

# A Spatial-Dependence Continuous-Time Model for Regional Unemployment in Germany

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## Abstract

This paper analyzes patterns of regional labour market development in Germany over the period 2000-2003 by means of a spatial-dependence continuous-time model. (Spatial) panel data are routinely modelled in discrete time. However, there are compelling arguments for continuous time modelling of (spatial) panel data. Particularly, most social processes evolve in continuous time such that analysis in discrete time is an oversimplification, gives a distorted representation of reality and leads to misinterpretation of estimation results. The most compelling reason for continuous time modelling is that, in contrast to discrete time modelling, it allows for adequate modelling of dynamic adjustment processes (see, for example, Special Issue 62:1, 2008, of *Statistica Neerlandica*). We introduce spatial dependence in a continuous time modelling framework and apply the unified framework to regional labour market development in Germany. The empirical results show substantial autoregressive effects for unemployment and population development, as well as a negative effect of unemployment development on population development. The reverse effect is not significant. Neither are the effects of the development of regional average wages and of the manufacturing sector on the development of unemployment and population.

**Keywords:** Continuous time modelling, structural equation modelling, spatial dependence, panel data, disattenuation, measurement errors, Germany.

**JEL codes:** C33, E24, O18, R11.

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## 1. Introduction

Socio-economic processes such as the development of unemployment are the outcomes of various decisions taken by different actors at different points in time. This basic feature gives rise to continuously evolving socio-economic dynamics, rather than to processes that change at discrete points in time. The analyst, however, only observes the processes at discrete points in time (for example, yearly observations of regional unemployment rates). The typical approach in conventional (that is, discrete) time series modelling and panel data analysis is to ignore the continuous nature of the processes underlying discrete time observations. Consequently, discrete time series and discrete panel data analysis are simplifications of reality and may lead to bias in the mapping of dynamic adjustment processes of socio-economic phenomena and to a misinterpretation of estimation results. Discrete analysis is at best a simplified approximation of real-world processes in continuous time (Oud and Singer 2008).

Continuous-time econometrics has been developed to model the continuous nature of social processes by means of systems of differential equations. It departs from the assumption that different agents take different actions at different points in time. This assumption implies that there is no obvious time interval that can serve as a natural unit. This is in contrast to discrete-time models (which are made up of systems of difference equations), which are necessarily formulated in relation to the data available (for example yearly or monthly data).

A discrete-time model estimated on the basis of, for example, monthly data will be different from a model estimated on the basis of annual data. For continuous-time approaches, however, the model is independent of the observation interval, and thus provides a common basis for accurate comparison of differently time-spaced models of the same process (Oud and Jansen 2000). These features enable the analyst to obtain predictions and simulations for any time interval, rather than for the time interval inherent to the data, as in the case of discrete-time modelling.

Continuous-time modelling is particularly useful for the analysis of dynamic adjustment processes (Gandolfo, 1993). Whereas in discrete-time models it may not be possible to obtain an estimate of the adjustment speed when the time lags are short compared to the observation period, in continuous-time models it is in general possible to obtain an asymptotically unbiased estimate of it. A continuous-time model may, therefore, allow a more satisfactory treatment of distributed-lag processes.

Continuous-time modelling has a long history in econometrics. Following the pioneering work by, amongst others, Bartlett (1946), Koopmans (1950) and Phillips (1959), continuous-time modelling has become quite common in applied econometric work (for an overview, see Bergstrom 1988). To our best knowledge, however, little attention has been paid to continuous-time modelling in spatial econometrics.<sup>2</sup> The reverse also holds: In continuous time modelling no attention has been paid to spatial dependence nor, more generally, to dependence among units of observation in cross section or panel data.

In this paper we introduce spatial dependence in a continuous time modelling framework to analyze the main determinants of regional unemployment development

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<sup>2</sup> For an interesting application we refer to Piras et al (2007). Note that in spatial analyses the units of observation usually are discrete. There is, however, theoretical work on continuous space modelling (see, for example, Puu 1997). It would be a great challenge in spatial modelling and spatial econometrics to explore the relationships between continuous time and continuous space modelling. See, amongst others Cressie 1993; Wackernagel 1995; Donaghy 2001).

in Germany, viz. the development of wages, population and industrial structure. This will be pursued in the framework of structural equations modelling (SEM). The German case is interesting and important because inflexible wages are often considered to be one of the main causes of unemployment development in Germany.

The paper is organized as follows. Section 2 presents the continuous-time model with spatial dependence and its estimation procedure by means of a nonlinear SEM procedure. Section 3 describes the model to analyse regional unemployment development in Germany, while in Section 4 estimation results are presented. Conclusions follow in Section 5.

## 2. Specification and Estimation of the Spatial-Dependence Continuous-Time Model<sup>3</sup>

Let  $\mathbf{x}(t)$  be the  $n$ -dimensional (endogenous) state vector,  $\mathbf{u}(t)$  the  $r$ -dimensional vector of (exogenous) fixed input variables,<sup>4</sup>  $\boldsymbol{\kappa}$  the  $n$ -dimensional vector of subject (region-) specific deviations (that is, a vector of ‘random subject effects’ or ‘unobserved unit heterogeneity’) and  $\mathbf{W}(t)$  the standard multivariate Wiener process. We consider the following spatial error model for regions  $i = 1, 2, \dots, N$ :

$$\frac{d\tilde{\mathbf{x}}(t)}{dt} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}\tilde{\mathbf{u}}(t) + \tilde{\boldsymbol{\kappa}} + \frac{d\tilde{\mathbf{z}}(t)}{dt}, \quad (1)$$

$$\frac{d\tilde{\mathbf{z}}(t)}{dt} = \tilde{\mathbf{R}}\tilde{\mathbf{C}} \frac{d\tilde{\mathbf{z}}(t)}{dt} + \tilde{\mathbf{G}} \frac{d\tilde{\mathbf{W}}(t)}{dt}, \quad (2)$$

where  $\tilde{\mathbf{x}}_{(Nn \times 1)} = \text{rowvec } \mathbf{X}_{(N \times n)}$  and  $\tilde{\mathbf{u}}_{(Nr \times 1)} = \text{rowvec } \mathbf{U}_{(N \times r)}$  vectorize the data matrices  $\mathbf{X}$  and  $\mathbf{U}$  (that is, for each region  $i$  there is a  $n$ -column vector of values for the  $n$  state variables and an  $r$ -column vector of values for the  $r$  fixed input variables). Similarly for the random subject effects  $\tilde{\boldsymbol{\kappa}}$ . Furthermore,  $\tilde{\mathbf{W}}$  has the same dimension ( $Nn \times 1$ ) as  $\tilde{\mathbf{x}}$ ,  $\tilde{\mathbf{A}} = \mathbf{I}_N \otimes \mathbf{A}$ ,  $\tilde{\mathbf{B}} = \mathbf{I}_N \otimes \mathbf{B}$ ,  $\tilde{\mathbf{G}} = \mathbf{I}_N \otimes \mathbf{G}$ , where the drift matrix  $\mathbf{A}$  represents the relationships among the state variables,  $\mathbf{B}$  represents the effects of the fixed input variables on the state vector, and the lower triangular matrix  $\mathbf{G}$  transforms the uncorrelated standard multivariate Wiener process with variance  $t$  into a process with variances  $\neq t$  and possible correlations  $\neq 0$ . Matrix  $\mathbf{C}$  is the spatial ( $N \times N$ ) connectivity matrix. For the multivariate case we specify the  $(Nn \times Nn)$  matrix  $\tilde{\mathbf{C}} = \mathbf{C} \otimes \mathbf{I}_n$ . Associated with  $\tilde{\mathbf{C}}$  are the spatial dependence parameters. In the general case of a different spatial dependence parameter for each state variable, we have the  $(n \times n)$  spatial parameter matrix  $\mathbf{R}$ , which for the  $N$  subjects becomes  $\tilde{\mathbf{R}} = \mathbf{I}_N \otimes \mathbf{R}$ . In

<sup>3</sup> This section is based on Oud and Folmer (2008a).

<sup>4</sup> We assume that the state variables are latent variables, that is, they cannot be directly observed due to measurement error. Latent variables are measured by one or more indicators. For instance, the concept of socioeconomic status is usually measured by means of more than one indicator, for example income, education, and profession. In the present paper, we only deal with latent variables that are measured by one indicator, though with error. The introduction of latent variables requires the use of a measurement model relating the latent variables to their indicators, and a structural model which presents the relationships between the latent variables. For further details see Oud and Folmer (2008b).

this paper, we assume one and the same spatial dependence parameter for the  $n$  state variables, that is,  $\mathbf{R} = \rho \mathbf{I}_n$ . This simplification safeguards the commuting property and has two advantages: (a) conventional procedures can be used to solve the stochastic differential equation implied by Equations (1) and (2); and (b) conventional spatial econometric methods can be applied. Observe that Model (1)–(2) includes three parameter matrices to be estimated ( $\mathbf{A}$ ,  $\mathbf{B}$ , and  $\mathbf{G}$ ) in addition to the spatial parameter  $\rho$ .

From Equations (1) and (2), we derive:

$$\frac{d\tilde{\mathbf{x}}(t)}{dt} = \tilde{\mathbf{A}}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}\tilde{\mathbf{u}}(t) + \tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathbf{C}})^{-1}\tilde{\mathbf{G}}\frac{d\tilde{\mathbf{W}}(t)}{dt}. \quad (3)$$

Equation (3) is solved over intervals  $[t - \Delta t, t)$  of length  $\Delta t$  by:

$$\begin{aligned} \tilde{\mathbf{x}}(t) = & \tilde{\mathbf{A}}_{\Delta t}\tilde{\mathbf{x}}(t - \Delta t) + \int_{t-\Delta t}^t \tilde{\mathbf{A}}_{t-s} ds \tilde{\mathbf{B}}\tilde{\mathbf{u}}(t - \Delta t) + \int_{t-\Delta t}^t \tilde{\mathbf{A}}_{t-s} ds \tilde{\boldsymbol{\kappa}} \\ & + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathbf{C}})^{-1} \int_{t-\Delta t}^t \tilde{\mathbf{A}}_{t-s} \tilde{\mathbf{G}} d\tilde{\mathbf{W}}(s), \end{aligned} \quad (4)$$

where  $\mathbf{A}_{\Delta t} = e^{\mathbf{A}\Delta t}$ ,  $\tilde{\mathbf{A}}_{\Delta t} = \mathbf{I}_N \otimes \mathbf{A}_{\Delta t}$ ,  $\mathbf{A}_{t-s} = e^{\mathbf{A}(t-s)}$ ,  $\tilde{\mathbf{A}}_{t-s} = \mathbf{I}_N \otimes \mathbf{A}_{t-s}$ , and where, for convenience sake, it is assumed that the input  $\tilde{\mathbf{u}}(t)$  can be approximated by constants over the relevant intervals  $[t - \Delta t, t)$  (for time-varying inputs, see Oud and Jansen 2000). Observe the important role of the matrix exponential  $\mathbf{A}_{\Delta t} = e^{\mathbf{A}[t-(t-\Delta t)]} = e^{\mathbf{A}\Delta t}$ , as well as the matrix exponential  $\mathbf{A}_{t-s} = e^{\mathbf{A}(t-s)}$ , which appears three times inside the integrals. Particularly,  $e^{\mathbf{A}\Delta t}$  computes the effect of  $\mathbf{x}(t-\Delta t)$  over the whole interval  $\Delta t$ , while  $e^{\mathbf{A}(t-s)}$  accounts for the fact that input, subject, and noise effects enter continuously over the interval. These effects (from each time point  $s$  to  $t$ ) must be added to obtain the total effect. For an explicit expression of the integral  $\int_{t-\Delta t}^t \mathbf{A}_{t-s} ds$  we refer to Oud and Jansen (2000).

We write Equation (4) in compact form as follows:

$$\begin{aligned} \tilde{\mathbf{x}}(t) = & \tilde{\mathbf{A}}_{\Delta t}\tilde{\mathbf{x}}(t - \Delta t) + \tilde{\mathbf{B}}_{\Delta t}\tilde{\mathbf{u}}(t - \Delta t) + \tilde{\mathbf{H}}_{\Delta t}\tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathbf{C}})^{-1}\tilde{\mathbf{w}}(t - \Delta t), \\ \text{where } \tilde{\mathbf{B}}_{\Delta t} = & \int_{t-\Delta t}^t \tilde{\mathbf{A}}_{t-s} ds \tilde{\mathbf{B}}, \\ \tilde{\mathbf{H}}_{\Delta t} = & \int_{t-\Delta t}^t \tilde{\mathbf{A}}_{t-s} ds, \\ \text{and } \tilde{\mathbf{w}}(t - \Delta t) = & \int_{t-\Delta t}^t \tilde{\mathbf{A}}_{t-s} \tilde{\mathbf{G}} d\tilde{\mathbf{W}}(s). \end{aligned} \quad (5)$$

For an explicit expression of the covariance matrix of  $\tilde{\mathbf{w}}(t - \Delta t)$  we again refer to Oud and Jansen (2000).

We now turn to the estimation of the continuous-time parameters on the basis of discrete-time observation time points  $t_i \in \{t_1, \dots, t_T\}$ . For this purpose, we specify, on the basis of Equation (5), the so-called exact discrete model (EDM) as follows (Oud and Jansen 2000):

$$\tilde{\mathbf{x}}_{t_i} = \tilde{\mathbf{A}}_{\Delta t_i} \tilde{\mathbf{x}}_{t_i - \Delta t_i} + \tilde{\mathbf{B}}_{\Delta t_i} \tilde{\mathbf{u}}_{t_i - \Delta t_i} + \tilde{\mathbf{H}}_{\Delta t_i} \tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathbf{C}})^{-1} \tilde{\mathbf{w}}_{t_i - \Delta t_i}. \quad (6)$$

Equations (5) and (6) look very similar. However, whereas (5) is a continuous time model defined for all  $t$  in continuous time, discrete time model (6) is defined for the discrete time observation points  $t_i \in \{t_1, \dots, t_T\}$  only. However, the continuous time matrices in (5) impose nonlinear restrictions on the discrete time matrices in (6). The EDM is called exact because the nonlinear restrictions it imposes ensure that the parameters estimated are exactly equal to the parameters of the underlying differential equation model. This is in contrast to several alternative estimation procedures in the literature that approximate the continuous-time parameter matrices in Equation (5) (see, for example, Singer 1990).

The continuous time parameters can be estimated by means of a nonlinear Structural Equation Model (SEM) procedure (using, for example, the Mx software, by Neale et al. 1999). For that purpose, we formulate the comprehensive SEM model, by first defining comprehensive state, input, and error vectors  $\bar{\mathbf{x}}$ ,  $\bar{\mathbf{u}}$ , and  $\bar{\mathbf{w}}$ , which contain all observations in constituting vectors at the successive observation time points  $t_i$ .

$$\begin{aligned} \bar{\mathbf{x}} &= [\tilde{\mathbf{x}}'_{t_0}, \dots, \tilde{\mathbf{x}}'_{t_{T-1}}]', \\ \bar{\mathbf{u}} &= [\tilde{\mathbf{u}}'_{t_0}, \dots, \tilde{\mathbf{u}}'_{t_{T-1}}]', \\ \bar{\mathbf{w}} &= [\tilde{\mathbf{x}}'_{t_0} - E(\tilde{\mathbf{x}}'_{t_0}), \tilde{\mathbf{w}}'_{t_0}, \dots, \tilde{\mathbf{w}}'_{t_{T-2}}]'. \end{aligned} \quad (7)$$

Next, we write Equation (6) in comprehensive form as:

$$\bar{\mathbf{x}} = \bar{B}\bar{\mathbf{x}} + \bar{\Gamma}_u \bar{\mathbf{u}} + \bar{\Gamma}_\kappa \tilde{\boldsymbol{\kappa}} + (\mathbf{I} - \bar{\mathbf{R}}\bar{\mathbf{C}})^{-1} \bar{\mathbf{w}}, \quad (8)$$

where we put all  $(Nn \times Nn)$  matrices  $\tilde{\mathbf{A}}_{\Delta t_i}$  at the appropriate places in the  $(TNn \times TNn)$  matrix  $\bar{B}$ , the  $(Nn \times Nr)$  matrices  $\tilde{\mathbf{B}}_{\Delta t_i}$  in  $(TNn \times TNr)$  matrix  $\bar{\Gamma}_u$ , and  $(Nn \times Nn)$  matrices  $\tilde{\mathbf{H}}_{\Delta t_i}$  in the  $(TNn \times Nn)$  matrix  $\bar{\Gamma}_\kappa$ . The block-diagonal  $(TNn \times TNn)$  matrix  $\bar{\mathbf{C}} = \mathbf{I}_T \otimes \tilde{\mathbf{C}}$  has (possibly asymmetric) blocks  $\tilde{\mathbf{C}}$  on its diagonal. Because of the assumption  $\mathbf{R} = \rho \mathbf{I}_n$ , we can write  $(\mathbf{I} - \bar{\mathbf{R}}\bar{\mathbf{C}})^{-1}$  as  $(\mathbf{I} - \rho \bar{\mathbf{C}})^{-1}$  with  $T$  blocks  $(\mathbf{I} - \rho \tilde{\mathbf{C}})^{-1}$  on its diagonal.<sup>5</sup>

If we reformulate Equation (8) in terms of the spatially lagged variables  $\bar{\mathbf{x}}_c = \bar{\mathbf{C}}\bar{\mathbf{x}}$  and  $\bar{\mathbf{u}}_c = \bar{\mathbf{C}}_u \bar{\mathbf{u}}$ , we obtain:

$$\bar{\mathbf{x}} = \bar{B}\bar{\mathbf{x}} + \rho(\mathbf{I} - \bar{B})\bar{\mathbf{x}}_c + \bar{\Gamma}_u \bar{\mathbf{u}} - \rho \bar{\Gamma}_u \bar{\mathbf{u}}_c + \bar{\Gamma}_\kappa \tilde{\boldsymbol{\kappa}}_c + \bar{\mathbf{w}}. \quad (9)$$

<sup>5</sup> The notation in (8) is a combination of the standard notations in state-space modelling and structural-equation modelling. Although it would be possible to introduce a new notation, we prefer to apply the combined notation so as to facilitate access to the constituting literatures.

where  $\bar{\mathbf{C}}\bar{\Gamma}_u\bar{\mathbf{u}} = \bar{\Gamma}_u\bar{\mathbf{C}}_u\bar{\mathbf{u}}$  for  $\bar{\mathbf{C}}_u = \mathbf{I}_r \otimes \mathbf{C} \otimes \mathbf{I}_r$ , and where the transformed unobserved heterogeneity  $\tilde{\boldsymbol{\kappa}}_c$  is related to the original  $\tilde{\boldsymbol{\kappa}}$  in Equation (6) as follows:

$$\tilde{\boldsymbol{\kappa}}_c = (\mathbf{I} - \tilde{\mathbf{R}}\tilde{\mathbf{C}})\tilde{\boldsymbol{\kappa}}.$$

Note that in the derivation of Equation (9) we have made use of the commuting property several times.

Equation (9) can be specified as a latent variables SEM as follows:

$$\begin{aligned} \bar{\boldsymbol{\eta}} &= \bar{\mathbf{B}}\bar{\boldsymbol{\eta}} + \bar{\Gamma}\bar{\boldsymbol{\xi}} + \bar{\boldsymbol{\zeta}} \\ \text{for} & \\ \bar{\boldsymbol{\eta}} = \bar{\mathbf{x}}, \bar{\boldsymbol{\xi}} &= [\bar{\mathbf{x}}'_c \quad \bar{\mathbf{u}}' \quad \bar{\mathbf{u}}'_c \quad \bar{\boldsymbol{\kappa}}'_c]', \bar{\boldsymbol{\zeta}} = \bar{\mathbf{w}}, \bar{\Gamma} = \begin{bmatrix} \rho(\mathbf{I} - \bar{\mathbf{B}}) & \bar{\Gamma}_u & -\rho\bar{\Gamma}_u & \bar{\Gamma}_\kappa \end{bmatrix}, \end{aligned} \quad (10)$$

which is conventionally written in variable form (rather than in terms of units of observation) as follows:

$$\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \Gamma\xi + \zeta. \quad (11)$$

If a SEM contains latent variables, in addition to structural equations, measurement equations are required which specify how the latent variables are measured, that is, how the observed variables  $\mathbf{y}$  are related to the latent variables  $[\boldsymbol{\eta}' \quad \boldsymbol{\xi}']'$ :

$$\mathbf{y} = \Lambda \begin{bmatrix} \boldsymbol{\eta} \\ \boldsymbol{\xi} \end{bmatrix} + \boldsymbol{\varepsilon}. \quad (12)$$

Matrix  $\Lambda$  in (12) contains the loadings, while the measurement errors are given by  $\boldsymbol{\varepsilon}$  (with covariance matrix  $\Theta$ ). The measurement model parameter matrices  $\Lambda$  and  $\Theta$  are estimated simultaneously with the other parameter matrices of Model (11). For reasons of interpretation and identification, it is customary to specify unifactorial observed variables only, which means that each observed variable in  $\mathbf{y}$  has a loading on only one single latent variable in  $[\boldsymbol{\eta}' \quad \boldsymbol{\xi}']'$ .

The vector  $\bar{\mathbf{y}}$  and its spatially lagged counterpart  $\bar{\mathbf{y}}_c$  are defined analogously to  $\bar{\mathbf{x}}$  and  $\bar{\mathbf{x}}_c$ . Therefore, we assume that their matrices of loadings and measurement intercepts  $\bar{\mathbf{d}}$  are equal. This gives the following measurement model for  $\bar{\mathbf{x}}$  and  $\bar{\mathbf{x}}_c$ .

$$\begin{aligned} \bar{\mathbf{y}} &= \bar{\mathbf{L}}\bar{\mathbf{x}} + \bar{\mathbf{d}} + \bar{\mathbf{v}}, \\ \bar{\mathbf{y}}_c &= \bar{\mathbf{L}}\bar{\mathbf{x}}_c + \bar{\mathbf{d}} + \bar{\mathbf{v}}_c. \end{aligned} \quad (13)$$

We impose no equality constraints between the measurement error variances of  $\bar{\mathbf{v}}$  and  $\bar{\mathbf{v}}_c$ , since the measurement errors  $\bar{\mathbf{v}}_c$  are linear combinations of the measurements errors  $\bar{\mathbf{v}}$  and therefore typically have lower variance. The repeated measurements of the state variables and of their lagged counterparts serve as the observed indicators of the underlying latent variables. The repeated measurements provide a larger number of observed indicators for each latent variables which is required for model identification.

Estimation of SEM models basically comes down to minimizing, in some metric, the distance between the theoretical variance-covariance or moment matrix of the observed variables (as determined by the model specifications) and the corresponding sample matrix. Oud and Folmer (2008b) show that in the case of maximum likelihood estimation the standard SEM likelihood function for a spatial dependency model is augmented by the Jacobian correction term  $\ln |\mathbf{I} - \rho\bar{\mathbf{C}}|$ , where  $\ln$  denotes the natural logarithm. The size of the Jacobian correction depends on the number of dependent variables. In a conventional spatial error model with only a single dependent variable and weights matrix  $\mathbf{C}$ , it is:

$$\ln |\mathbf{I} - \rho\mathbf{C}|. \quad (14)$$

In a multivariate model with equal spatial dependence parameter for  $n$  variables with  $(Nn \times Nn)$  matrix  $\tilde{\mathbf{C}}$ , the Jacobian correction is:

$$\ln |\mathbf{I} - \rho\tilde{\mathbf{C}}| = n \ln |\mathbf{I} - \rho\mathbf{C}|. \quad (15)$$

In a longitudinal analysis with  $T$  observations and  $\bar{\mathbf{C}}$  of order  $(TNn \times TNn)$ , the correction is:

$$\ln |\mathbf{I} - \rho\bar{\mathbf{C}}| = Tn \ln |\mathbf{I} - \rho\mathbf{C}|. \quad (16)$$

Finally, if each of the  $n$  latent variables is measured by  $m$  indicators, a  $(TNnm \times TNnm)$  matrix  $\tilde{\tilde{\mathbf{C}}}$  applies and the correction becomes (Oud and Folmer 2008b):

$$\ln |\mathbf{I} - \rho\tilde{\tilde{\mathbf{C}}}| = Tnm \ln |\mathbf{I} - \rho\mathbf{C}|. \quad (17)$$

We observe that nonlinear SEM programs like Mx (Neale et al. 1999) can be applied to estimate the Equations (11) and (12), including all linear and nonlinear restrictions implied by both the continuous time and the spatial dependence specification.

### 3. The Regional Unemployment Model

Following Elhorst (2003) and Blanchard and Katz (1992), we adopt a regional labour market model that relates regional unemployment rates (the result of job-matching) to regional labour supply, economic structure and wages. Elhorst points out that *'the regional unemployment rate both affects and is affected by regional factors of labour supply, labour demand, and wages'*. Therefore we adopt a simultaneous equations framework to study the reciprocal effects of regional unemployment development and regional labour supply development, as well as the impacts of the developments of economic structure and wages on both variables. The latter two variables are assumed exogenous. The rationale for considering the wage variable exogenous is based on the fact that in Germany, like in many other European countries, collective wage agreements are set at the national level on a sectoral basis rather than at the regional

level. This means that contractual wages may be considered exogenous for a given region. This view is supported by a large literature in labour economics (see, for example, Lommerud et al. 2000; Correa López and Naylor 2004). The fact that wages are set nationally rather than regionally does of course not mean that average wages are largely equal across regions. Wage differentials occur due to differences in regional economic structure.

The rationale for taking economic structure as exogenous follows from the fact that this variable evolves slowly, such that changes only show up in the long run. Moreover, its evolution depends on a large set of factors and definitely not only on the regional wage structure. Since the time span considered in this paper is seven years only, we consider economic structure exogenous. Due to lack of data, economic structure is measured in this paper as the proportion of the workforce employed in manufacturing (for a similar approach, see Jones and Manning 1992).

Formally, the regional unemployment model comprises two equations. In the first equation, the unemployment development is explained by population development, as well as by development of average daily wages of fulltime workers, and of economic structure. The expected effect of population development on unemployment development is ambiguous, since demographic development has an impact on both labour supply (positive or zero impact on unemployment development) and labour demand (negative impact on unemployment development). On the supply side, population development – via natural growth or immigration – may lead to a larger workforce and to changes in its age structure. Changes in the age structure leading to a younger average population – due to higher birth rates – have been shown to lead to higher, and more persistent, unemployment rates (Elhorst 1995), because of a larger workforce. However, natural change of population is known to be a slow, long-run process rather than a short-run one. Consequently, it is not expected to have a significant effect on unemployment in the short panel considered in the present paper.

With regard to immigration, the expected effect of an additional migrant on unemployment is zero, if the migrant fills a job opening for which no one in the home region qualifies or if the migrant does not join the workforce. However, the impact is expected to be positive (increasing unemployment) when the effect of additional migrants is worked out through the accounting identity<sup>6</sup> (Elhorst 2003). On the demand side, negative effects of net immigration on unemployment development may be identified, for example, because of increased productivity induced by migrants with different skill endowments (Ghatak et al. 1996), or greater investments attracted by an increase of higher-skilled labour, or higher consumption levels due to a larger population. We expect the labour supply side to dominate the demand side (and hence a positive effect of net immigration on unemployment development; see, for example, Pissarides and McMaster 1990). Since in Germany there is outmigration from high unemployment regions in the East (unemployment going down) to low unemployment regions in the West (unemployment going up or remaining constant), migration is likely to induce some convergence of regional unemployment.

Because of the negative impact of wages on labour demand (they represent greater costs for firms), higher wages are expected to increase unemployment. A positive coefficient of wage development on unemployment development may then be expected (as, for example, in Hall 1972; Layard et al. 1991). With regard to the relationship between economic structure (regional specialization) and unemployment, we expect, for regions with a relatively dominant manufacturing sector and a low-

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<sup>6</sup> The accounting identity is a deterministic method for computing regional unemployment levels.

skilled workforce (as in East Germany), that a decrease in the manufacturing sector will lead to an increase in the number of unemployed individuals. Workers laid off from the manufacturing sector are likely to experience difficulties in becoming re-employed, and will risk long-term unemployment and loss of skills. A negative effect of specialization in manufacturing on unemployment development may then be expected.

The second equation may be compared – with due differences – to the labour supply equation in the Blanchard and Katz (BK) model (Blanchard and Katz 1992). In this equation, population development is explained by the developments of unemployment, wages and manufacturing. We expect higher unemployment to increase out-migration and to lead to lower fertility rates. A negative effect of unemployment development on population development may then be expected. Inversely, higher wages will tend to attract individuals towards a region (in-migration). A positive value of the wage coefficient may be expected. With regard to regional economic structure, we note that, if decreasing specialization in manufacturing results in a greater number (or share) of long-term unemployed, a positive coefficient, though weak, may then be expected for the effect of change in specialization on population development (see, for example, Budd et al. 1987).

The above can be summarized as follows. The model is made up of:

- The state variables unemployment development and population development. For each state variable, an autoregressive effect is expected. Moreover, feedback relationships are hypothesized between unemployment development and population development: a positive effect of population development on unemployment development, and a negative effect for the opposite direction from unemployment development to population development.
- The fixed input variables wage development and development of the manufacturing sector. We assume a positive wage effect and a negative manufacturing effect on unemployment development. For population development we assume positive effects of both input variables.
- First-order spatial lags for each of the state variables. At first instance, we assume the spatial dependence parameters for the state variables to be equal such that  $\tilde{\mathbf{R}}$  in Equation (2) contains only a single spatial parameter  $\rho$ .

The simultaneous equation model is presented in Equation (18), where  $ud(t)$  is unemployment development,  $pd(t)$  is population development,  $wd(t)$  is wage development, and  $md(t)$  is manufacturing development:

$$\begin{aligned} \frac{dud(t)}{dt} &= a_{11}ud(t) + a_{12}pd(t) + b_{11}wd(t) + b_{12}md(t) + b_1 + \kappa_1 + \frac{dz_1(t)}{dt}, \\ \frac{dpd(t)}{dt} &= a_{21}ud(t) + a_{22}pd(t) + b_{21}wd(t) + b_{22}md(t) + b_2 + \kappa_2 + \frac{dz_2(t)}{dt}. \end{aligned} \quad (18)$$

Coefficients  $a_{11}$  and  $a_{22}$  represent the autoregressive effects of  $ud$  and  $pd$ ,  $a_{12}$  and  $a_{21}$  the cross-effects of  $pd$  on  $ud$  and of  $ud$  on  $pd$ , respectively, whereas the effects of the input variables  $wd$  and  $md$  are given by  $b_{11}$  and  $b_{12}$  (on  $ud$ ) and  $b_{21}$  and  $b_{22}$  (on  $pd$ ). Finally,  $b_1$  and  $b_2$  are the intercepts, and  $\kappa_1$  and  $\kappa_2$  are the region-specific random effects. Because the random subject effects  $\kappa_1$  and  $\kappa_2$  represent deviations from the

intercepts  $b_1$  and  $b_2$ ,  $E(\kappa_1) = E(\kappa_2) = 0$ . The squared deviations show up in the model as the variances  $\varphi_{\kappa_1}$  and  $\varphi_{\kappa_2}$  and covariance  $\varphi_{\kappa_1\kappa_2}$ .

Observe that continuous-time effects basically are the limits of the corresponding effects in discrete time for the observation interval going to zero. Due to the nonlinear relationship between continuous time and discrete time effects, the parameter values – and even their signs – may differ between continuous and discrete time. Particularly, the values of the continuous-time autoregressive effects (direct feedback-effects)  $a_{11}$  and  $a_{22}$  are to be interpreted differently from the corresponding discrete-time autoregressions  $a_{12\Delta t}$  and  $a_{21\Delta t}$ . A continuous-time autoregressive effect of 0 in  $\mathbf{A}$  (no change) corresponds to a discrete-time autoregression of 1 in  $\mathbf{A}_{\Delta t}$ , and a continuous time autoregressive effect of  $-\infty$  in  $\mathbf{A}$  (maximum negative feedback) to a discrete-time autoregression of 0 in  $\mathbf{A}_{\Delta t}$ . So, continuous-time autoregressive effects in the range  $(-\infty, 0)$  are transformed to discrete-time autoregressions in the range  $(0, 1)$ . The interpretation of the 'cross-effects'  $a_{12}$  and  $a_{21}$  is similar to the corresponding cross-lagged effect  $a_{12\Delta t}$  and  $a_{21\Delta t}$  in discrete time.

For the error components in Equation (18), Equation (2) applies with spatial dependence parameter  $\rho$  and the parameters  $g_{11}$ ,  $g_{22}$  and  $g_{21}$  in matrix  $\mathbf{G} = \begin{pmatrix} g_{11} & 0 \\ g_{21} & g_{22} \end{pmatrix}$ . This matrix transforms the two independent standard Wiener processes in

$$\frac{d\mathbf{W}(t)}{dt} = \begin{pmatrix} \frac{dW_1(t)}{dt} \\ \frac{dW_2(t)}{dt} \end{pmatrix}$$

into the correlated general Wiener processes for  $ud$  and  $pd$ .

Finally, we discuss the parameters relating to the initial time point when the process starts. First of all, there are the initial state means  $\mu_{x_{1t_0}}$  and  $\mu_{x_{2t_0}}$ , their variances  $\varphi_{x_{1t_0}}$ ,  $\varphi_{x_{2t_0}}$ , and covariance  $\varphi_{x_{1t_0}x_{2t_0}}$ . Because the initial means may differ in regions with different levels of  $wd$  and  $md$  at the initial point, regression coefficients  $b_{11t_0}$ ,  $b_{12t_0}$ ,  $b_{21t_0}$  and  $b_{22t_0}$  for the regression of  $ud(t_0)$  and  $pd(t_0)$  on  $wd(t_0)$  and  $md(t_0)$  are computed. Hence, as in standard regression analysis, the regression means  $b_{11t_0} \overline{wd(t_0)} + b_{12t_0} \overline{md(t_0)}$  and  $b_{21t_0} \overline{wd(t_0)} + b_{22t_0} \overline{md(t_0)}$  are subtracted from the initial means, and the initial conditional variances and covariance (conditional on the initial inputs) are taken. As the region specific random effects  $\kappa_1$  and  $\kappa_2$  are assumed to influence the state variables before as well as after initial time point  $t_0$ , the four covariances between the initial state variables and the random region effects  $\varphi_{\kappa_1 x_{1t_0}}$ ,  $\varphi_{\kappa_1 x_{2t_0}}$ ,  $\varphi_{\kappa_2 x_{1t_0}}$  and  $\varphi_{\kappa_2 x_{2t_0}}$  are estimated, since they cannot, in general, be taken as zero.

We estimate two types of models: one without (I), and one with (II) measurement errors for the state variables, as well as for their spatially lagged counterparts. Compared to Model I, there are four additional parameters in Model II: the measurement error variances  $\theta_{v1}$  and  $\theta_{v2}$  for the observed state variables, and measurement error variances  $\theta_{v1C}$  and  $\theta_{v2C}$  for their spatially lagged counterparts (see Equation (13)).

Both Model I and Model II are estimated by ML. Since  $T = 4$ ,  $n = 2$  and  $m = 1$  (a single indicator per latent variable), the Jacobian correction term added to the likelihood function in both models is  $\ln |\mathbf{I} - \rho\tilde{C}| = Tnm \ln |\mathbf{I} - \rho C| = 8 \ln |\mathbf{I} - \rho C|$ .

#### 4. Empirical Results

We analyse unemployment development in 439 German labour market regions over the period 2000–2003 by means of the continuous-time spatial-dependence modelling approach outlined above. All observed variables in the model (two endogenous (state) variables and two exogenous (input) variables) are defined as differences between subsequent years divided by 1,000. For instance, unemployment development in region  $r$  in 2000 is measured as  $(1/1000) * (\text{unemployed in region } r \text{ in 2000} - \text{unemployed in region } r \text{ in 1999})$ .

The estimation results are given in Table 1. First of all, we refer to the spatial dependence parameter  $\rho$ , which is 0.375 in Model I and 0.378 in Model II. In both models,  $\rho$  is highly significant. Since it is rather restrictive, we relaxed the assumption of equal spatial dependence for  $ud$  and  $pd$ . However, this did not lead to any significant improvement in fit, as measured by the  $\chi^2$ -difference test. We conclude that the restrictive assumption introduced with regard to  $\rho$  is not contradicted by the data.

Table 1 ML parameter estimates and associated t-values for model I (without measurement errors) and model II (with measurement errors)

<i>Par.</i>	Model I		Model II	
	<i>Est.</i>	<i>t</i>	<i>Est.</i>	<i>t</i>
Spatial parameter				
$\rho$	0.375	16.30*	0.378	14.00*
Measurement error variances				
$\theta_{v1}$			0.422	16.88*
$\theta_{v2}$			0.081	1.09
$\theta_{v1C}$			0.015	0.82
$\theta_{v2C}$			0.044	4.04*
State effects				
$a_{11}$	-1.664	-11.64*	-0.594	-6.83*
$a_{12}$	-0.009	-0.17	0.013	0.54
$a_{21}$	-0.252	-2.00*	-0.498	-4.64*
$a_{22}$	-1.363	-6.75*	-1.168	-5.01*

<i>Par.</i>	Model I		Model II	
	<i>Est.</i>	<i>t</i>	<i>Est.</i>	<i>t</i>
Input effects				
$b_{11}$	0.016	0.39	0.005	0.19
$b_{12}$	0.047	0.98	0.021	0.78
$b_{21}$	0.001	0.01	0.017	0.49
$b_{22}$	0.033	0.89	0.014	0.34
Fixed intercepts				
$b_1$	0.633	10.21*	0.463	11.87*
$b_2$	-0.136	-1.97*	-0.101	-1.65
Random intercept variance				
$\Phi_{\kappa_2}$	0.679	2.23*	0.439	1.64
Error parameters				
$g_{11}$	1.316	23.93*	0.156	1.61
$g_{22}$	1.196	19.93*	1.000	16.40*
$g_{21}$	-0.010	-0.18	-0.059	-0.31
Initial (reduced) state means				
$\mu_{x_{1t_0}}$	-0.656	-13.39*	-0.647	-12.68*
$\mu_{x_{2t_0}}$	-0.146	-2.24*	-0.148	-2.24*
Initial (conditional) state variances and covariance				
$\Phi_{x_{1t_0}}$	0.419	14.96*	0.052	1.85
$\Phi_{x_{2t_0}}$	1.270	14.76*	1.202	11.13*
$\Phi_{x_{1t_0} x_{2t_0}}$	-0.014	0.40	0.015	0.42
Initial time point regression coefficients				
$b_{11t_0}$	0.030	0.86	0.010	0.30
$b_{12t_0}$	-0.057	-1.63	-0.049	-1.44
$b_{21t_0}$	0.021	-0.51	-0.018	-0.42
$b_{22t_0}$	0.029	0.69	0.027	0.63
Covariances between random intercept and initial states				
$\Phi_{\kappa_2 x_{1t_0}}$	-0.085	-2.24*	-0.091	-2.52*
$\Phi_{\kappa_2 x_{2t_0}}$	0.940	5.25*	0.792	4.10*
Fit				
$\chi^2$	770.2		650.7	
<i>df</i>	206		202	
RMSEA	0.081		0.071	

From the significance of the measurement error variance of  $ud$  (0.422) and of the spatially lagged  $pd$  (0.044), it follows that Model II is more adequate than Model I. This conclusion is supported by the significant improvement of model fit when the assumption of no measurement errors is dropped, as shown by the difference test ( $\chi_{dif}^2 = 770.2 - 650.7 = 119.5$  for  $df = 206 - 202 = 4$ ). Finally, the RMSEA fit measure

of Model II (0.071) is smaller than for Model I (0.08), and meets the criterion of a ‘reasonable’ fit (Jöreskog and Sörbom 1996, p. 124). We conclude that Model II is preferable to Model I. For the remainder of this paper, we only consider Model II.

In order to facilitate interpretation, we present, in Table 2, the continuous-time state effect matrices  $\mathbf{A}$ , as well as the corresponding discrete-time effect matrices  $\mathbf{A}_{\Delta t}$  for observation interval  $\Delta t=1$  derived from the continuous time state effect matrices. Moreover, in Table 2 we present both the standardized (by the initial variances) and unstandardized effects in  $\mathbf{A}$  and  $\mathbf{A}_{\Delta t}$  (in Table 1, only unstandardized effects are presented).

Table 2 Continuous time state effect matrices  $\mathbf{A}$  and corresponding EDM autoregression matrices  $\mathbf{A}_{\Delta t} = e^{\mathbf{A}\Delta t}$  for Model I (without measurement errors) and Model II (with measurement errors), over the one-year observation interval  $\Delta t = 1$ , in unstandardized and standardized form

Par.	Model I		Model II	
	$\mathbf{A}$	$\mathbf{A}_{\Delta t=1}$	$\mathbf{A}$	$\mathbf{A}_{\Delta t=1}$
Unstandardized				
<i>ud</i>	$a_{11} = -1.664^*$	$a_{11\Delta t=1} = 0.190$	$a_{11} = -0.594^*$	$a_{11\Delta t=1} = 0.550$
<i>pd</i> → <i>ud</i>	$a_{12} = -0.009$	$a_{12\Delta t=1} = -0.002$	$a_{12} = 0.013$	$a_{12\Delta t=1} = 0.006$
<i>ud</i> → <i>pd</i>	$a_{21} = -0.252^*$	$a_{21\Delta t=1} = -0.056$	$a_{21} = -0.498^*$	$a_{21\Delta t=1} = -0.209$
<i>pd</i>	$a_{22} = -1.363^*$	$a_{22\Delta t=1} = 0.256$	$a_{22} = -1.168^*$	$a_{22\Delta t=1} = 0.310$
Standardized				
<i>ud</i>	$a_{11} = -1.664^*$	$a_{11\Delta t=1} = 0.190$	$a_{11} = -0.594^*$	$a_{11\Delta t=1} = 0.550$
<i>pd</i> → <i>ud</i>	$a_{12} = -0.016$	$a_{12\Delta t=1} = -0.003$	$a_{12} = 0.063$	$a_{12\Delta t=1} = 0.027$
<i>ud</i> → <i>pd</i>	$a_{21} = -0.145^*$	$a_{21\Delta t=1} = -0.032$	$a_{21} = -0.104^*$	$a_{21\Delta t=1} = -0.044$
<i>pd</i>	$a_{22} = -1.363^*$	$a_{22\Delta t=1} = 0.256$	$a_{22} = -1.168^*$	$a_{22\Delta t=1} = 0.310$

Comparison of Models I and II in Table 2 illustrates the disattenuation effect due to explicitly accounting for measurement errors. In general, the coefficients in Model II are larger in absolute value than the corresponding ones in Model I, indicating that the latter are attenuated by measurement errors. Therefore, we only consider Model II below.

From Table 1 and Table 2, it follows that, in accordance with our expectations (see Section 3), both *ud* and *pd* show substantial autoregressive effects ( $-0.594$  and  $-1.168$  in  $\mathbf{A}$ ;  $0.550$  and  $0.310$  in  $\mathbf{A}_{\Delta t}$ ). Since in both models the coefficients  $a_{11}$  and  $a_{22}$  are negative and significant, the model is stable. Moreover, as hypothesized, *ud* has a negative effect on *pd* (standardized value of  $-0.104$ ), which is highly significant. The cross-effect of *pd* on *ud*, however, though positive, is not significant (standardized value of  $0.063$ ). This result is in line with the dual population effect on unemployment (see Section 3), in that supply factors (such as a potentially larger workforce) are counterbalanced by demand factors (such as an increased demand for goods).

Table 1 shows that neither of the two input variables (wage development and development of the manufacturing sector) has a significant effect on either of the two state variables. The insignificant effect of wage development on unemployment

development is surprising. It could be due to the rigidity of the German wage structure (particularly national wage setting), such that regional wages insufficiently reflect the regional unemployment structure. Its insignificant effect on population development could be due to the many constraints on labour mobility (such as, for example, inefficiencies in the housing market). Longer time lags (and hence longer time series) may be needed for significant effects to show up.

The fixed intercept of  $ud$  is positive, substantial (0.463) and highly significant, whereas the fixed intercept for  $pd$  is negative (−0.101) and not significant at the 5 per cent level. The random intercept variance and covariances for  $\kappa_1$  ( $\phi_{\kappa_1}, \phi_{\kappa_1 x_{1t_0}}, \phi_{\kappa_1 x_{2t_0}}$ ) are not significant and have been left out of the final analysis because they affect the estimations of all other model parameters. However, because the covariances of  $\kappa_2$  with the initial state variables,  $\phi_{\kappa_2 x_{1t_0}}$  and  $\phi_{\kappa_2 x_{2t_0}}$ , are significant, the assumption of the presence of a random intercept for  $pd$  (variance  $\phi_{\kappa_2} = 0.439$ ) is supported by the data and it is therefore included in the model. It follows that regions resemble each other much more with regard to unemployment development than with regard to population development, where random intercepts are needed to give each region its own expected development curve. For unemployment development, on the other hand the data support only *one* single expected curve towards which each region in Germany regresses.

With regard to the error variances of the structural equations, we find that  $g_{11}$  and  $g_{21}$  are insignificant. However, we do not impose restrictions of the type  $g_{11} = 0$  and  $g_{21} = 0$ , since it is unrealistic to assume that the model explains all the variance in  $ud$  and all the covariance between  $ud$  and  $pd$ .

The initial state variance of  $ud$  is not significant. This result for  $ud$ , together with the insignificance of its random intercept variance, means that regression for  $ud$  is not only towards the same expected curve for all regions, but also that the variance of the regions around this common expected curve is quite small. The initial state variance of  $pd$ , however, is significant, meaning that from the start in 2000 the regions show clear differences in population development.

The initial means of the state variables over the 439 German labour market regions show that both unemployment and population decrease at the beginning of 2000 (−0.647<sup>7</sup> and −0.148, respectively). The initial mean for unemployment development is much larger (in absolute value) and has a higher  $t$ -value than the initial mean for population development ( $t = -12.68$  versus  $t = -2.24$ ). It should be noted that these means have been reduced by the (insignificant) regression means. The uncontrolled means are even larger in absolute value (−0.975 for  $ud$  and −0.288 for  $pd$ ).

#### 4.1 Autoregression Functions, Cross-Lagged Effect Functions, and Means Trajectories

The estimates of Model II will now be used to depict the autoregression functions of  $ud$  and  $pd$  (Figure 1), the standardized cross-lagged effect functions of  $ud \rightarrow pd$  and  $pd \rightarrow ud$  (Figure 2), and the means trajectories of  $ud$  and  $pd$  (Figure 3) in continuous time. Figure 1 shows the decay of the autoregressive effects of  $ud$  and  $pd$ . The decay is slower for  $pd$  than for  $ud$ . For  $pd$ , the decay is approximately 70 per cent after two years. For  $ud$ , the decay is approximately 90 per cent over the same period.

<sup>7</sup> Meaning a mean decrease of 647 unemployed persons over the previous year.

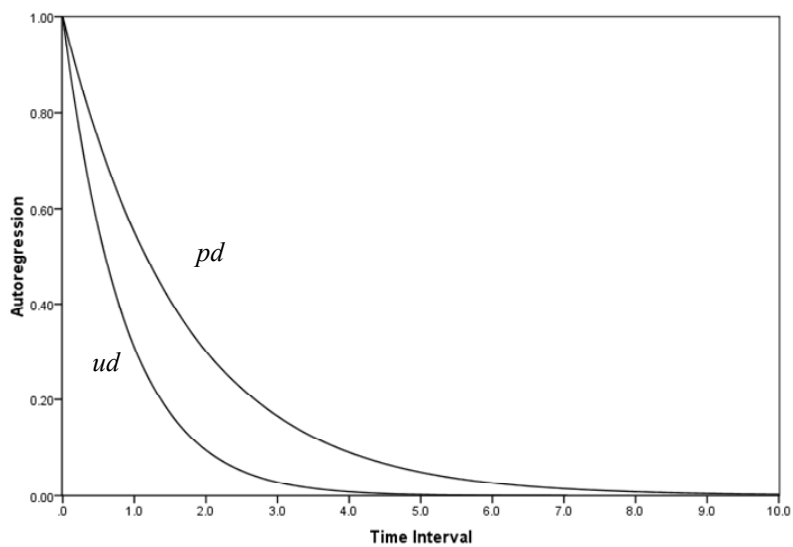


Figure 1 Autoregression functions based on Model II

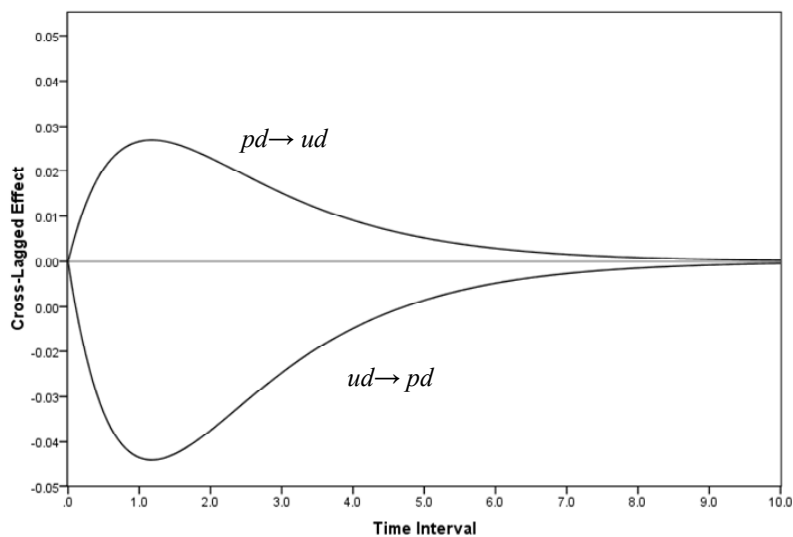


Figure 2 Standardized cross-lagged effect functions based on Model II

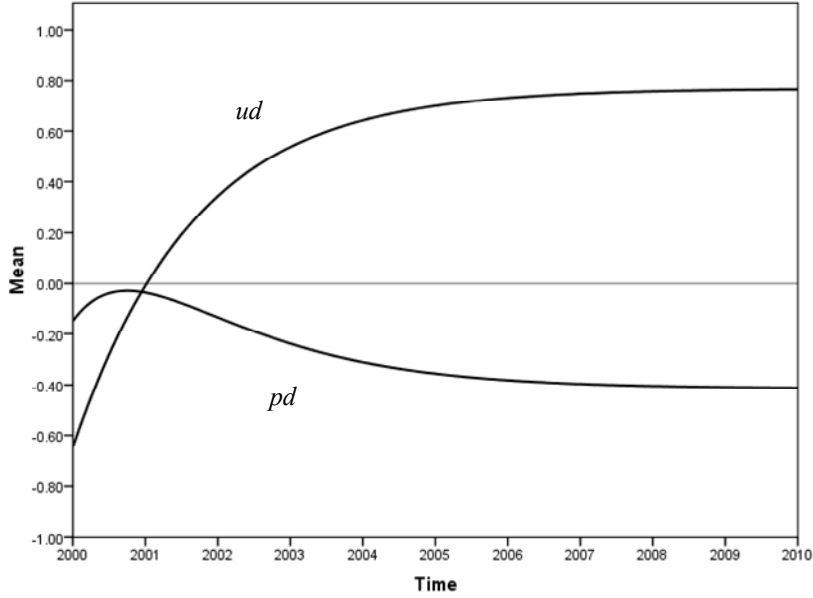


Figure 3 Means trajectories based on Model II

Because the model is asymptotically stable, both the autoregression functions and the cross-lagged effect functions go to zero. The standardized cross-lagged effect functions in Figure 2 show the effects in terms of standard deviation units of the dependent variable from a standard deviation unit increase in the explanatory variable. The cross-lagged effect functions start from zero, then reach a maximum ( $pd \rightarrow ud$ ), and a minimum ( $ud \rightarrow pd$ ), respectively, and finally die out towards zero. In both directions ( $ud \rightarrow pd$  and  $pd \rightarrow ud$ ), the effects are very small and in both cases the extreme values are reached after 1.15 years. A standard deviation increase in unemployment development diminishes population development by 0.044, while a standard deviation increase in population development increases employment development by 0.027 (standard deviations over regions). Observe that the effects die out rather slowly: in both cases, after four years, more than a quarter of the maximum impact is left.

Figure 3 depicts the autonomous developments of the means of  $ud$  and  $pd$ , independent from input effects. They are given by (see Oud and Jansen 2000):

$$E[\mathbf{x}(t)] = e^{\mathbf{A}(t-t_0)} E[\mathbf{x}(t_0)] + \mathbf{A}^{-1} [e^{\mathbf{A}(t-t_0)} - \mathbf{I}] \mathbf{b}, \quad (19)$$

where  $\mathbf{b}$  includes the fixed intercepts. The mean development is driven by two components: the autoregression effect of the initial means  $E[\mathbf{x}(t_0)]$  and the integrated effect of the intercepts  $\mathbf{b}$  over the time period. Model II estimates are used for the specification of  $\mathbf{A}$ ,  $\mathbf{b}$ , and  $\mathbf{x}(t_0)$  in (12). Figure 3 shows that, over the observation period 2000–2003, shortly after 2001, the mean unemployment decrease turned into an unemployment increase which started levelling off after 2003. The mean population decrease diminished until shortly before 2001, and then the downward trend increased again. Both trajectories tend to a stable equilibrium position. This stable equilibrium position implies for  $ud$  a mean unemployment increase of 770.1 per region and, for  $pd$ , a mean population decrease of 414.4.

## 5. Conclusions

In this paper, continuous-time modelling, as introduced in econometrics in the 1950s by, amongst others, Koopmans (1950) and Phillips (1959), and in sociology in the late 1960s by Coleman (1968), is used to analyse regional unemployment change in Germany on the basis of a data set for the period 2000–2003. For this purpose, we combine the continuous-time modelling approach developed by Oud and Jansen (2000) with the spatial dependence approach by Oud and Folmer (2008a).

Our results shed light on the determinants of regional unemployment and labour supply as measured by regional population development. We find that both unemployment development and population development have substantial autoregressive effects. Regarding cross-effects our results show that an increase of regional unemployment development leads to a decrease of local population development: an increase of unemployment in a given region leads to out-migration to which experience no or relatively less unemployment growth. This result is consistent with theoretical expectations. On the other hand, we do not find a significant effect of population increase (which usually generates pressure on the regional job market) on unemployment development. While this finding is worth further investigation, we may – at this stage – attribute it to demand factors (such as increased economic activity stimulated by increased population), which counterbalance the labour supply effect. Wages and specialization in manufacturing do not significantly impact on unemployment development nor population development. A possible explanation for this finding is that wage setting in Germany takes place at the national level and that there are only minor regional differences in wage development in Germany. The non-significance of specialization in manufacturing may be explained by the fact that changes in economic structure rarely happen in the short run.

Secondly, we find that regions resemble each other much more with regard to unemployment development than with regard to population development. Particularly, regression of unemployment development is not only towards the same expected curve for all regions, but also the variance of the regions around the common expected curve is quite small. This result confirms the uniform development of regional unemployment in Germany.

Thirdly, we find that, for both unemployment development and population development, regional shocks are absorbed rather fast (50 per cent is absorbed within 14 months), and have generally a short lifespan. They are slightly longer for population development than for unemployment development, most likely because of the many constraints to mobility, such as housing market imperfections. The reciprocal cross-lagged effects between unemployment and population development, though definitely small, are long-lasting. A peak is reached shortly after one year, and is reduced to 25 per cent after four years.

From a policy-making viewpoint, our findings suggest that wage development does not have a significant effect on regional unemployment development, which could be due to the fact that wages are set nationally on a sectoral basis in Germany. Therefore, locally set wages, which reflect regional unemployment development, might be considered as an instrument to reduce unemployment.

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## References

- Bartlett M.S. (1946) On the Theoretical Specification of Sampling Properties of Autocorrelated Time Series. *Journal of the Royal Statistical Society* 8 (1), 28-41
- Bergstrom A.R. (1988) The History of Continuous-Time Econometric Models. *Econometric Theory* 4 (3), 365-83
- Blanchard O.J. and L.F. Katz (1992) Regional Evolutions. *Brookings Papers on Economic Activity* 1, 1-75
- Budd P., R. Levine and P.N. Smith (1987) Long-term Unemployment and the Unemployment-Vacancies Relationship. *The Economic Journal* 98 (363), 1071-91
- Coleman J.S. (1968) The Mathematical Study of Change. In H.M. Blalock and A. Blalock (eds), *Methodology in social research* (pp. 428-78). New York: McGraw-Hill
- Correa López M. and R.A. Naylor (2004) The Cournot-Bertrand Profit Differential: A Reversal Result in a Differentiated Duopoly with Wage Bargaining. *European Economic Review* 48 (3), 681-96
- Cressie N.A.C. (1993) *Statistics for Spatial Data*. New York: Wiley
- Donaghy K.P. (2001) Solution and Econometric Estimation of Spatial Dynamic Models in Continuous Space and Continuous Time. *Journal of Geographical Systems* 3 (3), 257-70
- Elhorst J.P. (1995) Unemployment Disparities between Regions in the European Union. In H.W. Armstrong and R.W. Vickerman (eds), *Convergence and Divergence among European Unions* (pp. 209-21). London: Pion
- Elhorst J.P. (2003) The Mystery of Regional Unemployment Differentials: Theoretical and Empirical Explanations. *Journal of Economic Surveys* 17 (5), 709-48
- Ghatak S., P. Levine and S.W. Price (1996) Migration Theories and Evidence: An Assessment. *Journal of Economic Surveys* 10 (2), 159-98
- Hall R.E. (1972) Turnover in the Labour Force. *Brookings Papers on Economic Activity* 3, 709-64
- Jones D.R. and D.N. Manning (1992) Long Term Unemployment, Hysteresis and the Unemployment-Vacancy Relationship: A Regional Analysis. *Regional Studies* 26 (1), 17-29
- Jöreskog K.G. and D. Sörbom (1996) *LISREL 8: User's Reference Guide*. Chicago: Scientific Software International
- Koopmans T.C. (1950) Models Involving a Continuous Time Variable. In T.C. Koopmans (ed.), *Statistical Inference in Dynamic Economic Models* (pp. 384-92). New York: John Wiley & Sons
- Layard P.R.G., S.J. Nickell and R. Jackman (1991) *Unemployment: Macroeconomic Performance and the Labour Market*. Oxford: Oxford University Press
- Lommerud K.E., O.R. Straume and L. Sorgard (2000) *Merger Profitability in Unionized Oligopoly* (Departmental Working Papers). Santa Barbara: Department of Economics, UCSB

- Neale M.C., G. Boker, G. Xie and H.M. Maes (1999) *Mx: Statistical Modeling* (4th ed.). Richmond: Department of Psychiatry
- Oud J.H.L. and R.A.R.G. Jansen (2000) Continuous Time State Space Modeling of Panel Data by Means of SEM. *Psychometrika* 65 (2), 199-215
- Oud J.H.L. and H. Singer (2008) Continuous Time Modeling of Panel Data: SEM Versus Filter Techniques. *Statistica Neerlandica* 62 (1), 4-28
- Oud J.H.L. and H. Folmer (2008a) *Continuous Time Modeling and Spatial Dependence* (Manuscript under review)
- Oud J.H.L. and H. Folmer (2008b) A Structural Equation Approach to Models with Spatial Dependence. *Geographical Analysis* 40 (2), 152-66
- Phillips R. (1959) Dissipative Operators and Hyperbolic Systems of Partial Differential Equations. *Transactions of the American Mathematical Society* 90 (2), 193-254
- Piras G., K.P. Donaghy and G. Arbia (2007) Nonlinear Regional Economic Dynamics. *Journal of Geographical Systems* 9, 311-44
- Pissarides C.A. and I. McMaster (1990) Regional Migration, Wages and Unemployment: Empirical Evidence and Implications for Policy. *Oxford Economic Papers* 42 (4), 812-31
- Puu T. (1997) *Mathematical Location and Land Use Theory: An Introduction*. Berlin New York: Springer
- Singer H. (1990) *Parameter Schätzung in Zeitkontinuierlichen Dynamischen Systemen [Parameter Estimation in Continuous-Time Dynamic Systems]*. Konstanz: Hartung-Gorre
- Wackernagel H. (1995) *Multivariate Geostatistics: An Introduction with Applications*. Berlin New York: Springer