Spurious Long-Range Dependence in DAX Volatilities? A panel examination*

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Abstract

The long-memory properties of the daily volatility of the German stock market is analyzed in a panel of 30 DAX stocks. Since structural breaks can be mistaken as long memory, the paper tests the null of short memory while allowing for an unconditional shift in the mean at unknown time. For each individual stock, we compute the lag-augmented LM test for fractional integration (Demetrescu et al., 2008, Econometric Theory 24) for all possible break points and take as test statistic the smallest of the resulting values. The test statistic obtained this way is shown to have a standard normal asymptotic distribution. In small samples, the test is biased towards antipersistence, so a small-sample correction is provided. The panel test statistic is built by combining the significance of individual test statistics with a correction for cross-unit dependence; this way, breaks at different time can be considered without increasing the computational complexity of the overall procedure. Examining daily absolute returns in 2004 and 2005, the volatilities of DAX stock returns are found to have a shift in the mean *and* long memory.

Key words

Volatility of stock returns; Persistence; Unknown break point; Small-sample behavior; Cross-dependent panel

JEL classification

C22, C23, G10, G15

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1 Motivation

Series of absolute, or squared, stock returns (taken as a proxy for the volatility) often exhibit sample autocorrelations that decay slowly, possibly at a rate at which their population counterparts would not be summable. To appropriately model such long memory, the financial econometrics literature either extends specific volatility models to allow for it, as is the case with the FIGARCH model proposed by Baillie, Bollerslev and Mikkelsen (1996), or applies techniques first developed in the time series analysis literature – the most popular (linear) model used to capture long memory being integration of order d, I(d), with d not necessarily integer (Granger and Joyeux, 1980).

For the German stock market, Sibbertsen (2004) examines the absolute returns of 6 DAX stocks and of the DAX index with daily observations over almost 30 years. Using nonparametric methods for estimating the long-memory parameter d, he finds the high persistence in stock volatility to be caused by long memory. For the same period, Krämer, Sibbertsen and Kleiber (2002) find GARCH parameters that sum to almost 1 for 5 DAX stocks and the DAX index; this is an indication of long-range dependence, see Baillie, Bollerslev and Mikkelsen (1996).

But long memory may be spurious in the sense that other models than fractional integration can appear to have long memory as well. With similar stock market data, Lobato and Savin (1998) or Granger (1998) argue that an alternative cause of long-range dependence is the presence of structural breaks. Structural breaks are plausible as such data is examined in samples covering large periods of time. Granger and Hyung (2004) show how simple shifts in the mean of a short memory process[1] are easily confused with long memory; see also Davidson and Sibbertsen (2005) for a formal discussion. For the GARCH family of models, Hildebrand (2005) shows that structural changes lead to estimates of GARCH parameters that sum to almost 1, thus

[1] There is some dispute as to what rigorously defines short memory; see Davidson (2007) for a recent discussion.

mimicking long memory.

The issue of whether a particular volatility series exhibits “genuine” long memory, or the apparent persistence is caused by “just” structural breaks is one of relevance for practitioners, especially when forecasting. In the first case, forecasts can be improved by resorting to additional lagged values of the series, while, in the second case, a reliable identification of the break(s) is needed.

This paper hence examines in more detail the question of whether the long memory of of the German stock market volatility can be explained by an unconditional shift in the mean. Given that, since the work of Mikosch and Stărică (2004) at the latest, it has become widely recognized that stock volatilities do exhibit mean shifts, we test absolute returns from 2004 to 2005 for long memory *after* allowing for a break in the mean. We choose to work with a relatively short time span to minimize the possibility of multiple breaks in the examined period. To compensate the loss of information, we conduct a panel-level analysis with all 30 DAX stocks. In order to allow for possibly different individual break points across the panel, we combine unit-level evidence from single-unit test statistics into a panel test; the employed procedure, proposed by Hartung (1999), allows for very general types of cross-sectional dependence.

To obtain unit-level evidence, we draw on the existing time-series literature on discriminating long memory from short memory with breaks. Shimotsu (2006), or Ohanissian, Russell and Tsay (2008), use the invariance of nonparametric long memory parameter estimators w.r.t. subsampling, and aggregation, respectively, to distinguish between true and spurious long memory. Hsu (2005) modifies local Whittle estimation of the long memory parameter d to allow for changes in the mean. But frequency-domain approaches require the specification of a bandwidth parameter, and the respective tests react extremely sensitively to the choice of bandwidth; see Hassler and Olivares (2009) for recent experimental evidence. We therefore set on the time-domain approach to check for fractional integration after allowing for a break in mean.

In the time domain, Dolado, Gonzalo and Mayoral (2005) modify the fractional Dickey-Fuller test of Dolado, Gonzalo and Mayoral (2002) in the spirit of Zivot and Andrews (1992) to account for breaks at unknown time. But the fractional Dickey-Fuller test has two disadvantages. First, it is designed to test the *reverse* null hypothesis, I(d) with breaks, against a lower order of fractional integration. Thus, rejecting the null is not necessarily an indication of short memory with breaks (not to mention the thorny issue of how to choose an order of fractional integration to act as a null hypothesis). Second, it requires nonparametric estimation of d under the alternative; even without breaks, this is not an easy undertaking, see Hassler and Olivares (2009) again. The augmented Lagrange Multiplier [ALM] test due to Demetrescu, Kuzin and Hassler (2008) does not have these shortcomings, and is easily robustified against breaks at unknown time by taking the minimum statistic over all possible break points. Moreover, we are able to provide the asymptotic analysis of the case with lag augmentation.[2] Finally, the ALM test is robust to breaks in the variance; see Demetrescu (2009).

The remainder of the paper is structured as follows. To keep it self-contained, Section 2 begins by reviewing the ALM test. We then analyze the asymptotics of the break-robustified version, and present the method of building the panel-level statistic. The results of the empirical analysis are discussed in Section 3, and the final section concludes.

2 Methodology

This section describes and assesses the employed testing procedure. We briefly review the ALM test before incorporating structural breaks; presen-

[2]Dolado, Gonzalo and Mayoral (2005) only discuss the case of fractional white noise without short-run dynamics rigorously.

tation of the approach for the panel analysis completes the section.

2.1 The basic test

The starting point of the discussion is the process integrated of order d,

$$(1-L)^d x_t = u_t, \quad t = 1, \ldots, \infty,$$

where u_t is a short-memory process and L is the lag operator. The fractional difference operator $(1-L)^d$ is given by the usual series expansion and it is assumed that $x_t = 0$ for $t \leq 0$. We wish to test the null $d = 0$, i.e. $x_t \sim I(0)$, against the alternative $d > 0$.

The ALM test for fractional integration is a lag-augmented version of the regression-based Lagrange Multiplier test proposed by Breitung and Hassler (2002). They reformulate Robinson's (1991, 1994) test for fractional integration and work with the model

$$x_t = \phi\, x^*_{t-1} + e_t\,, \quad t = 1, \ldots, T\,, \tag{1}$$

where the regressor x^*_{t-1} is given by $x^*_{t-1} := \sum_{j=1}^{t-1} \frac{x_{t-j}}{j}$. These specific weights are derived from the LM principle.

Model (1) is estimated by ordinary least squares [OLS]; under the null hypothesis that x_t is an *iid* process with finite variance, it holds $\phi = 0$. Breitung and Hassler show that the t-type statistic of $\widehat{\phi}$ can be used to test for fractional integration; asymptotically, a standard normal distribution results under the null. One rejects in favor of $d > 0$ for large values of the test statistic, and in favor of $d < 0$ for small values.[3] The statistic can be justified by the Lagrange Multiplier principle if assuming Gaussian data, while the limiting normality does not require the assumption of Gaussianity.

[3] The null hypothesis could be any fractional order of integration, d_0; one simply builds fractional differences under the null $d = d_0$ – according to the Lagrange Multiplier principle – and applies the test to the differences.

Naturally, the *iid* assumption for x_t under H_0 is too restrictive in practice. Demetrescu, Kuzin and Hassler (2008) propose the following lag augmented regression

$$x_t = \widehat{\phi}\, x_{t-1}^* + \sum_{j=1}^{p} \widehat{a}_j x_{t-j} + \widehat{\varepsilon}_t\,, \quad t = p+1, \ldots, T \tag{2}$$

instead of the pre-whitening approach favored by Breitung and Hassler (2002). The regressor x_{t-1}^* is defined the same way, $x_{t-1}^* := \sum_{j=1}^{t-1} \frac{x_{t-j}}{j}$.

Demetrescu, Kuzin and Hassler (2008) work under the assumption that $x_t = e_t$ is an invertible linear process driven by conditionally heteroskedastic innovations. This allows for an autoregressive representation under the null, $x_t = \sum_{j=1}^{\infty} a_j x_{t-j} + \varepsilon_t$, which prompts the use of an approximative autoregressive model in (2) with the order p of lag augmentation increasing with the sample size T.

Furthermore, Demetrescu, Kuzin and Hassler (2008) suggest the use of White's heteroskedasticity consistent standard errors to compute a t-type statistic from (2) so that limiting standard normality remains unaffected even under conditional heteroskedasticity; White standard errors also robustify the ALM statistic against unconditional heteroskedasticity, see Demetrescu (2009). In our case it is important to allow for serial dependence and changing volatility of volatility, since these are common features of financial data.

Denote t_ϕ the t-type statistic from (2), computed with White standard errors. Its rigorous asymptotic analysis requires more precise assumptions on the short-memory component x_t:

Assumption 1 *Let x_t have an invertible linear representation $x_t = \sum_{j=0}^{\infty} b_j \varepsilon_{t-j}$ with $b_0 = 1$, where $\exists\, s > 2$ so that $\sum_{j=0}^{\infty} j^s |b_j| < \infty$. Let ε_t be a weakly stationary martingale difference series with absolutely summable 8^{th}-order cumulants.*

The statistic t_ϕ can then be shown to have a standard normal distribution as $T \to \infty$ if $p \to \infty$, yet at a slower rate that T. To also analyze the properties of the test under the alternative, we consider here a sequence of local alternatives of the form $d = c/\sqrt{T}$ with positive c. Setting $c \geq 0$ allows us to discuss the null and the local alternative at the same time. Arguably, asymptotic local power analyses can give information about the power against fixed alternatives in small samples.

Proposition 1 *Under Assumption 1 and $d = c/\sqrt{T}$ with $c \geq 0$, it holds as $T \to \infty$ and $p = T^\kappa$ with $\kappa \in (0, 0.25)$ that*

$$t_\phi \xrightarrow{d} \mathcal{N}(c/s_\phi, 1)$$

where "$\xrightarrow{d}$" stands for convergence in distribution and $s_\phi = \operatorname{plim} \sqrt{T}\,\widehat{s}(\widehat{\phi})$, with $\widehat{s}(\widehat{\phi})$ the standard error used to build t_ϕ.

Proof: *See Demetrescu, Kuzin and Hassler (2008).*

In what concerns the choice of the order of the autoregressive approximation, the Monte Carlo evidence provided by Demetrescu, Kuzin and Hassler suggests using the deterministic rule

$$p = \left[4(T/100)^{1/4}\right] \tag{3}$$

rather than information criteria or downward significance testing of a_j. This is because data-dependent methods of choosing the lag order fail in this setting with stationary regressors. See also Demetrescu, Hassler and Kuzin (2008), or, for a more rigorous review of the problems with post-model selection inference, Leeb and Pötscher (2005).

2.2 Dealing with structural breaks

Recently, Demetrescu (2009) showed that White standard errors robustify the ALM test not only against conditional heteroskedasticity (allowed for by

Assumption 1), but against unconditional heteroskedasticity as well; and in particular against breaks in the variance. But this is not true for breaks in the mean.

Let the data exhibit a shift in mean, i.e. let one observe

$$y_t = \mu + x_t + \gamma \mathbf{1}(t \geq [\lambda T]), \tag{4}$$

instead of the I(0) process x_t directly, where $\mathbf{1}(\cdot)$ is the usual indicator function and $[\cdot]$ denotes the integer part. Denote $t_{\widetilde{\phi}}$ the ALM test statistic computed with y_t from (4) instead of the (stationary) process x_t. As would have been expected with such a break, a test based on $t_{\widetilde{\phi}}$ rejects w.p.1, as formalized in the following Lemma.

Lemma 1 *Under the assumptions of Proposition 1 we have with y_t from (4) and $\gamma \neq 0$ that*

$$t_{\widetilde{\phi}} \xrightarrow{p} \infty,$$

where "$\xrightarrow{p}$" stands for convergence in probability.

Proof: *See the Appendix.*

Remark 1 *The lemma shows that the ALM test mistakes a break in the mean with long memory, too. It can easily be extended to the case of several breaks in the mean: t_{ϕ} will converge to ∞ as well.*

Taking into account deterministic components (including a break at known time $t = [\lambda T]$) is straightforward in the ALM framework. Following Robinson (1994), a two-step procedure arises. First, regress the data on a step dummy and a constant (accounting for a nonzero mean of the series),

$$y_t = \widetilde{\mu} + \widetilde{\gamma} D_t^{(\lambda)} + \widetilde{x}_t, \tag{5}$$

where $D_t^{(\lambda)} = \mathbf{1}(t \geq [\lambda T])$. In the second step, apply the ALM test to the residuals $\widetilde{x}_t$ from the "cleaning" regression (5):

$$\widetilde{x}_t = \widehat{\phi}\,\widetilde{x}^*_{t-1} + \sum_{j=1}^{p} \widehat{a}_j \widetilde{x}_{t-j} + \widehat{\varepsilon}_t\,, \quad t = p+1, \ldots, T\,. \tag{6}$$

The null distribution of the test statistic t_ϕ computed from (6) is not affected asymptotically, see Demetrescu, Kuzin and Hassler (2008, Proposition 4).

The assumption of a known break point, however, is only justified in a small number of cases, when reliable prior information is available. Following the work of Zivot and Andrews (1992) it has become common practice in time series econometrics to allow for a break at unknown time not by using an estimated breakpoint $\widehat{\lambda}$, but by computing the test for all break points $\lambda \in [\underline{\lambda}, \overline{\lambda}]$,[4] and taking as test statistic the minimum value resulting. For the case of unit root testing discussed by Zivot and Andrews (1992), this rule chooses as test statistic the value most favorable for the alternative hypothesis (stationarity), since the break mimics persistence (expected to be found under integration). Dolado, Gonzalo and Mayoral (2005) apply this procedure for the fractional Dickey-Fuller test and show its asymptotic null distribution to remain the standard normal if assuming that x_t is an *iid* sequence with finite variance.

This procedure can be equally well applied with the ALM test. But we shall establish the validity of the asymptotic normality under serial correlation, accounted for by lag augmentation.

Denote $t(\lambda_j)$ the ALM test statistic based on regression (6), i.e. computed with cleaned data from (5), where $\lambda_j = j/T$ for $j \in \{[\underline{\lambda}T], [\overline{\lambda}T]\}$. Also, let t_ϕ denote the (unfeasible) ALM test statistic based on the unobservable stochastic component x_t. The break-robustified ALM test statistic is then

$$t_{\min} = \min_j t(\lambda_j). \tag{7}$$

[4]Typical choices are $\underline{\lambda} = 1 - \overline{\lambda} = 0.15$, to which we stick as well. For the unit root case, one can weaken this to $\lambda \in (0,1)$.

The interpretation of the "min" rule, however, changes, since we test short memory with breaks against long memory: namely, the value least favorable to our alternative hypothesis is picked. This arguably ensures that spurious long memory has least chances of being detected. In order to establish the asymptotic properties of the min procedure from (7) under the null $d = 0$ and a sequence of local alternatives, we shall first present a useful lemma.

Lemma 2 *Under the data generating process (4) with $d = c/\sqrt{T}$ and $\gamma \neq 0$, and Assumption 1, it holds for fixed $\lambda^* \in [\underline{\lambda}, \overline{\lambda}]$ that*

1. *if $\lambda^* = \lambda$ then*
$$t(\lambda_{[\lambda^* T]}) - t_\phi \xrightarrow{p} 0$$
and

2. *if $\lambda^* \neq \lambda$ then*
$$t(\lambda_{[\lambda^* T]}) \xrightarrow{p} \infty$$

as $T \to \infty$. Furthermore, it holds that

3. *if $\gamma = 0$ then*
$$\sup_{\lambda^* \in [\underline{\lambda}, \overline{\lambda}]} \left| t(\lambda_{[\lambda^* T]}) - t_\phi \right| \xrightarrow{p} 0$$

as $T \to \infty$.

Proof: *See the Appendix.*

Remark 2 *The lemma shows that a misspecified break point has the same asymptotic consequences as an ignored break in the mean. It also shows that accounting for a non-existing break does not affect the behavior of the test asymptotically.*

Remark 3 *The behavior is different from the unit-root case, where the test statistic converges to a properly defined stochastic process indexed by λ^*.*

The fact that the misspecified break leads the test statistic to diverge if $\gamma \neq 0$ simplifies the proof of the following proposition establishing the asymptotic properties of the min procedure.

Proposition 2 *Under the data generating process (4) with $d = c/\sqrt{T}$, it holds if $\gamma \neq 0$ that*

$$\arg\min_{1\leq j\leq T} t(\lambda_j) \xrightarrow{p} \lambda,$$

and, irrespective of γ, that

$$\min_{1\leq j\leq T} t(\lambda_j) \xrightarrow{d} \mathcal{N}(c/s_\phi, 1)$$

as $T \to \infty$, where s_ϕ is defined in Proposition 1.

Proof: *See the Appendix.*

Remark 4 *The proof resorts to the theory of extremum estimators. As the limiting target function is discontinuous at the minimum when $\gamma \neq 0$ (see the proof for details), convergence of the break point estimator is faster that $\sqrt{T}$, most likely of order T; we do not further pursue this question here. See also Lavielle and Ludeña (2000).*

Remark 5 *The consistent determination of the break point λ requires, of course, that there is a break. Moreover, the break point is consistently estimated under the local alternative and not only under the null. The existence of the break is not critical for the asymptotic normality of the robustified test statistic, but makes a difference w.r.t. the distribution of the estimated break point.*

The finite-sample distribution of $t_{\min}$, however, turns out to be far from standard normality even for sample sizes common in financial econometrics.[5]

[5]The same was observed by Dolado, Gonzalo and Mayoral (2005) for their test, reason for which they provide a set of simulated critical values.

We examined the small-sample behavior of the test based on $t_{\min}$ in a small Monte Carlo experiment. For several sample sizes T, we generated 10 000 Gaussian *iid* samples, from which several characteristics of the small-sample distribution of $t_{\min}$ were computed, see Table 1.

	$T = 50$	$T = 100$	$T = 250$	$T = 500$	$T = 1000$
Mean	-2.043	-1.779	-1.491	-1.259	-1.177
Standard deviation	0.962	0.935	0.889	0.905	0.912
Skewness	-0.583	-0.401	-0.151	-0.034	-0.013
Excess kurtosis	1.306	0.417	0.120	-0.103	0.205
Jarque-Bera	318.9	85.12	10.99	1.582	4.447

Table 1: Characteristics of test statistic $t_{\min}$ in small samples (with lag order p from (3))

In particular, we observe a serious downward bias of the test statistic; in some relation to the bias, the distribution is skewed to the left. Also, the variance is smaller than 1. The test for normality rejects the null clearly for sample sizes up to $T = 250$, and, although the distribution appears to be normal for larger sample sizes, it is not standard normal. The reason for the observed bias is as follows. Demetrescu and Tarcolea (2008) show how accounting for deterministic components – a constant, in the case studied by them – induces downward bias in t_{ϕ}. Here, the effect is compounded with the fact that the break point has to be estimated, although the estimation is consistent when there is a break.

We hence propose a small-sample correction for $t_{\min}$. In the spirit of Demetrescu and Tarcolea (2008), who derive the analytical formula of the bias of the LM test statistic in the case without breaks, we propose to fit the bias and the standard deviation as a function of the sample size to standardize the test statistic correctly in small samples. An analytic determination of the bias, although possible, is in our opinion beyond the scope of the paper; it would also lead to very similar results, as the figures in Table 1 are invariant to the size of the break or the variance of x_t. Thus, the corresponding test statistic is

$$\widetilde{t}_{\min} = \frac{1}{\sigma_T} \left(t_{\min} - \mu_T \right), \tag{8}$$

where

$$\begin{aligned} \mu_T &= -\exp\left(1.454 - 0.191 \ln T\right), \\ \sigma_T &= 0.910; \end{aligned}$$

the values are fitted from Table 1. While μ_T describes the bias of the test statistic fairly well for samples larger than $T = 1000$, the correction for σ_T is only valid for sample sizes up to 1000; the cause is an "outlier" at $T = 250$, because of which the fitted standard deviation practically doesn't depend on T. Naturally, the correction factor μ_T approaches 0 as T increases.

A further experiment looks at the effectiveness of the suggested correction for $t_{\min}$. We generated 10 000 Monte Carlo replications of an *iid* sample at slightly different sample sizes, to check the validity of the correction factors. We tabulated rejection frequencies at the nominal level of 5%; the results are given in Table 2.

		d				
λ	T	-0.4	-0.2	0	0.2	0.4
0.25	200	0	0.8	6	14.2	16.1
	400	0	0.2	6.5	24.1	29.7
	800	0	0.1	8.8	45.2	62
0.5	200	0	1	6	9.3	11.7
	400	0	0	5	21.3	28.2
	800	0	0	7.8	40.8	58.2
0.75	200	0	0.7	6.1	16.3	16.2
	400	0	0.2	5.9	26	30
	800	0	0	7.1	47.6	61.3

Table 2: Rejection frequencies for $\widetilde{t}_{\min}$, nominal level 5%

The suggested correction works well for smaller sample sizes; the test is slightly liberal, especially for $T = 800$. Further work is required to extend the validity of the small-sample correction.

2.3 Panel approach

The main problem with the panel approach for examining the dynamic properties of stock volatilities is that of cross-unit dependence, violating a classic panel data assumption. Modeling joint dynamics of stock volatilities is a "persistent" topic in financial econometrics, see e.g. Silvennoinen and Teräsvirta (2008) for a recent review. In principle, the issue can be solved by treating our topic in a multivariate fractional integration framework. Dealing with 30 series with about 515 time observations, this approach may work satisfactorily. But a larger number of stocks could not be analyzed this way – and there are stock market indices with 500 stocks or more.

(Approximate) factor models following the work of Forni *et al.* (2000) or Bai and Ng (2002) have turned out to be a useful tool in dealing with a large number of series in general, and with cross-unit dependence in panels in particular. For our examination, however, the computational complexity increases as we would have to take the minimum statistic over all possible break point combinations in the panel.[6] For these reasons, we resort to the method of combining evidence from individual units proposed by Hartung (1999). Thus, we are able to allow for individual break points without increasing the computational burden of the procedure while still allowing for a potentially very large number of series.

Combining the significance of individual tests has a long tradition in statistics and can be traced back to Fisher (1954, p. 100). For instance, if the test statistics are independent, one transforms individual p values, say p_i, from each unit with the standard normal cumulative distribution function to

[6] An alternative would be to assume a common break point for all series of the panel. But we do not find such an assumption to be very plausible, as different industrial sectors may react differently to changes in the economic environment.

obtain standard normal test statistics, $t_i = \Phi^{-1}(p_i)$. These so-called probits are independent, and also standard normal by construction; building a panel statistic is thus straightforward. See Hartung (1999) and his references.

Should individual statistics be cross-dependent, as is the case here, Hartung points out that cross-unit dependence implies correlation of the probits t_i. He assumes a constant correlation ρ for all pairs $\{t_i, t_j\}$ and derives a correction for this type of correlation. The constant correlation can be estimated consistently as $N \to \infty$ based on the probits alone.

In our case it is not even necessary to transform the individual test statistics $\widetilde{t}_{\min,i}$, since they are approximately standard normal to begin with. Our panel test statistic is hence given by

$$t(\widehat{\rho}^*, \kappa) = \frac{\sum_{i=1}^{N} \widetilde{t}_{\min,i}}{\sqrt{N + [N^2 - N]\left[\widehat{\rho}^* + \kappa\sqrt{\frac{2}{N+1}}\,(1 - \widehat{\rho}^*)\right]}}, \tag{9}$$

where the parameter κ allows for a small-sample correction (made necessary by the plugging in of $\widehat{\rho}^*$) and is usually taken as $\kappa = 0.2$, and

$$\widehat{\rho}^* = \min\left(-\frac{1}{N-1}, \widehat{\rho}\right), \quad \text{with} \;\; \widehat{\rho} = 1 - \frac{1}{N-1}\sum_{i=1}^{N}\left(\widetilde{t}_{\min,i} - \frac{1}{N}\sum_{i=1}^{N}\widetilde{t}_{\min,i}\right)^2.$$

Different weights ω_i can be assigned to the units of the panel, see Hartung (1999). The distribution of the test statistic $t(\widehat{\rho}^*, \kappa)$ is approximately standard normal if the probits follow a joint multivariate normal distribution. Using this method, the panel statistic can be computed for unbalanced, cross-dependent panels as well; in contrast, factor models require balanced panels.

Under the assumption that the time series of the panel have a diagonal infinite-order moving average representation with (multivariate) martingale difference innovations, where each individual process satisfies Assumption 1 marginally and the joint cumulants of 8^{th} order are absolutely summable, joint multivariate normality with standardized marginals follows for the vector of robustified ALM test statistics asymptotically. This is only a sufficient condition for checking the conditions of validity of Hartung's method and can most likely be relaxed; but we do not pursue the topic here.

Although the weak point of this method appears to be the assumption of a constant correlation across the individual statistics, Hartung (1999) provides simulation evidence of robustness to departures from the constant-correlation specification. Moreover, Demetrescu, Hassler and Tarcolea (2006) give theoretical support to the robustness reported by Hartung. E.g., Westerlund and Costantini (2009) recently applied this method to examine the neutrality of money in a panel of 10 countries between 1870 and 1986.

3 Empirical analysis

3.1 The data

The data set consists of daily closing prices of 30 DAX stocks.[7] We thus worked with 515 observations for each stock, except for Deutsche Postbank, which has only been listed in the DAX since June 22nd, 2004, and only delivers 394 observations: the panel was unbalanced. The returns were computed as log-differences, and we examined the absolute returns series.

We first computed for each stock the ALM test without accounting for a possible break in the mean. The absolute returns are indeed highly persistent, thus confirming for the period 2004-2005 findings of long-range dependence in German stock market volatilities. We found 14 (one-sided) rejections at 5% in favor of long memory and only one in favor of antipersistence; see

[7] ADIDAS, ALLIANZ, BASF, BMW, BAYER, COMMERZBANK, CONTINENTAL, DAIMLER, DEUTSCHE BANK, DEUTSCHE BOERSE, DEUTSCHE POST, DEUTSCHE POSTBANK, DEUTSCHE TELEKOM, E-ON, FRESENIUS MED.CARE, HENKEL, HYPO REAL ESTATE, INFINEON TECHNOLOGIES, LINDE, LUFTHANSA, MAN, MERCK, METRO, MUENCHENER RUCK., RWE, SAP, SIEMENS, THYSSENKRUPP, TUI, and VOLKSWAGEN.

Table 3.

−0.9166	1.3382	1.1186	1.628	0.9077	0.907
1.7772	2.2724	2.2276	−2.1055	2.7461	2.8595
2.4028	0.7506	−0.4038	0.0195	2.2155	1.5169
0.7367	1.8801	1.2103	1.7192	2.7005	1.1279
1.1243	3.4483	3.0463	1.9817	1.903	1.3888

Table 3: Individual test results for testing against long memory without considering the possibility of a break; test statistics follow a standard normal distribution asymptotically

Computing a panel-level statistic with the method described in Section 2.3, the figures from Table 3 lead to a value of 6.4370, with a corresponding p value of 0 (based on the standard normal distribution). Subsequent tests, however, revealed that much of this could be explained by an unconditional shift in the mean; see below.

3.2 Breaks and long memory

When allowing for a break at unknown time, the evidence in favor of long memory became less pronounced. We used the test discussed in Section 2.2 (with small-sample correction) to compute robustified test statistics against long memory. The test results are reported for individual stocks in Table 4.

0.0714	0.1656	2.1487	0.9529	0.259	1.3107
0.5648	3.4697	0.5073	−1.1387	0.8679	1.139
−0.9536	1.2294	−0.5179	1.0116	0.7338	0.7263
−0.5775	−1.0438	0.3666	1.0801	1.1093	−1.1825
−1.3374	1.0768	0.3331	−0.5301	0.0902	2.2844

Table 4: Individual test results for testing against long memory with a break at unknown time; all 30 test statistics were computed with a small-sample correction

As can be observed, the null hypothesis of short memory was rejected in favor of long memory for 3 stocks only when allowing for a possible break in mean. The panel statistic from (9) is $t(\widehat{\rho}^*, 0.2) = 2.1024$. Although not as significantly as when not allowing for a break, the test still rejected at 5% significance level (with a p value of 0.0178).

Of course, this is no surprise, as allowing for a structural break is costly in terms of power. But a break seems to be substantiated by the data: when examining the estimated break points, we observe them to cluster in 2004, see Figure 1. The test procedure finds breaks in 2005 for four stocks only. Such clustering is not likely to happen if there is no break.

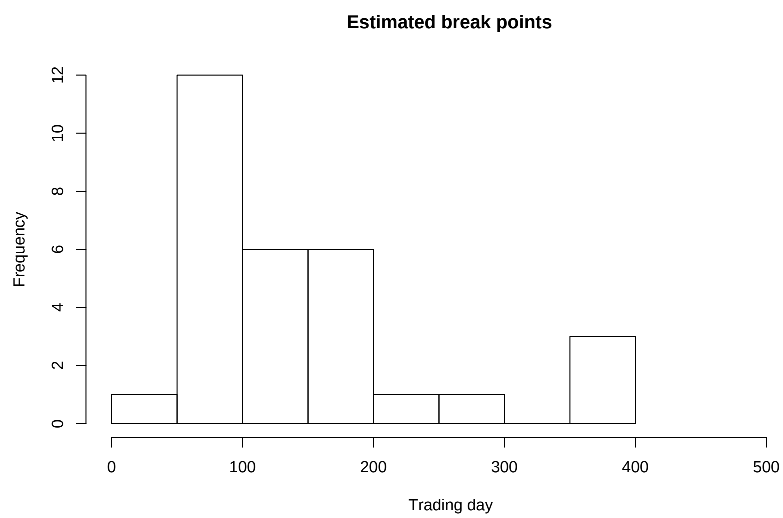

Figure 1: Absolute frequency of estimated break points

To wrap up the empirical analysis, we find the break in the volatility to lie in 2004 for 27 of 30 examined stocks. But this break does not entirely explain the long-range dependence of the volatility for the period 2004-2005. Of course, it could be the case that the data exhibit an additional break. A second break, however, does not seem plausible as the examined time span is rather short in absolute terms. Moreover, almost all break points were found to lie between March 2004 and September 2004, which is still consistent with an (almost) common break, considering the small-sample variability of the break point estimator.

4 Summary

The paper examined the long-memory property of a panel of 30 German stock volatilities. The empirical analysis employed a test for short memory against long memory based on the lag-augmented LM test that allows for unconditional shifts in the mean; this way, findings of long memory can't be spuriously caused by structural breaks. We showed the proposed robust test to have asymptotic normal distribution under the null, but to also have a distorted small-sample distribution. Thus, the paper also proposed a simple small-sample correction. To conduct the analysis at the panel level, we combined the significance of individual tests in such a way that cross-unit dependence was allowed for; combining p values has the advantage that unbalanced panels and individual break points are allowed for.

Rather than deciding between short memory plus break and long memory, our findings suggest a break plus some long memory to be more appropriate for modeling daily volatility of German stocks, at least in the period 2004-2005.

Appendix

Proof of Lemma 1

We begin by setting some notation. Let for convenience $b_t = \mathbf{1}(t \geq [\lambda T])$ and denote $\mathbf{y}_{t-1} = (y_{t-1}, \ldots, y_{t-p})'$ and $\mathbf{x}_{t-1} = (x_{t-1}, \ldots, x_{t-p})'$. As the regressor $y^*_{t-1} = \sum_{j\geq 1}^{t-1} j^{-1} y_{t-j}$ is obtained by a sample-size dependent filtering, let its asymptotic counterpart be $y^{**}_{t-1} = \sum_{j\geq 1} j^{-1} y_{t-j}$; denote also

$x^{**}_{t-1} = \sum_{j\geq 1} j^{-1} x_{t-j}$ and $b^{**}_{t-1} = \sum_{j\geq 1} j^{-1} b_{t-j}$. We have that

$$y^{**}_{t-1} = x^{**}_{t-1} + \gamma\, b^{**}_{t-1},$$

where

$$\begin{aligned} b^{**}_{t-1} &= 0, \quad t < [\lambda T] \\ b^{**}_{t-1} &= \sum_{j=[\lambda T]}^{t-[\lambda T]} \frac{1}{j} \\ &= O\left(\ln(t - [\lambda T])\right), \quad t > [\lambda T] \end{aligned}$$

(the order relies on the logarithmic behavior of the harmonic series, where $\sum_{j=1}^{t} j^{-1} - \ln t$ converges to Euler's constant).

Since the difference between y^{**}_{t-1} and y^{*}_{t-1} is that between x^{**}_{t-1} and x^{*}_{t-1}, which, under the null and the local alternative, does not affect the ALM test statistic asymptotically (see Demetrescu, Kuzin and Hassler, 2008, proof of Proposition 3), replacing y^{*}_{t-1} with y^{**}_{t-1} has no asymptotic effect, i.e. the test statistic $t_{\tilde{\phi}}$ is asymptotically equivalent to the quotient

$$\frac{\sum_{p+1}^{T} y^{**}_{t-1} y_t - \sum_{p+1}^{T} y^{**}_{t-1} \mathbf{y}'_{t-1} \left(\sum_{p+1}^{T} \mathbf{y}_{t-1} \mathbf{y}'_{t-1}\right)^{-1} \sum_{p+1}^{T} \mathbf{y}'_{t-1} y_t}{\sqrt{\sum_{p+1}^{T} \left(y^{**}_{t-1}\right)^2 - \sum_{p+1}^{T} y^{**}_{t-1} \mathbf{y}'_{t-1} \left(\sum_{p+1}^{T} \mathbf{y}_{t-1} \mathbf{y}'_{t-1}\right)^{-1} \sum_{p+1}^{T} \mathbf{y}'_{t-1} y^{**}_{t-1}}}. \quad (10)$$

To assess the behavior of the above quotient, we need to check the behavior of certain cross-product terms. We have for $0 \leq j \leq p$ with $p/T \to 0$ that

$$\begin{aligned}
\sum_{p+1}^{T} x_{t-1}^{**} b_{t-j} &= O_p(T), \\
\sum_{p+1}^{T} b_{t-1}^{**} x_t &= O_p(T \ln T), \\
\sum_{p+1}^{T} x_{t-1}^{**} x_{t-j} &= O_p(T)
\end{aligned}$$

uniformly in j, as x_t and x_{t-1}^{**} are uniformly bounded in probability, even under the local alternative.[8] Furthermore,

$$\sum_{p+1}^{T} \left(b_{t-1}^{**}\right)^2 = O\left(T \ln^2 T\right),$$

and, for $0 \leq j, k \leq p$,

$$\frac{1}{T} \sum_{p+1}^{T} b_{t-j} b_{t-k} \to 1 - \lambda$$

uniformly. (The derivations are straightforward and not given here to save space; as $p/T \to 0$, the sum terms with different values of j vanish asymptotically.) Some care is required, though, for showing that

$$\frac{1}{T \ln T} \sum_{p+1}^{T} b_{t-1}^{**} b_{t-j} \to 1 - \lambda$$

uniformly in j. To this end note first that $\beta_t = \frac{1}{\ln(T - [\lambda T])} b_{t-1}^{**} \to 1$. Then, the

[8]The properties of x_{t-1}^{**} are established in the proof of Proposition 3 of Demetrescu, Kuzin and Hassler (2008).

average of β_t will have the same limit, so

$$\frac{1}{T-[\lambda T]-j}\sum_{t=[\lambda T]+j}^{T}\beta_t \to 1.$$

The terms due to j are at most of $O(p/T)$, so they vanish uniformly. By replacing β_t, the required limit result is established.

The denominator of the quotient in (10) is the square root of the sum of squared residuals from the projection of y^{**}_{t-1} onto $\mathbf{y}_{t-1}$. The projection matrix is idempotent and its norm equals 1. Hence, the sum of squared projection residuals has the same order as the sum of $\left(y^{**}_{t-1}\right)^2$, whose dominant component is b^{**}_{t-1}. Summing up, the denominator is of order $O_p(\sqrt{T}\ln T)$

The numerator of the above statistic will be shown below to be of *exact* order $O_p(T\ln T)$; as such, $t_{\tilde{\phi}}$ diverges at rate $\sqrt{T}$. To establish the desired magnitude order of the numerator, we only need to show that its normalized dominant term,

$$\frac{1}{T\ln T}\sum_{p+1}^{T} b^{**}_{t-1}b_t - \frac{1}{T\ln T}\sum_{p+1}^{T} b^{**}_{t-1}\mathbf{b}'_{t-1}\left(\frac{1}{T}\sum_{p+1}^{T}\mathbf{x}_{t-1}\mathbf{x}'_{t-1}+\frac{1}{T}\sum_{p+1}^{T}\mathbf{b}_{t-1}\mathbf{b}'_{t-1}\right)^{-1}\frac{1}{T}\sum_{p+1}^{T}\mathbf{b}'_{t-1}b_t,$$

converges to a positive constant (the entire term is multiplied with γ^2, so the direction of the break does not matter). To this end, recall that

$$\frac{1}{T\ln T}\sum_{p+1}^{T} b^{**}_{t-1}b_t - \frac{1}{T\ln T}\sum_{p+1}^{T} b^{**}_{t-1}b_{t-j} \to 0$$

and

$$\frac{1}{T\ln T}\sum_{p+1}^{T} b^{**}_{t-1}b_{t-j} - \frac{1}{T}\sum_{p+1}^{T} b_t b_{t-j} \to 0$$

uniformly in $1 \leq j \leq p$ even as $p \to \infty$.

The dominant term of the numerator of the quotient in (10) thus has the same limit as the following expression

$$\frac{1}{T}\sum_{p+1}^{T} b_t^2 - \frac{1}{T}\sum_{p+1}^{T} b_t \mathbf{b}_{t-1}' \left(\frac{1}{T}\sum_{p+1}^{T} \mathbf{x}_{t-1}\mathbf{x}_{t-1}' + \frac{1}{T}\sum_{p+1}^{T} \mathbf{b}_{t-1}\mathbf{b}_{t-1}'\right)^{-1} \frac{1}{T}\sum_{p+1}^{T} \mathbf{b}_{t-1}' b_t.$$

To show its limit to be positive, as required for the result, we resort to classical OLS matrix notation and write it as a quadratic form, $\mathbf{b}'(I - B(X'X + B'B)^{-1}B')\mathbf{b}$, where $\mathbf{b}$, B and X are defined implicitly. The scalar $\mathbf{b}'(I - B(B'B)^{-1}B')\mathbf{b}$ is nonnegative as the projection matrix is positive semidefinite. Hence, the condition

$$\mathbf{b}'(I - B(X'X + B'B)^{-1}B')\mathbf{b} > \mathbf{b}'(I - B(B'B)^{-1}B')\mathbf{b}$$

implies the desired positive limit. The condition is equivalent to

$$\mathbf{b}'(B(B'B)^{-1}B' - B(X'X + B'B)^{-1}B')\mathbf{b} > 0,$$

which in turn is true if $(B'B)^{-1} - (X'X + B'B)^{-1}$ is positive definite. But this is fulfilled if $(X'X + B'B) - B'B$ is positive definite, which obviously holds true. The result follows.

Proof of Lemma 2

For simplicity, ignore the intercept; the extension for non-zero mean – and for other correctly specified deterministic components – is straightforward. The basic idea is that a misspecified break point induces omitted variable bias in the residuals $\widehat{x}_t^{(\lambda^*)}$, which affects the test statistic asymptotically. The residuals of projecting on the dummy variable $D_t^{(\lambda^*)} = \mathbf{1}(t \geq [\lambda^* T])$ are then

$$
\begin{aligned}
\widehat{x}_t^{(\lambda^*)} &= y_t - \widehat{\gamma}^{(\lambda^*)} D_t^{(\lambda^*)} = x_t + \gamma D_t^{(\lambda)} - \frac{\sum_{t=p+1}^T D_t^{(\lambda^*)} \left(x_t + \gamma D_t^{(\lambda)} \right)}{\sum_{t=p+1}^T \left(D_t^{(\lambda^*)} \right)^2} D_t^{(\lambda^*)} \\
&= x_t - \frac{\sum_{t=p+1}^T D_t^{(\lambda^*)} x_t}{\sum_{t=p+1}^T \left(D_t^{(\lambda^*)} \right)^2} D_t^{(\lambda^*)} + \gamma \left(D_t^{(\lambda)} - \frac{\sum_{t=p+1}^T D_t^{(\lambda^*)} \left(D_t^{(\lambda)} - D_t^{(\lambda^*)} \right)}{\sum_{t=p+1}^T \left(D_t^{(\lambda^*)} \right)^2} D_t^{(\lambda^*)} \right).
\end{aligned}
$$

Let

$$
\begin{aligned}
\widetilde{b}_t &= \widehat{x}_t^{(\lambda_j)} - x_t \\
&= \gamma \left(D_t^{(\lambda)} - \frac{\sum_{t=p+1}^T D_t^{(\lambda^*)} \left(D_t^{(\lambda)} - D_t^{(\lambda^*)} \right)}{\sum_{t=p+1}^T \left(D_t^{(\lambda^*)} \right)^2} D_t^{(\lambda^*)} \right) - \frac{\sum_{t=p+1}^T D_t^{(\lambda^*)} x_t}{\sum_{t=p+1}^T \left(D_t^{(\lambda^*)} \right)^2} D_t^{(\lambda^*)} \\
&= \gamma \left(D_t^{(\lambda)} - \frac{\sum_{t=p+1}^T D_t^{(\lambda^*)} \left(D_t^{(\lambda)} - D_t^{(\lambda^*)} \right)}{\sum_{t=p+1}^T \left(D_t^{(\lambda^*)} \right)^2} D_t^{(\lambda^*)} \right) + O_p\left(T^{-0.5}\right),
\end{aligned}
$$

where the order $O_p\left(T^{-0.5}\right)$ is uniform over $1, \ldots, T$ and in λ^* given that λ^* is restricted to belong to a compact subset of $(0, 1)$. Note that the behavior of the $O_p\left(T^{-0.5}\right)$ term does not depend on γ; in particular, the uniformity in t and λ^* holds for $\gamma = 0$ as well.

If $\lambda^* = \lambda$ or $\gamma = 0$, it follows that $\widetilde{b}_t = O_p\left(T^{-0.5}\right)$ with the $O_p\left(T^{-0.5}\right)$ being uniform. Following the arguments of Propositions 3 and 4 of Demetrescu, Kuzin and Hassler (2008), the first part of the lemma is established. The third follows from the fact that the order of b_t does not depend on λ^*.

For the second part, we examine $\lambda^* < \lambda$ and $\lambda^* > \lambda$ separately. For both cases, note that

$$\frac{\frac{1}{T}\sum_{t=p+1}^{T} D_t^{(\lambda^*)}\left(D_t^{(\lambda)} - D_t^{(\lambda^*)}\right)}{\frac{1}{T}\sum_{t=p+1}^{T}\left(D_t^{(\lambda^*)}\right)^2} \to -\frac{\lambda-\lambda^*}{1-\lambda^*}\mathbf{1}(\lambda^* < \lambda) > -1.$$

In the first case, we obtain by ignoring the $O_p\left(T^{-0.5}\right)$ term (since it does not matter asymptotically, see above) that

$$\widetilde{b}_t = 0, \quad t < [\lambda^* T],$$

$$\widetilde{b}_t \to \gamma\frac{\lambda-\lambda^*}{1-\lambda^*}, \quad [\lambda^* T] < t < [\lambda T],$$

$$\widetilde{b}_t \to \gamma\left(1+\frac{\lambda-\lambda^*}{1-\lambda^*}\right), \quad [\lambda T] < t,$$

where convergence is uniform in t. For the second, we have that $\widetilde{b}_t = \gamma D_t^{(\lambda)}$.

Summing up, $\widehat{x}_t^{(\lambda_j)}$ contains either one or two breaks not accounted for. In both cases, we can apply Lemma 1 to obtain the result.

Proof of Proposition 2

We examine the case $\gamma \neq 0$ first. Consider the target function $\mathcal{Q}_T(s) = t(\lambda_{[sT]})$ for $s \in (0,1)$; the function $\mathcal{Q}_T(s)$ is a step function, piecewise continuous. We shall now show that, for any $\varepsilon > 0$, we have

$$P\left(\left|\underset{s\in(0,1)}{\arg\min}\,\mathcal{Q}_T(s) - \lambda\right| < \varepsilon\right) \to 1,$$

as required for the result.

To this end, choose $T_0 = \varepsilon^{-1}$ and note that, for $s \in (0,\lambda)\cup[\lambda+T_0^{-1},1)$, we have $P\left(|\mathcal{Q}_T(s)| < |\mathcal{Q}_T(\lambda)|\right) \to 0$ as $T \to \infty$ (implying in particular $T > T_0$), since $\mathcal{Q}_T(\lambda) = O_p(1)$.

Hence, for any $\varepsilon > 0$, the probability of finding the minimum of $\mathcal{Q}_T(s)$

outside the interval $(\lambda - \varepsilon, \lambda + \varepsilon)$ vanishes as $T \to \infty$, from which it immediately follows that

$$\arg\min\, \mathcal{Q}_T(s) \xrightarrow{p} \lambda;$$

the unit root case requires some care in establishing uniform convergence to the limit process, but things are rather clear in our situation, as it diverges at all points but one. We obviously also have

$$\min\, \mathcal{Q}_T(s) \xrightarrow{p} t(\lambda).$$

If, on the other hand, $\gamma = 0$, the required convergence is a consequence of item 3 of Lemma 2 given that the convergence stated there is uniform in λ^*; this concludes the proof.

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