# Distinguishing between Models of Risk Sharing*

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January 2009

**Abstract**

This paper aims to statistically distinguish between the models of perfect risk sharing, autarky, and risk sharing with limited commitment in a structural manner. It extends the approach of Ligon, Thomas, and Worrall (2002) in several ways. Preferences may depend on both observable household characteristics and unobservable individual effects. Further, measurement error is dealt with in the structural models by using a simulated maximum likelihood estimator. Finally, Vuong's (1989) test is used to statistically compare how well the models are able to explain the consumption allocation. The data comes from an income-consumption panel collected by the International Food Policy Research Institute (IFPRI) in rural Pakistan.

**Keywords**: risk sharing, limited commitment, preference heterogeneity, dynamic panel, structural microeconometrics

**JEL codes**: D10, D80

*Preliminary, comments and suggestions welcome. This paper has greatly benefitted from discussions with Richard Blundell, Pierre Dubois, and Thierry Magnac. All errors are mine. I thank the International Food Policy Research Institute (IFPRI) for making the data available. The softwares Stata (www.stata.com) and R (www.r-project.org) have been used. The programs are available from the author.

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# 1 Introduction

Finding structural models of consumption behavior that fit survey data well is an important step both for explaining household behavior and for policy evaluation and design. This paper compares three structural models of the allocation of consumption within a community. In particular, it examines the models of perfect risk sharing, autarky, and risk sharing with limited commitment, and aims to compare how well these models are able to explain households' consumption. Since the estimation is done in a structural manner, the effects of policies on consumption can be simulated. Thus this research may provide guidance for the evaluation and design of micro-insurance programs, for example.

There exists ample empirical evidence that households do not fully share the risks they face, while they do achieve a remarkable amount of insurance without formal contracts (Cochrane, 1991, Mace, 1991, Townsend, 1994, Dercon and Krishnan, 2003, and many others), thereby moving away from the benchmark of autarky as well. The question is then how to model the observed partial insurance.

Assuming that limited commitment causes the imperfection has proved to be useful theoretically, and the model is also supported by some empirical evidence (Thomas and Worrall, 1988, Coate and Ravallion, 1993, Kocherlakota, 1996, Fafchamps, 1999, Ligon, Thomas, and Worrall, 2000, 2002, Attanasio and Ríos-Rull, 2000, Foster and Rosenzweig, 2001, Kehoe and Perri, 2002, Krueger and Perri, 2006, Mazzocco, 2007, Dubois, Jullien, and Magnac, 2008, and others). This means that informal insurance contracts are required to be self enforcing. In other words, insurance transfers have to be voluntary, and are rewarded by future insurance provided by the other members of the community. This is arguably a good description of informal insurance arrangements among households in rural communities, or members of a family. The model has also been used in a wide variety of other economic contexts, including risk sharing between an employee and an employer (Thomas and Worrall, 1988), and countries (Kehoe and Perri, 2002). Further, Schechter (2007) adopts the same model to examine the interaction between a farmer and a thief, and Dixit, Grossman, and Gul (2000) use a similar model to examine cooperation between opposing political parties.

This paper is mostly related to the work of Ligon, Thomas, and Worrall (2002), LTW hereafter, and extends their approach in several ways. First, it allows for preference heterogeneity across households and time. In particular households' utility may depend on some observable characteristics. To my knowledge, the risk sharing with limited commitment model has not yet been structurally tested allowing for preference heterogeneity. Second, this paper allows for unobserved individual effects. Third, it performs statistical tests to compare the models. In particular, I apply Vuong's (1989) test. LTW do not perform any statistical test to compare the different models of risk sharing. Finally, measurement error is dealt with explicitly in the structural model.

The extension to allow for heterogeneous preferences in the case of perfect risk sharing has been explored by several papers. Dubois (2000) specifies an isoelastic utility function, and allows the coefficient of relative risk aversion to depend on observables, as well as for multiplicative preference shocks. He performs a nondirectional test of perfect risk sharing based on overidentifying restrictions. Schulhofer-Wohl (2007) uses data on risk aversion from the Health and Retirement Study, and finds evidence that occupational choice is affected by risk preferences. He argues that this should be taken into account when evaluating how well people are able to mitigate the adverse effects of risk they face. He then constructs a new test of perfect risk sharing, where he captures the heterogeneity in risk preferences by a nuisance parameter. Mazzocco and Saini (2008) first test whether households' preferences are homogeneous, and find that heterogeneity is important in the ICRISAT villages in India. They then construct new, nonparametric tests of perfect risk sharing allowing for preference heterogeneity. The present paper contributes to this strand of literature by looking at the limited commitment case as well, but considers only parametric models.

This paper also contributes to the literature on explaining consumption inequality with respect to income inequality. Krueger and Perri (2006) show that, as a result of partial insurance, observed cross-sectional consumption inequality is smaller than income inequality. Partial insurance is modeled allowing for limited commitment, as here, but the model is then calibrated rather than estimated. The authors argue that consumption inequality increased less than income inequality in the United States over the last few decades because more income risk induces more informal insurance. On the other hand, Blundell, Pistaferri, and Preston (2008) document that income shocks have become less persistent, and thereby easier to be insured against. The present paper only allows for transitory shocks, but it builds and estimates a structural model of how consumption is allocated.

This paper first details the different theoretical models of risk sharing. Households are assumed to be infinitely lived and risk averse. They face some exogenous risk on their income each period. The distribution from which income is drawn is common knowledge ex ante, as well as income realizations ex post. The only way households may mitigate the adverse effects of risk they face is to insure one another. I examine insurance across states of the world, and not time, thus savings are assumed away. Perfect risk sharing means that all idiosyncratic risks are insured. In other words, households pool their income. However, formal insurance contracts are often not available in rural villages in developing countries, and the perfect risk sharing solution might not be self enforcing. That is, a household with a high income realization today may decide not to contribute, but to quit the risk sharing arrangement instead.

The risk sharing with limited commitment model characterizes the case where enforcement constraints may be binding. This paper talks in details about this model, and discusses how to solve it using numerical dynamic programming. The solution of the model is fully characterized by a set of state-dependent intervals on the relative Pareto-weights, or, the ratio of marginal utilities (LTW). Given preferences, today's income realizations, and the distribution from which incomes are drawn, these intervals can be solved for numerically. Then, the consumption allocation predicted by the model is computed trying to keep the ratio of marginal utilities as close as possible to the one from the previous period, while respecting the interval for the income state realized today.

Then, the empirical models are set up. These models are of the dynamic panel type. In particular, I am interested in how the consumption allocation today is explained by incomes today and in the past, household characteristics, and past consumption, according to the different models of risk sharing. More precisely, I model the changes of each household's consumption relative to mean consumption in the community. Maximum likelihood estimators are derived assuming that multiplicative errors in the measurement of consumption are log-normally distributed. The estimation of the perfect risk sharing model, as well as the benchmark of autarky, is relatively straightforward. The main difficulty in the limited commitment case is that the updating with respect to the ratio of marginal utilities depends on the unobservable individual effects and measurement error.

The data comes from an income-consumption survey conducted by the International Food Policy Research Institute (IFPRI) in rural Pakistan. Almost 1000 households have been interviewed over 12 rounds in 46 villages in 4 districts of Pakistan. The survey contains detailed information on household characteristics, consumption of food and other nondurable goods, and income from different sources, including crop production, wage labor, small businesses, and remittances from abroad. For the purposes of this paper, I only need a measure of consumption, income, and some household characteristics and wealth.

The rest of the paper is structured as follows. First, the theoretical models are discussed. Then, section 3 details the empirical models. Section 4 talks about the data used. Section 5 contains the estimation results for the structural models, both with and without preference heterogeneity, as well the statistical tests to compare the models. Concluding remarks are presented in section 6.

## 2 Models of risk sharing

Suppose that there are $n$ households in a community. The preferences of household $i$ at time $t$ are described by the utility function

$$u_{it}\left(c_{it}\right) = exp\left(\xi_{it}\right)\frac{c_{it}^{1-\sigma_{it}}-1}{1-\sigma_{it}}, \tag{1}$$

where $c_{it}$ is the consumption of household $i$ at time $t$, $\sigma_{it}$ is the coefficient of relative risk aversion, and $\xi_{it}$ accounts for preference heterogeneity unrelated to risk aversion.

Suppose that income is independently and identically distributed (i.i.d.) across time for each household, and is drawn from some discrete distribution $Y_i$ for household $i$. Let $s_t$ denote the state of the world that describes the income realizations of all households in the community at time $t$. Note that income is exogenous. In other words, the effects of risk on choices among different income generating processes are ignored. In addition, I study consumption smoothing across states of the world, not across time, thus savings are assumed away.

This section considers three models in turn. First, it talks about the model of perfect risk sharing. Second, it mentions the benchmark of autarky. Finally, the model of risk sharing with limited commitment is detailed.

## 2.1 Perfect risk sharing

Perfect risk sharing means that all idiosyncratic risks are insured away. To find the first best, or, Pareto optimal allocations, one may solve the social planner's problem. The (utilitarian) social planner maximizes a weighted sum of households' excepted lifetime utilities,

$$\max_{\{c_{it}(s_t)\}} \sum_i \lambda_i \sum_{t=1}^{\infty} \sum_{s_t} \delta^t \pi\left(s_t\right) u_{it}\left(c_{it}\left(s_t\right)\right),$$

where $\lambda_i$ is the (initial) weight of household $i$ in the social planner's objective, $\delta$ is the discount factor, and $\pi\left(s_t\right)$ is the probability of state $s_t$ occurring; subject to the resource constraint

$$\sum_i c_{it}\left(s_t\right) = \sum_i y_{it}\left(s_t\right), \forall s_t, \forall t,$$

where $y_{it}\left(s_t\right)$ is the income of household $i$ at time $t$ and state $s_t$.

The well-known result that

$$\frac{u'_{it}\left(c_{it}\left(s_t\right)\right)}{u'_{kt}\left(c_{kt}\left(s_t\right)\right)} = \frac{\lambda_k}{\lambda_i}, \forall s_t, \forall t, \tag{2}$$

that is, the ratio of marginal utilities for any two households $i$ and $k$ is constant over time and across states of the world, follows from the first order conditions of the social planner's problem. With the utility function (1), the first order condition (2), for any $s_t$, is

$$\lambda_i exp\left(\xi_{it}\right) c_{it}^{-\sigma_{it}} = \lambda_k exp\left(\xi_{kt}\right) c_{kt}^{-\sigma_{kt}}. \tag{3}$$

## 2.2 Autarky

When households stay in autarky, the problem is trivial, since savings have been assumed away. The model predicts that

$$c_{it}\left(s_t\right) = y_{it}\left(s_t\right), \forall s_t, \forall i, \forall t. \tag{4}$$

Let $U_i^{aut}\left(s_t\right)$ denote the expected lifetime utility, or, the value function, of household $i$ in autarky at state $s_t$ and time $t$. In mathematical terms,

$$U_i^{aut}\left(s_t\right) = u_{it}\left(y_{it}\left(s_t\right)\right) + \delta \sum_{s_{t+1}} \pi\left(s_{t+1}\right) U_i^{aut}\left(s_{t+1}\right). \tag{5}$$

## 2.3 Limited commitment

One may write the problem as follows. The (utilitarian) social planner maximizes a weighted sum of households' expected lifetime utilities,

$$\max_{\{c_i(s^t)\}} \sum_i \lambda_i \sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi\left(s^t\right) u_{it}\left(c_i\left(s^t\right)\right),$$

where $\pi\left(s^t\right)$ is the probability of history $s^t = \left(s_1, s_2, ..., s_t\right)$ occurring, and $c_i\left(s^t\right)$ denotes the consumption of individual $i$ when history $s^t$ has occurred; subject to the resource constraints

$$\sum_i c_i\left(s^t\right) \leq \sum_i y_i\left(s_t\right), \forall s^t, \forall t, \tag{6}$$

and the enforcement constraints,

$$\sum_{r=t}^{\infty} \sum_{s^r} \delta^{r-t} \pi\left(s^r\right) u_{ir}\left(c_i\left(s^r\right)\right) \geq U_i^{aut}\left(s_t\right), \forall s^t, \forall t, \forall i, \tag{7}$$

where the right hand side has been defined in equation (5). Note that even if income is i.i.d., consumption may depend on the whole history of states.

Denoting the multiplier on the enforcement constraint of household $i$ (7) by $\delta^t \pi\left(s^t\right) \mu_i\left(s^t\right)$, and the multiplier on the resource constraint (6) by $\delta^t \pi\left(s^t\right) \gamma\left(s^t\right)$ when history $s^t$ has occurred, the Lagrangian is

$$\sum_{t=1}^{\infty}\sum_{s^t}\delta^t\pi\left(s^t\right)\left[\sum_i \lambda_i u_{it}\left(c_i\left(s^t\right)\right)\right.$$
$$+\mu_i\left(s^t\right)\left(\sum_{r=t}^{\infty}\sum_{s^r}\delta^{r-t}\pi\left(s^r\right)u_{ir}\left(c_i\left(s^r\right)\right)-U_i^{aut}\left(s_t\right)\right)$$
$$\left.+\gamma\left(s^t\right)\left(\sum_i y_i\left(s_t\right)-c_i\left(s^t\right)\right)\right].$$

Using the ideas of Marcet and Marimon (1998), the Lagrangian can also be written in the form

$$\sum_{t=1}^{\infty}\sum_{s^t}\delta^t\pi\left(s^t\right)\left[\sum_i M_i\left(s^{t-1}\right)u_{it}\left(c_i\left(s^t\right)\right)\right.$$
$$\left.+\mu_i\left(s^t\right)\left(u_{it}\left(c_i\left(s^t\right)\right)-U_i^{aut}\left(s_t\right)\right)+\gamma\left(s^t\right)\left(\sum_i y_i\left(s_t\right)-c_i\left(s^t\right)\right)\right],$$

where $M_i\left(s^t\right)=M_i\left(s^{t-1}\right)+\mu_i\left(s^t\right)$ with $M_i\left(s^0\right)=\lambda_i$. In words, $M_i\left(s^t\right)$ is the initial weight of household $i$ plus the sum of the Lagrange multipliers on her enforcement constraints along the history $s^t$.

The first order condition with respect to $c_i\left(s^t\right)$ is

$$\delta^t\pi\left(s^t\right)M_i\left(s^t\right)u'(c_i\left(s^t\right))-\gamma\left(s^t\right)=0. \tag{8}$$

There are also standard first order conditions relating to the resource and enforcement constraints, with complementarity slackness conditions. Let us consider two households sharing risk, household $i$ and $k$. "Household" $k$ can be thought of as the rest of the community, as in LTW, or a typical household. Combining the first order conditions (8) for these two households for history $s^t$ at time $t$, we have

$$\frac{u'\left(c_k\left(s^t\right)\right)}{u'\left(c_i\left(s^t\right)\right)}=\frac{M_i\left(s^t\right)}{M_k\left(s^t\right)}=\frac{\lambda_i+\mu_i\left(s^1\right)+\mu_i\left(s^2\right)+...+\mu_i\left(s^t\right)}{\lambda_k+\mu_k\left(s^1\right)+\mu_k\left(s^2\right)+...+\mu_k\left(s^t\right)}\equiv x_i\left(s^t\right), \tag{9}$$

where $x_i\left(s^t\right)$ can be thought of as the relative Pareto-weight assigned to household $i$ when history $s^t$ has occurred.

The vector of relative weights $x\left(s^t\right)$, with elements $x_i\left(s^t\right)$ defined in (9), can be used as an additional co-state variable in order to rewrite the problem in a recursive form (Marcet and Marimon, 1998). The current income state $s_t$ does not tell us everything we need to know about the past, only $(s_t, x_{t-1})$ does, where $x_{t-1}$ is the relative weight, equal to the ratio of marginal utilities, inherited from the previous period. Denote by $x_t$ the new relative weight to be found at time $t$. The solution consists of policy functions for the consumption allocation and the new relative weight, with support over the extended state space $(s_t, x_{t-1})$. That is, $c_i(s_t, x_{t-1})$, $\forall i$, and $x_t(s_t, x_{t-1})$ are to be determined. At last, the value functions can be defined recursively as

$$V_i(s_t, x_{t-1}) = u(c_i(s_t, x_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1}) V_i(s_{t+1}, x_t(s_t, x_{t-1})). \quad (10)$$

The solution can be fully characterized by a set of state-dependent intervals on the relative weight, or, the ratio of marginal utilities, $x$, that give the possible relative weights in each income state. Denote the interval for state $s$ by $[\underline{x}^s, \overline{x}^s]$. Suppose that last period the ratio of marginal utilities was $x_{t-1}$, and today the income state is $s$. Today's ratio of marginal utilities, $x_t$, is determined by the following updating rule (LTW):

$$x_t = \begin{cases} \overline{x}^s & \text{if} \quad x_{t-1} > \overline{x}^s \\ x_{t-1} & \text{if} \quad x_{t-1} \in [\underline{x}^s, \overline{x}^s] \\ \underline{x}^s & \text{if} \quad x_{t-1} < \underline{x}^s \end{cases} \quad (11)$$

Numerical dynamic programming allows one to solve for the optimal intervals, and thereby the consumption allocation, given the income processes, utility functions and discount rates of the two households. After a sufficient number of periods, the initial relative weight in the social planner's objective only matters if perfect risk sharing is self enforcing (Kocherlakota, 1996).

Let us denote by $Y_i$ ($Y_k$) the distribution from which household $i$'s ($k$'s) income is drawn. Then, one may solve numerically for $x_{it}$ given $(y_{it}, y_{kt}, \xi_{it}, \xi_{kt}, \sigma_{it}, \sigma_{kt}, \delta, Y_i, Y_k, x_{it-1})$. Once we know $x_{it}$, the first order conditions (9) and the resource constraint (6) give the consumption allocation predicted by the model, that I denote $(\widehat{c}_{it}, \widehat{c}_{kt})$.

# 3 Empirical models

Let us first specify the elements of the utility function (1). $\xi_{it}$ is parametrized as a linear function of observables and an unobserved individual effect. In mathematical terms,

$$\xi_{it} = Z_{it}\alpha + \eta_i,$$

where $Z_{it}$ is a vector of observable characteristics of household $i$ at time $t$, $\alpha$ is a parameter vector to be estimated, and $\eta_i$ is the unobservable individual effect. Assume that $\sigma_{it}$ is a linear function of observables, that is,

$$\sigma_{it} = \beta_0 + W_{it}\beta,$$

where $(\beta_0, \beta)$ is a parameter vector to be estimated, and $W_{it}$ represents a vector of observable characteristics of household $i$ at time $t$. Note that preferences are only identified up to a multiplicative constant, thus a normalization is needed. In particular, let us normalize $\beta_0 = 1$, as in Dubois (2000). This particular normalization will prove useful to determine the distribution of the consumption allocation, not just the distribution of marginal utilities.

Assume that consumption is measured with a multiplicative measurement error that is log-normally distributed. Let $c^o_{it}$ denote observed consumption of household $i$ at time $t$, and let $exp\left(\varepsilon_{it}\right)$ be the multiplicative measurement error in household $i$'s consumption at time $t$. Then, one may write

$$c^o_{it} = exp\left(\varepsilon_{it}\right) c_{it},$$

where $\varepsilon_{it}$ is independently and identically distributed (i.i.d.) across households and time, and $\varepsilon_{it} \sim N\left(0, \gamma^2\right)$, where $\gamma^2$ is to be estimated.

I model the consumption allocation $c_t$, for $t = 2, ..., T$, determined by the history of income, the history of household characteristics, and consumption at time 1, $c_1$. In mathematical terms, it is of interest how the following conditional density could be specified based on the above models of risk sharing:

$$D\left(c_T, ..., c_2 \mid c_1, y_T, ..., y_1, Z, W; \alpha, \beta, \delta, \eta, \lambda, \gamma^2, Y\right), \tag{12}$$

where $Z$ and $W$ are stacked matrices of household observables for all households and all $t = 1, ..., T$, and $Y$ is a vector with elements $Y_i$, the discrete distribution from which household $i$'s income is drawn.

To deal with the unobservable, time-constant parameter $\eta$ and the initial Pareto-weights in the social planner's objective, $\lambda$, I first difference. This means trying to explain how the consumption allocation changes from one period to the next. This is similar to the "changes-in-shares" estimator of LTW, the estimator that was found to be the best at capturing how consumption reacts to income.

Household $i$'s consumption is modeled relative to mean consumption in the community. In other words, think of household $i$ sharing risk with the rest of the community, as in LTW. This results in important gains in computation time. The aim here is to explain the allocation of consumption in each community at each time $t$, but not how aggregate consumption changes over time.

I do not deal with aggregation issues related to heterogeneous preferences. Instead, for each household I subtract from each observable in the utility function its community-time mean. This then means that the preferences of a typical, or average agent in the community, household $k$, are described by a simple CRRA utility function, with coefficient of risk aversion equal to 1, that is, $u_k(c_{kt}) = ln(c_{kt})$. Normalize also the Pareto-weight of the household $k$ to 1, that is, $\lambda_k = 1$, and ignore measurement error in mean consumption in the community, that is, $c_{kt}$ is assumed to be well measured. Hereafter, think of explanatory variables in the utility function as deviations from their community-time mean, abusing notation.

## 3.1 Perfect risk sharing

In the case of perfect risk sharing, the current consumption allocation should only depend on current and not past exogenous variables. It does not depend neither on the discount factor, nor on the distribution from which incomes are drawn, and it only depends on today's income realizations through aggregate income. Thus (12) becomes

$$\prod_{t=2,...,T} D\left(c_t \mid c_{t-1}, y_t, Z_t, W_t; \alpha, \beta, \eta, \lambda, \gamma^2\right). \tag{13}$$

To fix ideas, let us consider household $i$ and the average household, $k$. Taking the logarithm of the first order conditions with respect to consumption for these two "households", equation (3), noting that $\sigma_{kt} = 1$, $\xi_{kt} = 0$, and $\lambda_k = 1$, we get

$$ln(\lambda_i) + \xi_{it} - \sigma_{it} ln(c_{it}) = -ln(c_{kt}).$$

Then, replacing for $\xi_{it}$ and $\sigma_{it}$ and rearranging gives

$$ln(c_{it}) - ln(c_{kt}) = -W_{it}\beta ln(c_{it}) + Z_{it}\alpha + \eta_i + ln(\lambda_i). \tag{14}$$

Now, let us take first differences. Doing so and rearranging yields

$$\begin{aligned} ln(c_{it}) - ln(c_{kt}) = {} & ln(c_{it-1}) - ln(c_{kt-1}) - W_{it}\beta ln(c_{it}) \\ & + W_{it-1}\beta ln(c_{it-1}) + Z_{it}\alpha - Z_{it-1}\alpha. \end{aligned} \tag{15}$$

In terms of measured consumption, $c_{it}^o$, (15) reads

$$\begin{aligned} ln(c_{it}^o) - ln(c_{kt}) = {} & ln\left(c_{it-1}^o\right) - ln(c_{kt-1}) - W_{it}\beta ln(c_{it}) + W_{it-1}\beta ln(c_{it-1}) \\ & + Z_{it}\alpha - Z_{it-1}\alpha - (1 + W_{it}\beta)\varepsilon_{it} + (1 + W_{it-1}\beta)\varepsilon_{it-1}. \end{aligned} \tag{16}$$

Let $\mu(\alpha, \beta) = ln\left(c_{it-1}^o\right) - ln(c_{kt-1}) - W_{it}\beta ln(c_{it}) + W_{it-1}\beta ln(c_{it-1}) + Z_{it}\alpha - Z_{it-1}\alpha$, and $\psi^2(\beta, \gamma^2) = \left((1 + W_{it}\beta)^2 + (1 + W_{it-1}\beta)^2\right)\gamma^2$. Then, the right hand side of (16) is distributed as $N(\mu(\alpha, \beta), \psi^2(\beta, \gamma^2))$. Let $d_{it}^{prs}(\alpha, \beta) = ln(c_{it}^o) - ln(c_{kt}) - \mu(\alpha, \beta)$. Now, we may write the likelihood of observation $d_{it}^{prs}(\alpha, \beta)$ as

$$L_{it}^{prs}\left(\alpha, \beta, \gamma^2\right) = \phi\left(\frac{d_{it}^{prs}(\alpha, \beta)}{\psi(\beta, \gamma^2)}\right),$$

where $\phi$ is the density of the standard normal distribution. First differencing introduces correlation between observation $it$ and $it-1$, because $\varepsilon_{it-1}$ appears for both, thus only quasi likelihoods can be written. The quasi log-likelihood function of the data in the perfect risk sharing case is

$$\ell^{prs}\left(\alpha, \beta, \gamma^2\right) = \sum_{t=2}^{T}\sum_{i=1}^{n} ln\left(\phi\left(\frac{d_{it}^{prs}(\alpha, \beta)}{\psi(\beta, \gamma^2)}\right)\right). \tag{17}$$

The parameters to be estimated are the vectors $\alpha$ and $\beta$ and the variance $\gamma^2$. The model is also estimated without preference heterogeneity for comparison. There the only parameter to be estimated is $\gamma^2$.

## 3.2 Autarky

Taking the logarithm of (4) and first differencing gives

$$ln\left(c_{it}\right)-ln\left(c_{it-1}\right)=ln\left(y_{it}\right)-ln\left(y_{it-1}\right),$$

which in terms of observable consumption is

$$ln\left(c_{it}^{o}\right)-ln\left(c_{it-1}^{o}\right)=ln\left(y_{it}\right)-ln\left(y_{it-1}\right)-\varepsilon_{it}+\varepsilon_{it-1}.$$

The consumption of household $i$ relative to mean consumption is

$$\begin{aligned} ln\left(c_{it}^{o}\right)-ln\left(c_{kt}\right)=&\, ln\left(c_{it-1}^{o}\right)-ln\left(c_{kt-1}\right)+ln\left(y_{it}\right)-ln\left(c_{kt}\right) \\ &-ln\left(y_{it-1}\right)+ln\left(c_{kt-1}\right)-\varepsilon_{it}+\varepsilon_{it-1}, \end{aligned}$$

where I have just added and subtracted $ln\left(c_{kt}\right)$ and $ln\left(c_{kt-1}\right)$ to have the same dependent variable in the equation to be estimated as for the other two models.

Let $d_{it}^{aut}=ln\left(c_{it}^{o}\right)-ln\left(c_{it-1}^{o}\right)-\left(ln\left(y_{it}\right)-ln\left(y_{it-1}\right)\right)$. The likelihood of observation $d_{it}^{aut}$ is

$$L_{it}^{aut}\left(\gamma^{2}\right)=\phi\left(\frac{d_{it}^{aut}}{\sqrt{2\gamma^{2}}}\right),$$

and the quasi log-likehood of the data is

$$\ell_{it}^{aut}\left(\gamma^{2}\right)=\sum_{t=2}^{T}\sum_{i=1}^{n}ln\left(\phi\left(\frac{d_{it}^{aut}}{\sqrt{2\gamma^{2}}}\right)\right). \tag{18}$$

The only parameter to be estimated is $\gamma^2$.

## 3.3 Limited commitment

In the limited commitment case, we know that the ratio of marginal utilities from last period, the co-state variable in the recursive version of the model, that I have denoted $x_{t-1}$, is a sufficient statistic for everything that happened in the past, including the initial condition. In other words, instead of conditioning on the whole history of income realizations, $y^t$ and the initial Pareto-weights in the social planner's objective, $\lambda$, it is sufficient to condition on $x_{t-1}$ and the current income realizations, $y_t$. However, unlike in the perfect risk sharing case, the consumption allocation depends also on the discount factor, $\delta$, and the distribution from which incomes are drawn, $Y$. Thus (12) becomes

$$\prod_{t=2,\ldots,T} D\left(c_t \mid c_{t-1}, y_t, Z_t, W_t; \alpha, \beta, \delta, \eta, x_{t-1}, \gamma^2, Y\right), \tag{19}$$

where $x_{t-1}$ ha elements $x_{it-1}$, which can be thought of as the relative weight of household $i$ with respect to the average household $k$ at time $t$.

Instead of (14), the first order condition can now be written as

$$ln\left(c_{it}\right) - ln\left(c_{kt}\right) = -W_{it}\beta ln\left(c_{it}\right) + Z_{it}\alpha + \eta_i + ln\left(x_{it}\right), \tag{20}$$

where I have just replaced $x_{it}$ for $\lambda_i$. Two questions now arise. First, how $x_{it}$ depends on $x_{it-1}$, the current exogenous variables, and the parameters of the model. Second, what $x_{it-1}$ is empirically.

According to the risk sharing with limited commitment model, given preferences and current income realizations, one can solve numerically for $x_{it}$, $\forall i$, see (11). Let the function $f()$ denote this relationship, that is, $x_{it} = f\left(x_{it-1}\right)$, where I do not make explicit the dependence on observables, unobservables, and parameters. The main econometric issue is that unobservables, namely, individual effects and measurement error influence the updating of the state variable, that is, they appear in $f\left(x_{it-1}\right)$.

Let us take first differences of (20). Then, replacing for $x_{it}$ and rearranging gives

$$\begin{aligned} ln\left(c_{it}\right) - ln\left(c_{kt}\right) = {} & ln\left(c_{it-1}\right) - ln\left(c_{kt-1}\right) - W_{it}\beta ln\left(c_{it}\right) + W_{it-1}\beta ln\left(c_{it-1}\right) \\ & + Z_{it}\alpha - Z_{it-1}\alpha + ln\left(f\left(x_{it-1}\right)\right) - ln\left(x_{it-1}\right). \end{aligned} \tag{21}$$

Instead of $x_{it-1}$, the econometrician only observes

$$x^o_{it-1} = \frac{\left(c^o_{it-1}\right)^{1+W_{it-1}\beta}}{exp\left(Z_{it-1}\alpha\right) c_{kt-1}} = \frac{\left(exp\left(\varepsilon_{it-1}\right) c_{it-1}\right)^{1+W_{it-1}\beta}}{exp\left(Z_{it-1}\alpha\right) c_{kt-1}} = \left(exp\left(\varepsilon_{it-1}\right)\right)^{1+W_{it-1}\beta} exp\left(\eta_i\right) x_{it-1}.$$

Then, (21) becomes

$$\begin{aligned} ln\left(c^o_{it}\right) - ln\left(c_{kt}\right) = {} & ln\left(c^o_{it-1}\right) - ln\left(c_{kt-1}\right) - W_{it}\beta ln\left(c^o_{it}\right) + W_{it-1}\beta ln\left(c^o_{it-1}\right) \\ & + Z_{it}\alpha - Z_{it-1}\alpha + ln\left(f\left(exp\left(-\eta_i\right)\left(exp\left(\varepsilon_{it-1}\right)\right)^{-1-W_{it-1}\beta} x^o_{it-1}\right)\right) \\ & - ln\left(x^o_{it-1}\right) + \eta_i + \left(1 + W_{it}\beta\right)\varepsilon_{it}. \end{aligned} \tag{22}$$

What remains to be determined is how to compute $f\left(exp\left(-\eta_i\right)\left(exp\left(\varepsilon_{it-1}\right)\right)^{-1-W_{it-1}\beta}x^o_{it-1}\right)$, that depends on the unobserved individual effects and measurement error at time $t-1$. Let us first consider measurement error. Suppose that the realization of the measurement error at time $t-1$ is $\widetilde{\varepsilon}_{it-1}$. That is, I draw some $\widetilde{\varepsilon}_{it-1}$ from the distribution of $\varepsilon_{it-1}$, $N\left(0,\gamma^2\right)$, and write the likelihood conditional on $\widetilde{\varepsilon}_{it-1}$ first. Then, averaging the likelihood over some number of draws, the simulated maximum likelihood estimator can be used.

To deal with the individual effects, I show that one may write

$$f\left(x_{it-1}\right)=f\left(exp\left(-\eta_i\right)\left(exp\left(\widetilde{\varepsilon}_{it-1}\right)\right)^{-1-W_{it-1}\beta}x^o_{it-1}\right)=exp\left(-\eta_i\right)f^o\left(\left(exp\left(\widetilde{\varepsilon}_{it-1}\right)\right)^{-1-W_{it-1}\beta}x^o_{it-1}\right),$$

where $f^o()$ gives the updating rule "forgetting" about the fact that the ratio of marginal utilities is influenced by the individual effect both at time $t$ and $t-1$. To see this, let us take a closer look at the enforcement constraint of some household $i$, that can be written as (7) in general. Replacing the utility function (1) in (7), with $\xi_{it}=Z_{it}\alpha+\eta_i$ and $\sigma_{it}=1+W_{it}\beta$, gives

$$\begin{aligned} & exp\left(Z_{it}\alpha+\eta_i\right)\frac{c_{it}^{-W_{it}\beta}-1}{-W_{it}\beta}+\sum_{r=t+1}^{\infty}\sum_{s_r}\delta^{r-t}\pi\left(s_r\right)exp\left(Z_{ir}\alpha+\eta_i\right)\frac{c_{ir}\left(s_r\right)^{-W_{ir}\beta}-1}{-W_{ir}\beta}\geq \\ & \geq exp\left(Z_{it}\alpha+\eta_i\right)\frac{y_{it}^{-W_{it}\beta}-1}{-W_{it}\beta}+\sum_{r=t+1}^{\infty}\sum_{s_r}\delta^{r-t}\pi\left(s_r\right)exp\left(Z_{ir}\alpha+\eta_i\right)\frac{y_{ir}\left(s_r\right)^{-W_{ir}\beta}-1}{-W_{ir}\beta}. \end{aligned} \quad (23)$$

Both sides can be divided by $exp\left(\eta_i\right)$, thereby eliminating the individual effects. Denote the consumption allocation predicted by the model by $\left(\widehat{c}_{it}\left(\widetilde{\varepsilon}_{it-1}\right),\widehat{c}_{kt}\left(\widetilde{\varepsilon}_{it-1}\right)\right)$, where I do not make explicit the dependence on parameters and observables. We have seen that it is independent of the individual effects, both when some enforcement constraints are binding and when none is binding.[1] Thus

$$x_{it}=f\left(x_{it-1}\right)=exp\left(-\eta_i\right)\frac{\widehat{c}_{it}\left(\alpha,\beta,\delta,\widetilde{\varepsilon}_{it-1},x^o_{it-1}\right)^{1+W_{it}\beta}}{exp\left(Z_{it}\alpha\right)\widehat{c}_{kt}\left(\alpha,\beta,\delta,\widetilde{\varepsilon}_{it-1},x^o_{it-1}\right)}.$$

[1] In this later case, we are back to perfect risk sharing, where $exp\left(\eta_i\right)$ appears multiplicatively on both sides $x_{it}=x_{it-1}$.

Then, (22) becomes

$$\begin{aligned} ln\left(c^o_{it}\right) - ln\left(c_{kt}\right) = ln\left(c^o_{it-1}\right) - ln\left(c_{kt-1}\right) - W_{it}\beta ln\left(c^o_{it}\right) + W_{it-1}\beta ln\left(c^o_{it-1}\right) \\ + Z_{it}\alpha - Z_{it-1}\alpha + ln\left(f^o\left(\left(exp\left(\widetilde{\varepsilon}_{it-1}\right)\right)^{-1-W_{it-1}\beta} x^o_{it-1}\right)\right) \\ - ln\left(x^o_{it-1}\right) + \left(1 + W_{it}\beta\right)\varepsilon_{it}. \end{aligned} \tag{24}$$

Finally, assume that, when determining the informal insurance contract, households' observable characteristics (relative to the community average) are not expected to change, that is, $Z_{ir} = Z_{it}$ and $W_{ir} = W_{it}$, $\forall r > t$, in (23). This is admittedly a strong assumption. While one could model how household size, for example, is expected to evolve over time, given today's household structure, this would increase the state space and thereby computation time a lot. The enforcement constraint for household $i$ at time $t$, that the predicted consumption allocation has to satisfy, can now be written as

$$\begin{aligned} \frac{\widehat{c}_{it}^{-W_{it}\beta} - 1}{-W_{it}\beta} + \sum_{r=t+1}^{\infty}\sum_{s_r}\delta^{r-t}\pi\left(s_r\right)\frac{\widehat{c}_{ir}\left(s_r\right)^{-W_{it}\beta} - 1}{-W_{it}\beta} \geq \\ \geq \frac{y_{it}^{-W_{it}\beta} - 1}{-W_{it}\beta} + \sum_{r=t+1}^{\infty}\sum_{s_r}\delta^{r-t}\pi\left(s_r\right)\frac{y_{ir}\left(s_r\right)^{-W_{it}\beta} - 1}{-W_{it}\beta}, \end{aligned} \tag{25}$$

which is to be used in the numerical solution of the model.

The estimation is done in three steps. A preliminary step involves estimating the distribution from which income is drawn, $Y_i$, for each household $i$. Then, the inside optimization computes the consumption allocation predicted by the model, given observable covariates and parameters. Finally, the log-likelihood is maximized over the remaining parameters, $\alpha$, $\beta$, $\delta$, and $\gamma^2$. Now I turn to the details of each of these steps.

First, the discrete distribution from which income is drawn, $Y_i$, $\forall i$, has to be estimated. This cannot be done in general, because the time dimension of the panel is not large enough (the sample size is maximum 12 for each household). Thus, I assume that all households face the same multiplicative risk. In particular, I divide by individual mean income for each household, then I discretize the multiplicative risk by creating 6 quantiles. The income state inputted to the model when computing the consumption of household $i$ is the mean in each quantile multiplied by the household's mean income. Household $k$ is assumed to have mean income equal to that of the community.

Second, one has to solve the inside optimization to find the consumption allocation predicted my the model. With measurement error, this step first involves drawing a realization, denoted $\widetilde{\varepsilon}_{it-1}$. The algorithm to solve for the constrained-efficient risk sharing contract, given the realization $\widetilde{\varepsilon}_{it-1}$, observables, and parameters, does not impose much additional difficulty relative to the case without preference heterogeneity, except for computational time. The Bellman equation (10) is solved by iteration. A 31-point grid is defined over the continuous state variable $x_t$, $\forall t$. At iteration $h$, one solves for the new consumption values in states where an enforcement constraint is binding using (25) with equality, while the ratio of marginal utilities stays constant in other states. The consumption values from iteration $h-1$ are kept for $\widehat{c}_{ir}\left(s_r\right)$, $\forall s_r$, $\forall r > t$ in (25). At the first iteration the consumption values of perfect risk sharing can be used. I keep iterating until the policy function converges, that is, the optimal intervals on $x_{it}$, $\forall i$, do not change[2], but allow for maximum 20 iterations only. This step leads to the predicted consumption values, $\left(\widehat{c}_{it}\left(\widetilde{\varepsilon}_{it-1}\right), \widehat{c}_{kt}\left(\widetilde{\varepsilon}_{it-1}\right)\right)$, as a function of the preference parameters, $\alpha$, $\beta$, and $\delta$, and the data. Then we also have

$$f^o\left(\left(exp\left(\widetilde{\varepsilon}_{it-1}\right)\right)^{-1-W_{it-1}\beta} x^o_{it-1}\right) = exp\left(-Z_{it}\alpha\right) \frac{\widehat{c}_{it}\left(\widetilde{\varepsilon}_{it-1}\right)^{1+W_{it}\beta}}{\widehat{c}_{kt}\left(\widetilde{\varepsilon}_{it-1}\right)},$$

to be replaced in (24). Doing so yields

$$\begin{aligned} ln\left(c^o_{it}\right) - ln\left(c_{kt}\right) = {} & ln\left(c^o_{it-1}\right) - ln\left(c_{kt-1}\right) - W_{it}\beta ln\left(c^o_{it}\right) + W_{it-1}\beta ln\left(c^o_{it-1}\right) - Z_{it-1}\alpha \\ & + ln\left(\frac{\widehat{c}_{it}\left(\widetilde{\varepsilon}_{it-1}\right)^{1+W_{it}\beta}}{\widehat{c}_{kt}\left(\widetilde{\varepsilon}_{it-1}\right)}\right) - ln\left(x^o_{it-1}\right) - \left(1 + W_{it}\beta\right)\varepsilon_{it}. \end{aligned} \tag{26}$$

Now, let us turn to the third and last step, the outside optimization. This involves maximizing the (quasi) log-likelihood of the data. To write the likelihood of an observation

[2]More precisely, the solution has been found, if the length of the difference between the endpoints of the optimal intervals at iteration $h-1$ and $h$ is less than 0.001.

$it$ given $\widetilde{\varepsilon}_{it-1}$, let

$$\mu^{lc}\left(\widetilde{\varepsilon}_{it-1},\alpha,\beta,\delta,\gamma^2\right)=ln\left(c^o_{it-1}\right)-ln\left(c_{kt-1}\right)-W_{it}\beta ln\left(c^o_{it}\right)+W_{it-1}\beta ln\left(c^o_{it-1}\right)$$
$$-Z_{it-1}\alpha+ln\left(\frac{\widehat{c}_{it}\left(\widetilde{\varepsilon}_{it-1}\right)^{1+W_{it}\beta}}{\widehat{c}_{kt}\left(\widetilde{\varepsilon}_{it-1}\right)}\right)-ln\left(x^o_{it-1}\right),$$

and $\psi^{lc}\left(\beta,\gamma^2\right)=\left(1+W_{it}\beta\right)\gamma$. Then, the right hand side of (26) is distributed as

$$N\left(\mu^{lc}\left(\widetilde{\varepsilon}_{it-1},\alpha,\beta,\delta,\gamma^2\right),\psi^{lc}\left(\beta,\gamma^2\right)^2\right).$$

Let $d^{lc}_{it}\left(\widetilde{\varepsilon}_{it-1},\alpha,\beta,\delta,\gamma^2\right)=ln\left(c^o_{it}\right)-ln\left(c_{kt}\right)-\mu^{lc}\left(\widetilde{\varepsilon}_{it-1},\alpha,\beta,\delta,\gamma^2\right)$. The likelihood of observation $d^{lc}_{it}\left(\widetilde{\varepsilon}_{it-1},\alpha,\beta,\delta,\gamma^2\right)$ is

$$L^{lc}_{it}\left(\alpha,\beta,\delta,\gamma^2\mid\widetilde{\varepsilon}_{it-1}\right)=\phi\left(\frac{d^{lc}_{it}\left(\widetilde{\varepsilon}_{it-1},\alpha,\beta,\delta,\gamma^2\right)}{\psi^{lc}\left(\beta,\gamma^2\right)}\right),$$

where $\phi$ is the density of the standard normal distribution. Making $J$ draws for $\widetilde{\varepsilon}_{it-1}$, the simulated quasi log-likelihood function of the data is

$$\ell^{lc}\left(\alpha,\beta,\delta,\gamma^2\right)=\sum_{t=2}^{T}\sum_{i=1}^{n}ln\left(\frac{1}{J}\sum_{j=1}^{J}\phi\left(\frac{d^{lc}_{it}\left(\widetilde{\varepsilon}^j_{it-1},\alpha,\beta,\delta,\gamma^2\right)}{\psi^{lc}\left(\beta,\gamma^2\right)}\right)\right), \tag{27}$$

where $\widetilde{\varepsilon}^j_{it-1}$ denotes the $j^{\text{th}}$ draw. In practice I take $J=10$.

The parameters to be estimated are the vectors $\alpha$ and $\beta$, the discount factor $\delta$, and the variance $\gamma^2$. For comparison, the model is also estimated without preference heterogeneity. There only $\delta$ and $\gamma^2$ are to be estimated. The outside optimization over the preference parameters $(\alpha,\beta,\delta)$ and $\gamma^2$ is done by the function "mle" available in R, that uses a standard optimization algorithm, namely BFGS with bounds ("L-BFGS-B"), which is a quasi-Newton method.

# 4 Data

The data comes from an income-consumption survey conducted by the International Food Policy Research Institute (IFPRI) in rural Pakistan between July 1986 and September 1989. Almost 1000 households have been interviewed over 12 rounds in 46 villages in 4 districts of Pakistan. The districts were not chosen randomly: 3 are the least-developed districts in their respective provinces (Attock in Punjab, Badin in Sind, and Dir in North-West Frontier Province), while the $4^{th}$ is a more prosperous district (Faisalabad in Punjab). Then, in each district, two markets were chosen, and villages were randomly selected from a stratified sample based on distance from these markets. Finally, households were chosen randomly within each village. Attrition seems to be due to administrative problems, and not households' self selection, and I assume that attrition is random. In each household, both the male and female heads were interviewed. In addition, village questionnaires were also administered, that give information about prices, for example. For further details, see Alderman and Garcia (1993).

This dataset is attractive for the purposes of the present paper for several reasons. First of all, both the cross-sectional and the time dimension is relatively big compared to other similar datasets. Second, one can examine both small and larger communities (villages and districts). Further, consumption data was collected from the female head of the household, while income data from the male head. Thus the assumption that measurement error in consumption is independent of the income measure is more compelling than usual.

## 4.1 Variables used

For the purposes of this paper, I need measures of consumption, income, and some household characteristics and wealth. For consumption, I consider two different measures, (i) food consumption, and (ii) nondurable consumption, which is constructed as the sum of food consumption, expenditures on clothing, hygiene items, tobacco, and cinema.

Income is constructed as the sum of net income from crop production, net income from poultry and livestock, net income from craft work, net income from produce from orchards, income from assets (hiring out bullocks, tractor, thresher, land, income from mills owned, and selling water), wage income minus wages paid for hired labor (that is not used in agricultural production), transfers from the state, and transfers from abroad, that is, all transfers from outside of the community (transfers from within Pakistan from friends, relatives, or religious organizations, typically the local mosque, are thus excluded). Medical expenditures and education investment are subtracted (as in Gourinchas and Parker, 2002).

Table 1: Summary statistics

| Variable | Mean | Sd | Min | Max |
|---|---|---|---|---|
| Food consumption (rupees per week) | 368.8 | 185.4 | 50.6 | 2926.7 |
| Log(food consumption) | 5.798 | 0.4807 | 3.924 | 7.982 |
| nondurable consumption (rupees per week) | 872.4 | 758.8 | 54.2 | 9839.4 |
| Log(nondurable consumption) | 6.508 | 0.7146 | 3.992 | 9.194 |
| Income (rupees per week) | 218.4 | 627.7 | −1574.6 | 7242.7 |
| Log(income>0) | 5.077 | 1.559 | NA | 8.888 |
| Household size | 9.11 | 4.11 | 1 | 25 |
| Log(household size) | 2.11 | 0.456 | 0 | 3.22 |
| Proportion of children | 0.357 | 0.197 | 0 | 0.818 |
| Age of head | 47.79 | 13.88 | 16 | 87 |
| Land owned (acres) | 10.20 | 22.93 | 0 | 200 |
| Log(land owned+1) | 1.40 | 1.34 | 0 | 5.30 |
| Illness (days per week) | 0.183 | 0.680 | 0 | 14.4 |

I consider five variables in specifying preference heterogeneity, namely, household size ("hhsize"), the proportion of children under 12 ("propch"), age of the head of the household ("age"), (the logarithm of ) acres of land owned at time 1 as a measure of wealth ("land"), and days of illness, more precisely, average number of days of work per week lost due to illness by adult members of the household ("ill").

I delete an observation *it* if consumption is missing, income is missing, or any income component is outside some reasonable range. I delete the household if mean income over the 12 periods is negative. I also delete households whose size is bigger than 25 in any period, whose head is below 16 years of age, or if the head changes over the three-year period of the interviews. Thereafter I am left with a sample size of 654 households and 7133 observations. Table 1 presents summary statistics.

For the structural estimation, I further delete the 1% highest consumption and income observations by district. Further, when logarithms are taken, negative income observations cannot be used. Finally, aggregate income should be equal to aggregate consumption in the community, since savings have been assumed away. To achieve this, income is rescaled (divided by mean income and multiplied by mean consumption in the community) so that aggregate income be equal to aggregate consumption at each $t$.

## 4.2 Existing evidence

Perfect risk sharing has been tested and rejected using the present dataset. Dubois (2000) constructs a nondirectional test of perfect risk sharing based on overidentifying restrictions implied by the model. Allowing for preference heterogeneity he is able to reject that households share risk perfectly. He also shows that sharecropping contracts are used to achieve better insurance. Dubois, Jullien, and Magnac (2008) also provide evidence that perfect risk sharing is not achieved, and both short-term formal and informal insurance contracts are important.

Ogaki and Zhang (2001) use both the ICRISAT Indian data and the dataset from Pakistan used in this paper. They argue that earlier tests of perfect risk sharing do not take into account the possibility of decreasing relative risk aversion. In other words, the coefficient of relative risk aversion may depend on wealth. Taking this heterogeneity into account, the authors cannot reject perfect risk sharing for the vast majority of villages examined. The present paper allows risk aversion to depend on a wider range of observables, and compares the perfect risk sharing model to a well-specified alternative, namely the model of risk sharing with limited commitment.

# 5 Structural estimation results

This section first looks at food consumption, then, in subsection 5.2, at nondurable consumption. Both subsections first detail the structural estimation of the perfect risk sharing model, both with and without preference heterogeneity depending on observables. Then, the benchmark of autarky is estimated. Afterwards, both subsections deal with the model of risk sharing with limited commitment, with and without allowing for heterogeneity in risk preferences. Finally, Vuong's (1989) test is performed to statistically compare the five models.

Let us specify

$$Z_{it}\alpha = \alpha \mathrm{ill}_{it},$$

and

$$W_{it}\beta = \beta_1 log\left(\text{hhsize}_{it}\right) + \beta_2\text{propch}_{it} + \beta_3\text{age}_i + \beta_4 log\left(\text{land}_i + 1\right),$$

where $log\left(\text{hhsize}_{it}\right)$ aims to measure risk sharing opportunities already present within bigger household, and $log\left(\text{land}_i + 1\right)$ controls for self-insurance possibilities of wealthier households. Thus I expect both $\beta_1$ and $\beta_4$ to be negative. Households with more children should be more risk averse, since children are more vulnerable, and their long-term life prospects can be greatly affected by shocks, thus I expect $\beta_2$ to be positive. The effect of age could go either way.

The community is taken to be a district to keep sample size and statistical power higher, therefore I report results for four districts. The computations have been done using the software R, see www.r-project.org. In all the tables of this section standard errors are in parentheses, and $^{*}$ indicates significance at the 10% level, $^{**}$ at 5%, and $^{***}$ at 1%.

## 5.1 Food consumption

Let us first consider food consumption. Table 2 shows the results for the perfect risk sharing model with preference heterogeneity. Equation (16) is estimated by maximizing the log-likelihood function (17). For comparison, the model of perfect risk sharing without preference heterogeneity is also estimated. This means restricting $\alpha = \beta = 0$ in (16). Note that individual effects are still accommodated by the model. The results are shown in Table 3. The benchmark model of autarky is also estimated maximizing the log-likelihood function (18). Table 4 shows the results. Then, Tables 5 and 6 contain the estimation results for the risk sharing with limited commitment model, with and without preference heterogeneity, respectively.

## 5.2 Nondurable consumption

This subsection presents the estimation results for nondurable consumption. Tables 7 and 8 deal with the perfect risk sharing model, Table 9 with autarky, and Tables 10 and 11 show the results for the model of risk sharing with limited commitment.

Table 2: Perfect risk sharing with preference heterogeneity, food consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| $\alpha$ | 0.0420 | 0.1532 | $-0.0098$ | $-0.0278$ |
| | (0.0572) | (0.1316) | (0.0427) | (0.0238) |
| $\beta_1$ | $-0.0698^{*}$ | 0.0368 | $-0.0431$ | 0.0057 |
| | (0.0413) | (0.0240) | (0.0369) | (0.0188) |
| $\beta_2$ | 0.0153 | $-0.0341$ | $-0.0188$ | 0.0528 |
| | (0.0630) | (0.0466) | (0.0544) | (0.0325) |
| $\beta_3$ | 0.0028 | $0.0221^{***}$ | $0.0035^{**}$ | $-0.0004$ |
| | (0.0035) | (0.0016) | (0.0017) | (0.0013) |
| $\beta_4$ | $-0.0271^{*}$ | $-0.0545^{***}$ | $-0.0505^{***}$ | $-0.0421^{**}$ |
| | (0.0165) | (0.0082) | (0.0162) | (0.0188) |
| $\gamma^2$ | $0.0857^{***}$ | $0.0858^{***}$ | $0.0571^{***}$ | $0.0319^{***}$ |
| | (0.0046) | (0.0048) | (0.0029) | (0.0014) |
| Log-likelihood | $-369.5$ | $-342.5$ | $-262.7$ | $-43.7$ |
| N | 826 | 854 | 965 | 1240 |

Table 3: Perfect risk sharing without preference heterogeneity, food consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| $\gamma^2$ | 0.0871 | 0.0837 | 0.0597 | 0.0343 |
| | (0.0047) | (0.0044) | (0.0030) | (0.0015) |
| Log-likelihood | $-857.0$ | $-865.1$ | $-836.6$ | $-813.2$ |
| N | 826 | 854 | 965 | 1240 |

Table 4: Autarky, food consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| $\gamma^2$ | $1.396^{***}$ | $1.298^{***}$ | $1.458^{***}$ | $1.281^{***}$ |
| | (0.0750) | (0.0689) | (0.0730) | (0.0558) |
| Log-likelihood | $-1337.1$ | $-1346.2$ | $-1557.3$ | $-1989.5$ |
| N | 826 | 854 | 965 | 1240 |

Table 5: Limited commitment with preference heterogeneity, food consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| to be computed for the current specification | | | | |

Table 6: Limited commitment without preference heterogeneity, food consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| to be computed for the current specification | | | | |

Table 7: Perfect risk sharing with preference heterogeneity, nondurable consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| $\alpha$ | $-0.1243$ | $-0.2236$ | $-0.0747$ | $-0.0332$ |
| | (0.0804) | (0.2045) | (0.1178) | (0.0388) |
| $\beta_1$ | $-0.1656^{***}$ | $-0.0707^{*}$ | 0.0555 | $-0.0495^{*}$ |
| | (0.0475) | (0.0374) | (0.0944) | (0.0278) |
| $\beta_2$ | 0.0006 | $0.1969^{***}$ | 0.0533 | 0.0664 |
| | (0.0787) | (0.0709) | (0.1285) | (0.0488) |
| $\beta_3$ | 0.0037 | $0.0203^{***}$ | $-0.0001$ | $-0.0056^{***}$ |
| | (0.0027) | (0.0016) | (0.0055) | (0.0016) |
| $\beta_4$ | $-0.0357^{**}$ | $-0.0373$ | $-0.0288^{***}$ | $-0.0691^{***}$ |
| | (0.0168) | (0.0125) | (0.0713) | (0.0199) |
| $\gamma^2$ | $0.1800^{***}$ | $0.2029^{***}$ | $0.2680^{***}$ | $0.0943^{***}$ |
| | (0.0097) | (0.0113) | (0.0160) | (0.0041) |
| Log-likelihood | $-622.7$ | $-657.2$ | $-632.2$ | $-615.1$ |
| N | 825 | 854 | 688 | 1253 |

Table 8: Perfect risk sharing without preference heterogeneity, nondurable consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| $\gamma^2$ | $0.2010^{***}$ | $0.2052^{***}$ | $0.2740^{***}$ | $0.1147^{***}$ |
| | (0.0108) | (0.0109) | (0.0162) | (0.0050) |
| Log-likelihood | $-1144.6$ | $-1183.4$ | $-1032.5$ | $-1465.2$ |
| N | 825 | 854 | 688 | 1253 |

Table 9: Autarky, nondurable consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| $\gamma^2$ | $1.427^{***}$ | $1.352^{***}$ | $1.602^{***}$ | $1.316^{***}$ |
| | (0.0768) | (0.0718) | (0.0949) | (0.0570) |
| Log-likelihood | $-1342.9$ | $-1360.6$ | $-1140.6$ | $-2026.3$ |
| N | 825 | 854 | 688 | 1253 |

Table 10: Limited commitment with preference heterogeneity, nondurable consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| to be computed for the current specification | | | | |

Table 11: Limited commitment without preference heterogeneity, nondurable consumption

| District | Faisalabad | Attock | Badin | Dir |
|---|---|---|---|---|
| to be computed for the current specification | | | | |

# 6 Concluding remarks

This paper has aimed to perform statistical tests to distinguish between five models of risk sharing, namely, perfect risk sharing with and without preference heterogeneity, no risk sharing, or, autarky, and risk sharing with limited commitment with and without preference heterogeneity. Preliminary results suggests that both heterogeneity in preferences and limitations to the enforcement of informal insurance contracts are important.

This research on the structural modeling of the consumption allocation between households in poor communities can serve as an input for policy design. The effects of redistributive policies, or micro-insurance programs could be simulated. Thus policy makers and members of non-governmental organizations could have a better understanding of the effects of programs taking into account existing informal insurance arrangements.

A next step could be to incorporate other models into the analysis, for example, the model of risk sharing with private information (Wang, 1995). Another important extension in future work would be to allow for savings, as in Ligon, Thomas, and Worrall (2000).

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