Trends in earnings inequality among Canadian men, 1985–2005

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Abstract

I consider two flexible models of earnings dynamics suggested in recent literature and alternative approaches to the treatment of left-censored observations to examine trends in the permanent and transitory variances of earnings of Canadian male workers from 1985 to 2005. I find that both permanent and transitory variances were higher in the 2000s than in the late 1980s or 1990s. In contrast to the late 1980s and the recession period of the early 1990s, both components of variance grew at a similar pace during the postrecession period, and the share of each component in the total variance remained fairly stable. The results are robust to the choice of a model. The study is based on a large sample from a uniquely rich longitudinal administrative dataset.

Keywords: income distribution, earnings inequality, earnings instability, permanent-transitory decomposition, GMM

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1 Introduction

Numerous studies have examined trends in earnings inequality during the past two decades, leading to a much improved understanding of the changes in the earnings structure amid rapidly changing social and economic conditions. These studies have also highlighted the limitations of traditional methods used in the analysis of inequality and the related assumptions that such methods often require. One of the most common assumptions in most such studies is that the changes in the dispersion of annual earnings reflect changes in the dispersion of *permanent* earnings; therefore, the growth in cross-sectional earnings inequality is often interpreted as the growth in long-term earnings inequality. Such an assumption is often made out of necessity: most inequality studies are based on cross-sectional data or use conventional cross-sectional measures of inequality, such as the Gini coefficient, coefficient of variation, or percentile ratios derived from cross-sectional distributions of earnings. Although informative in many ways, this approach ignores the possibility that the changes in the dispersion of annual earnings may—at least partially—be explained by the changes in *transitory* earnings (or changes in the magnitude of yearly deviations from the long-run earnings), which are usually associated with changes in earnings *instability* rather than inequality. The distinction between the variations in permanent and transitory earnings is essential: whereas the former is directly related to the demand for and supply of human capital, the latter is a reflection of market stability. The volatility of transitory earnings may increase, for instance, if there is a market shift toward (i) temporary, part-time, or seasonal employment, (ii) increase in global competition, or (iii) change in industry regulations.

Recently, studies by Gottschalk and Moffitt (1994), Moffitt and Gottschalk (2002, 2008), Haider (2001), Baker and Solon (2003), Shin and Solon (2008) emphasize the importance of the dynamic approach to the analysis of earnings inequality. These studies analyze changes in the annual variance of earnings using flexible dynamic functional forms of earnings and models that distinguish between persistent and transitory variance components. In a study based on employers' reports obtained from the *Canada Revenue Agency*, Baker and Solon (2003) (henceforth, BAS) develop a particularly flexible model that incorporates several important aspects of earnings dynamics and apply it to the analysis of male earnings inequality in Canada from 1976 to 1992. Apart from making an important methodological contribution to literature, BAS show that the upward trends in both permanent and transitory variance components contributed to the growth in the annual earnings variance during that period and that the increase in the permanent variance was proportionally larger than the increase in the transitory variance. Their findings provide valuable insights into the dynamics of both variance components during the business cycles, reaffirm the substantial role of the transitory variance in the total earnings variance, and highlight important differences and similarities between the trends in variances of male earnings in Canada and the US in the 1980s and early 1990s.

Although BAS offer a very thorough and thoughtful analysis of the earnings dynamics among Canadian men, one important shortcoming of the study is that the analysis in it does not extend beyond 1992. As Figure1 based on administrative records shows, after a sharp increase during the recession years of the early 1990s, the variance of annual earnings remained high in the postrecession years and continued to grow during the late 1990s and early 2000s, albeit at a slower pace than in the early 1990s. Other studies report similar trends[2]. Therefore, considering the strong evidence that the dispersion of the annual earnings of male workers continued to grow

[2] There is a general consensus that the cross-sectional earnings inequality among individuals and families in Canada was substantially higher in the 1990s and early 2000s compared to the previous decades (Card and Lemieux, 2001; Beach, Finnie, and Gray, 2003; Card, Lemieux, and Riddell, 2004; Johnson and Kuhn, 2004; Morissette and Ostrovsky, 2005; Frenette, Green, and Picot, 2006; Boudarbat, Lemieux, and Riddell, 2006; Frenette, Green, and Milligan, 2007). Some studies are based on administrative data (Beach et al., 2003; Morissette and Ostrovsky, 2005), whereas others report similar trends using other data sources, such as the Survey of Consumer Finances and Census (Frenette *et al.,* 2006, 2007).

in the 1990s and 2000s, the question regarding whether such a trend reflects changes in the distribution of permanent or transitory earnings remains pertinent and warrants further examination.

The contribution of this study to the current literature is twofold. First, I reexamine earnings inequality and earnings instability among Canadian males from the 'permanent vs. transitory' point of view and extend the analysis in BAS (2003) to the period from 1985 to 2005. An essential goal of the analysis is to create a continuous profile of earnings inequality in Canada for the period from 1976 to 2005. To that end, I use sample selection criteria similar to the ones reported in BAS and apply them to the period from 1985 to 2005. Our analysis samples overlap in 1985-1992; consequently, the results corresponding to this period are directly comparable. Second, although the BAS model is one of the most flexible models in literature, I consider an alternative model specification suggested by Moffitt and Gottschalk (henceforth, MG) (2008), who argue that the conventional treatment of age-specific first-year transitory variances as parameters may be problematic because it implies a "built-in" time-trend in the estimated variances and covariances. MG suggest an alternative treatment of left-censored observations based on the approximation of the transitory component for the period before the year in which they are first observed. The results from such models are compared to the results obtained using the original BAS specification.

It should also be noted that my analysis is based on a truly unique longitudinal administrative dataset that spans a period of almost 25 years and contains ample income data. The very high tax-filing rates, thanks to strong filing incentives in Canada, and the accuracy of information related to earnings make administrative data an attractive alternative to survey data in studies of inequality in male earnings (Frenette *et al.,* 2006). My sample size is much larger than the sample size in most similar studies, including that in BAS, which should also lead to more precise estimates.

I find that the increase in annual earnings inequality from 1985 to 2005 resulted from increases in both permanent and transitory components, although, in terms of levels, the transitory component was much smaller in magnitude. Both permanent and transitory variances were higher in the 2000s than they were in the late 1980s or 1990s. Overall, it appears that, in contrast to the late 1980s and early 1990s, the dynamics of earnings inequality and earnings instability in the postrecession years were fairly similar. I also find that the estimation results are robust to the choice of a specification and that both the original BAS model and the alternative Moffitt-Gottschalk specification yield very similar results.

2 Data and sample

I have used data from the Statistics Canada's Longitudinal Administrative Databank (LAD), which is a random 20% sample of all Canadian tax-filers who filed tax returns between 1982 and 2006 (the last available year at the time of writing) and their families. There are no age restrictions; and once selected, individuals are in the LAD whenever they file a tax return. Each year, the LAD is augmented with new tax-filers to remain representative of 20% of all tax-filers for that year. To maintain the LAD up to date, a part of each year's sample consists of individuals who file their returns for the first time. Such a selection scheme allows annual increases in the LAD to parallel the annual increases in the Canadian population. A unique LAD identification number generated from the Social Insurance Number makes it possible to link individuals (and families) over time to construct longitudinal profiles[3].

[3] Detailed information about LAD is available at http://www.statcan.gc.ca/bsolc/olc-cel/olc-cel?lang=eng&catno=12-585-X.

The variable ‘earnings’ in the study is constructed as a sum of two LAD variables: T4E__I and OEI__I. The former is the total employment income from T4 slips[4], which includes all paid-employment incomes, such as wages, salaries, and commissions, before deductions. The latter variable is the ‘other employment incomes’, which includes any taxable receipts from employments excluding wages, salaries, and commissions, such as tips, gratuities, and director fees. This definition of earnings is consistent with the definition of earnings in the BAS study and other studies of earnings dynamics that use LAD (Beach et al., 2003, Morissette and Ostrovsky, 2005).

One important difference between the data sets used here and in BAS is that their data originate from reports of employers and may be hence less subjected to incentives to file a tax return. This concern, however, is assuaged by two considerations. First, the introduction of the Federal Sales Tax (FST) credit in 1986 (replaced by the GST credit in 1991) has proven to provide a strong incentive for filing tax returns even for families with very low income. The Child Tax Benefit and refundable provincial tax credits provide further incentives for such families to file a tax return. Second, the focus of this study is on male workers with positive earnings. For this group, the filing rates are expected to approach 100%[5].

For consistency with BAS, I have attempted to replicate their sample-selection rules. I begin by identifying nineteen two-year birth cohorts, starting with those born in 1933/34 and ending with those born in 1973/74. I then identify male members of these birth cohorts, who were between 25 and 59 years of age and had earnings greater than $500 (dollar values as in 2007) in all years in which they satisfied the age conditions. For instance, men born in 1933/34

[4] The T4 form issued by employers in Canada closely resembles the W-2 form in the United States.
[5] According to the estimates of Statistics Canada, the LAD-coverage rate for *all men and women* aged 25–59 in 2005 was between 88% and 96%, depending on the age group. It can be expected to be substantially higher for men aged 25–59 with a positive employment income.

who had positive earnings in each year from 1984 to 1992 (when they were between 51 and 59 years of age[6]) would satisfy these criteria. Similarly, men born in 1973/74 with positive earnings in each year from 1998 to 2006 (when they were between 25 and 33 years of age) would also satisfy such conditions. Finally, I discard the first and the last observations for the members of all birth cohorts to avoid the effects of the first and last years in their working spells. As BAS point out, the variance of earnings in these years may be affected by entry into and exit from the labor market, in addition to immigration and emigration. The resulting sample is essentially an unbalanced panel that consists, however, of stacked balanced subsamples for each birth cohort (Appendix 1).

BAS discuss the advantages of such a sample structure in comparison to a fully balanced or unbalanced panel. In a fully balanced panel that only includes cohorts for the full length of the period from 1985–2005, age would be perfectly collinear with time, rendering it difficult to separate their effects. An unbalanced panel, alternatively, in which any years of positive earnings would be included, would lead to compositional differences within each cohort for the different years during which this cohort is observed. Therefore, an unbalanced panel, consisting of stacked balanced subpanels for each cohort, has the advantage of being consistent with reference to the cohort composition, while simultaneously avoiding a perfect collinearity between age and time.

The resulting sample is summarized in Table 1. The birth-cohort sample sizes, which range from 18,430 to 35,410, are by an order of magnitude larger than those in BAS, which range from 877 to 3049. There are 21 birth cohorts in the sample. The cohorts in the middle—1947/48 to 1959/60—were observed for the longest period, 21 years, whereas the 1933/34 and 1973/74 cohorts were observed for only 7 years. Considering that the focus of the study is on the variances and covariances of earnings, it is also worth pointing out that the total number of

[6] The age of a two-year birth cohort is defined by the first year.

distinct elements in all autocovariance matrices for all 21 birth cohorts (sample moments) is 3003, compared to 2077 in BAS. Because the sample size and number of sample moments is larger in the present study, the accuracy of my estimates should be expected to be higher.

Figure 2 shows the variance of log earnings in the analysis samples of both the author and BAS. Although there are slight differences in the overlapping portion of the graph, there is also sufficient similarity in shape and magnitude between the two profiles for this period to view it as a continuous profile. There is a clear decline in the variance of log earnings in the late 1980s and a sharp increase during the first years of the recession. The variance of annual earnings remained high, albeit fairly constant, after the recession years, but the upward trend resumed in the early 2000s, which is somewhat different from the more general variance profile (Figure 1), showing that inequality grew fairly steadily throughout the late 1990s and early 2000s. Nevertheless, a general upward trend during the period spanning 1985–2005 is evident in both profiles.

The increasing inequality of annual earnings in recent years raises the same questions BAS attempt to answer in their study of the inequality trends from 1976 to 1992: does the increase in inequality of annual earnings in the 1990s and 2000s reflect the growth in permanent or transitory variance components? Was the contribution of one of the components to the variance growth proportionally larger during this period? These questions need to be answered before proceeding with an inquiry into the role of skills and other aspects of human capital in further studies on earnings inequality.

3 Empirical specification

As in most similar studies, I begin by assuming that

$$Y_{bit} = m_{bt} + y_{bit}, \tag{1}$$

where Y_{bit} is the log earnings of individual i, born in year b and observed in year t; m_{bt} is the mean earnings of birth cohort b in year t; and y_{bit} is an individual-specific deviation from the cohort's mean in year t. A basic permanent-transitory model stipulates that

$$y_{bit} = \mu_{bi} + v_{bit}\,, \tag{2}$$

where μ_{bi} is a time-invariant permanent-earnings component with population variance σ_μ^2; and v_{bit} is a transitory component, with population variance σ_v^2, orthogonal to μ_{bi}. Assuming that v_{bit} is serially uncorrelated, it follows that

$Var(y_{bit}) = \sigma_\mu^2 + \sigma_v^2$; and

$Cov(y_{bit}, y'_{bis}) = \sigma_\mu^2$.

Several studies, including those by Gottschalk and Moffitt (1994), Baker (1997), Haider (2001), and MG (2002, 2008), have added several important elements to this basic model; for instance, factor loadings p_t and λ_t have been included, so that Eq. (2) is modified as follows:

$$y_{bit} = p_t\mu_{bi} + \lambda_t v_{bit}\,, \tag{3}$$

where the first term now represents the permanent-earnings component and the second term represents the transitory-earnings component. The addition of factor loadings allows both the earnings components to change over time. Such changes can be easily related to the changes in the permanent- and transitory-variance components as follows:

$$Var(y_{bit}) = p_t^2\sigma_\mu^2 + \lambda_t^2\sigma_v^2\,. \tag{4}$$

The identification of p_t and λ_t is derived from the fact that although $Var(y_{bit})$ depends on both p_t and λ_t, $Cov(y_{bit}, y'_{bis})$ does not depend on λ_t, because $Cov(v_{bit}, v'_{bis}) = 0$. In other words, a change in p_t (assuming no change in λ_t) will lead to changes in both $Var(y_{bit})$ and $Cov(y_{bit}, y'_{bis})$, but a change in λ_t (assuming no change in p_t) will only lead to a change in $Var(y_{bit})$.

The BAS study adds more flexibility to the model by incorporating heterogeneous growth rates in earnings (β_{bi} in (5a)), a random-walk component (5b), serial correlation in the transitory component (5c), and age-related heteroskedasticity of the transitory variance (5d), so that the final model assumes the following structure:

$$y_{bit} = p_t[\alpha_{bi} + \beta_{bi} X_{bt} + u_{bit}] + \varepsilon_{bit}, \tag{5a}$$

where

$$u_{bit} = u_{bi,t-1} + r_{bit}; \tag{5b}$$

$$\varepsilon_{bit} = \rho\varepsilon_{bi,t-1} + \lambda_t v_{bit} \text{; and} \tag{5c}$$

$$Var(v_{bit}) = \gamma_0 + \gamma_1 X_{bt} + \gamma_2 X_{bt}^2 + \gamma_3 X_{bt}^3 + \gamma_4 X_{bt}^4 = g(X_{bt}), \tag{5d}$$

where X_{bt} is the potential experience of birth cohort b in the year t computed as t-b-26[7].

At the first stage of the estimation process, y_{bit} is obtained by de-meaning the individual log earnings within each $b \times t$ cell to adjust for the effects of age and year on the average earnings of each cohort. Next, for each birth cohort, a sample autocovariance matrix of y_{bit} is computed. The size of the matrix depends on the number of years for which the birth cohort is observed. For instance, for a birth cohort present in the sample for 15 years, it will be a 15×15 autocovariance matrix, with $(15 \times 16)/2 = 120$ distinct elements.

Once the autocovariance matrices for each birth cohort are obtained, all distinct elements of such matrices, both diagonal and off-diagonal, are stacked into an aggregate vector Ω that contains the number of sample moments equal to the number of all distinct elements in all

[7] Baker and Solon provide a detailed justification for this model. For brevity, I do not repeat their argument here. Refer Baker and Solon (2003) for more details.

autocovariance matrices. My sample produced 3,003 moments based on 21 variance-covariance matrices for 21 birth cohorts (Table 1). Each element in Ω has a functional representation based on the underlying earnings function in (5a)–(5d). Therefore, for instance, if the first year in the sample is 1985, the element in Ω representing the fifth diagonal element in the autocovariance matrix for the 1945/46 birth cohort—the variance for this cohort in the year 1989—can be represented as follows:

$$Var(y_{1945/1946,i,1989}) = p_{1989}^2(\sigma_\alpha^2 + \sigma_\beta^2 18^2 + 2\sigma_{\alpha\beta}18 + \sigma_r^2 18) + \rho^2 Var(\varepsilon_{1945/1946,i,1988}) + \\ + \lambda_{1989}^2(\gamma_0 + \gamma_1 18 + \gamma_2 18^2 + \gamma_3 18^3 + \gamma_4 18^4).$$

As evident from the example above, because of its recursive structure, the cohort-specific transitory variance, $Var(\varepsilon_{bit})$, depends on the transitory variance in the first year in which the cohort is observed. BAS consider the initial cohort-specific variances as unrestricted and treat them as parameters $(\sigma_{24/25}^2, \sigma_{26/27}^2, ..., \sigma_{60/61}^2)$ to be estimated. Hence, the BAS model makes no distinction between left-censored observations (cohorts that were 26 before 1985) and non-left-censored observations; and the values of σ_b^2 are assumed to absorb the transitory variance "accumulated" before 1985 for the left-censored observations.

In the next stage, the set of parameters

$$\theta = (p_t^2, \sigma_\alpha^2, \sigma_\beta^2, \sigma_{\alpha\beta}, \sigma_r^2, \rho, \lambda_t^2, \gamma_0 ... \gamma_4, \sigma_{24/25}^2, ..., \sigma_{60/61}^2)$$

is estimated by the generalized method-of-moments (GMM), so that $\hat{\theta}$ is chosen to minimize

$$\Delta = \left[\Omega - f(\hat{\theta})\right]' W \left[\Omega - f(\hat{\theta})\right], \tag{6}$$

where W is a (positive definite) weighting matrix. Although the econometric theory suggests that the optimal W is the inverse of a matrix that consistently estimates the variance-covariance matrix, BAS and most similar studies use the identity matrix as the weighting matrix following the results of the studies by Altonji and Segal (1996) and Clark (1996), who argue that using the optimal W suggested by the theory can produce seriously biased estimates in finite samples[8]. Finally, the estimates of θ can be used to decompose the predicted total variance into permanent and transitory components and to construct predicted profiles of these components for workers of a certain age (for instance, the mean age in the sample). Contrary to the profiles of factor loadings by themselves, such profiles show the magnitude of each component and its share in the total variance.

4 An alternative approach to the left-censoring problem

MG (2008) point out that the treatment of the initial transitory variances of each cohort as parameters σ_b^2 may be problematic, because it introduces a 'built-in' time trend for the left-censored observations, and suggest an alternative treatment of left-censored observations. They note that for the left-censored observations, the unobserved portion of the evolution of variance from a certain age (the beginning of the life cycle) to the age at which the cohort is first observed can be approximated using a hypothetical (but known) variance process that assumes $\lambda_t=1$ in all unobserved years. For instance, if the transitory component follows an ARMA (1,1) process, then in period $t=0$ (the period first observed in a sample)

$$\varepsilon_{bi0} = \rho\varepsilon_{bi,-1} + \lambda_0 v_{bi0} + \vartheta\lambda_{-1} v_{bi,-1},$$

where $\rho\varepsilon_{bi,-1} + \vartheta\lambda_{-1} v_{bi,-1}$ is unobserved. MG assume that

$$Var(\rho\varepsilon_{bi,-1} + \vartheta\lambda_{-1} v_{bi,-1}) = \left[1 + \eta \cdot X_{b,0}\right] \cdot \Lambda_{bi,0}, \tag{7}$$

where $X_{b,0}$ is the potential experience of the birth cohort in the first year of observation (or alternatively, the number of censored years), η is a modifying parameter, and

[8] In an earlier version of the study, Baker and Solon also use a weighting matrix that weights sample moments in proportion to their sample sizes but conclude that this approach does not produce more accurate estimates.

$$\Lambda_{bi,0} = \sum_{h=0}^{X_{b,0}-1} \rho^{2h} (\rho+\theta)^2 Var(v_{bi,0-h-1}) = \sum_{h=0}^{X_{b,0}-1} \rho^{2h} (\rho+\theta)^2 \cdot g(X_{b,0}-h-1) \,.^{9} \tag{8}$$

$\Lambda_{bi,0}$ captures the evolution of the cohort's transitory variance from the beginning of its life cycle to *t=0,* assuming *λ=1* in all unobserved years, whereas $[1+\eta \cdot X_{b,0}]$ captures the cumulative deviation from such a hypothetical process, which is expected to be greater for the older birth cohorts (or cohorts with larger $X_{b,0}$), for which more years are censored (more details in Appendix B; refer also Appendix B in the study by MG). In the limiting case of ($\eta = 0$), the unobserved portion of the variance accumulated before the first observed period is identical to $\Lambda_{bi,0}$.

For comparison with the BAS study, I estimate a model with the same treatment of the permanent component as in BAS, but which in addition to (5a), (5b), and (5d), assumes that

$$\varepsilon_{bit} = \rho\varepsilon_{bi,t-1} + \lambda_t v_{bit} + \vartheta\lambda_{t-1} v_{bi,t-1} \tag{9}$$

and thereafter approximates the left-censored observations using the MG method. In particular, for the birth cohorts that are not left-censored, $Var(\varepsilon_{bit})$ takes the form of

$$Var(\varepsilon_{bit}) = l \cdot \sum_{h=0}^{X_{bt}-1} \rho^{2h} (\rho+\theta)^2 \lambda_{t-h-1}^2 \cdot Var(v_{bi,t-h-1}) + \lambda_t^2 \cdot Var(v_{bit}) \,, \tag{10a}$$

whereas for the left-censored cohorts, the expression becomes

$$\begin{aligned} Var(\varepsilon_{bit}) = \rho^{2(t-85)} [1+\eta \cdot X_{b,85}] \sum_{h=0}^{X_{b,85}-1} \rho^{2h} (\rho+\theta)^2 \cdot Var(v_{bi,85-h-1}) + \\ + k \cdot \sum_{h=0}^{t-86} \rho^{2h} (\rho+\theta)^2 \lambda_{t-h-1}^2 \cdot Var(v_{bi,t-h-1}) + \lambda_t^2 \cdot Var(v_{bit}), \end{aligned} \tag{10b}$$

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[9] The formula provided by MG is slightly different because in their study, age (potential experience) is counted from 1, whereas I count it from 0.

where $k=1$, if $t \geq 86$, and 0, otherwise; and $l=1$, if $X_{bt} \geq 1$, and 0, otherwise. There are no initial variance parameters σ_b^2 in this specification, and the life cycle is assumed to begin at age 26; for instance, $Var(\varepsilon_{69/70,i,95}) = \lambda_{95}^2 Var(v_{69/70,i,95}) = \lambda_{95}^2 \gamma_0$.

The MG study argues that their approach to left-censoring offers an important advantage compared to a specification that treats the initial transitory variances as parameters. Considering the evolution of $Var(\varepsilon_{bit})$ across the life cycle as a continuous process, they point out that for the left-censored observations, the values of $Var(\varepsilon_{bit})$ in the observed portion of this process will depend not only on the age of the birth cohort in period *t* and effects of the calendar time, but also on the number of years that have elapsed since the first year in which the cohort is observed. This creates a problem of a "built-in" time trend, which manifests itself as erroneous estimates of λ_t. The estimates of λ_t will depend not only on the effects of calendar time, but also on the number of years in which each censored cohort is observed. Rolling the transitory variance of *each* cohort back to the beginning of the life cycle ($X_{bt} = 0$) permits the netting out of the "built-in" time trends from $\hat{\lambda}_t$. In the next section, I present the results based on the original BAS model and compare them to the results obtained from the proposed model that uses an MG treatment of left-censored observations in the transitory variance.

5 Estimation results

I define the age of each two-year cohort by the age of the older cohort; thus, for instance, the 1949/50 cohort would be 41 years old in 1990. Following (1), I begin by subtracting the mean log earnings of each $b \times t$ cell (μ_{bt}) from the log of the individual's annual earnings (Y_{bit}) to net out the effects of life cycle and year. The deviations from the mean, y_{bit}, are then used to construct the empirical autocovariance matrices for each cohort. An example of such an autocovariance matrix—the autocovariance matrix for the 1951/52 cohort observed for the full period from 1985 to 2005, when members of this cohort were between 34 and 54 years of age—is provided in Table 2. The choice of the cohort is not accidental: BAS furnish an example of the autocovariance matrix for the 1942/43 cohort, which was also 34-year-old in 1976, the first year in their sample.

It is interesting to note the differences and similarities between the autocovariance matrices in both studies. Both variance profiles are upward-sloping, with the estimated annual variances increasing during the recession years; however, the variances for the 1951/52 cohort are generally larger. The variance for 1985—the year in which the 1951/52 cohort was first observed—is considerably larger than the variance reported in BAS for 1976, when the 1942/43 cohort in that study was the same age as the 1951/52 cohort in 1985 (0.321 compared to 0.225). It is also higher than the variance reported in BAS for 1985 (0.283); however, their cohort was 45-year-old in that year, which is the age when, as the BAS study shows, the U-shaped age-variance profile is at its lowest point. I observe a much smaller variance in 1992 (0.342 compared to 0.433 in BAS), but the variance BAS report in the previous year, 0.345, is almost identical to the one in this study and consistent with their age-variance profile, which suggests that variances at 42 and 49 should be similar.

To gauge the magnitude of the growth in variance of y_{bit} from 1985 to 2005, it may be more informative to pull all variances from all matrixes together (329 variances) and regress the logs of variances on a linear time trend, a polynomial in potential experience and unemployment rate with period t. Using a quartic in age, the coefficient on a time-trend variable is positive, 0.017, and strongly significant (the estimated standard error is 0.001), indicating a fairly strong variance growth from 1985 to 2005.

As mentioned in Section 3, from the analysis of total variances, it is impossible to gauge the degree of persistence in the earnings distribution from one period to another that can be associated with the relative importance of permanent- and transitory-variance components. It is the trend in the covariances that allows us to estimate the relative contributions of the permanent and transitory components to the changes in the total variance. One way to net out such a trend is to estimate

$$\phi_{bts} = \psi_0 + \tau \cdot T + \psi_1 \cdot U_t + \psi_2 \cdot U_s + \psi_3 \cdot X_{bt} + \psi_4 \cdot X_{bt}^2 + \psi_5 \cdot X_{bt}^3 + \sum \xi_d \cdot D_d + \omega_{ts}, \quad (11)$$

where ϕ_{bts} is a vector of the logs of all autocorrelations for all cohorts pulled together, X_{bt} is the potential experience of the birth cohort b in year t, T represents a linear time trend, U_t and U_s are unemployment rates in t and s, respectively[10], and D_d represents the dummy variables (fixed effects) for the orders of autocorrelation, $d = s - t$, $s > t$. The sign and magnitude of τ indicate the relative contributions of the permanent and transitory components to the changes in the total variance. If the autocorrelations increase with time, it implies that the autocovariances grow faster than the variances and that p_t is growing faster than λ_t. Intuitively, even as the annual earnings become more dispersed with time (variance growth), the degree of persistence in the distribution of earnings is increasing even quicker (stronger autocovariance growth). In other words, whereas there is increasing rigidity in the ranking of individuals in the earnings distribution over time, the distribution itself is becoming more spread out. Considered together, these trends imply increasing long-term earnings inequality. Alternatively, if the variances are increasing with time but the autocorrelations are declining, then the variances grow faster than the covariances. This trend in turn implies more short-term (year-to-year) variations, i.e. more instability. Therefore, a positive τ would suggest that the permanent-variance component played

[10] U_t and U_s are unemployment rates for male workers of age 15 and above.

a greater role in the growth of total variance; a negative τ, conversely, would indicate a proportionally greater increase in the transitory component.

The estimate of τ is positive but very small (0.00014, only about 1/10th of the estimate in the BAS study) and not significant at the 95% level (standard error equals 0.00041)[11]. In other words, when the entire period from 1985 to 2005 is considered, there appears to be no significant time trend in the autocorrelations controlling for age, business cycle effects, and the order of the autocorrelations. The implication of this result is that both permanent and transitory components contributed to the increase of total annual variance of earnings during the period 1985–2005, but neither component played a dominant role.

I now address the GMM estimation outlined in Section 3 (refer Appendix B for a detailed description of the models). In the first two columns of Table 3 (Model I), I report the coefficients and standard errors from the BAS model defined by the equations (5a)–(5d). For identification, the first-factor loading p_{1985} is set to 1. Similar to the BAS study, I observe a strong cyclical effect on p_t: the factor loadings on the permanent component decrease during the expansion years of the late 1980s and subsequently increase during the recession years of the early 1990s. Although there is a fair degree of fluctuation in p_t after this period, it remains at levels close to those prevalent during the recession years when the late 1990s are considered, and increases during the early 2000s. The overall increase in p_t is consistent with our analysis of the autocovariance matrices above: the increment in annual variances is produced by the increases in both components. The profile of p_t, however, suggests that the growth in the permanent component may have been particularly strong in the early 2000s.

[11] I also tried a quadratic and a quartic in age, with very similar results.

Compared to the estimates in the BAS study, my estimate of σ_{α}^{2}—the variance of intercepts in (5a)—is slightly higher (0.186 compared to 0.134), whereas the estimate of σ_{β}^{2},—the variance of slopes in the same equation—is considerably lower (0.000025 compared to 0.00009). Both estimates, however, are strongly significant. The relatively low estimate of σ_{β}^{2} is surprising; however, it may be to some degree explained by the relatively larger estimate of the random-walk variance. My estimate of the random-walk variance is 0.010 compared to 0.007 in the BAS study. The estimates of $\sigma_{\alpha\beta}$ are very similar in both studies (−0.003): those who start out with higher earnings can generally expect lower growth rates as they gain more experience.

Considering the estimates related to the transitory component, my estimate of ρ is slightly higher compared to the one in BAS (0.586 compared to 0.533). There are some differences in the estimates of the age quartic $\gamma_0, \ldots, \gamma_4$; the first two, however, are fairly similar in magnitude and have the same sign. The estimates of the birth cohort-specific transitory innovations are not fully comparable because they correspond to different points in time. Nevertheless, similar to BAS, I find that younger cohorts appear to have larger transitory innovations. This is fully consistent with the U-shaped age profile of the transitory variance (Figure 2 in BAS). The figure shows that younger workers, many of whom are just entering the labor force, experience much higher levels of earnings instability. The transitory variance gradually declines to the lowest levels at around the 45-year-old mark but increases again thereafter. As more recent cohorts are observed at younger ages, their transitory innovations can be expected to be substantially higher.

Finally, I report the estimates of the factor loadings on the transitory component, λ_t. For identification, the first-factor loading λ_{1986} is set to one. The trend in $\hat{\lambda}_t$ is similar to the trend in $\hat{p}_t$. As noted above, both components are expected to have contributed to the increase in the annual variance in the 1990s and 2000s. The similarity in the profiles of both factor loadings is consistent with the very small and insignificant estimate of τ in (11). Just as $\hat{p}_t$ increases sharply in the 2000s, $\hat{\lambda}_t$ also increases in the 2000s to levels higher than those during the recession years of the early 1990s.

In the third and fourth columns of Table 3 (Model II), I report the coefficients and standards errors from the model with an ARMA (1,1) transitory component and the MG approximation for the left-censored observations (equations (5a), (5b), (5d), (9), (10a), and (10b)). Most of the estimates related to the transitory component are very similar to the estimates from Model I, with a possible exception for $\hat{\sigma}_\beta^2$, which is about 40% larger in Model II than in Model I. Among other estimates, $\hat{\rho}$ is somewhat larger, 0.67 compared to 0.59 in the first model. My estimate of θ is -0.21, which is less than half the magnitude reported in the MG study. In the first model, λ_{1986} is set to 1 for identification (because initial variances in 1985 are parameters σ_b^2), whereas in the second model λ_{1985} is set to 1. Both profiles of λ_t, however, appear to be quite similar.

The profiles of $\hat{p}_t$ and $\hat{\lambda}_t$ by themselves do not fully convey information about the trends in the permanent and transitory components during the period analyzed in the study because they say little about the relative magnitude of each factor. One way to assess the full contribution of each component is to estimate each of these components holding X_{bt} in (5a) and (5d) constant at a specific value. I follow BAS and compute the relative shares of the permanent and transitory components for male workers with 14 to 15 years of potential experience in each year[12]. The

[12] As in BAS, the initial variances in the transitory component change every two years, whereas the potential experience changes annually. For each two-year cohort, I set the variable of potential experience equal to 14 in the first year and 15 in the second year. For instance, the 1955/56 cohort had 14 years of potential experience in 1995

results are shown in Table 4. Both models produce very similar profiles of the permanent and transitory components, and the profile of the sum of the two components—total variance—is actually quite close to the actual annual variance of y_{bit} computed for 40- to 41-year-old workers (Figure 3). The share of the permanent component is much larger than that of the transitory component.

How did the growth in the permanent variance during the period 1985–2005 compare to the growth in the transitory variance? Figure 4 shows the changes in the ratio of the permanent and transitory components during this period. The ratio increased sharply in the late 1980s, suggesting that the permanent variance grew more rapidly than the transitory variance; however, it rapidly declined during the early 1990s. From 1993, it changed little until the early 2000s, declined during the initial years of the new century, but mounted again toward the middle of the decade. The relatively faster growth of the permanent variance in the past three years is apparent from Figures 3 and 4.

The dynamics of the permanent and transitory variances shown in Figures 3 and 4 do not consider the changes in the labor-market conditions during the period 1985–2005. One approach to controlling for such changes is to regress the estimates of the permanent- and transitory-variance components on the linear time trend and U_t (unemployment rate for men of age 15 years and above in year t). The estimated effects of the time trend are positive and significant for both variances, in both the models (Table 5). A Wald test rejects the null hypothesis of the statistical equality of the time-trend coefficients for the permanent and transitory variances at the 95% level, albeit weakly, particularly in Model II, indicating a slightly larger contribution of the permanent variance to the growth in total variance during this period when the effects of the

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and 15 years in 1996. Subsequently, the 1957/58 cohort had 14 years of experience in 1997 and 15 years in 1998, and so on.

business cycles are also considered. However, when only the post-1992 period is considered, the null hypothesis of statistical equality of the time-trend coefficients cannot be rejected although the time-trend coefficients are positive and significant.

In sum, three central findings emerge from the analysis of autocorrelations and GMM estimation. First, in a sample of male workers with a fairly strong attachment to the labor market, the annual variance of earnings increased sharply during the recession of the early 1990s and remained high in the postrecession years. Second, after a relatively stable period during the late 1990s, the annual variance of earnings resumed the upward trend in the early 2000s. Third, changes in both permanent- and transitory-variance components contributed to the growth in total variance. The analysis of autocorrelations further suggests that the contribution of each component during the period 1985–2005 was proportionally fairly equal. A more formal analysis using a GMM estimator also shows that in the postrecession period, both variance components contributed roughly equally to the upward trend of the annual variance.

6 Conclusions

In this study, based on a uniquely rich longitudinal administrative dataset, I examined earnings dynamics of male workers in Canada during the period from 1985 to 2005 and decomposed the changes in the variance of annual earnings into permanent- and transitory-variance components. This analysis is an extension of the analysis of earnings dynamics of men carried out by BAS (2003) for the period 1976–1992. An essential element of this study is a new approach to the problem of left-censoring recently suggested by MG (2008). The results obtained by using the MG approach were compared to the results from the original BAS model. The study was based on a very large sample size and used administrative data usually considered to be more accurate than survey data.

I found that the variance of log earnings remained high in the postrecession years and continued to increase throughout the 1990s and into the 2000s and that both variance components contributed to the growth in the total variance. However, contrary to what BAS found for the period 1976–1992, I found little evidence that the growth in the permanent component was proportionally larger, particularly during the postrecession period from 1992 to 2005. Both the original BAS model and the model with the MG treatment of the left-censored observations provide good fit and very similar estimates of the permanent and transitory variances. The robustness of the results confers a degree of reassurance on the reliability of previous studies that model the transitory variance with cohort-specific initial variances as parameters.

The parallel increases in the annual variance of male earnings and the permanent component of such variance in the 1990s and 2000s provide more credence to the argument that increasing annual-earnings inequality reflects the growing dispersion of permanent earnings related to changes in the demand for and supply of human capital in the Canadian labor market. My findings also add to the substantial body of evidence that permanent and transitory variances often grow in tandem and that the increases in earnings inequality are often accompanied by proportionally similar increases in earnings instability. The overall share of the permanent variance in the total annual variance, however, remains much larger than the share of the transitory variance, although not as large as it was in the late 1980s. Hence, the focus in future analyses of earnings inequality and related policy debates is expected to remain on long-term inequality and the reasons behind the widening dispersion of permanent earnings.

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Table 1. The summary statistics of the sample

Birth year	sample size	years observed	# of years	age in initial year	sample moments
1933/34	21,430	1985-1991	7	52	28
1935/36	19,570	1985-1993	9	50	45
1937/38	18,640	1985-1995	11	48	66
1939/40	18,430	1985-1997	13	46	91
1941/42	19,500	1985-1999	15	44	120
1943/44	20,380	1985-2001	17	42	153
1945/46	22,470	1985-2003	19	40	190
1947/48	25,470	1985-2005	21	38	231
1949/50	28,220	1985-2005	21	36	231
1951/52	31,060	1985-2005	21	34	231
1953/54	33,450	1985-2005	21	32	231
1955/56	34,610	1985-2005	21	30	231
1957/58	34,750	1985-2005	21	28	231
1959/60	33,910	1985-2005	21	26	231
1961/62	34,400	1987-2005	19	26	190
1963/64	35,160	1989-2005	17	26	153
1965/66	32,030	1991-2005	15	26	120
1967/68	30,300	1993-2005	13	26	91
1969/70	32,180	1995-2005	11	26	66
1971/72	33,330	1997-2005	9	26	45
1973/74	35,410	1999-2005	7	26	28
				total:	3003

Source: Statistics Canada, Longitudinal Administrative Databank

Note: men 26 to 58, born between 1933 and 1974, with annual earnings above $500 in constant 2007 dollars in each year in which they satisfy the age criteria

Table 2. The autocovariance matrix of log-earnings residuals $y_{1951/52,it}$, 1951/52 birth cohort aged 34 in 1985

	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994
1985	0.321	0.800	0.719	0.675	0.643	0.622	0.601	0.574	0.554	0.553
	(.006)									
1986	0.249	0.303	0.804	0.731	0.693	0.663	0.633	0.609	0.592	0.584
	(.004)	(.005)								
1987	0.216	0.235	0.281	0.813	0.742	0.698	0.657	0.626	0.609	0.604
	(.003)	(.004)	(.005)							
1988	0.200	0.211	0.225	0.273	0.826	0.750	0.702	0.659	0.638	0.633
	(.003)	(.003)	(.004)	(.005)						
1989	0.187	0.196	0.202	0.221	0.263	0.827	0.747	0.696	0.673	0.662
	(.003)	(.003)	(.003)	(.003)	(.005)					
1990	0.184	0.191	0.194	0.205	0.222	0.274	0.825	0.749	0.713	0.699
	(.003)	(.003)	(.003)	(.003)	(.003)	(.005)				
1991	0.194	0.199	0.199	0.209	0.218	0.246	0.325	0.809	0.745	0.716
	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.006)			
1992	0.190	0.196	0.194	0.202	0.209	0.229	0.270	0.342	0.820	0.753
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.006)		
1993	0.186	0.193	0.191	0.197	0.204	0.221	0.251	0.284	0.350	0.821
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)	(.006)	
1994	0.181	0.186	0.185	0.191	0.196	0.211	0.236	0.254	0.280	0.333
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)	(.006)
1995	0.180	0.184	0.183	0.190	0.195	0.209	0.230	0.244	0.261	0.281
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)	(.004)
1996	0.179	0.184	0.182	0.190	0.194	0.208	0.228	0.240	0.255	0.266
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)	(.004)
1997	0.179	0.182	0.182	0.189	0.192	0.205	0.224	0.234	0.248	0.256
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)	(.004)
1998	0.178	0.181	0.181	0.187	0.192	0.204	0.222	0.231	0.243	0.249
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)
1999	0.173	0.179	0.179	0.185	0.189	0.200	0.217	0.226	0.237	0.242
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)
2000	0.174	0.179	0.178	0.185	0.188	0.200	0.217	0.224	0.234	0.239
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)
2001	0.176	0.182	0.181	0.187	0.190	0.202	0.220	0.226	0.234	0.239
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)
2002	0.173	0.178	0.177	0.183	0.187	0.199	0.216	0.221	0.229	0.234
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)
2003	0.169	0.176	0.174	0.180	0.185	0.196	0.214	0.218	0.226	0.230
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)
2004	0.165	0.171	0.170	0.177	0.181	0.192	0.210	0.214	0.221	0.225
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)
2005	0.162	0.169	0.166	0.175	0.181	0.191	0.207	0.211	0.219	0.223
	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)	(.004)

Source: Statistics Canada, Longitudinal Administrative Databank.

Table 2 (cont'd). The autocovariance matrix of log-earnings residuals $y_{1951/52,it}$

1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
0.549	0.531	0.525	0.524	0.514	0.511	0.505	0.483	0.465	0.429	0.399
0.577	0.562	0.550	0.550	0.548	0.542	0.539	0.511	0.496	0.457	0.428
0.597	0.578	0.571	0.570	0.569	0.560	0.555	0.527	0.510	0.472	0.439
0.627	0.609	0.601	0.599	0.596	0.589	0.582	0.552	0.536	0.497	0.467
0.657	0.635	0.625	0.624	0.621	0.613	0.604	0.578	0.560	0.520	0.492
0.690	0.667	0.653	0.650	0.642	0.638	0.627	0.601	0.581	0.539	0.509
0.699	0.673	0.656	0.649	0.642	0.634	0.627	0.600	0.583	0.542	0.508
0.722	0.690	0.666	0.660	0.650	0.638	0.628	0.597	0.580	0.537	0.505
0.764	0.723	0.699	0.687	0.675	0.660	0.646	0.613	0.594	0.548	0.517
0.840	0.774	0.739	0.720	0.705	0.692	0.674	0.640	0.620	0.573	0.540
0.334	0.841	0.785	0.761	0.740	0.724	0.700	0.663	0.644	0.597	0.567
(.006)										
0.289	0.354	0.845	0.789	0.761	0.741	0.720	0.682	0.656	0.607	0.577
(.004)	(.006)									
0.273	0.302	0.361	0.855	0.807	0.782	0.751	0.710	0.683	0.633	0.603
(.004)	(.005)	(.006)								
0.264	0.281	0.308	0.359	0.867	0.818	0.783	0.741	0.711	0.660	0.626
(.004)	(.004)	(.005)	(.006)							
0.254	0.269	0.288	0.308	0.353	0.871	0.816	0.771	0.738	0.682	0.644
(.004)	(.004)	(.004)	(.005)	(.006)						
0.251	0.264	0.282	0.294	0.310	0.360	0.873	0.804	0.765	0.704	0.669
(.004)	(.004)	(.004)	(.005)	(.005)	(.006)					
0.249	0.263	0.277	0.288	0.298	0.322	0.377	0.870	0.813	0.740	0.702
(.004)	(.004)	(.004)	(.004)	(.004)	(.005)	(.006)				
0.242	0.257	0.270	0.281	0.290	0.305	0.338	0.400	0.866	0.777	0.728
(.004)	(.004)	(.004)	(.004)	(.004)	(.005)	(.005)	(.007)			
0.240	0.251	0.264	0.274	0.282	0.295	0.321	0.352	0.414	0.858	0.777
(.004)	(.004)	(.004)	(.004)	(.004)	(.004)	(.005)	(.005)	(.007)		
0.235	0.246	0.259	0.269	0.275	0.287	0.309	0.334	0.375	0.463	0.851
(.004)	(.004)	(.004)	(.004)	(.004)	(.005)	(.005)	(.005)	(.006)	(.008)	
0.235	0.246	0.259	0.268	0.274	0.287	0.309	0.329	0.358	0.414	0.512
(.004)	(.004)	(.004)	(.004)	(.004)	(.005)	(.005)	(.005)	(.006)	(.007)	(.009)

Note: Estimated standard errors are in the parentheses. Correlation coefficients are reported above the diagonal.

Table 3. GMM estimates from Models I and II (see Appendix B)

	Model I		Model II	
	coefficient	st. error	coefficient	st. error
Persistent component:				
p_1985	1.000	n/a	1.000	n/a
p_1986	0.995	0.007	0.999	0.008
p_1987	0.973	0.007	0.979	0.008
p_1988	0.972	0.008	0.983	0.008
p_1989	0.970	0.008	0.981	0.008
p_1990	0.982	0.008	0.997	0.009
p_1991	1.034	0.008	1.042	0.009
p_1992	1.043	0.008	1.056	0.009
p_1993	1.048	0.008	1.058	0.009
p_1994	1.027	0.008	1.043	0.009
p_1995	1.027	0.008	1.036	0.009
p_1996	1.039	0.008	1.053	0.009
p_1997	1.043	0.008	1.056	0.009
p_1998	1.046	0.008	1.063	0.009
p_1999	1.037	0.008	1.050	0.009
p_2000	1.039	0.008	1.055	0.009
p_2001	1.058	0.008	1.074	0.009
p_2002	1.055	0.008	1.069	0.009
p_2003	1.063	0.008	1.078	0.009
p_2004	1.083	0.008	1.095	0.009
p_2005	1.110	0.008	1.123	0.009
σ^2_r	0.010	0.000	0.009	0.000
σ^2_a	0.186	0.003	0.180	0.004
σ^2_b	2.48E-05	6.65E-06	3.48E-05	6.84E-06
σ_ab	-3.45E-03	1.62E-04	-3.22E-03	1.95E-04
Transitory component:				
λ_1985	n/a	n/a	1.000	n/a
λ_1986	1.000	n/a	1.085	0.036
λ_1987	0.958	0.052	1.092	0.034
λ_1988	0.896	0.043	0.998	0.035
λ_1989	0.922	0.043	1.030	0.032
λ_1990	0.970	0.040	1.044	0.033
λ_1991	1.059	0.041	1.221	0.033
λ_1992	1.184	0.044	1.287	0.035
λ_1993	1.142	0.043	1.313	0.034
λ_1994	1.120	0.043	1.230	0.035
λ_1995	1.123	0.042	1.292	0.034
λ_1996	1.163	0.043	1.275	0.035
λ_1997	1.111	0.042	1.252	0.033

λ_1998	1.121	0.042	1.216	0.034
λ_1999	1.141	0.043	1.271	0.033
λ_2000	1.171	0.043	1.279	0.035
λ_2001	1.213	0.044	1.320	0.036
λ_2002	1.266	0.046	1.401	0.038
λ_2003	1.254	0.046	1.391	0.038
λ_2004	1.203	0.046	1.372	0.039
λ_2005	1.245	0.046	1.363	0.039
rho	0.586	0.006	0.668	0.010
theta	n/a	n/a	-0.208	0.013
gamma	n/a	n/a	0.210	0.025
σ^2_1933/34	0.158	0.012	n/a	n/a
σ^2_1935/36	0.152	0.012	n/a	n/a
σ^2_1937/38	0.153	0.012	n/a	n/a
σ^2_1939/40	0.141	0.012	n/a	n/a
σ^2_1941/42	0.127	0.012	n/a	n/a
σ^2_1943/44	0.111	0.012	n/a	n/a
σ^2_1945/46	0.098	0.012	n/a	n/a
σ^2_1947/48	0.110	0.012	n/a	n/a
σ^2_1949/50	0.073	0.012	n/a	n/a
σ^2_1951/52	0.080	0.012	n/a	n/a
σ^2_1953/54	0.097	0.012	n/a	n/a
σ^2_1955/56	0.139	0.012	n/a	n/a
σ^2_1957/58	0.184	0.012	n/a	n/a
σ^2_1959/60	0.237	0.012	n/a	n/a
σ^2_1961/62	0.205	0.012	n/a	n/a
σ^2_1963/64	0.189	0.012	n/a	n/a
σ^2_1965/66	0.307	0.012	n/a	n/a
σ^2_1967/68	0.329	0.011	n/a	n/a
σ^2_1969/70	0.334	0.011	n/a	n/a
σ^2_1971/72	0.280	0.012	n/a	n/a
σ^2_1973/74	0.296	0.012	n/a	n/a
γ0	0.139	0.010	0.172	0.008
γ1	-0.022	0.002	-0.035	0.002
γ2	2.15E-03	1.90E-04	0.003535	1.91E-04
γ3	-1.02E-04	8.85E-06	-1.57E-04	8.58E-06
γ4	1.83E-06	1.48E-07	2.52E-06	1.36E-07

Source: Statistics Canada, Longitudinal Administrative Databank

Table 4. A log-earnings variance decomposition; male workers, 40-41 years old

Year	Permanent		Transitory		Total		Actual annual variance
	Model I	Model II	Model I	Model II	Model I	Model II	
1985	0.235	0.224	0.098	0.114	0.333	0.339	0.339
1986	0.236	0.228	0.078	0.093	0.314	0.321	0.331
1987	0.222	0.215	0.072	0.081	0.294	0.296	0.310
1988	0.225	0.220	0.060	0.067	0.286	0.288	0.300
1989	0.221	0.216	0.061	0.068	0.282	0.284	0.276
1990	0.230	0.227	0.063	0.068	0.293	0.295	0.295
1991	0.251	0.244	0.075	0.087	0.326	0.330	0.325
1992	0.259	0.255	0.088	0.095	0.348	0.350	0.342
1993	0.258	0.251	0.093	0.102	0.351	0.354	0.346
1994	0.251	0.248	0.088	0.093	0.339	0.341	0.333
1995	0.247	0.241	0.091	0.105	0.338	0.345	0.332
1996	0.258	0.253	0.091	0.103	0.349	0.356	0.340
1997	0.255	0.250	0.091	0.108	0.346	0.358	0.351
1998	0.261	0.258	0.087	0.103	0.348	0.361	0.346
1999	0.252	0.247	0.092	0.100	0.345	0.347	0.333
2000	0.258	0.254	0.093	0.101	0.351	0.355	0.341
2001	0.263	0.259	0.102	0.107	0.365	0.366	0.364
2002	0.266	0.261	0.107	0.117	0.372	0.378	0.380
2003	0.265	0.261	0.112	0.120	0.377	0.381	0.391
2004	0.280	0.274	0.103	0.117	0.383	0.391	0.399
2005	0.289	0.283	0.110	0.117	0.399	0.400	0.415

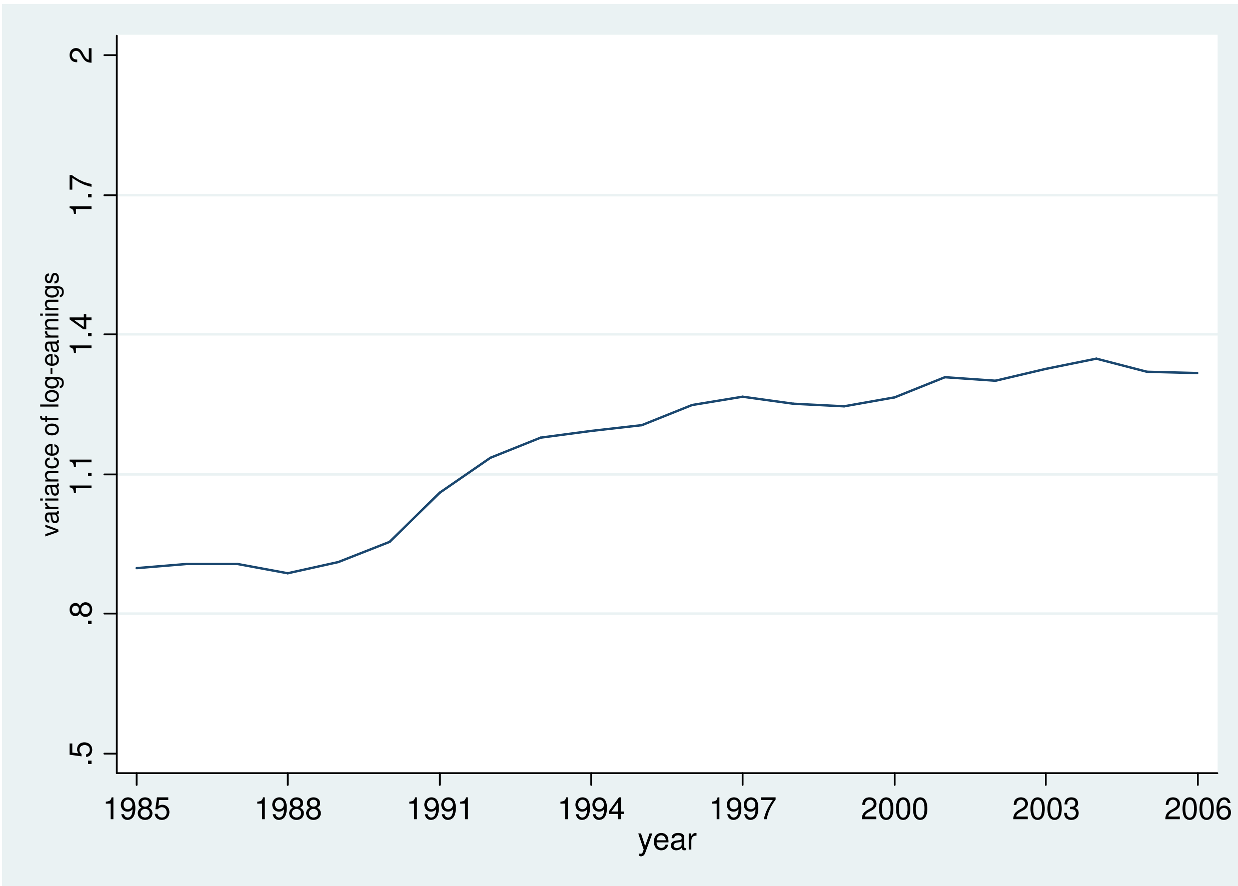
Source: Statistics Canada, Longitudinal Administrative Database

Table 5. Test for the presence of trends in permanent and transitory variances controlling for business cycles; male workers 40-41 years old

	Model I		Model II	
	Permanent	Transitory	Permanent	Transitory
	Period: 1985-2005			
Time trend	0.0031	0.0023	0.0032	0.0023
	(0.0003)	(0.0004)	(0.0002)	(0.0004)
Unemployment	0.0044	0.0036	0.0042	0.0042
	(0.0010)	(0.0014)	(0.0009)	(0.0017)
Constant	0.1833	0.0335	0.1785	0.0379
	(0.0105)	(0.0145)	(0.0090)	(0.0177)
R-sq	0.89	0.70	0.92	0.61
Wald test (permanent=transitory) "time trend":				
	F(1,36) = 5.1; prob>F = 0.030		F(1,36) = 4.26; prob>F = 0.046	
	Period: 1992-2005			
Time trend	0.0049	0.0036	0.0046	0.0035
	(0.0010)	(0.0006)	(0.0009)	(0.0006)
Unemployment	0.0073	0.0050	0.0067	0.0047
	(0.0024)	(0.0014)	(0.0022)	(0.0015)
Constant	0.1325	0.0022	0.1363	0.0179
	(0.0339)	(0.0193)	(0.0321)	(0.0218)
R-sq	0.77	0.87	0.79	0.83
Wald test (permanent=transitory) "time trend":				
	F(1,22) = 0.95; prob> F = 0.340		F(1,22) = 0.79; prob>F = 0.384	

Source: Statistics Canada, Longitudinal Administrative Databank

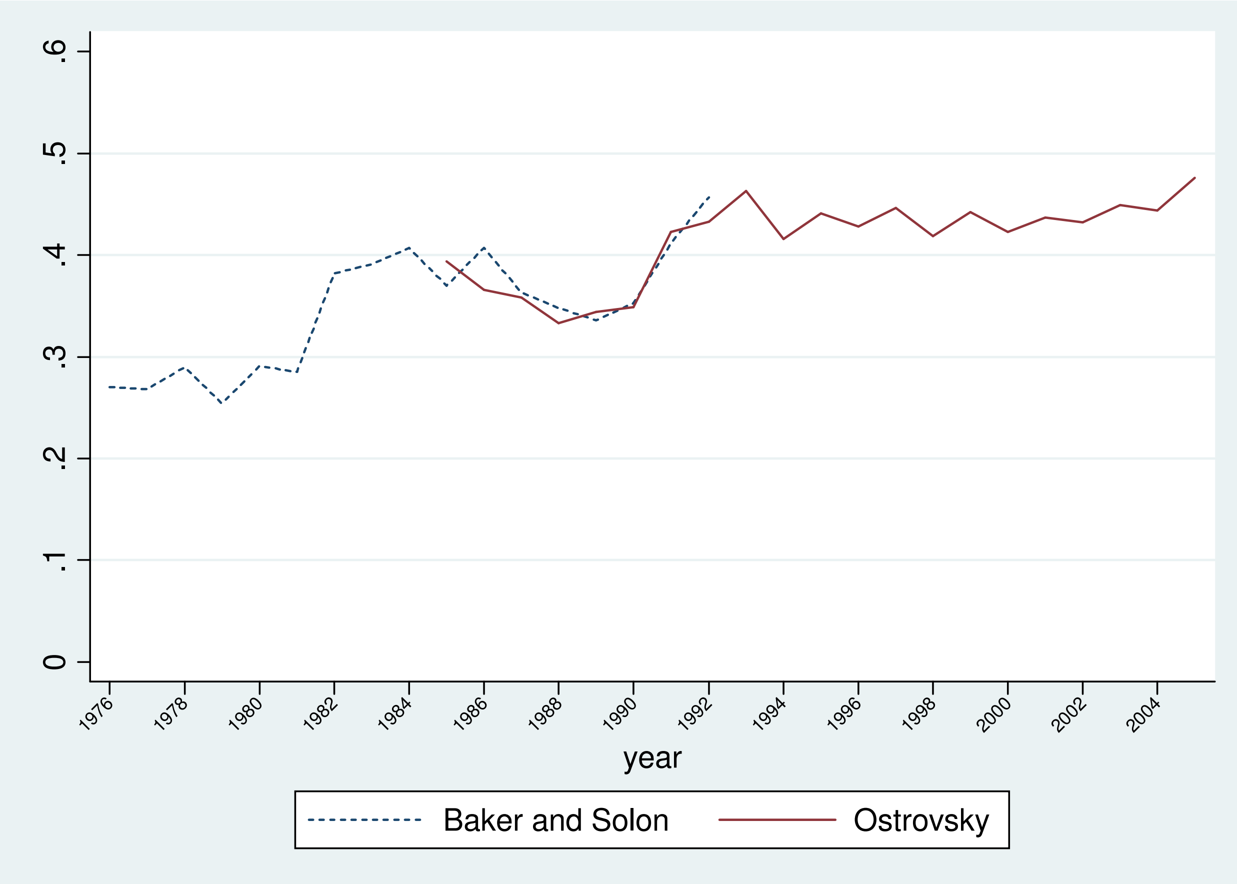
Figure 1. The annual variance of log-earnings of Canadian men, age 25-59



Source: Statistics Canada, Longitudinal Administrative Databank

Note: based an all men 25-59 in LAD with positive earnings in a given year.

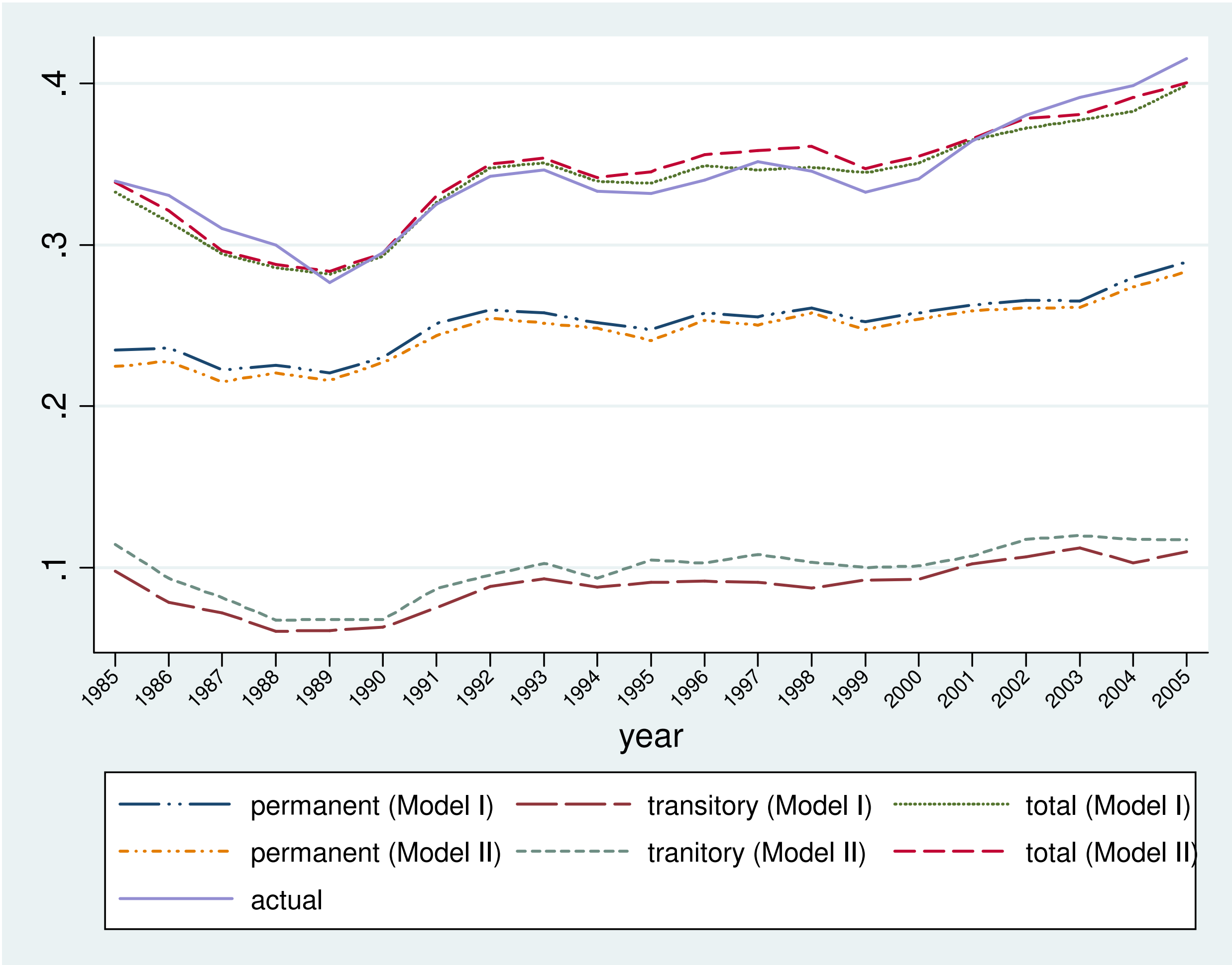
Figure 2. The variance of log-earnings in the analysis sample in this study and in Baker and Solon (2003)



Source: Statistics Canada, Longitudinal Administrative Databank

Note: The men in both samples were between 26 and 52 in the first year of the panel and 32 and 58 in the last year of the panel and had earnings above $500 (in $2007) in each year.

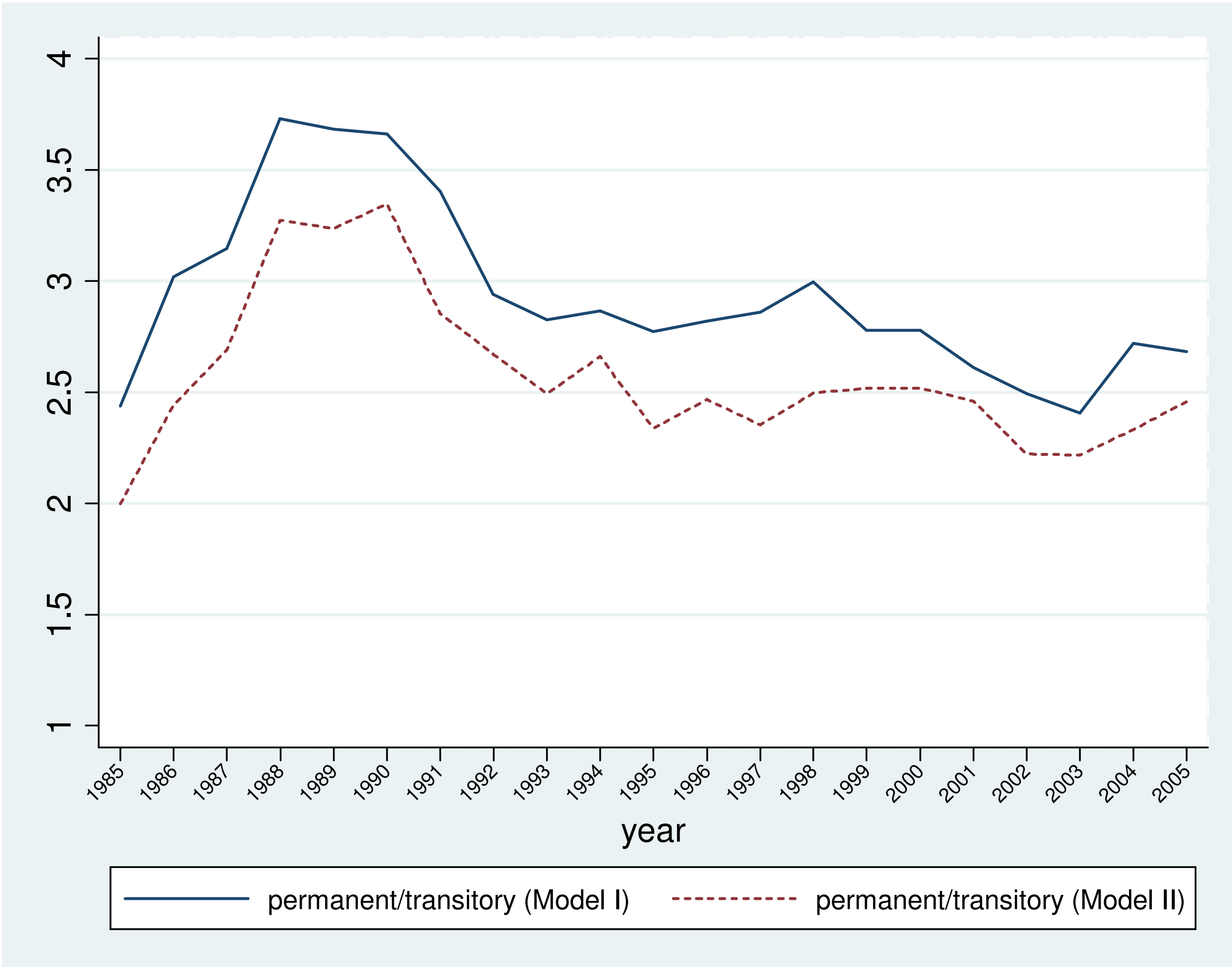
Figure 3. The variance decomposition of individual log-earnings, *Var(y_{bit});* male workers 40-41 years old, 1985-2005.



Source: Statistics Canada, Longitudinal Administrative Databank

Note: The permanent and transitory components are computed using the estimated parameters in Models I and II and holding potential experience constant across all years in the panel.

Figure 4. The estimated ratio of the permanent to transitory log-earnings variance component; male workers 40-41 years old.



Source: Statistics Canada, Longitudinal Administrative Databank

Appendix 1. Main sample: male workers between ages 25 and 59 with positive earnings in each year they satisfy age conditions. The first and last years in the series (shaded areas) are discarded.

Birth cohort	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
1933/34	51	52	53	54	55	56	57	58	59														
1935/36	49	50	51	52	53	54	55	56	57	58	59												
1937/38	47	48	49	50	51	52	53	54	55	56	57	58	59										
1939/40	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59								
1941/42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59						
1943/44	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59				
1945/46	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59		
1947/48	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59
1949/50	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
1951/52	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
1953/54	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53
1955/56	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51
1957/58	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
1959/60	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
1961/62			25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
1963/64					25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
1965/66							25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
1967/68									25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1969/70											25	26	27	28	29	30	31	32	33	34	35	36	37
1971/72													25	26	27	28	29	30	31	32	33	34	35
1973/74															25	26	27	28	29	30	31	32	33

Year

Birth cohort's age

Birth cohort's age

discarded observations

Appendix B

Model I

$y_{bit} = p_t[\alpha_{bi} + \beta_{bi} X_{bt} + u_{bit}] + \varepsilon_{bit}$.

$u_{bit} = u_{bi,t-1} + r_{bit}$, r is "white noise" and $Var(r_{bit}) = \sigma_r^2$.

$\varepsilon_{bit} = \rho\varepsilon_{bi,t-1} + \lambda_t v_{bit}$, $Cov(v_{bit}, v_{bis}) = 0$.

$Var(v_{bit}) = \gamma_0 + \gamma_1 X_{bt} + \gamma_2 X_{bt}^2 + \gamma_3 X_{bt}^3 + \gamma_4 X_{bt}^4 = g(X_{bt})$.

- Left-censored observations:

$$\varepsilon_{bit} = \rho\varepsilon_{bi,t-1} + \lambda_t v_{bit} = \rho^2 \varepsilon_{bi,t-2} + \rho\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit} = \rho^3 \varepsilon_{bi,t-3} + \rho^2 \lambda_{t-2} v_{bi,t-2} + \rho\lambda_{t-1} v_{bi,t-1} +$$
$$+ \lambda_t v_{bit} = \ldots = \rho^{t^*} \varepsilon_{bi,85} + \rho^{t^*-1} \lambda_{86} v_{bi,86} + \rho^{t^*-2} \lambda_{87} v_{bi,87} + \ldots + \rho^2 \lambda_{t-2} v_{bi,t-2} + \rho\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit}.$$

$$\varepsilon_{bit} = \rho^{t^*} \varepsilon_{bi,85} + k \cdot \sum_{h=0}^{t^*-1} \rho^h \lambda_{t-h} v_{bi,t-h}, \quad t^* = t - 1985, \quad k = 1(t \geq 1986), \quad t \in [1985, 2005].$$

- Non left-censored observations:

$$\varepsilon_{bit} = \rho\varepsilon_{bi,t-1} + \lambda_t v_{bit} = \rho^2 \varepsilon_{bi,t-2} + \rho\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit} = \rho^3 \varepsilon_{bi,t-3} + \rho^2 \lambda_{t-2} v_{bi,t-2} + \rho\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit} = \ldots$$
$$= \rho^{X_{bt}} \varepsilon_{bi,t-X_{bt}} + \rho^{X_{bt}-1} \lambda_{t-X_{bt}+1} v_{bi,t-X_{bt}+1} + \rho^{X_{bt}-2} \lambda_{t-X_{bt}+2} v_{bi,t-X_{bt}+2} + \ldots + \rho^2 \lambda_{t-2} v_{bi,t-2} + \rho\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit}.$$

$$\varepsilon_{bit} = \rho^{X_{bt}} \varepsilon_{bi,t-X_{bt}} + l \cdot \sum_{h=0}^{X_{bt}-1} \rho^h \lambda_{t-h} v_{bi,t-h}, \quad l = 1(X_{bt} \geq 1), \quad X_{bt} \in [0, 26].$$

A. Total variance: $Var(y_{bit}) = p_t^2 Var(\alpha_{bi} + \beta_{bi} X_{bt} + u_{bit}) + Var(\varepsilon_{bit})$.

Permanent variance component

$$p_t^2 Var(\alpha_{bi} + \beta_{bi} X_{bt} + u_{bit}) = p_t^2 \left[\sigma_\alpha^2 + \sigma_\beta^2 X_{bt}^2 + 2\sigma_{\alpha\beta} X_{bt} + \sigma_r^2 X_{bt}\right].$$

Transitory variance component

- Left-censored observations: $Var(\varepsilon_{bit}) = \rho^{2t^*} \sigma_{\varepsilon_b}^2 + l \cdot \sum_{h=0}^{t^*-1} \rho^{2h} \lambda_{t-h}^2 \cdot g(X_{b,t-h})$.
- Non left-censored observations: $Var(\varepsilon_{bit}) = \rho^{2X_{bt}} \sigma_{\varepsilon_b}^2 + l \cdot \sum_{h=0}^{X_{bt}-1} \rho^{2h} \lambda_{t-h}^2 \cdot g(X_{b,t-h})$.

B. Covariance:

$$Cov(y_{bit}, y_{bis}) = p_t p_s \left[\sigma_\alpha^2 + \sigma_\beta^2 X_{bt} X_{bs} + \sigma_{\alpha\beta}(X_{bt} + X_{bis}) + \sigma_r^2 X_{bt}\right] + \rho^{(s-t)} Var(\varepsilon_{bit}), \quad t < s.$$

Model II

$y_{bit} = p_t[\alpha_{bi} + \beta_{bi} X_{bt} + u_{bit}] + \varepsilon_{bit}$,

$u_{bit} = u_{bi,t-1} + r_{bit}$,

$\varepsilon_{bit} = \rho\varepsilon_{bi,t-1} + \lambda_t v_{bit} + \theta\lambda_{t-1} v_{bi,t-1}$,

$Var(v_{bit}) = \gamma_0 + \gamma_1 X_{bt} + \gamma_2 X_{bt}^2 + \gamma_3 X_{bt}^3 + \gamma_4 X_{bt}^4 = g(X_{bt})$.

Left-censored observations:

$$\varepsilon_{bit} = \rho\varepsilon_{bi,t-1} + \lambda_t v_{bit} + \theta\lambda_{t-1} v_{bi,t-1} = \rho^2 \varepsilon_{bi,t-2} + \rho\lambda_{t-1} v_{bi,t-1} + \rho\theta\lambda_{t-2} v_{bi,t-2} + \lambda_t v_{bit} + \theta\lambda_{t-1} v_{bi,t-1} =$$
$$= \rho^3 \varepsilon_{bi,t-3} + \rho^2\theta\lambda_{t-3} v_{bi,t-3} + \rho(\rho+\theta)\lambda_{t-2} v_{bi,t-2} + (\rho+\theta)\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit} = \ldots$$
$$= \rho^{X_{bt}} \varepsilon_{bi,t-X_{bt}} + \rho^{X_{bt}-1}\theta\lambda_{t-X_{bt}} v_{bi,t-X_{bt}} + \rho^{X_{bt}-2}(\rho+\theta)\lambda_{t-X_{bt}+1} v_{bi,t-X_{bt}+1} + \ldots + \rho(\rho+\theta)\lambda_{t-2} v_{bi,t-2} +$$
$$+ (\rho+\theta)\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit}.$$

Assuming $\varepsilon_{bi,85-X_{b,85}} = \lambda_{85-X_{b,85}} v_{bi,85-X_{b,85}} = \lambda_{t-X_{bt}} v_{bi,t-X_{bt}}$,

$$\varepsilon_{bit} = \rho^{X_{bt}-1}(\rho+\theta)\lambda_{t-X_{bt}} v_{bi,t-X_{bt}} + \rho^{X_{bt}-2}(\rho+\theta)\lambda_{t-X_{bt}+1} v_{bi,t-X_{bt}+1} + \ldots + \rho(\rho+\theta)\lambda_{t-2} v_{bi,t-2} +$$
$$+ (\rho+\theta)\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit} = \rho^{(X_{bt}-X_{85})}\Big[\rho^{X_{85}-1}(\rho+\theta)\lambda_{85-X_{b,85}} v_{bi,85-X_{b,85}} +$$
$$+ \rho^{X_{b,85}-2}(\rho+\theta)\lambda_{85-X_{b,85}+1} v_{bi,85-X_{b,85}+1} + \ldots + \rho^2(\rho+\theta)\lambda_{82} v_{bi,82} + \rho(\rho+\theta)\lambda_{83} v_{bi,83} +$$
$$+ (\rho+\theta)\lambda_{84} v_{bi,84}\Big] + \rho^{X_t - X_{85}-1}(\rho+\theta)\lambda_{85} v_{bi,85} + \rho^{X_t - X_{85}-2}(\rho+\theta)\lambda_{86} v_{bi,86} + \ldots$$
$$\ldots + \rho(\rho+\theta)\lambda_{t-2} v_{bi,t-2} + (\rho+\theta)\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit}.$$

$$\varepsilon_{bit} = \rho^{(t-85)} \sum_{h=0}^{X_{b,85}-1} \rho^h (\rho+\theta)\lambda_{85-h-1} v_{bi,85-h-1} + k \cdot \sum_{h=0}^{t-86} \rho^h (\rho+\theta)\lambda_{t-h-1} v_{t-h-1} + \lambda_t v_t,$$

where λ_{85-h-1} are unobserved ($X_{bt} - X_{b,85} = t-85$ and $X_{bt} - X_{b,85} - 1 = t-86$).

Transitory variance component (left-censored observations)

$$Var(\varepsilon_{bit}) = \rho^{2(t-85)} \sum_{h=0}^{X_{b,85}-1} \rho^{2h} (\rho+\theta)^2 \lambda_{85-h-1}^2 \cdot g(X_{b,85-h-1}) +$$
$$+ k \cdot \sum_{h=0}^{t-86} \rho^{2h} (\rho+\theta)^2 \lambda_{t-h-1}^2 \cdot g(X_{b,t-h-1}) + \lambda_t^2 \cdot g(X_{bt})$$

Moffitt and Gottschalk (2008) suggest

$\sum_{h=0}^{X_{b,85}-1} \rho^{2h} (\rho+\theta)^2 \lambda_{85-h-1}^2 \cdot g(X_{b,85-h-1}) = [1+\eta \cdot X_{b,85}] \cdot \sum_{h=0}^{X_{b,85}-1} \rho^{2h} (\rho+\theta)^2 \cdot g(X_{b,85-h-1})$, where η is a "deviation" parameter, so the transitory component of the left-censored observations takes the following form

$$Var(\varepsilon_{bit}) = \rho^{2(t-85)} [1+\eta \cdot X_{b,85}] \sum_{h=0}^{X_{b,85}-1} \rho^{2h} (\rho+\theta)^2 g(X_{b,85-h-1}) +$$
$$+ k \cdot \sum_{h=0}^{t-86} \rho^{2h} (\rho+\theta)^2 \lambda_{t-h-1}^2 \cdot g(X_{b,t-h-1}) + \lambda_t^2 \cdot g(X_{bt})$$

Non left-censored observations:

$$\varepsilon_{bit} = \rho^{X_{bt}-1}(\rho+\theta)\lambda_{t-X_{bt}} v_{bi,t-X_{bt}} + \rho^{X_{bt}-2}(\rho+\theta)\lambda_{t-X_{bt}+1} v_{bi,t-X_{bt}+1} + \ldots + \rho(\rho+\theta)\lambda_{t-2} v_{bi,t-2} +$$
$$+ (\rho+\theta)\lambda_{t-1} v_{bi,t-1} + \lambda_t v_{bit}.$$

$$\varepsilon_{bit} = l \cdot \sum_{h=0}^{X_{bt}-1} \rho^s (\rho+\theta)\lambda_{t-h-1} v_{bi,t-h-1} + \lambda_t v_{bit}.$$

Transitory variance component (non left-censored observations)

$$Var(\varepsilon_{bit}) = l \cdot \sum_{h=0}^{X_{bt}-1} \rho^{2h} (\rho + \theta)^2 \lambda_{t-h-1}^2 \cdot g(X_{b,t-h-1}) + \lambda_t^2 \cdot g(X_{bt}) .$$

B. Covariance

$$Cov(y_{bit}, y_{bis}) = p_t p_s \left[\sigma_\alpha^2 + \sigma_\beta^2 X_{bt} X_{bs} + \sigma_{\alpha\beta} (X_{bt} + X_{bis}) + \sigma_r^2 X_{bt} \right] +$$

$$\rho^{(s-t)} Var(\varepsilon_{bit}) + \rho^{(s-t-1)} \theta \lambda_t^2 \cdot g(X_{bt}), \quad t < s.$$