j=2}^{t}\hat{z}_{i,j}$.

Cointegration analysis can be then performed focusing on the idiosyncratic component once the effects of the common factors have been removed. This gives further insight on the cointegration analysis, since as we shown in the Monte Carlo analysis, the inference on the cointegrating rank can be distorted if common factors are not accounted for.

In this paper we propose to determine the rank with a test statistic that is based on the multivariate version of the square of the modified Sargan-Bhargava (MSB) statistic proposed in Stock (1999). The definition of the testing procedure builds upon the different rates of convergence of the elements on the $Q_{\hat{e}_i\hat{e}_i} = T^{-1}\hat{e}_i'\hat{e}_i$ matrix under the null hypothesis. Without loss of generality, let us assume that the rank of the cointegrating space is $0 < r < k$. We can define the orthogonal matrix $A = [A_1 : A_2]$ with $A_1$ a $(k \times r)$ matrix and $A_2$ a $(k \times m)$ matrix, $m = k - r$, such that the first $r$ elements of the rotated vector $e_{i,t}^A = A' e_{i,t} = \left((A_1' e_{i,t})', (A_2' e_{i,t})'\right)'$ are I(0) and the other $m$ elements are I(1). Accordingly, we partition the long-run variance matrix as

$$\Omega_{\Delta e_i^A \Delta e_i^A} = \begin{bmatrix} \Omega_{11,i} & \Omega_{12,i} \\ \Omega_{21,i} & \Omega_{22,i} \end{bmatrix}.$$

We have $\pi\left(T^{-1}Q_{\hat{e}_i\hat{e}_i}\hat{\Omega}^{-1}_{\Delta\hat{e}_i\Delta\hat{e}_i}\right) = \pi\left(T^{-1}Q_{\hat{e}_i^A\hat{e}_i^A}\hat{\Omega}^{-1}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A}\right) = \pi\left(T^{-1}\hat{\Omega}^{-1/2}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A}Q_{\hat{e}_i^A\hat{e}_i^A}\hat{\Omega}^{-1/2}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A}\right)$, where $\pi(\cdot)$ denotes the eigenvalues of the matrix between parentheses. The determination of the number of stochastic trends in the system relies on the following sequential testing procedure:

1. First, assume that the cointegrating rank is zero, i.e., set $m = k$.

2. Specify the null hypothesis that there are $l = m$ common stochastic trends $(H_0 : l = m)$ against the alternative hypothesis that there are $l < m$ common stochastic trends $(H_1 : l < m)$.

3. Estimate $A_2$ as the $m$ eigenvectors that corresponds with the $m$ largest eigenvalues of $T^{-1}Q_{\hat{e}_i\hat{e}_i}$.

4. Define the univariate MSB statistic as

$$\begin{aligned} MSB_{j,i}(m) &= \pi^{\min}\left(T^{-1}Q_{\hat{e}_i^{A_2}\hat{e}_i^{A_2}}\hat{\Omega}^{-1}_{\Delta\hat{e}_i^{A_2}\Delta\hat{e}_i^{A_2}}\right) \\ &= \pi^{\min}\left(T^{-1}\hat{\Omega}^{-1/2}_{\Delta\hat{e}_i^{A_2}\Delta\hat{e}_i^{A_2}}Q_{\hat{e}_i^{A_2}\hat{e}_i^{A_2}}\hat{\Omega}^{-1/2}_{\Delta\hat{e}_i^{A_2}\Delta\hat{e}_i^{A_2}}\right) \\ &= \hat{\pi}_{i,1}, \end{aligned} \quad (6)$$

   where the subscript $j = \{\mu, \tau\}$ refers to the deterministic component that is used in the model – $\mu$ for the $D_{i,t} = \mu_i$ deterministic specification and $\tau$ for the $D_{i,t} = \mu_i + \delta_i t$ one – being $\pi_{i,1} < \cdots < \pi_{i,m}$ the eigenvalues of $T^{-1}Q_{\hat{e}_i^{A_2}\hat{e}_i^{A_2}}\hat{\Omega}^{-1}_{\Delta\hat{e}_i^{A_2}\Delta\hat{e}_i^{A_2}}$ sorted in ascending order, and $\pi^{\min}(\cdot)$ denoting the minimum eigenvalue operator.

5. Compare the value of the $MSB_{j,i}(m)$ statistic with the corresponding critical values from the left tail of the distribution – i.e., the null hypothesis is rejected if $MSB_{j,i}(m)$ is smaller than the critical value.

6. If the null hypothesis of $H_0 : l = m$ common stochastic trends is rejected, specify $l = m - 1$ and return to step 2. The process continues till either the null hypothesis is not rejected or when $l = 0$ is achieved.

The estimation of $\Omega_{\Delta e_i^{A_2} \Delta e_i^{A_2}}$ can be obtained in a parametric way from the estimation of the VECM model specification. Expressed in matrix notation, we have

$$\begin{aligned} \Delta e_i^{A_2} &= e_{i,-1}^{A_2} \Pi_i + \Delta e_i^{A_2} \Gamma_{i,p_i}(L) + \varepsilon_i \\ \Delta e_i^{A_2} (I - \Gamma_{i,p_i}(L)) &= e_{i,-1}^{A_2} \Pi_i + \varepsilon_i \\ \Delta e_i^{A_2} &= e_{i,-1}^{A_2} \Pi_i (I - \Gamma_{i,p_i}(L))^{-1} + \varepsilon_i (I - \Gamma_{i,p_i}(L))^{-1}, \end{aligned}$$

where $p_i$ denotes the number of lags of $\Delta e_{i,i}^{A_2}$ that are considered. Following Ng and Perron (2001), we define $\hat{\Omega}_{\Delta \hat{e}_i^{A_2} \Delta \hat{e}_i^{A_2}} = \left( \left( I - \hat{\Gamma}_{i,p_i}(1) \right)^{-1} \right)' T^{-1} \hat{\varepsilon}_i' \hat{\varepsilon}_i \left( I - \hat{\Gamma}_{i,p_i}(1) \right)^{-1}$, where the lag order of the model $p_i$ is estimated using the modified information criterion in Perron and Qu (2007) assuming that the cointegrating rank is zero – note that under the null hypothesis we assume that there are $m$ stochastic trends in the system defined by the $m$ variables of $e_{i,t}^{A_2}$.

The limiting distribution of the $MSB_{j,i}(m)$ statistic, $j = \{\mu, \tau\}$, is established in the following Theorem.

**Theorem 1** *Let $Y_{i,t}$, $i = 1, \ldots, N$, $t = 1, \ldots, T$, be a $(k \times 1)$ vector of stochastic processes with the DGP given by (1) to (4) and satisfying Assumptions A to E. Under the null hypothesis that there are $m = k - r$ common stochastic trends, with $p_i \to \infty$ and $p_i^3 / \min[T, N] \to 0$ as $T \to \infty$, $N \to \infty$, $N/T \to 0$, the MSB statistic given in (6) converges to:*

*(a) For the only constant model:* $MSB_{\mu,i}(m) \Rightarrow \pi^{\min} \left( \int_0^1 W_i(s) W_i(s)' ds \right)$

*(b) For the linear trend model:* $MSB_{\tau,i}(m) \Rightarrow \pi^{\min} \left( \int_0^1 V_i(s) V_i(s)' ds \right)$,

*where $\Rightarrow$ denotes weak convergence, $W_i(s)$ is an $(m \times 1)$ vector of independent standard Brownian motions, and $V_i(s) = W_i(s) - sW_i(1)$ is an $(m \times 1)$ vector of independent Brownian bridges.*

The proof of Theorem 1 is given in the Appendix. It has to be stressed that our framework treats as a special case the situation in which the time series are assumed to be cross-section independent – in this case we only need to impose $\lambda_i = 0$ $\forall i$ in (2). The estimation of the parameters of the deterministic component can be done specifying the model in first differences so that $y_i = z_i$, where $y_i = M \Delta Y_i$ and $z_i = M \Delta e_i$ – as before, $M = I_{T-1}$ for the constant and $M = I_{T-1} - \iota (\iota' \iota)^{-1} \iota'$ for the time trend deterministic specifications. Then, defining $\hat{e}_{i,t} = \sum_{j=2}^{t} y_{i,j}$ the computation of the MSB statistic proceeds as above, giving rise to test statistics with the limiting distribution given in Theorem 1. The critical values for the $MSB_j(m)$ statistic, $j = \{\mu, \tau\}$, are reported in Table 1 for different sample sizes. These finite sample critical values are computed using the autoregressive spectral density estimator $\hat{\Omega}_{\Delta \hat{e}_i^{A_2} \Delta \hat{e}_i^{A_2}}$ defined above. Following Ng and Perron (1998), we have specified the upper bound for $p_i$ as $p_i^{max} = T^{1/3}$ for $T = \{100, 200, 500\}$. The asymptotic critical values are obtained using $T = 1,000$ observations assuming that the disturbance terms are *iid*.

The MSB statistic that is presented in the paper is consistent under the alternative hypothesis that there are less common stochastic trends than the ones specified under the null hypothesis. The following Theorem presents the rate at which the MSB statistic diverges under the alternative hypothesis.

**Theorem 2** *Let $Y_{i,t}$, $i = 1, \ldots, N$ , $t = 1, \ldots, T$, be a $(k \times 1)$ vector of stochastic processes with the DGP given by (1) to (4) and satisfying Assumptions A to E. Under the alternative hypothesis that there are $l < m$ common stochastic trends we have that $MSB_{j,i}(m) = O_p(T^{-1})$, $j = \{\mu, \tau\}$.*

The proof is given in the Appendix. The result in Theorem 2 shows that the MSB statistic converges to zero at rate $T$ under the alternative hypothesis so that it is consistent.

# 3 Panel data cointegrating rank tests

The individual MSB statistics can be pooled to define panel data statistics, which are expected to increase the performance of the statistical inference when estimating the cointegrating rank. In this section we define up to four different panel data statistics depending on the way in which the individual information is combined. In all cases, the null hypothesis is that all $N$ individual systems have the same common stochastic trends $(m)$ while the alternative hypothesis is that there are less than $m$ stochastic trends, i.e.

$$\left\{ \begin{array}{lll} H_0: & l_i = m & \forall i = 1, \ldots, N \\ H_1: & l_i < m & \text{for some } i \end{array} \right. .$$

The performance of these four alternatives is analyzed by simulation in the next section.

The first panel data MSB (PMSB) statistic is based on the standardized mean of the individual statistics

$$PMSB_j^Z(m) = \frac{\sqrt{N}(\overline{MSB}_j(m) - E(MSB_j(m)))}{\sqrt{Var(MSB_j(m))}}, \tag{7}$$

where $\overline{MSB}_j(m) = N^{-1} \sum_{i=1}^{N} MSB_{j,i}(m)$, and $E(MSB_j(m))$ and $Var(MSB_j(m))$ are the mean and the variance of the $MSB_j(m)$ statistic computed from (6). The limiting distribution of the PMSB statistic is given in the following Theorem.

**Theorem 3** *Let $Y_{i,t}$, $i = 1, \ldots, N$ , $t = 1, \ldots, T$, be a $(k \times 1)$ vector of stochastic processes with the DGP given by (1) to (4) and satisfying Assumptions A to E. Under the null hypothesis that there are $m = k - r$ common stochastic trends, with $p_i \to \infty$ and $p_i^3 / \min[T, N] \to 0$ as $T \to \infty$, $N \to \infty$, $N/T \to 0$, the PMSB statistic given in (7) converges to:*

$$PMSB_j^Z(m) \Rightarrow N(0,1), \ j = \{\mu, \tau\}.$$

The proof is given in the appendix. The mean and the variance of the $MSB_j(m)$ statistic, $j = \{\mu, \tau\}$, that have been computed by simulation are presented in Table 2.

It is possible to define panel data statistics based on the combination of the individual p-values. Maddala and Wu (1999) defines the panel data Fisher-type statistic $PMSB_j^F(m) = -2\sum_{i=1}^N \ln\varphi_i \sim \chi^2_{2N}$, where $\varphi_i$ denotes the p-value of the $MSB_{j,i}(m)$ statistic, $j = \{\mu, \tau\}$. Although the $PMSB_j^F(m)$ statistic is valid for finite $N$, Choi (2001) suggests to compute the following tests when $N \to \infty$:

$$
\begin{aligned}
PMSB_j^{C1}(m) &= \frac{-2\sum_{i=1}^N \ln\varphi_i - 2N}{\sqrt{4N}} \Rightarrow N(0,1) \\
PMSB_j^{C2}(m) &= \frac{1}{\sqrt{N}}\sum_{i=1}^N \Phi^{-1}(\varphi_i) \Rightarrow N(0,1),
\end{aligned}
$$

where $\Phi(\cdot)$ denotes the standard cumulative distribution function.

# 4 Monte Carlo simulation

We now analyse the small sample performance of the MSB panel cointegration rank test for the two deterministic specifications that are considered in this paper. The DGP that is used in this section is based on the ones in Toda (1995) and Saikkonen and Lütkepohl (2000) and has the following form:

$$
Y_{i,t} = D_{i,t} + \lambda_i F_t + e_{i,t} \tag{8}
$$

$$
\begin{pmatrix} e_{1,i,t} \\ e_{2,i,t} \end{pmatrix} = \begin{pmatrix} \psi_i & 0 \\ 0 & I_{k-r} \end{pmatrix} \begin{pmatrix} e_{1,i,t-1} \\ e_{2,i,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,i,t} \\ \varepsilon_{2,i,t} \end{pmatrix} \tag{9}
$$

$$
F_j = \rho F_{j-1} + \sigma_F w_t \tag{10}
$$

$$
\varepsilon_{i,t} = iid\ N\left(0, \begin{pmatrix} I_r & \theta_i \\ \theta_i & I_{k-r} \end{pmatrix}\right), \tag{11}
$$

where $i = 1, \ldots, N$, $t = 1, \ldots, T$ and $j = 1, \ldots, q$. Note that when there is not cointegration among any of the time series ($rank = 0$) equation (9) reduces to $e_{i,t} = e_{i,t-1} + \varepsilon_{i,t}$ with $\varepsilon_{i,t} \sim iid\ N(0, I_k)$. On the other hand, when all time series are stationary ($rank = k$) equation (9) reduces to $e_{i,t} = \psi_i e_{i,t-1} + \varepsilon_{i,t}$.

The parameter sets are defined as follows. We consider a system defined by $k = 3$ variables. For the deterministic component, we have set $D_{i,t} = \mu_i + \delta_i t$, with $\mu_i \sim U[-1, 1]$ and $\delta_i \sim U[-0.5, 0.5]$, where $U$ denotes the uniform distribution. The idiosyncratic cointegrating rank is investigated using $\psi_i = aI_r$ with $a = \{0.5, 0.8, 1\}$. Furthermore, we define $\theta_i = b1_{r\times(k-r)}$ where $1_{r\times(k-r)}$ denotes a $r \times (k-r)$ matrix of ones, and $b = \{0, 0.4, 0.8\}$. Note that $\theta_i$ controls the correlation among the $\varepsilon_{i,t}$ disturbance terms. As for the common factor component, we specify $\lambda_i \sim N(1, 1)$, $\rho = \{0.9, 0.95, 1\}$, $\sigma_F^2 = \{0.5, 1, 10\}$ and $w_t \sim N(0, 1)$, $j = 1, \ldots, q$, with $q = \{1, 3\}$ common factors. The number of common factors is estimated using the panel Bayesian information criterion (BIC) in Bai and Ng (2002) using $q_{\max} = 6$ as the maximum number of common factors.

The simulations are performed in GAUSS using the COINT 2.0 library. The empirical size and power of the statistics are obtained using 1,000 replications with the nominal size set at the 5% level of significance for all different combinations of individuals $N = \{1, 20, 40\}$ and time series observations $T = \{100, 200, 500\}$. For conciseness, we report only the results for $N = 1$ and for $N = 20$. The results for $N = 40$ are qualitatively similar to those for $N = 20$ so they are not reported to save space – these results are available upon request. The simulations use either the critical values or the mean and the variance in Tables 1 and 2.

Our analysis focuses on two different situations. First, we analyse the effects on the cointegrating rank determination when we fail to consider the presence of common factors. This experiment shows the extent that this misspecification error can have on the statistical inference. Second, we proceed to study the case where common factors are taken into account.

## 4.1 Cross-section dependence is not considered

### 4.1.1 Unit-by-unit analysis

In this section we investigate the performance of the statistic when $N = 1$. To motivate the importance of the presence of common factors, we first simulate the $MSB$ statistic for both the only constant and the linear time trend models when the common factors are not accounted for. In order to save space, we only report the results for $a = 0.5$ for the deterministic specification given by a constant term $D_{i,t} = \mu_i$, although the complete set of results is available upon request – similar conclusions are obtained for the other cases that have been analysed.

In Panel A of Table 3 we present the results when the true number of stochastic trends is 0. When the common factor is stationary $\left(\rho_j < 1\right)$, the most frequently estimated number of stochastic trends coincides with the true one, although this depends on $T$ and on the importance of the common factors $\left(\sigma^2_{F,j}\right)$. Thus, note that for $T = 100$ and for a given value of $\rho_j$, the $MSB$ statistic tends to increase the frequency that $\hat{r} = 1$ is selected as $\sigma^2_{F,j}$ increases. This missclassification error reduces as the sample size increases, so that the procedure leads to the right classification when $T = 500$ regardless of the values of $\rho_j < 1$ and $\sigma^2_{F,j}$ that are used. The picture changes when the common factor is non-stationary $\left(\rho_j = 1\right)$. Now the $MSB$ statistic tends to detect more stochastic trends than there exist, leading to incorrect conclusions.

Panel B of Table 3 presents the results when the true number of stochastic trends is 1. Again the results depend on whether the common factor is stationary and on the sample size. When $\rho_j < 1$ $r = 1$ is selected most of the times, although the presence of stationary common factors tends to bias the analysis to conclude that there are fewer stochastic trends than there exist. For instance, when $\rho_j = 0.95$ and $T = 100$ the percentage of times that $r = 0$ is selected for $\sigma^2_{F,j} = 0.5$ is 7.3, for $\sigma^2_{F,j} = 1$ is 9.5 and for $\sigma^2_{F,j} = 10$ is 22.7. When $T = 200$ the percentages are 13, 23.8 and 56.8 respectively, whereas for $T = 500$ they are 23.1, 26.6 and 67.4, respectively. Therefore, we can see that as the importance of the stationary common factor and the sample size increases, the $MSB$ statistic tends to detect fewer stochastic trends than there exist. This problem does not appear if the common factor is non-stationary, since in all cases $r = 1$ is selected in 95% of cases.

Similar results are obtained if we look at Panels C and D of Table 3. As the sample size increase we observe a tendency to select fewer stochastic trends than exist. The novelty now is that this feature is obtained even if the common factor is non-stationary.

The overall conclusion that is obtained from this analysis indicates that the presence of common factors biases the analysis leading to conclude that there are fewer stochastic trends than there really are, i.e., to conclude that there are more cointegration relationships than there exist.

#### 4.1.2 Panel data analysis

Table 4 presents the results for the $PMSB^{Z}$ statistic when $N = 20$, $a = 0.5$ and the deterministic component is given a constant term. As before, the complete set of results are available upon request, although the conclusions are similar to the ones reported below.

Panel A of Table 4 indicates that the presence of stationary factors does not affect the statistical inference, since in almost all cases the proportion that the true number of stochastic trends is selected is 1. However, this does not happen when the common factor is non-stationary. In this case the statistic tends approximately to indicate half of the times that $r = 0$ and the other half that $r = 1$.

Panel B of Table 4 presents the results when $r = 1$. If the common factor is stationary, it can be seen that the proportion of times that $r = 0$ is erroneously selected increases with $T$ and $\sigma^2_{F,j}$. For instance, when $\rho_j = 0.95$ and $T = 100$ the percentage of times that $r = 0$ is selected for $\sigma^2_{F,j} = 0.5$ is 41.4, for $\sigma^2_{F,j} = 1$ is 71.1 and for $\sigma^2_{F,j} = 10$ is 95.9. For $T = 200$ these proportions are 51.9, 91.3 and 99.8, respectively. Finally, for $T = 500$ we have 60.5, 86.1 and 100, respectively. Only in those cases where the common factor is non-stationary we obtain that the percentage of selecting that $r = 1$ instead of zero is larger, although the $PMSB^{Z}$ statistic can lead to indicate half of the times that $r = 0$ and the other half that $r = 1$ when $\sigma^2_{F,j} = 10$. Panels C and D of Table 4 reveal that as $T$ and $\sigma^2_{F,j}$ increases, the tendency is to indicate that there are fewer stochastic trends than there exist, even if the common factor is non-stationary.

Overall, we observe that the presence of unattended common factors can lead to obtain distorted conclusions, since we have obtained that there is a bias to indicate that there are more cointegrating relationships than there exist in our system. This evidences the importance of accounting for cross-section dependence both in unit-by-unit and panel data analyses.

### 4.2 Cross-section dependence is considered

#### 4.2.1 Unit-by-unit analysis

Table 5 reports the results for the only constant case with $a = 0.5$ and $N = 1$. Before discussing the simulation results it should be stated that the use of the panel BIC always detected the true number of common factors. The simulations reveal some important features, regardless of the deterministic component that is used.

First, we can see that the results do not depend on the stochastic properties of the common factor, provided that the performance of the $MSB$ statistics is similar regardless of whether the common factor is I(0) or I(1), and regardless of the magnitude of the disturbance variance that participates in the generation of the common factors ($\sigma^2_F$). Second, the performance of the $MSB$ statistic depends on how close the autoregressive parameter $a$ is to one. Thus, for $a = 0.5$ we can see in Table 5 that the statistical procedure that has been proposed in this paper selects the correct number of stochastic trends in most cases. As expected, the behaviour of the procedure worsens for $a = 0.8$, although it tends to select the correct number of stochastic trends as $T$ increases – these results are not presented in the paper, however they are available upon request.

Finally, note that this picture is in sharp contrast with the one that was obtained above when the common factors are not considered. Therefore, we can see that even in the case that the practitioner is only interested in the unit-by-unit analysis, the presence of cross-section dependence should be taken into account provided that there is a risk to conclude that there are more cointegration relationships than there exist.

### 4.2.2 Panel data analysis

Table 6 collects the simulation results for the $PMSB^Z$ statistic when the common factors are accounted with $N = 20$. Here we only focus on the $PMSB^Z$ statistic, provided that similar results have been obtained when we have analysed the statistics that are based on the combination of the p-values, i.e., the $PMSB^F$, $PMSB^{C1}$ and $PMSB^{C2}$ statistics. We can see that the statistic has substantially better performance than the one exhibited for univariate situation. This improvement is noticeable for the smaller sample size that we have considered, where the use of the panel data based statistic reduces the tendency shown by the individual $MSB$ statistic to overestimate the number of common stochastic trends. Another interesting issue is that these results do not depend on the stochastic properties of the common factor and the importance of the common factor. These conclusions are also reached for the other configurations that have been essayed in the Monte Carlo experiment, and for the linear time trend deterministic component.

# 5 Conclusion

In this paper we propose a new test statistic to estimate the cointegrating rank both in a unit-by-unit analysis and in a panel data framework. Our proposal covers the case of cross-section dependence through the specification of approximate common factor models, which is a relevant situation from both theoretical and empirical point of views. This setup allows to cover strong cross-section dependence cases, i.e., cases where the time series of one individual systems are cointegrated with times series of other individual systems. This situation can be considered as the multivariate extension of the cross-cointegration concept defined earlier in the literature.

The performance of the proposal is investigated with Monte Carlo simulations. In general, the panel data based MSB statistic provides better estimation of the number of stochastic trends that are present in each individual system. More interestingly, the simulations have revealed that the presence of common factors can lead to misleading conclusions even if the analysis is carried out unit-by-unit. This is relevant from an empirical point of view if we think that in most cases cointegration analysis is conducted focusing on one country whose economic system is related to that of other countries or ruled by international organizations such as in the case of the European Union. Therefore, the theoretical proposal presented in this paper has a significant contribution from an empirical point of view.

# A Mathematical appendix

## A.1 Proof of Theorem 1

### A.1.1 The only constant case

Note that the estimated difference of the idiosyncratic stochastic term is:

$$\hat{z}_{i,t} = z_{i,t} + \lambda_i f_t - \hat{\lambda}_i \hat{f}_t.$$

Following Bai and Ng (2004), we can express the model as

$$\begin{aligned}
\hat{z}_{i,t} &= z_{i,t} + \lambda_i H^{-1} H f_t - \lambda_i H^{-1} \hat{f}_t + \lambda_i H^{-1} \hat{f}_t - \hat{\lambda}_i \hat{f}_t \\
&= z_{i,t} + \lambda_i H^{-1} \left( H f_t - \hat{f}_t \right) - \left( \hat{\lambda}_i - \lambda_i H^{-1} \right) \hat{f}_t \\
&= z_{i,t} + \lambda_i H^{-1} v_t - d_i \hat{f}_t, \qquad (12)
\end{aligned}$$

where $v_t = \left( H f_t - \hat{f}_t \right)$ and $d_i = \left( \hat{\lambda}_i - \lambda_i H^{-1} \right)$. Let us define the partial sum process using the estimated residuals as $\hat{e}_{i,t} = \sum_{j=2}^{t} \hat{z}_{i,j} = \sum_{j=2}^{t} \left( [M \; \Delta \hat{e}_i]_j \right)'$, where $[\cdot]_j$ denotes the $j$-th row of the matrix between brackets. The idiosyncratic disturbance terms can be expressed as $\hat{e}_{i,t} = e_{i,t} + A_{i,t}$ with $A_{i,t} = -e_{i,1} + \lambda_i H^{-1} V_t - d_i \hat{F}_t$. Denote by $\hat{e}_{i,t}(l)$ the $l$-th element of the $(k \times 1)$-vector $\hat{e}_{i,t}$, $l = 1, \ldots, k$. If $\hat{e}_{i,t}(l) \sim I(0)$ then $T^{-2} \sum_{t=2}^{T} \hat{e}_{i,t}^2(l) = O_p(T^{-1})$, whereas if $\hat{e}_{i,t}(l) \sim I(1)$ then we have

$$T^{-2} \sum_{t=2}^{T} \hat{e}_{i,t}^2(l) = T^{-2} \sum_{t=2}^{T} e_{i,t}^2(l) + 2T^{-2} \sum_{t=2}^{T} e_{i,t}(l) A_{i,t}(l) + T^{-2} \sum_{t=2}^{T} A_{i,t}^2(l) = I + II + III$$

Part $I$ is $O_p(1)$ and, from Bai and Ng (2004), $III$ is $O_p\left(C_{NT}^{-2}\right)$ for all $i$, with $C_{NT} = \min\left[\sqrt{N}, \sqrt{T}\right]$. Let us now focus on $II$:

$$\begin{aligned}
T^{-2} \sum_{t=2}^{T} e_{i,t}(l) A_{i,t}(l) &= -T^{-2} \sum_{t=2}^{T} e_{i,t}(l) e_{i,1}(l) + T^{-2} \sum_{t=2}^{T} e_{i,t}(l) \lambda_i(l) H^{-1} V_t \\
&\quad - T^{-2} \sum_{t=2}^{T} e_{i,t}(l) d_i(l) \hat{F}_t \\
&= a + b + c,
\end{aligned}$$

where $\lambda_i(l)$ and $d_i(l)$ denote the $l$-th row of the $(k \times q)$ matrices $\lambda_i$ and $d_i$. Element $a$ is $O_p\left(T^{-1/2}\right) = O_p\left(C_{NT}^{-1}\right)$, while $b$ is

$$\begin{aligned}
\|b\| &\leq \left( T^{-2} \sum_{t=2}^{T} \| e_{i,t}(l) \lambda_i(l) \|^2 \right)^{1/2} \left( T^{-2} \sum_{t=2}^{T} \left\| H^{-1} V_t \right\|^2 \right)^{1/2} \\
&= O_p(1) O_p\left(N^{-1/2}\right) = O_p\left(C_{NT}^{-1}\right).
\end{aligned}$$

Finally, element $c$

$$\begin{aligned} \|c\| &\leq \left(T^{-2}\sum_{t=2}^{T}\|e_{i,t}(l)\,d_i(l)\|^2\right)^{1/2}\left(T^{-2}\sum_{t=2}^{T}\left\|\hat{F}_t\right\|^2\right)^{1/2} \\ &= O_p\left(T^{-1/2}\right)O_p(1) = O_p\left(C_{NT}^{-1}\right) \end{aligned}$$

given that $d_i(l) = O_p\left(\min\left[N, T^{-1/2}\right]\right)$ – see Lemma 1(c) in Bai and Ng (2004). Taking all these elements together we have that $II$ is $O_p\left(C_{NT}^{-1}\right)$. Consequently, if $\hat{e}_{i,t}(l) \sim I(1)$

$$T^{-2}\sum_{t=2}^{T}\hat{e}_{i,t}^2(l) = T^{-2}\sum_{t=2}^{T}e_{i,t}^2(l) + O_p\left(C_{NT}^{-1}\right).$$

For subsequent results we note that Bai and Ng (2007) showed that averaging part $II$ across $N$ gives $N^{-1}\sum_{i=1}^{N}T^{-2}\sum_{t=2}^{T}e_{i,t}(l)\,A_{i,t}(l) = O_p\left(C_{NT}^{-2}\right)$ provided that $N/T \to 0$.

If we rotate the vector $\hat{e}_{i,t}$ and define $\hat{e}_{i,t}^A = A'\hat{e}_{i,t} = \left((A_1'\hat{e}_{i,t})', (A_2'\hat{e}_{i,t})'\right)'$ we have that $T^{-1/2}A_1'\hat{e}_{i,t} = o_p(1)$ provided that $A_1'\hat{e}_{i,t}$ defines the stationary relationships and $T^{-1/2}A_2'\hat{e}_{i,t} = O_p(1)$ given that $A_2'\hat{e}_{i,t}$ defines the I(1) stochastic trends. Therefore, as $T \to \infty$

$$\begin{aligned} T^{-1}Q_{\hat{e}_i^A\hat{e}_i^A} &= T^{-2}\hat{e}_i^{A\prime}\hat{e}_i^A \\ &\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & \Omega_{22,i}^{1/2}\int_0^1 W_i(s)\,W_i(s)'\,ds\;\Omega_{22,i}^{1/2} \end{bmatrix}, \end{aligned}$$

where $W_i(s)$ denotes a $k-r$ vector of independent standard Brownian motions with $W_i(0) = 0$ and given that $T^{-2}A_1'e_i'e_iA_1 = o_p(1)$, $T^{-2}A_1'e_i'e_iA_2 = o_p(1)$ and $T^{-2}A_2'e_i'e_iA_2 = O_p(1)$. Therefore, using these elements the limiting distribution of the multivariate MSB statistic is given by:

$$\begin{aligned} MSB_{\mu,i}(m) &= \pi^{\min}\left(T^{-1}Q_{\hat{e}_i\hat{e}_i}\hat{\Omega}_{\Delta\hat{e}_i\Delta\hat{e}_i}^{-1}\right) \\ &= \pi^{\min}\left(T^{-1}Q_{\hat{e}_i^A\hat{e}_i^A}\hat{\Omega}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A}^{-1}\right) \\ &= \pi^{\min}\left(T^{-1}\hat{\Omega}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A}^{-1/2}Q_{\hat{e}_i^A\hat{e}_i^A}\hat{\Omega}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A}^{-1/2}\right) \\ &\Rightarrow \pi^{\min}\left(\int_0^1 W_i(s)\,W_i(s)'\,ds\right), \end{aligned}$$

provided that $\hat{\Omega}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A} \xrightarrow{p} \Omega_{\Delta e_i^A\Delta e_i^A}$, where $\xrightarrow{p}$ denotes convergence in probability.

### A.1.2 The linear time trend case

The proof for this deterministic component follows the one for the constant case, but now $\hat{e}_{i,t} = e_{i,t} - \frac{t-1}{T-1}e_{i,T} + A_{i,t}$ with $A_{i,t} = -e_{i,1} + \frac{t-1}{T-1}e_{i,1} + \lambda_i H^{-1}V_t - d_i\hat{F}_t = -\frac{T-t}{T-1}e_{i,1} + \lambda_i H^{-1}V_t - d_i\hat{F}_t$. If $\hat{e}_{i,t}(l) \sim I(0)$ then $T^{-2}\sum_{t=2}^{T}\hat{e}_{i,t}^2(l) = O_p\left(T^{-1}\right)$, whereas if $\hat{e}_{i,t}(l) \sim I(1)$ then we have

$$
\begin{aligned}
T^{-2}\sum_{t=2}^{T}\hat{e}_{i,t}^{2}(l) &= T^{-2}\sum_{t=2}^{T}\left(e_{i,t}(l)-\frac{t-1}{T-1}e_{i,T}(l)\right)^{2}+2T^{-2}\sum_{t=2}^{T}\left(e_{i,t}(l)-\frac{t-1}{T-1}e_{i,T}(l)\right)A_{i,t}(l) \\
&\quad +T^{-2}\sum_{t=2}^{T}A_{i,t}^{2}(l) \\
&= I+II+III
\end{aligned}
$$

Part $I$ is $O_p(1)$ and, as before, $III$ is $O_p\left(C_{NT}^{-2}\right)$ for all $i$. Part $II$ is given by

$$
\begin{aligned}
II &= -T^{-2}\sum_{t=2}^{T}\frac{T-t}{T-1}e_{i,t}(l)\,e_{i,1}(l)+T^{-2}\sum_{t=2}^{T}e_{i,t}(l)\,\lambda_i(l)\,H^{-1}V_t-T^{-2}\sum_{t=2}^{T}e_{i,t}(l)\,d_i(l)\,\hat{F}_t \\
&\quad +T^{-2}\sum_{t=2}^{T}\frac{T-t}{T-1}\frac{t-1}{T-1}e_{i,T}(l)\,e_{i,1}(l)-T^{-2}\sum_{t=2}^{T}\frac{t-1}{T-1}e_{i,T}(l)\,\lambda_i(l)\,H^{-1}V_t \\
&\quad +T^{-2}\sum_{t=2}^{T}\frac{t-1}{T-1}e_{i,T}(l)\,d_i(l)\,\hat{F}_t \\
&= a+b+c+d+e+f.
\end{aligned}
$$

The first component is $a=O_p\left(T^{-1/2}\right)=O_p\left(C_{NT}^{-1}\right)$, and from the previous proof, $b=O_p\left(C_{NT}^{-1}\right)$ and $c=O_p\left(C_{NT}^{-1}\right)$. Consider $d$

$$
\begin{aligned}
d &= (T-1)^{-2}T^{-2}\sum_{t=2}^{T}\left(-t^2+(T+1)t-T\right)e_{i,T}(l)\,e_{i,1}(l) \\
&= O_p\left(T^{-1/2}\right)+O_p\left(T^{-1/2}\right)+O_p\left(T^{-3/2}\right)=O_p\left(C_{NT}^{-1}\right).
\end{aligned}
$$

Component $e$ is given by

$$
\begin{aligned}
\|e\| &\le \left(T^{-2}\sum_{t=2}^{T}\left\|\frac{t-1}{T-1}e_{i,T}(l)\,\lambda_i(l)\right\|^2\right)^{1/2}\left(T^{-2}\sum_{t=2}^{T}\left\|H^{-1}V_t\right\|^2\right)^{1/2} \\
&= O_p(1)\,O_p\left(N^{-1/2}\right)=O_p\left(C_{NT}^{-1}\right).
\end{aligned}
$$

Finally, component $f$ is

$$
\begin{aligned}
\|f\| &\le \left(T^{-2}\sum_{t=2}^{T}\left\|\frac{t-1}{T-1}e_{i,T}(l)\,d_i(l)\right\|^2\right)^{1/2}\left(T^{-2}\sum_{t=2}^{T}\left\|\hat{F}_t\right\|^2\right)^{1/2} \\
&= O_p\left(T^{-1/2}\right)O_p(1)=O_p\left(C_{NT}^{-1}\right).
\end{aligned}
$$

Therefore, part $II=O_p\left(C_{NT}^{-1}\right)$, so that if $\hat{e}_{i,t}(l)\sim I(1)$

$$
T^{-2}\sum_{t=2}^{T}\hat{e}_{i,t}^{2}(l)=T^{-2}\sum_{t=2}^{T}\left(e_{i,t}(l)-\frac{t-1}{T-1}e_{i,T}(l)\right)^{2}+O_p\left(C_{NT}^{-1}\right).
$$

As already mentioned above, Bai and Ng (2007) showed that averaging part $II$ across $N$ gives $N^{-1}\sum_{i=1}^{N} T^{-2}\sum_{t=2}^{T} e_{i,t}(l)\, A_{i,t}(l) = O_p\left(C_{NT}^{-2}\right)$ provided that $N/T \rightarrow 0$.

As before, we define $\hat{e}_{i,t}^{A} = A'\hat{e}_{i,t} = \left((A_1'\hat{e}_{i,t})', (A_2'\hat{e}_{i,t})'\right)'$, with $T^{-1/2}A_1'\hat{e}_{i,t} = o_p(1)$ and $T^{-1/2}A_2'\hat{e}_{i,t} = O_p(1)$ provided that $A_2'\hat{e}_{i,t}$ defines the I(1) stochastic trends. Then,

$$T^{-1/2}A'\hat{e}_{i,t} \Rightarrow \left(0_r', \Omega_{22,i}^{1/2}\left(W_i(s) - sW_i(1)\right)'\right)',$$

which implies that

$$\begin{aligned} T^{-1}Q_{\hat{e}_i^A\hat{e}_i^A} &= T^{-2}\hat{e}_i^{A\prime}\hat{e}_i^A \\ &\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & \Omega_{22,i}^{1/2}\int_0^1 V_i(s)\,V_i(s)'\,ds\;\Omega_{22,i}^{1/2} \end{bmatrix}, \end{aligned}$$

where $V_i(s) = W_i(s) - sW_i(1)$ is a vector of independent Brownian bridges. Therefore,

$$MSB_{\tau,i}(m) \Rightarrow \pi^{\min}\left(\int_0^1 V_i(s)\,V_i(s)'\,ds\right),$$

given that $\hat{\Omega}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A} \xrightarrow{p} \Omega_{\Delta e_i^A\Delta e_i^A}$.

## A.2 Proof of Theorem 2

Let us consider the null hypothesis that there are $m = k - r$ stochastic trends. From the proof of Theorem 1 we have that $T^{-1}A_1'e_i'e_iA_1 = O_p(1)$, $T^{-1}A_1'e_i'e_iA_2 = O_p(1)$ and $T^{-2}A_2'e_i'e_iA_2 = O_p(1)$, so that $T^{-2}A_1'e_i'e_iA_1 = O_p(T^{-1})$ and $T^{-2}A_1'e_i'e_iA_2 = O_p(T^{-1})$. Consequently, under the alternative hypothesis that there are $l < m$ stochastic trends the rank of the matrix $T^{-1}Q_{\hat{e}_i^A\hat{e}_i^A}$ will be $l < m$. Using these elements, we can see that the cross-products involving I(0) stochastic processes in $T^{-1}Q_{\hat{e}_i^A\hat{e}_i^A}$ tend to zero at rate $O_p(T^{-1})$.

Let us now focus on the estimate of the long-run covariance matrix. Note that under both the null and the alternative hypotheses $T^{-1}\hat{\varepsilon}_i'\hat{\varepsilon}_i = O(1)$, with $T^{-1}\hat{\varepsilon}_i'\hat{\varepsilon}_i \xrightarrow{p} \Sigma_{\varepsilon_i}$. Since all roots of the determinant of $\left(I - \hat{\Gamma}_{i,p_i}(L)\right)$ lie outside the unit circle interval, we can define $\hat{\Xi}_{i,\infty}(L) = \left(I - \hat{\Gamma}_{i,p_i}(L)\right)^{-1}$, with $\Xi_{i,\infty}(L) = \left(I + \Xi_{i,1}L + \Xi_{i,2}L^2 + \cdots\right)$ and where the sequence of matrix coefficients $\{\Xi_{i,s}\}_{s=0}^{\infty}$ is absolutely summable. Then, $\Xi_{i,\infty}(1) < \infty$ so that $\hat{\Omega}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A} \xrightarrow{p} \Xi_{i,\infty}'(1)\,\Sigma_{\varepsilon_i}\Xi_{i,\infty}(1) = \Omega_{\Delta e_i^A\Delta e_i^A}$. Therefore, the long-run covariance matrix estimator converges to a positive definite matrix under both the null and the alternative hypotheses – note that this result can be seen as the generalization of the one in Perron and Ng (1998) and Stock (1999).

Finally, note that under the alternative hypothesis that there are $l\,(< m)$ stochastic trends $rank\left(T^{-1}Q_{\hat{e}_i^A\hat{e}_i^A}\hat{\Omega}_{\Delta\hat{e}_i^A\Delta\hat{e}_i^A}^{-1}\right) = l$, where the elements that cause rank deficiency tend to zero at rate $O_p(T^{-1})$. This proves the consistency of the MSB statistic under the alternative hypothesis.

## A.3 Proof of Theorem 3

Let us first focus on the only constant specification. Note that for any real symmetric $(k \times k)$ matrix, say $B_i = \frac{1}{T^2} e_i' e_i$, it is true that $\pi_i^{\min}(B_i) \leq T^{-2} \sum_{t=2}^{T} e_{i,t}^2(l)$ for $l = 1, \ldots, k$. Using this inequality we have

$$\hat{\pi}_{i,1} - \pi_{i,1} \leq T^{-2} \sum_{t=2}^{T} \hat{e}_{i,t}^2(l) - T^{-2} \sum_{t=2}^{T} e_{i,t}^2(l) = O_p\left(C_{NT}^{-1}\right), \tag{13}$$

where $\hat{\pi}_{i,1} = \pi^{\min}\left(\frac{1}{T^2} \hat{e}_i' \hat{e}_i\right)$ and $\pi_{i,1} = \pi^{\min}\left(\frac{1}{T^2} e_i' e_i\right)$, given that $T^{-2} \sum_{t=2}^{T} \hat{e}_{i,t}^2(l) = T^{-2} \sum_{t=2}^{T} e_{i,t}^2(l) + O_p\left(C_{NT}^{-1}\right)$. Therefore,

$$\hat{\pi}_{i,1} \leq \pi_{i,1} + O_p\left(C_{NT}^{-1}\right). \tag{14}$$

The same result is achieved for the time trend specification, but replacing $T^{-2} \sum_{t=2}^{T} e_{i,t}^2(l)$ by $T^{-2} \sum_{t=2}^{T} \left(e_{i,t}(l) - \frac{t-1}{T-1} e_{i,T}(l)\right)^2$.

Bai and Ng (2007) show that averaging the $O_p\left(C_{NT}^{-1}\right)$ component in (13) and (14) across $N$ produces a term that is $O_p\left(C_{NT}^{-2}\right)$ provided that $N/T \to 0$ – see the order of magnitude of part $II$ in equation (6) of Bai and Ng (2007). Consequently, under the null hypothesis that there are $m$ stochastic trends

$$\begin{aligned}
PMSB_j^Z(m) &= \frac{\sqrt{N}(\overline{MSB}_j(m) - E(MSB_j(m)))}{\sqrt{Var(MSB_j(m))}} \\
&= \frac{\sqrt{N}\left[N^{-1} \sum_{i=1}^{N} \left(\pi_{i,1} + O_p\left(C_{NT}^{-1}\right)\right) - E(MSB_j(m))\right]}{\sqrt{Var(MSB_j(m))}} \\
&= \frac{\sqrt{N}\left[N^{-1} \sum_{i=1}^{N} \pi_{i,1} - E(MSB_j(m))\right]}{\sqrt{Var(MSB_j(m))}} + \frac{\sqrt{N}}{\min[N, T]} \\
&= \frac{\sqrt{N}\left[N^{-1} \sum_{i=1}^{N} \pi_{i,1} - E(MSB_j(m))\right]}{\sqrt{Var(MSB_j(m))}} + o_p(1),
\end{aligned}$$

for $j = \{\mu, \tau\}$. Therefore, as $T \to \infty$, $N \to \infty$, with $N/T \to 0$, and assuming finite second moments of the random variables characterized as Brownian motion functionals $\Upsilon \equiv \left(\pi^{\min}\left(\int_0^1 W_i(s) W_i(s)' ds\right), \pi^{\min}\left(\int_0^1 V_i(s) V_i(s)' ds\right)\right)'$, $PMSB_j^Z(m) \Rightarrow N(0, 1)$, $j = \{\mu, \tau\}$, by the Lindberg-Levy Central Limit Theorem. Theorem 3 has been proved.

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Table 1: Critical values for the $MSB_\mu$ and $MSB_\tau$ statistics

| | $MSB_\mu$ statistic | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $T=100$ | | | $T=200$ | | | $T=500$ | | | $T=1000$ | | |
| $k$ | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| 1 | 0.0387 | 0.0629 | 0.0853 | 0.0375 | 0.0610 | 0.0841 | 0.0365 | 0.0593 | 0.0810 | 0.0334 | 0.0562 | 0.0787 |
| 2 | 0.0246 | 0.0318 | 0.0377 | 0.0224 | 0.0298 | 0.0357 | 0.0204 | 0.0280 | 0.0336 | 0.0198 | 0.0274 | 0.0329 |
| 3 | 0.0188 | 0.0227 | 0.0257 | 0.0163 | 0.0203 | 0.0234 | 0.0147 | 0.0188 | 0.0218 | 0.0144 | 0.0186 | 0.0213 |
| 4 | 0.0158 | 0.0186 | 0.0203 | 0.0133 | 0.0160 | 0.0179 | 0.0117 | 0.0147 | 0.0165 | 0.0117 | 0.0142 | 0.0160 |
| 5 | 0.0139 | 0.0158 | 0.0170 | 0.0113 | 0.0133 | 0.0145 | 0.0101 | 0.0120 | 0.0132 | 0.0096 | 0.0115 | 0.0128 |
| 6 | 0.0129 | 0.0141 | 0.0150 | 0.0102 | 0.0116 | 0.0124 | 0.0088 | 0.0102 | 0.0111 | 0.0081 | 0.0097 | 0.0106 |
| | $MSB_\tau$ statistic | | | | | | | | | | | |
| | $T=100$ | | | $T=200$ | | | $T=500$ | | | $T=1000$ | | |
| $k$ | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% | 1% | 5% | 10% |
| 1 | 0.0297 | 0.0420 | 0.0520 | 0.0276 | 0.0397 | 0.0496 | 0.0269 | 0.0389 | 0.0480 | 0.0257 | 0.0377 | 0.0463 |
| 2 | 0.0213 | 0.0268 | 0.0307 | 0.0188 | 0.0243 | 0.0282 | 0.0174 | 0.0228 | 0.0267 | 0.0167 | 0.0221 | 0.0260 |
| 3 | 0.0172 | 0.0206 | 0.0226 | 0.0145 | 0.0178 | 0.0200 | 0.0133 | 0.0166 | 0.0189 | 0.0127 | 0.0161 | 0.0182 |
| 4 | 0.0147 | 0.0172 | 0.0186 | 0.0122 | 0.0146 | 0.0162 | 0.0108 | 0.0131 | 0.0145 | 0.0104 | 0.0128 | 0.0143 |
| 5 | 0.0133 | 0.0150 | 0.0160 | 0.0109 | 0.0125 | 0.0135 | 0.0093 | 0.0109 | 0.0120 | 0.0087 | 0.0106 | 0.0116 |
| 6 | 0.0123 | 0.0135 | 0.0143 | 0.0096 | 0.0109 | 0.0117 | 0.0081 | 0.0095 | 0.0103 | 0.0078 | 0.0091 | 0.0099 |

$k$ denotes the number of stochastic trends under the null hypothesis. Simulations are based on 10,000 replications. The maximum number of lags used to obtain the critical values is 5 for $T=100$, 6 for $T=200$, 8 for $T=500$ and 10 for $T=1000$. The data generating process is given by (1) to (4).

Table 2: Simulated mean and variance of the $MSB_\mu$ and $MSB_\tau$ statistics

| | $MSB_\mu$ statistic | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 100$ | | $T = 200$ | | $T = 500$ | | $T = 1000$ | |
| $k$ | Mean | Variance | Mean | Variance | Mean | Variance | Mean | Variance |
| 1 | 0.50363233 | 0.29973853 | 0.50453666 | 0.32442068 | 0.49960175 | 0.31145873 | 0.49538422 | 0.29598126 |
| 2 | 0.09293090 | 0.00387378 | 0.09048697 | 0.00386357 | 0.08805936 | 0.00377656 | 0.08736662 | 0.00383574 |
| 3 | 0.04653255 | 0.00042435 | 0.04356099 | 0.00041983 | 0.04190975 | 0.00040145 | 0.04115824 | 0.00038487 |
| 4 | 0.03106941 | 0.00010149 | 0.02863743 | 0.00010062 | 0.02708473 | 0.00009719 | 0.02668884 | 0.00009769 |
| 5 | 0.02378946 | 0.00003807 | 0.02115253 | 0.00003491 | 0.01995204 | 0.00003616 | 0.01942875 | 0.00003591 |
| 6 | 0.01969598 | 0.00001723 | 0.01710484 | 0.00001636 | 0.01570673 | 0.00001578 | 0.01518595 | 0.00001599 |

| | $MSB_\tau$ statistic | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $T = 100$ | | $T = 200$ | | $T = 500$ | | $T = 1000$ | |
| $k$ | Mean | Variance | Mean | Variance | Mean | Variance | Mean | Variance |
| 1 | 0.17829789 | 0.02079461 | 0.17637393 | 0.02194526 | 0.16824715 | 0.02122115 | 0.16492404 | 0.02053408 |
| 2 | 0.06172200 | 0.00108132 | 0.05942748 | 0.00109100 | 0.05704523 | 0.00106186 | 0.05664139 | 0.00102857 |
| 3 | 0.03708937 | 0.00019160 | 0.03430545 | 0.00018197 | 0.03281475 | 0.00017868 | 0.03238186 | 0.00018095 |
| 4 | 0.02696152 | 0.00005735 | 0.02454057 | 0.00005863 | 0.02269910 | 0.00005478 | 0.02249813 | 0.00005645 |
| 5 | 0.02160916 | 0.00002413 | 0.01900110 | 0.00002412 | 0.01744459 | 0.00002292 | 0.01706004 | 0.00002290 |
| 6 | 0.01831139 | 0.00001183 | 0.01570302 | 0.00001185 | 0.01426576 | 0.00001166 | 0.01384474 | 0.00001141 |

$k$ denotes the number of stochastic trends under the null hypothesis. Simulations are based on 10,000 replications. The maximum number of lags used to obtain the critical values is 5 for $T = 100$, 6 for $T = 200$, 8 for $T = 500$ and 10 for $T = 1000$. The data generating process is given by (1) to (4).

Table 3: Proportions for $MSB_\mu$ with $a = 0.5$ and $N = 1$ without the presence of common factors

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.967 | 0.033 | 0.000 | 0.000 | 0.110 | 0.890 | 0.000 | 0.000 | 0.002 | 0.088 | 0.910 | 0.000 | 0.000 | 0.004 | 0.060 | 0.936 |
| | | 0.95 | 0.695 | 0.305 | 0.000 | 0.000 | 0.073 | 0.927 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.005 | 0.053 | 0.942 |
| | | 1 | 0.192 | 0.808 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.002 | 0.060 | 0.937 | 0.001 | 0.000 | 0.001 | 0.056 | 0.943 |
| | 1 | 0.90 | 0.848 | 0.152 | 0.000 | 0.000 | 0.141 | 0.859 | 0.000 | 0.000 | 0.004 | 0.132 | 0.864 | 0.000 | 0.000 | 0.006 | 0.081 | 0.913 |
| | | 0.95 | 0.581 | 0.419 | 0.000 | 0.000 | 0.095 | 0.905 | 0.000 | 0.000 | 0.003 | 0.089 | 0.908 | 0.000 | 0.000 | 0.006 | 0.058 | 0.936 |
| | | 1 | 0.158 | 0.842 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.084 | 0.916 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 |
| | 10 | 0.90 | 0.745 | 0.255 | 0.000 | 0.000 | 0.460 | 0.540 | 0.000 | 0.000 | 0.077 | 0.238 | 0.685 | 0.000 | 0.018 | 0.102 | 0.219 | 0.661 |
| | | 0.95 | 0.310 | 0.690 | 0.000 | 0.000 | 0.227 | 0.773 | 0.000 | 0.000 | 0.073 | 0.295 | 0.632 | 0.000 | 0.003 | 0.049 | 0.163 | 0.785 |
| | | 1 | 0.071 | 0.929 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.009 | 0.263 | 0.728 | 0.000 | 0.000 | 0.110 | 0.301 | 0.589 |
| 200 | 0.5 | 0.90 | 0.999 | 0.001 | 0.000 | 0.000 | 0.115 | 0.885 | 0.000 | 0.000 | 0.003 | 0.079 | 0.918 | 0.000 | 0.000 | 0.009 | 0.124 | 0.867 |
| | | 0.95 | 0.967 | 0.033 | 0.000 | 0.000 | 0.130 | 0.870 | 0.000 | 0.000 | 0.007 | 0.110 | 0.883 | 0.000 | 0.000 | 0.003 | 0.062 | 0.935 |
| | | 1 | 0.164 | 0.836 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.001 | 0.069 | 0.930 |
| | 1 | 0.90 | 0.999 | 0.001 | 0.000 | 0.000 | 0.299 | 0.701 | 0.000 | 0.000 | 0.007 | 0.169 | 0.824 | 0.000 | 0.001 | 0.056 | 0.168 | 0.775 |
| | | 0.95 | 0.887 | 0.113 | 0.000 | 0.000 | 0.238 | 0.762 | 0.000 | 0.000 | 0.007 | 0.111 | 0.882 | 0.000 | 0.002 | 0.010 | 0.109 | 0.879 |
| | | 1 | 0.127 | 0.873 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.080 | 0.920 | 0.000 | 0.000 | 0.001 | 0.052 | 0.947 |
| | 10 | 0.90 | 0.999 | 0.001 | 0.000 | 0.000 | 0.822 | 0.178 | 0.000 | 0.000 | 0.262 | 0.386 | 0.352 | 0.000 | 0.108 | 0.324 | 0.181 | 0.387 |
| | | 0.95 | 0.767 | 0.233 | 0.000 | 0.000 | 0.568 | 0.432 | 0.000 | 0.000 | 0.175 | 0.273 | 0.552 | 0.000 | 0.027 | 0.150 | 0.215 | 0.608 |
| | | 1 | 0.067 | 0.933 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.015 | 0.285 | 0.700 | 0.000 | 0.000 | 0.036 | 0.183 | 0.781 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.115 | 0.885 | 0.000 | 0.000 | 0.003 | 0.100 | 0.897 | 0.000 | 0.000 | 0.015 | 0.183 | 0.802 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.231 | 0.769 | 0.000 | 0.000 | 0.003 | 0.125 | 0.872 | 0.000 | 0.000 | 0.014 | 0.169 | 0.817 |
| | | 1 | 0.133 | 0.867 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.001 | 0.059 | 0.940 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.246 | 0.754 | 0.000 | 0.000 | 0.013 | 0.349 | 0.638 | 0.000 | 0.000 | 0.007 | 0.144 | 0.849 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.266 | 0.734 | 0.000 | 0.000 | 0.038 | 0.314 | 0.648 | 0.000 | 0.000 | 0.016 | 0.147 | 0.837 |
| | | 1 | 0.119 | 0.881 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.002 | 0.072 | 0.926 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.900 | 0.100 | 0.000 | 0.000 | 0.618 | 0.353 | 0.029 | 0.000 | 0.321 | 0.603 | 0.061 | 0.015 |
| | | 0.95 | 0.999 | 0.001 | 0.000 | 0.000 | 0.674 | 0.326 | 0.000 | 0.000 | 0.179 | 0.451 | 0.370 | 0.000 | 0.368 | 0.384 | 0.090 | 0.158 |
| | | 1 | 0.083 | 0.917 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.014 | 0.225 | 0.761 | 0.000 | 0.003 | 0.075 | 0.237 | 0.685 |

Note: $MSB_\mu$ is the statistic for the only constant model for $N = 1$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 4: Proportions for $PMSB_{\mu}^{Z}$ with $a = 0.5$ and $N = 20$ without the presence of common factors

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.507 | 0.493 | 0.000 | 0.000 | 0.000 | 0.423 | 0.577 | 0.000 | 0.000 | 0.000 | 0.228 | 0.772 |
| | | 0.95 | 0.991 | 0.009 | 0.000 | 0.000 | 0.414 | 0.586 | 0.000 | 0.000 | 0.000 | 0.248 | 0.752 | 0.000 | 0.000 | 0.000 | 0.137 | 0.863 |
| | | 1 | 0.522 | 0.478 | 0.000 | 0.000 | 0.200 | 0.800 | 0.000 | 0.000 | 0.000 | 0.081 | 0.919 | 0.000 | 0.000 | 0.000 | 0.068 | 0.932 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.812 | 0.188 | 0.000 | 0.000 | 0.006 | 0.689 | 0.305 | 0.000 | 0.007 | 0.002 | 0.507 | 0.484 |
| | | 0.95 | 0.975 | 0.025 | 0.000 | 0.000 | 0.711 | 0.289 | 0.000 | 0.000 | 0.005 | 0.457 | 0.538 | 0.000 | 0.011 | 0.000 | 0.339 | 0.650 |
| | | 1 | 0.540 | 0.460 | 0.000 | 0.000 | 0.286 | 0.714 | 0.000 | 0.000 | 0.000 | 0.181 | 0.819 | 0.000 | 0.000 | 0.000 | 0.191 | 0.809 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.999 | 0.001 | 0.000 | 0.000 | 0.921 | 0.061 | 0.018 | 0.000 | 0.817 | 0.086 | 0.058 | 0.039 |
| | | 0.95 | 0.967 | 0.033 | 0.000 | 0.000 | 0.959 | 0.041 | 0.000 | 0.000 | 0.741 | 0.105 | 0.154 | 0.000 | 0.648 | 0.059 | 0.215 | 0.078 |
| | | 1 | 0.519 | 0.481 | 0.000 | 0.000 | 0.463 | 0.537 | 0.000 | 0.000 | 0.251 | 0.423 | 0.326 | 0.000 | 0.307 | 0.113 | 0.494 | 0.086 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.517 | 0.483 | 0.000 | 0.000 | 0.001 | 0.653 | 0.346 | 0.000 | 0.000 | 0.000 | 0.497 | 0.503 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.519 | 0.481 | 0.000 | 0.000 | 0.000 | 0.376 | 0.624 | 0.000 | 0.000 | 0.000 | 0.235 | 0.765 |
| | | 1 | 0.496 | 0.504 | 0.000 | 0.000 | 0.207 | 0.793 | 0.000 | 0.000 | 0.000 | 0.112 | 0.888 | 0.000 | 0.000 | 0.000 | 0.089 | 0.911 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.905 | 0.095 | 0.000 | 0.000 | 0.005 | 0.938 | 0.057 | 0.000 | 0.009 | 0.001 | 0.774 | 0.216 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.913 | 0.087 | 0.000 | 0.000 | 0.003 | 0.664 | 0.333 | 0.000 | 0.010 | 0.002 | 0.544 | 0.444 |
| | | 1 | 0.503 | 0.497 | 0.000 | 0.000 | 0.311 | 0.689 | 0.000 | 0.000 | 0.001 | 0.174 | 0.825 | 0.000 | 0.003 | 0.000 | 0.217 | 0.780 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.909 | 0.091 | 0.000 | 0.000 | 0.731 | 0.269 | 0.000 | 0.000 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.998 | 0.002 | 0.000 | 0.000 | 0.943 | 0.046 | 0.011 | 0.000 | 0.892 | 0.045 | 0.036 | 0.027 |
| | | 1 | 0.486 | 0.514 | 0.000 | 0.000 | 0.455 | 0.545 | 0.000 | 0.000 | 0.281 | 0.392 | 0.327 | 0.000 | 0.292 | 0.066 | 0.529 | 0.113 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.469 | 0.531 | 0.000 | 0.000 | 0.000 | 0.718 | 0.282 | 0.000 | 0.000 | 0.001 | 0.822 | 0.177 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.605 | 0.395 | 0.000 | 0.000 | 0.001 | 0.611 | 0.388 | 0.000 | 0.000 | 0.000 | 0.620 | 0.380 |
| | | 1 | 0.513 | 0.487 | 0.000 | 0.000 | 0.214 | 0.786 | 0.000 | 0.000 | 0.000 | 0.102 | 0.898 | 0.000 | 0.000 | 0.000 | 0.099 | 0.901 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.834 | 0.166 | 0.000 | 0.000 | 0.003 | 0.987 | 0.010 | 0.000 | 0.024 | 0.032 | 0.935 | 0.009 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.861 | 0.139 | 0.000 | 0.000 | 0.001 | 0.898 | 0.101 | 0.000 | 0.020 | 0.003 | 0.877 | 0.100 |
| | | 1 | 0.510 | 0.490 | 0.000 | 0.000 | 0.359 | 0.641 | 0.000 | 0.000 | 0.002 | 0.186 | 0.812 | 0.000 | 0.000 | 0.000 | 0.184 | 0.816 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.853 | 0.147 | 0.000 | 0.000 | 0.277 | 0.723 | 0.000 | 0.000 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.949 | 0.051 | 0.000 | 0.000 | 0.666 | 0.334 | 0.000 | 0.000 |
| | | 1 | 0.536 | 0.464 | 0.000 | 0.000 | 0.462 | 0.538 | 0.000 | 0.000 | 0.334 | 0.384 | 0.282 | 0.000 | 0.364 | 0.066 | 0.505 | 0.065 |

Note: $PMSB_{\mu}^{Z}$ is the statistic for the only constant model for $N = 20$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 5: Proportions for $MSB_\mu$ with $a = 0.5$ and $N = 1$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.054 | 0.945 | 0.001 | 0.000 | 0.001 | 0.048 | 0.951 | 0.000 | 0.000 | 0.002 | 0.028 | 0.970 |
| | | 0.95 | 0.986 | 0.014 | 0.000 | 0.000 | 0.046 | 0.953 | 0.001 | 0.000 | 0.001 | 0.050 | 0.946 | 0.003 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.752 | 0.248 | 0.000 | 0.000 | 0.040 | 0.899 | 0.061 | 0.000 | 0.000 | 0.038 | 0.934 | 0.028 | 0.000 | 0.000 | 0.046 | 0.954 |
| | 1 | 0.90 | 0.999 | 0.001 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.037 | 0.962 | 0.001 | 0.000 | 0.001 | 0.047 | 0.952 |
| | | 0.95 | 0.991 | 0.009 | 0.000 | 0.000 | 0.050 | 0.948 | 0.002 | 0.000 | 0.000 | 0.044 | 0.954 | 0.002 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 1 | 0.753 | 0.247 | 0.000 | 0.000 | 0.041 | 0.903 | 0.056 | 0.000 | 0.001 | 0.047 | 0.925 | 0.027 | 0.000 | 0.001 | 0.046 | 0.953 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.959 | 0.001 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 0.989 | 0.011 | 0.000 | 0.000 | 0.062 | 0.935 | 0.003 | 0.000 | 0.000 | 0.045 | 0.953 | 0.002 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 1 | 0.762 | 0.238 | 0.000 | 0.000 | 0.030 | 0.927 | 0.043 | 0.000 | 0.001 | 0.039 | 0.932 | 0.028 | 0.000 | 0.000 | 0.034 | 0.966 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.001 | 0.034 | 0.965 | 0.000 | 0.000 | 0.001 | 0.051 | 0.948 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 1 | 0.791 | 0.209 | 0.000 | 0.000 | 0.038 | 0.954 | 0.008 | 0.000 | 0.001 | 0.049 | 0.946 | 0.004 | 0.000 | 0.000 | 0.050 | 0.950 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.002 | 0.043 | 0.955 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.003 | 0.046 | 0.951 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.774 | 0.226 | 0.000 | 0.000 | 0.034 | 0.957 | 0.009 | 0.000 | 0.000 | 0.037 | 0.955 | 0.008 | 0.000 | 0.001 | 0.046 | 0.953 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.001 | 0.048 | 0.951 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.001 | 0.051 | 0.948 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 |
| | | 1 | 0.793 | 0.207 | 0.000 | 0.000 | 0.045 | 0.944 | 0.011 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.001 | 0.047 | 0.952 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.001 | 0.050 | 0.949 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 1 | 0.805 | 0.195 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.001 | 0.050 | 0.949 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 0.826 | 0.174 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.001 | 0.036 | 0.963 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.060 | 0.940 | 0.000 | 0.000 | 0.002 | 0.050 | 0.948 | 0.000 | 0.000 | 0.001 | 0.052 | 0.947 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.001 | 0.040 | 0.959 |
| | | 1 | 0.834 | 0.166 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |

Note: $MSB_\mu$ is the statistic for the only constant model for $N = 1$ and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 6: Proportions for $PMSB_{\mu}^{Z}$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.000 | 0.013 | 0.987 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 1 | 0.956 | 0.044 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.018 | 0.982 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.018 | 0.982 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | | 1 | 0.950 | 0.050 | 0.000 | 0.000 | 0.017 | 0.983 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.014 | 0.986 | 0.000 | 0.000 | 0.000 | 0.015 | 0.985 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.954 | 0.046 | 0.000 | 0.000 | 0.016 | 0.984 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.988 | 0.012 | 0.000 | 0.000 | 0.018 | 0.982 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | | 1 | 0.984 | 0.016 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | | 1 | 0.983 | 0.017 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |

Note: $PMSB_{\mu}^{Z}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

# B Tables that are available upon request (included here only for the referees)

These tables collect the simulation results for those configurations of the Monte Carlo experiment that are mentioned in the paper, but for which we have not included their results in the paper. However, they have been made available upon request to those interested readers.

Table 7: Proportions for $MSB_\mu$ with $a = 0.8$ and $N = 1$ without the presence of common factors

| | | | Panel A: true $r = 0$ | | | | Panel B: true $r = 1$ | | | | Panel C: true $r = 2$ | | | | Panel D: true $r = 3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.820 | 0.085 | 0.095 | 0.000 | 0.128 | 0.695 | 0.177 | 0.000 | 0.003 | 0.052 | 0.408 | 0.537 | 0.000 | 0.004 | 0.060 | 0.936 |
| | | 0.95 | 0.562 | 0.308 | 0.130 | 0.000 | 0.066 | 0.723 | 0.210 | 0.001 | 0.001 | 0.032 | 0.426 | 0.541 | 0.000 | 0.005 | 0.053 | 0.942 |
| | | 1 | 0.154 | 0.669 | 0.177 | 0.000 | 0.044 | 0.750 | 0.206 | 0.000 | 0.000 | 0.030 | 0.421 | 0.549 | 0.000 | 0.001 | 0.056 | 0.943 |
| | 1 | 0.90 | 0.866 | 0.065 | 0.069 | 0.000 | 0.165 | 0.646 | 0.189 | 0.000 | 0.003 | 0.063 | 0.425 | 0.509 | 0.000 | 0.006 | 0.081 | 0.913 |
| | | 0.95 | 0.426 | 0.420 | 0.154 | 0.000 | 0.064 | 0.733 | 0.203 | 0.000 | 0.002 | 0.041 | 0.405 | 0.552 | 0.000 | 0.006 | 0.058 | 0.936 |
| | | 1 | 0.124 | 0.686 | 0.190 | 0.000 | 0.040 | 0.763 | 0.197 | 0.000 | 0.000 | 0.030 | 0.462 | 0.508 | 0.000 | 0.000 | 0.053 | 0.947 |
| | 10 | 0.90 | 0.659 | 0.259 | 0.082 | 0.000 | 0.343 | 0.541 | 0.116 | 0.000 | 0.066 | 0.220 | 0.420 | 0.294 | 0.018 | 0.102 | 0.219 | 0.661 |
| | | 0.95 | 0.299 | 0.615 | 0.086 | 0.000 | 0.203 | 0.668 | 0.129 | 0.000 | 0.065 | 0.216 | 0.482 | 0.237 | 0.003 | 0.049 | 0.163 | 0.785 |
| | | 1 | 0.055 | 0.799 | 0.146 | 0.000 | 0.037 | 0.835 | 0.128 | 0.000 | 0.009 | 0.189 | 0.496 | 0.306 | 0.000 | 0.110 | 0.301 | 0.589 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.140 | 0.860 | 0.000 | 0.000 | 0.000 | 0.100 | 0.896 | 0.004 | 0.000 | 0.009 | 0.124 | 0.867 |
| | | 0.95 | 0.925 | 0.075 | 0.000 | 0.000 | 0.129 | 0.871 | 0.000 | 0.000 | 0.001 | 0.068 | 0.921 | 0.010 | 0.000 | 0.003 | 0.062 | 0.935 |
| | | 1 | 0.196 | 0.804 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.001 | 0.047 | 0.942 | 0.010 | 0.000 | 0.001 | 0.069 | 0.930 |
| | 1 | 0.90 | 0.998 | 0.002 | 0.000 | 0.000 | 0.238 | 0.762 | 0.000 | 0.000 | 0.022 | 0.243 | 0.730 | 0.005 | 0.001 | 0.056 | 0.168 | 0.775 |
| | | 0.95 | 0.915 | 0.085 | 0.000 | 0.000 | 0.145 | 0.855 | 0.000 | 0.000 | 0.001 | 0.094 | 0.898 | 0.007 | 0.002 | 0.010 | 0.109 | 0.879 |
| | | 1 | 0.104 | 0.896 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.002 | 0.091 | 0.898 | 0.009 | 0.000 | 0.001 | 0.052 | 0.947 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.879 | 0.121 | 0.000 | 0.000 | 0.276 | 0.383 | 0.339 | 0.002 | 0.108 | 0.324 | 0.181 | 0.387 |
| | | 0.95 | 0.763 | 0.237 | 0.000 | 0.000 | 0.571 | 0.429 | 0.000 | 0.000 | 0.242 | 0.249 | 0.507 | 0.002 | 0.027 | 0.150 | 0.215 | 0.608 |
| | | 1 | 0.059 | 0.941 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.009 | 0.241 | 0.744 | 0.006 | 0.000 | 0.036 | 0.183 | 0.781 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.843 | 0.000 | 0.000 | 0.003 | 0.182 | 0.815 | 0.000 | 0.000 | 0.015 | 0.183 | 0.802 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.263 | 0.737 | 0.000 | 0.000 | 0.002 | 0.112 | 0.886 | 0.000 | 0.000 | 0.014 | 0.169 | 0.817 |
| | | 1 | 0.137 | 0.863 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.002 | 0.059 | 0.939 | 0.000 | 0.000 | 0.001 | 0.059 | 0.940 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.402 | 0.598 | 0.000 | 0.000 | 0.018 | 0.338 | 0.644 | 0.000 | 0.000 | 0.007 | 0.144 | 0.849 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.274 | 0.726 | 0.000 | 0.000 | 0.018 | 0.316 | 0.666 | 0.000 | 0.000 | 0.016 | 0.147 | 0.837 |
| | | 1 | 0.106 | 0.894 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.001 | 0.081 | 0.918 | 0.000 | 0.000 | 0.002 | 0.072 | 0.926 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.948 | 0.052 | 0.000 | 0.000 | 0.240 | 0.597 | 0.163 | 0.000 | 0.321 | 0.603 | 0.061 | 0.015 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.698 | 0.302 | 0.000 | 0.000 | 0.566 | 0.289 | 0.145 | 0.000 | 0.368 | 0.384 | 0.090 | 0.158 |
| | | 1 | 0.060 | 0.940 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.008 | 0.240 | 0.752 | 0.000 | 0.003 | 0.075 | 0.237 | 0.685 |

Note: $MSB_\mu$ is the statistic for the only constant model for $N = 1$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 8: Proportions for $MSB_\tau$ with $a = 0.5$ and $N = 1$ without the presence of common factors

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.597 | 0.382 | 0.021 | 0.000 | 0.053 | 0.881 | 0.066 | 0.000 | 0.004 | 0.069 | 0.888 | 0.039 | 0.000 | 0.000 | 0.058 | 0.942 |
| | | 0.95 | 0.342 | 0.612 | 0.045 | 0.001 | 0.045 | 0.892 | 0.063 | 0.000 | 0.000 | 0.040 | 0.914 | 0.046 | 0.000 | 0.002 | 0.055 | 0.943 |
| | | 1 | 0.329 | 0.634 | 0.036 | 0.001 | 0.036 | 0.889 | 0.075 | 0.000 | 0.001 | 0.053 | 0.916 | 0.030 | 0.000 | 0.000 | 0.049 | 0.951 |
| | 1 | 0.90 | 0.468 | 0.499 | 0.033 | 0.000 | 0.099 | 0.855 | 0.046 | 0.000 | 0.005 | 0.091 | 0.869 | 0.035 | 0.000 | 0.003 | 0.075 | 0.922 |
| | | 0.95 | 0.196 | 0.756 | 0.047 | 0.001 | 0.078 | 0.867 | 0.055 | 0.000 | 0.009 | 0.087 | 0.863 | 0.041 | 0.000 | 0.003 | 0.063 | 0.934 |
| | | 1 | 0.084 | 0.858 | 0.058 | 0.000 | 0.039 | 0.888 | 0.073 | 0.000 | 0.001 | 0.067 | 0.889 | 0.043 | 0.000 | 0.000 | 0.051 | 0.949 |
| | 10 | 0.90 | 0.328 | 0.645 | 0.027 | 0.000 | 0.227 | 0.753 | 0.020 | 0.000 | 0.125 | 0.301 | 0.557 | 0.017 | 0.032 | 0.126 | 0.262 | 0.580 |
| | | 0.95 | 0.128 | 0.825 | 0.047 | 0.000 | 0.082 | 0.876 | 0.042 | 0.000 | 0.035 | 0.212 | 0.732 | 0.021 | 0.026 | 0.150 | 0.304 | 0.520 |
| | | 1 | 0.047 | 0.908 | 0.045 | 0.000 | 0.040 | 0.921 | 0.039 | 0.000 | 0.023 | 0.322 | 0.645 | 0.010 | 0.003 | 0.096 | 0.266 | 0.635 |
| 200 | 0.5 | 0.90 | 0.943 | 0.057 | 0.000 | 0.000 | 0.192 | 0.806 | 0.002 | 0.000 | 0.006 | 0.127 | 0.867 | 0.000 | 0.000 | 0.005 | 0.083 | 0.912 |
| | | 0.95 | 0.652 | 0.347 | 0.001 | 0.000 | 0.100 | 0.900 | 0.000 | 0.000 | 0.005 | 0.077 | 0.916 | 0.002 | 0.000 | 0.008 | 0.065 | 0.927 |
| | | 1 | 0.146 | 0.851 | 0.003 | 0.000 | 0.048 | 0.951 | 0.001 | 0.000 | 0.004 | 0.048 | 0.948 | 0.000 | 0.000 | 0.002 | 0.043 | 0.955 |
| | 1 | 0.90 | 0.918 | 0.082 | 0.000 | 0.000 | 0.187 | 0.813 | 0.000 | 0.000 | 0.016 | 0.140 | 0.842 | 0.002 | 0.000 | 0.005 | 0.086 | 0.909 |
| | | 0.95 | 0.711 | 0.288 | 0.001 | 0.000 | 0.086 | 0.911 | 0.003 | 0.000 | 0.016 | 0.140 | 0.844 | 0.000 | 0.001 | 0.016 | 0.117 | 0.866 |
| | | 1 | 0.162 | 0.837 | 0.001 | 0.000 | 0.047 | 0.952 | 0.001 | 0.000 | 0.001 | 0.088 | 0.911 | 0.000 | 0.000 | 0.005 | 0.071 | 0.924 |
| | 10 | 0.90 | 0.802 | 0.197 | 0.001 | 0.000 | 0.668 | 0.332 | 0.000 | 0.000 | 0.208 | 0.256 | 0.536 | 0.000 | 0.166 | 0.202 | 0.210 | 0.422 |
| | | 0.95 | 0.345 | 0.653 | 0.002 | 0.000 | 0.288 | 0.712 | 0.000 | 0.000 | 0.113 | 0.250 | 0.636 | 0.001 | 0.079 | 0.206 | 0.323 | 0.392 |
| | | 1 | 0.044 | 0.956 | 0.000 | 0.000 | 0.041 | 0.957 | 0.002 | 0.000 | 0.027 | 0.317 | 0.655 | 0.001 | 0.005 | 0.077 | 0.275 | 0.643 |
| 500 | 0.5 | 0.90 | 0.997 | 0.003 | 0.000 | 0.000 | 0.367 | 0.633 | 0.000 | 0.000 | 0.004 | 0.138 | 0.858 | 0.000 | 0.000 | 0.014 | 0.164 | 0.822 |
| | | 0.95 | 0.974 | 0.026 | 0.000 | 0.000 | 0.114 | 0.886 | 0.000 | 0.000 | 0.012 | 0.168 | 0.820 | 0.000 | 0.000 | 0.005 | 0.086 | 0.909 |
| | | 1 | 0.276 | 0.724 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.001 | 0.059 | 0.940 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 |
| | 1 | 0.90 | 0.998 | 0.002 | 0.000 | 0.000 | 0.464 | 0.536 | 0.000 | 0.000 | 0.011 | 0.224 | 0.765 | 0.000 | 0.000 | 0.032 | 0.215 | 0.753 |
| | | 0.95 | 0.963 | 0.037 | 0.000 | 0.000 | 0.190 | 0.810 | 0.000 | 0.000 | 0.007 | 0.117 | 0.876 | 0.000 | 0.004 | 0.050 | 0.187 | 0.759 |
| | | 1 | 0.106 | 0.894 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.006 | 0.065 | 0.929 | 0.000 | 0.000 | 0.005 | 0.070 | 0.925 |
| | 10 | 0.90 | 0.995 | 0.005 | 0.000 | 0.000 | 0.960 | 0.040 | 0.000 | 0.000 | 0.815 | 0.126 | 0.059 | 0.000 | 0.290 | 0.445 | 0.152 | 0.113 |
| | | 0.95 | 0.915 | 0.085 | 0.000 | 0.000 | 0.691 | 0.309 | 0.000 | 0.000 | 0.340 | 0.268 | 0.392 | 0.000 | 0.084 | 0.207 | 0.227 | 0.482 |
| | | 1 | 0.065 | 0.935 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.021 | 0.219 | 0.760 | 0.000 | 0.016 | 0.135 | 0.314 | 0.535 |

Note: $MSB_\tau$ is the statistic for the linear time trend model for $N = 1$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 9: Proportions for $MSB_\tau$ with $a = 0.8$ and $N = 1$ without the presence of common factors

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.397 | 0.289 | 0.303 | 0.011 | 0.047 | 0.412 | 0.502 | 0.039 | 0.001 | 0.013 | 0.239 | 0.747 | 0.000 | 0.000 | 0.058 | 0.942 |
| | | 0.95 | 0.187 | 0.364 | 0.429 | 0.020 | 0.027 | 0.407 | 0.529 | 0.037 | 0.001 | 0.023 | 0.273 | 0.703 | 0.000 | 0.002 | 0.055 | 0.943 |
| | | 1 | 0.107 | 0.428 | 0.433 | 0.032 | 0.028 | 0.406 | 0.535 | 0.031 | 0.000 | 0.014 | 0.275 | 0.711 | 0.000 | 0.000 | 0.049 | 0.951 |
| | 1 | 0.90 | 0.312 | 0.297 | 0.375 | 0.016 | 0.067 | 0.408 | 0.502 | 0.023 | 0.004 | 0.045 | 0.288 | 0.663 | 0.000 | 0.003 | 0.075 | 0.922 |
| | | 0.95 | 0.219 | 0.380 | 0.377 | 0.024 | 0.042 | 0.418 | 0.507 | 0.033 | 0.004 | 0.032 | 0.276 | 0.688 | 0.000 | 0.003 | 0.063 | 0.934 |
| | | 1 | 0.087 | 0.435 | 0.459 | 0.019 | 0.019 | 0.468 | 0.485 | 0.028 | 0.001 | 0.013 | 0.272 | 0.714 | 0.000 | 0.000 | 0.051 | 0.949 |
| | 10 | 0.90 | 0.232 | 0.520 | 0.242 | 0.006 | 0.204 | 0.526 | 0.264 | 0.006 | 0.050 | 0.147 | 0.356 | 0.447 | 0.032 | 0.126 | 0.262 | 0.580 |
| | | 0.95 | 0.080 | 0.566 | 0.344 | 0.010 | 0.073 | 0.668 | 0.251 | 0.008 | 0.024 | 0.230 | 0.394 | 0.352 | 0.026 | 0.150 | 0.304 | 0.520 |
| | | 1 | 0.050 | 0.588 | 0.349 | 0.013 | 0.021 | 0.617 | 0.349 | 0.013 | 0.016 | 0.267 | 0.409 | 0.308 | 0.003 | 0.096 | 0.266 | 0.635 |
| 200 | 0.5 | 0.90 | 0.923 | 0.075 | 0.002 | 0.000 | 0.225 | 0.716 | 0.059 | 0.000 | 0.006 | 0.101 | 0.767 | 0.126 | 0.000 | 0.005 | 0.083 | 0.912 |
| | | 0.95 | 0.618 | 0.355 | 0.027 | 0.000 | 0.091 | 0.850 | 0.059 | 0.000 | 0.002 | 0.041 | 0.809 | 0.148 | 0.000 | 0.008 | 0.065 | 0.927 |
| | | 1 | 0.245 | 0.715 | 0.040 | 0.000 | 0.041 | 0.884 | 0.075 | 0.000 | 0.002 | 0.040 | 0.814 | 0.144 | 0.000 | 0.002 | 0.043 | 0.955 |
| | 1 | 0.90 | 0.857 | 0.137 | 0.006 | 0.000 | 0.343 | 0.612 | 0.045 | 0.000 | 0.037 | 0.197 | 0.682 | 0.084 | 0.000 | 0.005 | 0.086 | 0.909 |
| | | 0.95 | 0.446 | 0.516 | 0.038 | 0.000 | 0.171 | 0.788 | 0.041 | 0.000 | 0.004 | 0.090 | 0.781 | 0.125 | 0.001 | 0.016 | 0.117 | 0.866 |
| | | 1 | 0.133 | 0.821 | 0.046 | 0.000 | 0.040 | 0.906 | 0.054 | 0.000 | 0.001 | 0.044 | 0.820 | 0.135 | 0.000 | 0.005 | 0.071 | 0.924 |
| | 10 | 0.90 | 0.780 | 0.208 | 0.012 | 0.000 | 0.643 | 0.343 | 0.014 | 0.000 | 0.321 | 0.233 | 0.411 | 0.035 | 0.166 | 0.202 | 0.210 | 0.422 |
| | | 0.95 | 0.304 | 0.656 | 0.040 | 0.000 | 0.245 | 0.718 | 0.037 | 0.000 | 0.104 | 0.239 | 0.614 | 0.043 | 0.079 | 0.206 | 0.323 | 0.392 |
| | | 1 | 0.053 | 0.919 | 0.028 | 0.000 | 0.041 | 0.924 | 0.035 | 0.000 | 0.018 | 0.199 | 0.691 | 0.092 | 0.005 | 0.077 | 0.275 | 0.643 |
| 500 | 0.5 | 0.90 | 0.995 | 0.005 | 0.000 | 0.000 | 0.278 | 0.722 | 0.000 | 0.000 | 0.006 | 0.180 | 0.813 | 0.001 | 0.000 | 0.014 | 0.164 | 0.822 |
| | | 0.95 | 0.977 | 0.023 | 0.000 | 0.000 | 0.174 | 0.825 | 0.001 | 0.000 | 0.007 | 0.151 | 0.841 | 0.001 | 0.000 | 0.005 | 0.086 | 0.909 |
| | | 1 | 0.188 | 0.812 | 0.000 | 0.000 | 0.052 | 0.947 | 0.001 | 0.000 | 0.001 | 0.040 | 0.957 | 0.002 | 0.000 | 0.000 | 0.044 | 0.956 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.489 | 0.511 | 0.000 | 0.000 | 0.051 | 0.296 | 0.653 | 0.000 | 0.000 | 0.032 | 0.215 | 0.753 |
| | | 0.95 | 0.967 | 0.033 | 0.000 | 0.000 | 0.395 | 0.605 | 0.000 | 0.000 | 0.007 | 0.143 | 0.848 | 0.002 | 0.004 | 0.050 | 0.187 | 0.759 |
| | | 1 | 0.132 | 0.868 | 0.000 | 0.000 | 0.045 | 0.954 | 0.001 | 0.000 | 0.001 | 0.089 | 0.910 | 0.000 | 0.000 | 0.005 | 0.070 | 0.925 |
| | 10 | 0.90 | 0.998 | 0.002 | 0.000 | 0.000 | 0.842 | 0.158 | 0.000 | 0.000 | 0.688 | 0.225 | 0.087 | 0.000 | 0.290 | 0.445 | 0.152 | 0.113 |
| | | 0.95 | 0.917 | 0.083 | 0.000 | 0.000 | 0.723 | 0.277 | 0.000 | 0.000 | 0.607 | 0.156 | 0.237 | 0.000 | 0.084 | 0.207 | 0.227 | 0.482 |
| | | 1 | 0.068 | 0.932 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.011 | 0.166 | 0.823 | 0.000 | 0.016 | 0.135 | 0.314 | 0.535 |

Note: $MSB_\tau$ is the statistic for the linear time trend model for $N = 1$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 10: Proportions for $PMSB_{\mu}^{Z}$ with $a = 0.8$ and $N = 20$ without the presence of common factors

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.395 | 0.605 | 0.000 | 0.000 | 0.110 | 0.139 | 0.751 | 0.000 | 0.000 | 0.000 | 0.228 | 0.772 |
| | | 0.95 | 0.982 | 0.018 | 0.000 | 0.000 | 0.357 | 0.643 | 0.000 | 0.000 | 0.104 | 0.090 | 0.806 | 0.000 | 0.000 | 0.000 | 0.137 | 0.863 |
| | | 1 | 0.511 | 0.489 | 0.000 | 0.000 | 0.191 | 0.809 | 0.000 | 0.000 | 0.019 | 0.045 | 0.936 | 0.000 | 0.000 | 0.000 | 0.068 | 0.932 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.781 | 0.219 | 0.000 | 0.000 | 0.363 | 0.247 | 0.390 | 0.000 | 0.007 | 0.002 | 0.507 | 0.484 |
| | | 0.95 | 0.969 | 0.031 | 0.000 | 0.000 | 0.713 | 0.287 | 0.000 | 0.000 | 0.212 | 0.156 | 0.632 | 0.000 | 0.011 | 0.000 | 0.339 | 0.650 |
| | | 1 | 0.530 | 0.470 | 0.000 | 0.000 | 0.358 | 0.642 | 0.000 | 0.000 | 0.070 | 0.070 | 0.860 | 0.000 | 0.000 | 0.000 | 0.191 | 0.809 |
| | 10 | 0.90 | 0.999 | 0.001 | 0.000 | 0.000 | 0.999 | 0.001 | 0.000 | 0.000 | 0.955 | 0.021 | 0.024 | 0.000 | 0.817 | 0.086 | 0.058 | 0.039 |
| | | 0.95 | 0.973 | 0.027 | 0.000 | 0.000 | 0.954 | 0.046 | 0.000 | 0.000 | 0.805 | 0.049 | 0.146 | 0.000 | 0.648 | 0.059 | 0.215 | 0.078 |
| | | 1 | 0.480 | 0.520 | 0.000 | 0.000 | 0.444 | 0.556 | 0.000 | 0.000 | 0.387 | 0.318 | 0.295 | 0.000 | 0.307 | 0.113 | 0.494 | 0.086 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.502 | 0.498 | 0.000 | 0.000 | 0.000 | 0.570 | 0.430 | 0.000 | 0.000 | 0.000 | 0.497 | 0.503 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.455 | 0.545 | 0.000 | 0.000 | 0.001 | 0.390 | 0.609 | 0.000 | 0.000 | 0.000 | 0.235 | 0.765 |
| | | 1 | 0.532 | 0.468 | 0.000 | 0.000 | 0.181 | 0.819 | 0.000 | 0.000 | 0.000 | 0.108 | 0.892 | 0.000 | 0.000 | 0.000 | 0.089 | 0.911 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.814 | 0.186 | 0.000 | 0.000 | 0.003 | 0.875 | 0.122 | 0.000 | 0.009 | 0.001 | 0.774 | 0.216 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.861 | 0.139 | 0.000 | 0.000 | 0.004 | 0.680 | 0.316 | 0.000 | 0.010 | 0.002 | 0.544 | 0.444 |
| | | 1 | 0.493 | 0.507 | 0.000 | 0.000 | 0.312 | 0.688 | 0.000 | 0.000 | 0.001 | 0.209 | 0.790 | 0.000 | 0.003 | 0.000 | 0.217 | 0.780 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.917 | 0.083 | 0.000 | 0.000 | 0.731 | 0.269 | 0.000 | 0.000 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.925 | 0.060 | 0.015 | 0.000 | 0.892 | 0.045 | 0.036 | 0.027 |
| | | 1 | 0.497 | 0.503 | 0.000 | 0.000 | 0.431 | 0.569 | 0.000 | 0.000 | 0.286 | 0.390 | 0.324 | 0.000 | 0.292 | 0.066 | 0.529 | 0.113 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.491 | 0.509 | 0.000 | 0.000 | 0.000 | 0.769 | 0.231 | 0.000 | 0.000 | 0.001 | 0.822 | 0.177 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.412 | 0.588 | 0.000 | 0.000 | 0.000 | 0.745 | 0.255 | 0.000 | 0.000 | 0.000 | 0.620 | 0.380 |
| | | 1 | 0.536 | 0.464 | 0.000 | 0.000 | 0.222 | 0.778 | 0.000 | 0.000 | 0.000 | 0.122 | 0.878 | 0.000 | 0.000 | 0.000 | 0.099 | 0.901 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.883 | 0.117 | 0.000 | 0.000 | 0.003 | 0.984 | 0.013 | 0.000 | 0.024 | 0.032 | 0.935 | 0.009 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.951 | 0.049 | 0.000 | 0.000 | 0.003 | 0.901 | 0.096 | 0.000 | 0.020 | 0.003 | 0.877 | 0.100 |
| | | 1 | 0.475 | 0.525 | 0.000 | 0.000 | 0.352 | 0.648 | 0.000 | 0.000 | 0.002 | 0.218 | 0.780 | 0.000 | 0.000 | 0.000 | 0.184 | 0.816 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.845 | 0.155 | 0.000 | 0.000 | 0.277 | 0.723 | 0.000 | 0.000 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.948 | 0.052 | 0.000 | 0.000 | 0.666 | 0.334 | 0.000 | 0.000 |
| | | 1 | 0.491 | 0.509 | 0.000 | 0.000 | 0.464 | 0.536 | 0.000 | 0.000 | 0.303 | 0.379 | 0.318 | 0.000 | 0.364 | 0.066 | 0.505 | 0.065 |

Note: $PMSB_{\mu}^{Z}$ is the statistic for the only constant model for $N = 20$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 11: Proportions for $PMSB_{\tau}^{Z}$ with $a = 0.5$ and $N = 20$ without the presence of common factors

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.961 | 0.039 | 0.000 | 0.000 | 0.485 | 0.515 | 0.000 | 0.000 | 0.001 | 0.266 | 0.733 | 0.000 | 0.000 | 0.000 | 0.207 | 0.793 |
| | | 0.95 | 0.871 | 0.129 | 0.000 | 0.000 | 0.312 | 0.688 | 0.000 | 0.000 | 0.001 | 0.134 | 0.865 | 0.000 | 0.000 | 0.000 | 0.121 | 0.879 |
| | | 1 | 0.631 | 0.369 | 0.000 | 0.000 | 0.177 | 0.823 | 0.000 | 0.000 | 0.000 | 0.067 | 0.933 | 0.000 | 0.000 | 0.000 | 0.090 | 0.910 |
| | 1 | 0.90 | 0.963 | 0.037 | 0.000 | 0.000 | 0.744 | 0.256 | 0.000 | 0.000 | 0.099 | 0.488 | 0.413 | 0.000 | 0.020 | 0.002 | 0.424 | 0.554 |
| | | 0.95 | 0.851 | 0.149 | 0.000 | 0.000 | 0.527 | 0.473 | 0.000 | 0.000 | 0.020 | 0.223 | 0.757 | 0.000 | 0.003 | 0.000 | 0.238 | 0.759 |
| | | 1 | 0.591 | 0.409 | 0.000 | 0.000 | 0.306 | 0.694 | 0.000 | 0.000 | 0.005 | 0.148 | 0.847 | 0.000 | 0.002 | 0.000 | 0.188 | 0.810 |
| | 10 | 0.90 | 0.937 | 0.063 | 0.000 | 0.000 | 0.929 | 0.071 | 0.000 | 0.000 | 0.863 | 0.079 | 0.058 | 0.000 | 0.797 | 0.021 | 0.146 | 0.036 |
| | | 0.95 | 0.739 | 0.261 | 0.000 | 0.000 | 0.722 | 0.278 | 0.000 | 0.000 | 0.603 | 0.202 | 0.195 | 0.000 | 0.492 | 0.047 | 0.394 | 0.067 |
| | | 1 | 0.516 | 0.484 | 0.000 | 0.000 | 0.438 | 0.562 | 0.000 | 0.000 | 0.340 | 0.310 | 0.350 | 0.000 | 0.331 | 0.049 | 0.545 | 0.075 |
| 200 | 0.5 | 0.90 | 0.999 | 0.001 | 0.000 | 0.000 | 0.804 | 0.196 | 0.000 | 0.000 | 0.003 | 0.615 | 0.382 | 0.000 | 0.000 | 0.000 | 0.429 | 0.571 |
| | | 0.95 | 0.975 | 0.025 | 0.000 | 0.000 | 0.636 | 0.364 | 0.000 | 0.000 | 0.000 | 0.294 | 0.706 | 0.000 | 0.000 | 0.000 | 0.199 | 0.801 |
| | | 1 | 0.647 | 0.353 | 0.000 | 0.000 | 0.242 | 0.758 | 0.000 | 0.000 | 0.000 | 0.100 | 0.900 | 0.000 | 0.000 | 0.000 | 0.068 | 0.932 |
| | 1 | 0.90 | 0.997 | 0.003 | 0.000 | 0.000 | 0.922 | 0.078 | 0.000 | 0.000 | 0.056 | 0.820 | 0.124 | 0.000 | 0.129 | 0.008 | 0.679 | 0.184 |
| | | 0.95 | 0.964 | 0.036 | 0.000 | 0.000 | 0.821 | 0.179 | 0.000 | 0.000 | 0.019 | 0.535 | 0.446 | 0.000 | 0.019 | 0.000 | 0.426 | 0.555 |
| | | 1 | 0.625 | 0.375 | 0.000 | 0.000 | 0.337 | 0.663 | 0.000 | 0.000 | 0.016 | 0.207 | 0.777 | 0.000 | 0.001 | 0.000 | 0.135 | 0.864 |
| | 10 | 0.90 | 0.997 | 0.003 | 0.000 | 0.000 | 0.993 | 0.007 | 0.000 | 0.000 | 0.995 | 0.005 | 0.000 | 0.000 | 0.988 | 0.005 | 0.007 | 0.000 |
| | | 0.95 | 0.960 | 0.040 | 0.000 | 0.000 | 0.925 | 0.075 | 0.000 | 0.000 | 0.884 | 0.067 | 0.049 | 0.000 | 0.867 | 0.009 | 0.113 | 0.011 |
| | | 1 | 0.492 | 0.508 | 0.000 | 0.000 | 0.468 | 0.532 | 0.000 | 0.000 | 0.378 | 0.348 | 0.274 | 0.000 | 0.316 | 0.038 | 0.574 | 0.072 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.830 | 0.170 | 0.000 | 0.000 | 0.001 | 0.775 | 0.224 | 0.000 | 0.002 | 0.000 | 0.773 | 0.225 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.718 | 0.282 | 0.000 | 0.000 | 0.001 | 0.585 | 0.414 | 0.000 | 0.000 | 0.000 | 0.537 | 0.463 |
| | | 1 | 0.566 | 0.434 | 0.000 | 0.000 | 0.215 | 0.785 | 0.000 | 0.000 | 0.000 | 0.115 | 0.885 | 0.000 | 0.000 | 0.000 | 0.113 | 0.887 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.991 | 0.009 | 0.000 | 0.000 | 0.081 | 0.917 | 0.002 | 0.000 | 0.089 | 0.043 | 0.862 | 0.006 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.986 | 0.014 | 0.000 | 0.000 | 0.028 | 0.895 | 0.077 | 0.000 | 0.044 | 0.003 | 0.819 | 0.134 |
| | | 1 | 0.557 | 0.443 | 0.000 | 0.000 | 0.330 | 0.670 | 0.000 | 0.000 | 0.003 | 0.182 | 0.815 | 0.000 | 0.001 | 0.000 | 0.212 | 0.787 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.994 | 0.006 | 0.000 | 0.000 | 0.934 | 0.066 | 0.000 | 0.000 |
| | | 0.95 | 0.999 | 0.001 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.999 | 0.001 | 0.000 | 0.000 | 0.982 | 0.014 | 0.004 | 0.000 |
| | | 1 | 0.481 | 0.519 | 0.000 | 0.000 | 0.468 | 0.532 | 0.000 | 0.000 | 0.368 | 0.340 | 0.292 | 0.000 | 0.345 | 0.032 | 0.545 | 0.078 |

Note: $PMSB_{\tau}^{Z}$ is the statistic for the linear time trend model for $N = 20$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 12: Proportions for $PMSB_{\tau}^{Z}$ with $a = 0.8$ and $N = 20$ without the presence of common factors

| | | | Panel A: true $r = 0$ | | | | Panel B: true $r = 1$ | | | | Panel C: true $r = 2$ | | | | Panel D: true $r = 3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.955 | 0.045 | 0.000 | 0.000 | 0.449 | 0.551 | 0.000 | 0.000 | 0.149 | 0.007 | 0.844 | 0.000 | 0.000 | 0.000 | 0.207 | 0.793 |
| | | 0.95 | 0.869 | 0.131 | 0.000 | 0.000 | 0.332 | 0.668 | 0.000 | 0.000 | 0.061 | 0.002 | 0.937 | 0.000 | 0.000 | 0.000 | 0.121 | 0.879 |
| | | 1 | 0.669 | 0.331 | 0.000 | 0.000 | 0.191 | 0.809 | 0.000 | 0.000 | 0.049 | 0.003 | 0.948 | 0.000 | 0.000 | 0.000 | 0.090 | 0.910 |
| | 1 | 0.90 | 0.953 | 0.047 | 0.000 | 0.000 | 0.805 | 0.195 | 0.000 | 0.000 | 0.397 | 0.015 | 0.588 | 0.000 | 0.020 | 0.002 | 0.424 | 0.554 |
| | | 0.95 | 0.847 | 0.153 | 0.000 | 0.000 | 0.547 | 0.453 | 0.000 | 0.000 | 0.171 | 0.010 | 0.819 | 0.000 | 0.003 | 0.000 | 0.238 | 0.759 |
| | | 1 | 0.611 | 0.389 | 0.000 | 0.000 | 0.285 | 0.715 | 0.000 | 0.000 | 0.080 | 0.004 | 0.916 | 0.000 | 0.002 | 0.000 | 0.188 | 0.810 |
| | 10 | 0.90 | 0.950 | 0.050 | 0.000 | 0.000 | 0.925 | 0.075 | 0.000 | 0.000 | 0.889 | 0.040 | 0.071 | 0.000 | 0.797 | 0.021 | 0.146 | 0.036 |
| | | 0.95 | 0.724 | 0.276 | 0.000 | 0.000 | 0.691 | 0.309 | 0.000 | 0.000 | 0.666 | 0.089 | 0.245 | 0.000 | 0.492 | 0.047 | 0.394 | 0.067 |
| | | 1 | 0.495 | 0.505 | 0.000 | 0.000 | 0.455 | 0.545 | 0.000 | 0.000 | 0.435 | 0.144 | 0.421 | 0.000 | 0.331 | 0.049 | 0.545 | 0.075 |
| 200 | 0.5 | 0.90 | 0.998 | 0.002 | 0.000 | 0.000 | 0.690 | 0.310 | 0.000 | 0.000 | 0.008 | 0.442 | 0.550 | 0.000 | 0.000 | 0.000 | 0.429 | 0.571 |
| | | 0.95 | 0.975 | 0.025 | 0.000 | 0.000 | 0.518 | 0.482 | 0.000 | 0.000 | 0.009 | 0.240 | 0.751 | 0.000 | 0.000 | 0.000 | 0.199 | 0.801 |
| | | 1 | 0.644 | 0.356 | 0.000 | 0.000 | 0.217 | 0.783 | 0.000 | 0.000 | 0.000 | 0.091 | 0.909 | 0.000 | 0.000 | 0.000 | 0.068 | 0.932 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.934 | 0.066 | 0.000 | 0.000 | 0.266 | 0.649 | 0.085 | 0.000 | 0.129 | 0.008 | 0.679 | 0.184 |
| | | 0.95 | 0.971 | 0.029 | 0.000 | 0.000 | 0.800 | 0.200 | 0.000 | 0.000 | 0.136 | 0.416 | 0.448 | 0.000 | 0.019 | 0.000 | 0.426 | 0.555 |
| | | 1 | 0.622 | 0.378 | 0.000 | 0.000 | 0.330 | 0.670 | 0.000 | 0.000 | 0.033 | 0.153 | 0.814 | 0.000 | 0.001 | 0.000 | 0.135 | 0.864 |
| | 10 | 0.90 | 0.995 | 0.005 | 0.000 | 0.000 | 0.998 | 0.002 | 0.000 | 0.000 | 0.989 | 0.010 | 0.001 | 0.000 | 0.988 | 0.005 | 0.007 | 0.000 |
| | | 0.95 | 0.954 | 0.046 | 0.000 | 0.000 | 0.924 | 0.076 | 0.000 | 0.000 | 0.901 | 0.050 | 0.049 | 0.000 | 0.867 | 0.009 | 0.113 | 0.011 |
| | | 1 | 0.517 | 0.483 | 0.000 | 0.000 | 0.440 | 0.560 | 0.000 | 0.000 | 0.419 | 0.303 | 0.278 | 0.000 | 0.316 | 0.038 | 0.574 | 0.072 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.781 | 0.219 | 0.000 | 0.000 | 0.002 | 0.884 | 0.114 | 0.000 | 0.002 | 0.000 | 0.773 | 0.225 |
| | | 0.95 | 0.999 | 0.001 | 0.000 | 0.000 | 0.821 | 0.179 | 0.000 | 0.000 | 0.002 | 0.702 | 0.296 | 0.000 | 0.000 | 0.000 | 0.537 | 0.463 |
| | | 1 | 0.591 | 0.409 | 0.000 | 0.000 | 0.184 | 0.816 | 0.000 | 0.000 | 0.000 | 0.134 | 0.866 | 0.000 | 0.000 | 0.000 | 0.113 | 0.887 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.989 | 0.011 | 0.000 | 0.000 | 0.032 | 0.953 | 0.015 | 0.000 | 0.089 | 0.043 | 0.862 | 0.006 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.986 | 0.014 | 0.000 | 0.000 | 0.067 | 0.885 | 0.048 | 0.000 | 0.044 | 0.003 | 0.819 | 0.134 |
| | | 1 | 0.597 | 0.403 | 0.000 | 0.000 | 0.357 | 0.643 | 0.000 | 0.000 | 0.000 | 0.177 | 0.823 | 0.000 | 0.001 | 0.000 | 0.212 | 0.787 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.992 | 0.008 | 0.000 | 0.000 | 0.934 | 0.066 | 0.000 | 0.000 |
| | | 0.95 | 0.999 | 0.001 | 0.000 | 0.000 | 0.999 | 0.001 | 0.000 | 0.000 | 0.998 | 0.002 | 0.000 | 0.000 | 0.982 | 0.014 | 0.004 | 0.000 |
| | | 1 | 0.494 | 0.506 | 0.000 | 0.000 | 0.462 | 0.538 | 0.000 | 0.000 | 0.320 | 0.300 | 0.380 | 0.000 | 0.345 | 0.032 | 0.545 | 0.078 |

Note: $PMSB_{\tau}^{Z}$ is the statistic for the linear time trend model for $N = 20$ when we fail to account for the presence of common factors and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 13: Proportions for $MSB_\mu$ with $a = 0.8$ and $N = 1$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.992 | 0.008 | 0.000 | 0.000 | 0.028 | 0.746 | 0.226 | 0.000 | 0.000 | 0.021 | 0.391 | 0.588 | 0.000 | 0.002 | 0.028 | 0.970 |
| | | 0.95 | 0.986 | 0.014 | 0.000 | 0.000 | 0.039 | 0.726 | 0.230 | 0.005 | 0.001 | 0.023 | 0.411 | 0.565 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.803 | 0.180 | 0.017 | 0.000 | 0.031 | 0.648 | 0.297 | 0.024 | 0.000 | 0.015 | 0.367 | 0.618 | 0.000 | 0.000 | 0.046 | 0.954 |
| | 1 | 0.90 | 0.994 | 0.006 | 0.000 | 0.000 | 0.034 | 0.727 | 0.238 | 0.001 | 0.000 | 0.018 | 0.421 | 0.561 | 0.000 | 0.001 | 0.047 | 0.952 |
| | | 0.95 | 0.981 | 0.018 | 0.001 | 0.000 | 0.042 | 0.719 | 0.234 | 0.005 | 0.000 | 0.016 | 0.392 | 0.592 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 1 | 0.820 | 0.165 | 0.015 | 0.000 | 0.028 | 0.656 | 0.299 | 0.017 | 0.000 | 0.027 | 0.371 | 0.602 | 0.000 | 0.001 | 0.046 | 0.953 |
| | 10 | 0.90 | 0.991 | 0.009 | 0.000 | 0.000 | 0.027 | 0.748 | 0.225 | 0.000 | 0.001 | 0.023 | 0.391 | 0.585 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 0.982 | 0.017 | 0.001 | 0.000 | 0.038 | 0.730 | 0.231 | 0.001 | 0.000 | 0.015 | 0.382 | 0.603 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 1 | 0.812 | 0.175 | 0.013 | 0.000 | 0.021 | 0.669 | 0.290 | 0.020 | 0.000 | 0.012 | 0.354 | 0.634 | 0.000 | 0.000 | 0.034 | 0.966 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.057 | 0.928 | 0.015 | 0.000 | 0.001 | 0.051 | 0.948 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.048 | 0.936 | 0.016 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 1 | 0.879 | 0.121 | 0.000 | 0.000 | 0.059 | 0.924 | 0.017 | 0.000 | 0.000 | 0.047 | 0.923 | 0.030 | 0.000 | 0.000 | 0.050 | 0.950 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.001 | 0.056 | 0.934 | 0.009 | 0.000 | 0.002 | 0.043 | 0.955 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.045 | 0.946 | 0.009 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.892 | 0.108 | 0.000 | 0.000 | 0.044 | 0.927 | 0.029 | 0.000 | 0.000 | 0.045 | 0.922 | 0.033 | 0.000 | 0.001 | 0.046 | 0.953 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.066 | 0.934 | 0.000 | 0.000 | 0.001 | 0.046 | 0.941 | 0.012 | 0.000 | 0.001 | 0.048 | 0.951 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 | 0.000 | 0.000 | 0.000 | 0.050 | 0.940 | 0.010 | 0.000 | 0.000 | 0.049 | 0.951 |
| | | 1 | 0.904 | 0.096 | 0.000 | 0.000 | 0.040 | 0.940 | 0.020 | 0.000 | 0.000 | 0.039 | 0.936 | 0.025 | 0.000 | 0.000 | 0.048 | 0.952 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.069 | 0.931 | 0.000 | 0.000 | 0.003 | 0.040 | 0.957 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 1 | 0.909 | 0.091 | 0.000 | 0.000 | 0.048 | 0.950 | 0.002 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.001 | 0.043 | 0.956 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 0.890 | 0.110 | 0.000 | 0.000 | 0.046 | 0.952 | 0.002 | 0.000 | 0.001 | 0.048 | 0.951 | 0.000 | 0.000 | 0.001 | 0.036 | 0.963 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.001 | 0.052 | 0.947 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.001 | 0.046 | 0.953 | 0.000 | 0.000 | 0.001 | 0.040 | 0.959 |
| | | 1 | 0.927 | 0.073 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.002 | 0.051 | 0.947 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |

Note: $MSB_\mu$ is the statistic for the only constant model for $N = 1$ and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 14: Proportions for $MSB_\tau$ with $a = 0.5$ and $N = 1$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.834 | 0.166 | 0.000 | 0.000 | 0.040 | 0.889 | 0.071 | 0.000 | 0.000 | 0.063 | 0.881 | 0.056 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 0.95 | 0.813 | 0.186 | 0.001 | 0.000 | 0.038 | 0.871 | 0.091 | 0.000 | 0.001 | 0.048 | 0.896 | 0.055 | 0.000 | 0.001 | 0.051 | 0.948 |
| | | 1 | 0.792 | 0.206 | 0.002 | 0.000 | 0.027 | 0.861 | 0.110 | 0.002 | 0.000 | 0.049 | 0.867 | 0.084 | 0.000 | 0.002 | 0.040 | 0.958 |
| | 1 | 0.90 | 0.840 | 0.160 | 0.000 | 0.000 | 0.043 | 0.881 | 0.076 | 0.000 | 0.003 | 0.045 | 0.901 | 0.051 | 0.000 | 0.001 | 0.049 | 0.950 |
| | | 0.95 | 0.822 | 0.178 | 0.000 | 0.000 | 0.038 | 0.877 | 0.084 | 0.001 | 0.001 | 0.032 | 0.923 | 0.044 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 1 | 0.774 | 0.225 | 0.001 | 0.000 | 0.036 | 0.859 | 0.102 | 0.003 | 0.000 | 0.047 | 0.878 | 0.075 | 0.000 | 0.001 | 0.048 | 0.951 |
| | 10 | 0.90 | 0.860 | 0.140 | 0.000 | 0.000 | 0.039 | 0.888 | 0.073 | 0.000 | 0.000 | 0.052 | 0.902 | 0.046 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 0.95 | 0.817 | 0.182 | 0.001 | 0.000 | 0.052 | 0.859 | 0.088 | 0.001 | 0.001 | 0.057 | 0.891 | 0.051 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 0.791 | 0.207 | 0.002 | 0.000 | 0.050 | 0.845 | 0.102 | 0.003 | 0.001 | 0.052 | 0.870 | 0.077 | 0.000 | 0.002 | 0.042 | 0.956 |
| 200 | 0.5 | 0.90 | 0.917 | 0.083 | 0.000 | 0.000 | 0.060 | 0.933 | 0.007 | 0.000 | 0.001 | 0.053 | 0.944 | 0.002 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 0.95 | 0.889 | 0.111 | 0.000 | 0.000 | 0.047 | 0.951 | 0.002 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 1 | 0.862 | 0.138 | 0.000 | 0.000 | 0.046 | 0.945 | 0.009 | 0.000 | 0.000 | 0.051 | 0.945 | 0.004 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 0.888 | 0.112 | 0.000 | 0.000 | 0.044 | 0.954 | 0.002 | 0.000 | 0.000 | 0.043 | 0.956 | 0.001 | 0.000 | 0.001 | 0.040 | 0.959 |
| | | 0.95 | 0.893 | 0.107 | 0.000 | 0.000 | 0.053 | 0.946 | 0.001 | 0.000 | 0.001 | 0.048 | 0.948 | 0.003 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 1 | 0.844 | 0.156 | 0.000 | 0.000 | 0.053 | 0.941 | 0.006 | 0.000 | 0.001 | 0.053 | 0.941 | 0.005 | 0.000 | 0.000 | 0.042 | 0.958 |
| | 10 | 0.90 | 0.888 | 0.112 | 0.000 | 0.000 | 0.045 | 0.953 | 0.002 | 0.000 | 0.000 | 0.048 | 0.951 | 0.001 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 0.95 | 0.893 | 0.107 | 0.000 | 0.000 | 0.051 | 0.948 | 0.001 | 0.000 | 0.001 | 0.051 | 0.947 | 0.001 | 0.000 | 0.001 | 0.039 | 0.960 |
| | | 1 | 0.831 | 0.169 | 0.000 | 0.000 | 0.048 | 0.945 | 0.007 | 0.000 | 0.001 | 0.053 | 0.944 | 0.002 | 0.000 | 0.001 | 0.046 | 0.953 |
| 500 | 0.5 | 0.90 | 0.890 | 0.110 | 0.000 | 0.000 | 0.068 | 0.932 | 0.000 | 0.000 | 0.001 | 0.040 | 0.959 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 |
| | | 0.95 | 0.891 | 0.109 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 1 | 0.819 | 0.181 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.001 | 0.036 | 0.963 |
| | 1 | 0.90 | 0.917 | 0.083 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 |
| | | 0.95 | 0.901 | 0.099 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.003 | 0.051 | 0.946 |
| | | 1 | 0.845 | 0.155 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.001 | 0.050 | 0.949 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | 10 | 0.90 | 0.904 | 0.096 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 0.888 | 0.112 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.001 | 0.054 | 0.945 |
| | | 1 | 0.835 | 0.165 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |

Note: $MSB_\tau$ is the statistic for the linear time trend model for $N = 1$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 15: Proportions for $MSB_\tau$ with $a = 0.8$ and $N = 1$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 0.627 | 0.343 | 0.030 | 0.000 | 0.013 | 0.385 | 0.568 | 0.034 | 0.000 | 0.004 | 0.255 | 0.741 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 0.95 | 0.610 | 0.349 | 0.039 | 0.002 | 0.014 | 0.383 | 0.565 | 0.038 | 0.000 | 0.008 | 0.265 | 0.727 | 0.000 | 0.001 | 0.051 | 0.948 |
| | | 1 | 0.551 | 0.390 | 0.057 | 0.002 | 0.019 | 0.376 | 0.544 | 0.061 | 0.000 | 0.011 | 0.245 | 0.744 | 0.000 | 0.002 | 0.040 | 0.958 |
| | 1 | 0.90 | 0.625 | 0.345 | 0.029 | 0.001 | 0.015 | 0.416 | 0.533 | 0.036 | 0.000 | 0.010 | 0.249 | 0.741 | 0.000 | 0.001 | 0.049 | 0.950 |
| | | 0.95 | 0.611 | 0.357 | 0.031 | 0.001 | 0.020 | 0.399 | 0.540 | 0.041 | 0.000 | 0.007 | 0.247 | 0.746 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 1 | 0.558 | 0.390 | 0.052 | 0.000 | 0.013 | 0.386 | 0.549 | 0.052 | 0.001 | 0.008 | 0.246 | 0.745 | 0.000 | 0.001 | 0.048 | 0.951 |
| | 10 | 0.90 | 0.609 | 0.355 | 0.036 | 0.000 | 0.011 | 0.392 | 0.555 | 0.042 | 0.000 | 0.021 | 0.220 | 0.759 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 0.95 | 0.617 | 0.343 | 0.040 | 0.000 | 0.015 | 0.373 | 0.566 | 0.046 | 0.000 | 0.008 | 0.266 | 0.726 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 0.569 | 0.375 | 0.054 | 0.002 | 0.012 | 0.388 | 0.544 | 0.056 | 0.000 | 0.011 | 0.258 | 0.731 | 0.000 | 0.002 | 0.042 | 0.956 |
| 200 | 0.5 | 0.90 | 0.944 | 0.056 | 0.000 | 0.000 | 0.044 | 0.889 | 0.067 | 0.000 | 0.000 | 0.036 | 0.811 | 0.153 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 0.95 | 0.935 | 0.065 | 0.000 | 0.000 | 0.033 | 0.871 | 0.096 | 0.000 | 0.000 | 0.039 | 0.813 | 0.148 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 1 | 0.914 | 0.086 | 0.000 | 0.000 | 0.036 | 0.863 | 0.101 | 0.000 | 0.001 | 0.039 | 0.794 | 0.166 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 0.949 | 0.051 | 0.000 | 0.000 | 0.054 | 0.886 | 0.060 | 0.000 | 0.001 | 0.040 | 0.812 | 0.147 | 0.000 | 0.001 | 0.040 | 0.959 |
| | | 0.95 | 0.939 | 0.061 | 0.000 | 0.000 | 0.041 | 0.887 | 0.072 | 0.000 | 0.000 | 0.034 | 0.817 | 0.149 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 1 | 0.923 | 0.077 | 0.000 | 0.000 | 0.043 | 0.867 | 0.090 | 0.000 | 0.002 | 0.033 | 0.793 | 0.172 | 0.000 | 0.000 | 0.042 | 0.958 |
| | 10 | 0.90 | 0.944 | 0.056 | 0.000 | 0.000 | 0.042 | 0.884 | 0.074 | 0.000 | 0.000 | 0.035 | 0.818 | 0.147 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 0.95 | 0.944 | 0.056 | 0.000 | 0.000 | 0.037 | 0.883 | 0.080 | 0.000 | 0.000 | 0.026 | 0.823 | 0.151 | 0.000 | 0.001 | 0.039 | 0.960 |
| | | 1 | 0.904 | 0.096 | 0.000 | 0.000 | 0.048 | 0.861 | 0.091 | 0.000 | 0.001 | 0.034 | 0.799 | 0.166 | 0.000 | 0.001 | 0.046 | 0.953 |
| 500 | 0.5 | 0.90 | 0.982 | 0.018 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.001 | 0.046 | 0.951 | 0.002 | 0.000 | 0.000 | 0.056 | 0.944 |
| | | 0.95 | 0.988 | 0.012 | 0.000 | 0.000 | 0.044 | 0.955 | 0.001 | 0.000 | 0.001 | 0.048 | 0.951 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 1 | 0.955 | 0.045 | 0.000 | 0.000 | 0.044 | 0.955 | 0.001 | 0.000 | 0.002 | 0.040 | 0.957 | 0.001 | 0.000 | 0.001 | 0.036 | 0.963 |
| | 1 | 0.90 | 0.977 | 0.023 | 0.000 | 0.000 | 0.056 | 0.943 | 0.001 | 0.000 | 0.001 | 0.046 | 0.953 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 |
| | | 0.95 | 0.982 | 0.018 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.003 | 0.051 | 0.946 |
| | | 1 | 0.958 | 0.042 | 0.000 | 0.000 | 0.043 | 0.956 | 0.001 | 0.000 | 0.001 | 0.040 | 0.956 | 0.003 | 0.000 | 0.000 | 0.052 | 0.948 |
| | 10 | 0.90 | 0.978 | 0.022 | 0.000 | 0.000 | 0.053 | 0.946 | 0.001 | 0.000 | 0.001 | 0.042 | 0.957 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 0.985 | 0.015 | 0.000 | 0.000 | 0.051 | 0.948 | 0.001 | 0.000 | 0.001 | 0.053 | 0.944 | 0.002 | 0.000 | 0.001 | 0.054 | 0.945 |
| | | 1 | 0.962 | 0.038 | 0.000 | 0.000 | 0.059 | 0.940 | 0.001 | 0.000 | 0.001 | 0.034 | 0.963 | 0.002 | 0.000 | 0.000 | 0.048 | 0.952 |

Note: $MSB_\tau$ is the statistic for the linear time trend model for $N = 1$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 16: Proportions for $PMSB_{\mu}^{Z}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.003 | 0.007 | 0.990 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.004 | 0.008 | 0.988 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 1 | 0.968 | 0.032 | 0.000 | 0.000 | 0.017 | 0.983 | 0.000 | 0.000 | 0.003 | 0.004 | 0.993 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.004 | 0.007 | 0.989 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.014 | 0.986 | 0.000 | 0.000 | 0.009 | 0.010 | 0.981 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | | 1 | 0.986 | 0.014 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.006 | 0.006 | 0.988 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.006 | 0.005 | 0.989 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.008 | 0.014 | 0.978 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.984 | 0.016 | 0.000 | 0.000 | 0.018 | 0.982 | 0.000 | 0.000 | 0.005 | 0.007 | 0.988 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.014 | 0.986 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.996 | 0.004 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | | 1 | 0.997 | 0.003 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.017 | 0.983 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |

Note: $PMSB_{\mu}^{Z}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 17: Proportions for $PMSB_\tau^Z$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 | 0.000 | 0.000 | 0.000 | 0.012 | 0.988 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | | 0.95 | 0.998 | 0.002 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 |
| | | 1 | 0.971 | 0.029 | 0.000 | 0.000 | 0.021 | 0.979 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |
| | | 0.95 | 0.996 | 0.004 | 0.000 | 0.000 | 0.017 | 0.983 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 |
| | | 1 | 0.975 | 0.025 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.017 | 0.983 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.966 | 0.034 | 0.000 | 0.000 | 0.015 | 0.985 | 0.000 | 0.000 | 0.000 | 0.015 | 0.985 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.016 | 0.984 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |

Note: $PMSB_\tau^Z$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag(0.5)$.

Table 18: Proportions for $PMSB_{\tau}^{Z}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.002 | 0.000 | 0.998 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.006 | 0.000 | 0.994 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 |
| | | 1 | 0.993 | 0.007 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.003 | 0.001 | 0.996 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.005 | 0.000 | 0.995 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.006 | 0.000 | 0.994 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 |
| | | 1 | 0.993 | 0.007 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.002 | 0.001 | 0.997 | 0.000 | 0.000 | 0.000 | 0.017 | 0.983 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 | 0.000 | 0.000 | 0.001 | 0.002 | 0.997 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.006 | 0.001 | 0.993 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.987 | 0.013 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.001 | 0.000 | 0.999 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.018 | 0.982 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.018 | 0.982 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.016 | 0.984 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.019 | 0.981 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |

Note: $PMSB_{\tau}^{Z}$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 19: Proportions for $PMSB_{\mu}^{F}$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r = 0$ | | | | Panel B: true $r = 1$ | | | | Panel C: true $r = 2$ | | | | Panel D: true $r = 3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.976 | 0.024 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 1 | 0.977 | 0.023 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.001 | 0.039 | 0.960 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.976 | 0.024 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 0.989 | 0.011 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.988 | 0.012 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.070 | 0.930 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |

Note: $PMSB_{\mu}^{F}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 20: Proportions for $PMSB_{\mu}^{F}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.013 | 0.010 | 0.977 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.007 | 0.009 | 0.984 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.986 | 0.014 | 0.000 | 0.000 | 0.083 | 0.917 | 0.000 | 0.000 | 0.004 | 0.008 | 0.988 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.006 | 0.012 | 0.982 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.012 | 0.011 | 0.977 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 1 | 0.993 | 0.007 | 0.000 | 0.000 | 0.117 | 0.883 | 0.000 | 0.000 | 0.008 | 0.007 | 0.985 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.008 | 0.013 | 0.979 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.006 | 0.016 | 0.978 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.990 | 0.010 | 0.000 | 0.000 | 0.105 | 0.895 | 0.000 | 0.000 | 0.009 | 0.010 | 0.981 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.001 | 0.037 | 0.962 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.001 | 0.039 | 0.960 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |

Note: $PMSB_{\mu}^{F}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 21: Proportions for $PMSB_\tau^F$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.001 | 0.046 | 0.953 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.003 | 0.040 | 0.957 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.008 | 0.048 | 0.944 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.004 | 0.049 | 0.947 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.002 | 0.047 | 0.951 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.003 | 0.048 | 0.949 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.003 | 0.053 | 0.944 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.004 | 0.052 | 0.944 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.001 | 0.049 | 0.950 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.072 | 0.928 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.060 | 0.940 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |

Note: $PMSB_\tau^F$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 22: Proportions for $PMSB_{\tau}^{F}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.171 | 0.829 | 0.000 | 0.000 | 0.000 | 0.000 | 0.999 | 0.001 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.810 | 0.000 | 0.000 | 0.003 | 0.003 | 0.994 | 0.000 | 0.000 | 0.000 | 0.019 | 0.981 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.263 | 0.737 | 0.000 | 0.000 | 0.002 | 0.003 | 0.992 | 0.003 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.157 | 0.843 | 0.000 | 0.000 | 0.001 | 0.003 | 0.996 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.814 | 0.000 | 0.000 | 0.000 | 0.002 | 0.996 | 0.002 | 0.000 | 0.000 | 0.032 | 0.968 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.247 | 0.753 | 0.000 | 0.000 | 0.002 | 0.002 | 0.994 | 0.002 | 0.000 | 0.000 | 0.023 | 0.977 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.136 | 0.864 | 0.000 | 0.000 | 0.001 | 0.001 | 0.995 | 0.003 | 0.000 | 0.000 | 0.019 | 0.981 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.802 | 0.000 | 0.000 | 0.001 | 0.001 | 0.996 | 0.002 | 0.000 | 0.000 | 0.023 | 0.977 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.242 | 0.758 | 0.000 | 0.000 | 0.000 | 0.002 | 0.996 | 0.002 | 0.000 | 0.000 | 0.034 | 0.966 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.010 | 0.026 | 0.964 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.008 | 0.020 | 0.972 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 | 0.000 | 0.000 | 0.009 | 0.019 | 0.972 | 0.000 | 0.000 | 0.000 | 0.022 | 0.978 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.008 | 0.022 | 0.970 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.003 | 0.021 | 0.976 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.005 | 0.016 | 0.979 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.009 | 0.025 | 0.966 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.004 | 0.021 | 0.975 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.009 | 0.022 | 0.969 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |

Note: $PMSB_{\tau}^{F}$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 23: Proportions for $PMSB_{\mu}^{C1}$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.976 | 0.024 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.001 | 0.041 | 0.958 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 0.977 | 0.023 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.001 | 0.043 | 0.956 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.977 | 0.023 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.001 | 0.044 | 0.955 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.068 | 0.932 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.074 | 0.926 | 0.000 | 0.000 | 0.000 | 0.067 | 0.933 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | | 1 | 0.989 | 0.011 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.065 | 0.935 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.068 | 0.932 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 | 0.000 | 0.000 | 0.000 | 0.071 | 0.929 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 0.988 | 0.012 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.078 | 0.922 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.060 | 0.940 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.070 | 0.930 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.079 | 0.921 | 0.000 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.068 | 0.932 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 1 | 0.995 | 0.005 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 |

Note: $PMSB_{\mu}^{C1}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 24: Proportions for $PMSB_{\mu}^{C1}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.021 | 0.000 | 0.979 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.015 | 0.000 | 0.985 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.987 | 0.013 | 0.000 | 0.000 | 0.100 | 0.900 | 0.000 | 0.000 | 0.009 | 0.000 | 0.991 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.013 | 0.000 | 0.987 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.018 | 0.000 | 0.982 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 1 | 0.993 | 0.007 | 0.000 | 0.000 | 0.134 | 0.866 | 0.000 | 0.000 | 0.010 | 0.000 | 0.990 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.019 | 0.000 | 0.981 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.011 | 0.000 | 0.989 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.990 | 0.010 | 0.000 | 0.000 | 0.120 | 0.880 | 0.000 | 0.000 | 0.017 | 0.000 | 0.983 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.065 | 0.935 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.001 | 0.049 | 0.950 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.060 | 0.940 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.001 | 0.049 | 0.950 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.060 | 0.940 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.002 | 0.046 | 0.952 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.067 | 0.933 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.074 | 0.926 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.065 | 0.935 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 |

Note: $PMSB_{\mu}^{C1}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 25: Proportions for $PMSB_{\tau}^{C1}$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.003 | 0.053 | 0.944 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.004 | 0.048 | 0.948 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.014 | 0.054 | 0.932 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.006 | 0.052 | 0.942 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.002 | 0.056 | 0.942 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.005 | 0.058 | 0.937 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.004 | 0.062 | 0.934 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.013 | 0.051 | 0.936 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.006 | 0.049 | 0.945 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.079 | 0.921 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.065 | 0.935 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.066 | 0.934 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.065 | 0.935 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.069 | 0.931 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 |

Note: $PMSB_{\tau}^{C1}$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 26: Proportions for $PMSB_{\tau}^{C1}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.184 | 0.816 | 0.000 | 0.000 | 0.001 | 0.000 | 0.998 | 0.001 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.213 | 0.787 | 0.000 | 0.000 | 0.005 | 0.000 | 0.995 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.286 | 0.714 | 0.000 | 0.000 | 0.004 | 0.000 | 0.994 | 0.002 | 0.000 | 0.000 | 0.039 | 0.961 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.178 | 0.822 | 0.000 | 0.000 | 0.003 | 0.000 | 0.997 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.213 | 0.787 | 0.000 | 0.000 | 0.001 | 0.000 | 0.997 | 0.002 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.272 | 0.728 | 0.000 | 0.000 | 0.003 | 0.000 | 0.996 | 0.001 | 0.000 | 0.000 | 0.027 | 0.973 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.148 | 0.852 | 0.000 | 0.000 | 0.002 | 0.000 | 0.996 | 0.002 | 0.000 | 0.000 | 0.034 | 0.966 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.220 | 0.780 | 0.000 | 0.000 | 0.001 | 0.000 | 0.997 | 0.002 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.267 | 0.733 | 0.000 | 0.000 | 0.000 | 0.000 | 0.999 | 0.001 | 0.000 | 0.000 | 0.042 | 0.958 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.021 | 0.015 | 0.964 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.019 | 0.012 | 0.969 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.021 | 0.011 | 0.968 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.011 | 0.020 | 0.969 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.008 | 0.014 | 0.978 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.012 | 0.009 | 0.979 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.020 | 0.015 | 0.965 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.011 | 0.015 | 0.974 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.016 | 0.014 | 0.970 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.070 | 0.930 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.062 | 0.938 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.068 | 0.932 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.065 | 0.935 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 |

Note: $PMSB_{\tau}^{C1}$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.

Table 27: Proportions for $PMSB_{\mu}^{C2}$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 1 | 0.969 | 0.031 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 0.973 | 0.027 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 0.974 | 0.026 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 0.991 | 0.009 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.993 | 0.007 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 0.987 | 0.013 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 0.997 | 0.003 | 0.000 | 0.000 | 0.059 | 0.941 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.066 | 0.934 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 1 | 0.996 | 0.004 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |

Note: $PMSB_{\mu}^{C2}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on $1,000$ replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 28: Proportions for $PMSB_{\mu}^{C2}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r = 0$ | | | | Panel B: true $r = 1$ | | | | Panel C: true $r = 2$ | | | | Panel D: true $r = 3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r = 0$ | $r = 1$ | $r = 2$ | $r = 3$ | $r = 0$ | $r = 1$ | $r = 2$ | $r = 3$ | $r = 0$ | $r = 1$ | $r = 2$ | $r = 3$ | $r = 0$ | $r = 1$ | $r = 2$ | $r = 3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.018 | 0.001 | 0.981 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.013 | 0.000 | 0.987 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 1 | 0.987 | 0.013 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.009 | 0.000 | 0.991 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.016 | 0.000 | 0.984 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.019 | 0.000 | 0.981 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 0.992 | 0.008 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.015 | 0.000 | 0.985 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.017 | 0.000 | 0.983 | 0.000 | 0.000 | 0.000 | 0.020 | 0.980 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.020 | 0.000 | 0.980 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |
| | | 1 | 0.988 | 0.012 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.015 | 0.000 | 0.985 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.054 | 0.946 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.063 | 0.937 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | | 1 | 0.996 | 0.004 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.058 | 0.942 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.055 | 0.945 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.061 | 0.939 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.064 | 0.936 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.053 | 0.947 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 |

Note: $PMSB_{\mu}^{C2}$ is the statistic for the only constant model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag(0.8)$.

Table 29: Proportions for $PMSB_{\tau}^{C2}$ with $a = 0.5$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma^2_{F,j}$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.024 | 0.976 | 0.000 | 0.000 | 0.001 | 0.043 | 0.956 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.025 | 0.975 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.998 | 0.002 | 0.000 | 0.000 | 0.018 | 0.982 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.066 | 0.934 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.018 | 0.982 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.051 | 0.949 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.049 | 0.951 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |

Note: $PMSB_{\tau}^{C2}$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.5, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.5, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.5)$.

Table 30: Proportions for $PMSB_{\tau}^{C2}$ with $a = 0.8$ and $N = 20$

| | | | Panel A: true $r=0$ | | | | Panel B: true $r=1$ | | | | Panel C: true $r=2$ | | | | Panel D: true $r=3$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $T$ | $\sigma_{F,j}^2$ | $\rho_j$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ | $r=0$ | $r=1$ | $r=2$ | $r=3$ |
| 100 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.071 | 0.929 | 0.000 | 0.000 | 0.000 | 0.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.082 | 0.918 | 0.000 | 0.000 | 0.006 | 0.000 | 0.994 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.103 | 0.897 | 0.000 | 0.000 | 0.005 | 0.000 | 0.995 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.004 | 0.000 | 0.996 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.077 | 0.923 | 0.000 | 0.000 | 0.002 | 0.000 | 0.998 | 0.000 | 0.000 | 0.000 | 0.026 | 0.974 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.102 | 0.898 | 0.000 | 0.000 | 0.003 | 0.000 | 0.997 | 0.000 | 0.000 | 0.000 | 0.021 | 0.979 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.057 | 0.943 | 0.000 | 0.000 | 0.002 | 0.000 | 0.998 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.093 | 0.907 | 0.000 | 0.000 | 0.002 | 0.000 | 0.998 | 0.000 | 0.000 | 0.000 | 0.030 | 0.970 |
| | | 1 | 0.999 | 0.001 | 0.000 | 0.000 | 0.108 | 0.892 | 0.000 | 0.000 | 0.002 | 0.000 | 0.998 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| 200 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.002 | 0.023 | 0.975 | 0.000 | 0.000 | 0.000 | 0.028 | 0.972 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.001 | 0.022 | 0.977 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.005 | 0.019 | 0.976 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.005 | 0.030 | 0.965 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.001 | 0.020 | 0.979 | 0.000 | 0.000 | 0.000 | 0.043 | 0.957 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.029 | 0.971 | 0.000 | 0.000 | 0.003 | 0.017 | 0.980 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.003 | 0.026 | 0.971 | 0.000 | 0.000 | 0.000 | 0.018 | 0.982 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.038 | 0.962 | 0.000 | 0.000 | 0.001 | 0.030 | 0.969 | 0.000 | 0.000 | 0.000 | 0.027 | 0.973 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.023 | 0.977 | 0.000 | 0.000 | 0.003 | 0.026 | 0.971 | 0.000 | 0.000 | 0.000 | 0.033 | 0.967 |
| 500 | 0.5 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.056 | 0.944 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.039 | 0.961 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.048 | 0.952 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 |
| | 1 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.035 | 0.965 | 0.000 | 0.000 | 0.000 | 0.044 | 0.956 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.036 | 0.964 | 0.000 | 0.000 | 0.000 | 0.042 | 0.958 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.032 | 0.968 | 0.000 | 0.000 | 0.000 | 0.045 | 0.955 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 |
| | 10 | 0.90 | 1.000 | 0.000 | 0.000 | 0.000 | 0.050 | 0.950 | 0.000 | 0.000 | 0.000 | 0.034 | 0.966 | 0.000 | 0.000 | 0.000 | 0.052 | 0.948 |
| | | 0.95 | 1.000 | 0.000 | 0.000 | 0.000 | 0.041 | 0.959 | 0.000 | 0.000 | 0.000 | 0.046 | 0.954 | 0.000 | 0.000 | 0.000 | 0.040 | 0.960 |
| | | 1 | 1.000 | 0.000 | 0.000 | 0.000 | 0.037 | 0.963 | 0.000 | 0.000 | 0.000 | 0.047 | 0.953 | 0.000 | 0.000 | 0.000 | 0.031 | 0.969 |

Note: $PMSB_{\tau}^{C2}$ is the statistic for the linear time trend model for $N = 20$ and $r$ denotes the number of stochastic trends. Simulations are based on 1,000 replications using the 5% critical values from Table 1 and the mean and the variance from Table 2. For *Panel A*, $\psi_i = diag\,(0.8, 0.5, 0.5)$, for *Panel B*, $\psi_i = diag\,(0.8, 0.5)$ and for *Panel C*, $\psi_i = diag\,(0.8)$.