Seemingly Unrelated Regressions With Spatial Error Components

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Abstract

This paper considers various estimators using panel data seemingly unrelated regressions (SUR) with spatial error correlation. The true data generating process is assumed to be SUR with spatial error of the autoregressive or moving average type. Moreover, the remainder term of the spatial process is assumed to follow an error component structure. Both maximum likelihood and generalized moments (GM) methods of estimation are used. Using Monte Carlo experiments, we check the performance of these estimators and their forecasts under misspecifiation of the spatial error process, various spatial weight matrices, and heterogeneous versus homogeneous panel data models.

Keywords: Seemingly unrelated regressions; Panel data; Spatial dependence; Heterogeneity; Forecasting.

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1 Introduction

Since Zellner's (1962) seminal paper on seemingly unrelated regressions (SUR) analyzing multiple equations with correlated disturbances, various extensions have been proposed, for e.g., to deal with the serially correlated case, the nonlinear case, the misspecified case, and SUR with unequal number of observations, see Srivastava and Dwivedi (1979).[1] Of particular interest for this paper are the extensions of SUR to panel data utilizing the error component model, see Avery (1977), Baltagi (1980), Magnus (1982) and Prucha (1984) to mention a few. Some applications of SUR panel data with error components include Verbon (1980) who estimated a set of four labor demand equations, using data from the Netherlands on 18 industries over 10 semiannual periods covering the period 1972-79; Beierlein, Dunn and McConnon (1981) who estimated six equations describing the demand for electricity and natural gas in the northeastern United States using data on nine states comprising the Census Bureau's northeastern region of the USA for the period 1967-77; Brown, Kleidon and Marsh (1983) who studied the size-related anomalies in stock returns using a panel of 566 firms observed quarterly over the period June 1967 to December 1975; Howrey and Varian (1984) who estimated a system of demand equations for electricity by time of day. Their data were based on the records of 60 households whose electricity usage was recorded over a five-month period in 1976 by the Arizona Public Service Company; Sickles (1985) who modeled the technology and specific factor productivity growth in the US airline industry; Wan, Griffiths and Anderson (1992) who estimated production functions for rice, maize and wheat production using panel data on 28 regions of China over the period 1980-83; Baltagi, Griffin and Rich (1995) who estimated a SUR model consisting of a translog variable cost function and its corresponding input share equations for labor, fuel and material using panel data of 24 U.S. airlines over the period 1971-1986; Egger and Pfaffermayr (2004) who used industry-level data of bilateral outward FDI stocks and exports of the U.S. and Germany to other countries between 1989 and 1999 to study the effects of distance as a common determinant of exports and foreign direct investment (FDI) in a three-factor New Trade Theory model; and more recently, Baltagi and Rich (2005) who estimated a SUR model consisting of a translog cost function and its corresponding in-

[1] For a monograph dedicated to SUR models and their extensions, see Srivastava and Giles (1987), also, the chapter by Fiebig (2001).

put share equations for production workers, nonproduction workers, energy, materials, and capital utilizing the National Bureau of Economic Research (NBER) manufacturing productivity database file. The panel data covered 459 manufacturing industries at the SIC 4-digit level over the period 1959–1996.

In addition, SUR models have been extended to allow for spatial autocorrelation, see Anselin (1988a,b). In fact, Anselin (1988a) derived a Lagrange Multiplier test for spatial autocorrelation in a SUR context. This paper extends Anselin's (1988a,b) SUR spatial model to the panel data case. This more general model allows for correlation across space, time and equations. It combines the simplicity of dealing with heterogeneity in the panel using an error component model and spatial correlation using a spatial autoregressive (SAR) or spatial moving average (SMA) disturbances. In this context, Wang and Kockelman (2007) derived the maximum likelihood estimator (under the normality assumption) of a SUR error component panel data model with SAR disturbances. They applied it to estimation of crash rates in 169 cities in China over the period 1999-2002.

The next section presents the seemingly unrelated regressions (SUR) panel model with spatial correlated error components. Section 3 presents the various estimators considered including maximum likelihood and generalized moments (GM) methods. We propose extensions of the Kapoor, et al. (2007) GM method to deal with SUR panel with SAR error component structure. Also, extensions of the Fingleton (2007a) GM method and Wang and Kockelman (2007) maximum likelihood (ML) method to deal with SUR panel with SMA error component structure. Section 4 gives the Monte Carlo design. The true data generating process is assumed to be SUR with spatial error of the autoregressive (SAR) or moving average (SMA) type. Moreover, the remainder term of the spatial process is assumed to follow an error component structure. Section 5 gives the Monte Carlo results along with sensitivity checks of these results to misspecifiation of the spatial error process, various spatial weight matrices, heterogeneous versus homogeneous spatial and panel estimators, and their performance in out of sample prediction. Section 6 concludes.

2 SUR with spatially correlated error components

Consider the set of M equations:

$$y_j = X_j\beta_j + \varepsilon_j \ , \ j = 1, ..., M \tag{1}$$

where y_j is $(TN \times 1)$, X_j is $(TN \times k_j)$, β_j is $(k_j \times 1)$, and the $(TN \times 1)$ error vector ε_j follows a spatial autoregressive (SAR) or a spatial moving average (SMA) process. Those processes can be expressed as :

$$\varepsilon_j = \begin{cases} \left(I_T \otimes \rho_j W_{jN}\right) \varepsilon_j + u_j & \text{SAR} \\ \left(I_T \otimes \lambda_j W_{jN}\right) u_j + u_j & \text{SMA} \end{cases} \tag{2}$$

where I_T is an identity matrix of order T, W_{jN} is an $(N \times N)$ known spatial weights matrix, ρ_j is the spatial autoregressive parameter and λ_j is the spatial moving average parameter for equation $j = 1, ..., M$. The diagonal elements of the spatial weight matrices W_{jN} are zero. We assume that the matrices $\left(I_N - \rho_j W_{jN}\right)$ are non-singular, and that the row and column sums of the matrices W_{jN} are bounded uniformly in absolute value for $j = 1, ..., M$. The matrix of exogenous regressors X_j has full column rank and its elements are uniformly bounded in absolute value. In contrast to much of the classical literature on panel data, we group the data by periods rather than units. This grouping is more convenient for modelling spatial correlation via (2). The disturbance term u_j of the processes (2) follows an additive error components structure:

$$u_j = Z_\mu \mu_j + v_j \tag{3}$$

where $Z_\mu = \iota_T \otimes I_N$, ι_T is a $(T \times 1)$ vector of ones; $\mu_j = \left(\mu_{1j}, ..., \mu_{Nj}\right)'$ and $v_j = \left(v'_{Nj}(1), ..., v'_{Nj}(T)\right)'$ are random vectors with 0 means and covariance matrix

$$E \begin{pmatrix} \mu_j \\ v_j \end{pmatrix} \begin{pmatrix} \mu'_l & v'_l \end{pmatrix} = \begin{pmatrix} \sigma^2_{\mu_{jl}} I_N & 0 \\ 0 & \sigma^2_{v_{jl}} I_{TN} \end{pmatrix} \tag{4}$$

for j and $l = 1, 2, ..., M$, see Baltagi (1980). $v_{Nj}(t)$ denotes the $(N \times 1)$ vector of remainder disturbances. We note that the specification of u_j corresponds to that of classical one-way error component model, see Baltagi

(2008). In fact, if $\rho_j = 0$ (resp. $\lambda_j = 0$), $\forall j = 1, 2, .., M$, so that there is no spatial autocorrelation, then this reduces to the usual SUR panel model with error components suggested by Avery (1977) and Baltagi (1980).

Following Baltagi (1980), the covariance matrix of u is given by

$$\Omega_u = E\left(uu^{'}\right) = [\Omega_{jl}] \tag{5}$$

where Ω_{jl} is a typical submatrix of Ω_u given by

$$\Omega_{jl} = E\left(u_j u_l^{'}\right) = \sigma^2_{\mu_{jl}} (J_T \otimes I_N) + \sigma^2_{v_{jl}} I_{TN} \tag{6}$$

where $J_T = \iota_T \iota_T^{'}$ is a $(T \times T)$ matrix of ones. (6) can also be written as

$$\Omega_{jl} = \sigma^2_{1_{jl}} Q_1 + \sigma^2_{v_{jl}} Q_2 \tag{7}$$

with

$$Q_1 = \overline{J}_T \otimes I_N \tag{8}$$
$$Q_2 = \left(I_T - \overline{J}_T\right) \otimes I_N \tag{9}$$

where $\overline{J}_T = J_T/T$ and $\sigma^2_{1_{jl}} = \sigma^2_{v_{jl}} + T\sigma^2_{\mu_{jl}}$. The matrices Q_1 and Q_2 are symmetric, idempotent and orthogonal to each other. Furthermore, $Q_1 + Q_2 = I_{TN}$, tr$Q_1 = N$ and tr$Q_2 = N(T-1)$. Replacing Ω_{jl} in (5) by its value, given in (7) we get

$$\Omega_u = \Sigma_u \otimes I_N \tag{10}$$

where $\Sigma_u = \Omega_\mu \otimes J_T + \Omega_v \otimes I_T$. Alternatively,

$$\Omega_u = \Omega_1 \otimes Q_1 + \Omega_v \otimes Q_2 \tag{11}$$

where $\Omega_\mu = \left[\sigma^2_{\mu_{jl}}\right]$, $\Omega_1 = \left[\sigma^2_{1_{jl}}\right]$ and $\Omega_v = \left[\sigma^2_{v_{jl}}\right]$, all of dimension $(M \times M)$. Then, the inverse of that covariance matrix is given by

$$\Omega_u^{-1} = \Sigma_u^{-1} \otimes I_N \tag{12}$$

or

$$\Omega_u^{-1} = \Omega_1^{-1} \otimes Q_1 + \Omega_v^{-1} \otimes Q_2 \tag{13}$$

see Baltagi (1980). From (2) and (3), the spatial-RE specification of the $(TN \times 1)$ error vector ε_j of equation j can be expressed as:

$$\varepsilon_j = (\iota_T \otimes H_{Nj})\, \mu_j + (I_T \otimes H_{Nj})\, v_j \tag{14}$$

with $H_{Nj} = B_{Nj}^{-1} = \left(I_N - \rho_j W_{jN}\right)^{-1}$ for SAR-RE and $H_{Nj} = D_{Nj} = (I_N + \lambda_j W_{jN})$ for SMA-RE. The corresponding $(TN \times TN)$ covariance matrix of (14) is given by:

$$\Lambda_{jl} = E\left(\varepsilon_j \varepsilon_l^{'}\right) = \sigma^2_{\mu_{jl}} \left(J_T \otimes H_{Nj} H'_{Nl}\right) + \sigma^2_{v_{jl}} \left[I_T \otimes H_{Nj} H'_{Nl}\right] \tag{15}$$

or

$$\Lambda_{jl} = \left(\sigma^2_{1_j} \overline{J}_T + \sigma^2_{v_j} \left(I_T - \overline{J}_T\right)\right) \otimes H_{Nj} H'_{Nl} \tag{16}$$

with $H_{Nj} H'_{Nl} = \left(B'_{Nl} B_{Nj}\right)^{-1}$ for SAR-RE and $H_{Nj} H'_{Nl} = \left(D_{Nj} D'_{Nl}\right)$ for SMA-RE. Combining the set of M equations, we get

$$y = X\beta + \varepsilon \tag{17}$$

with

$$\Lambda_\varepsilon = E\left(\varepsilon \varepsilon^{'}\right) = A \Omega_u A^{'} \tag{18}$$

where A is a block-diagonal matrix defined as

$$A = \begin{pmatrix} A_{11} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & A_{MM} \end{pmatrix} \tag{19}$$

with typical block matrix $A_{jj} = I_T \otimes H_{Nj}$ for $j = 1, ..., M$. Following the properties of the matrices Ω_u and A, we obtain the inverse covariance matrix of ε defined as

$$\Lambda_\varepsilon^{-1} = \left(A^{'}\right)^{-1} \Omega_u^{-1} A^{-1} \tag{20}$$

or

$$\Lambda_\varepsilon^{-1} = \left(A^{'}\right)^{-1} \left(\Sigma_u^{-1} \otimes I_N\right) A^{-1} \tag{21}$$

3 SUR spatial panel estimation

Consider the SUR spatial panel model given in (1) - (3). The true generalized least squares (GLS) estimator of β is given by

$$\begin{aligned}\widehat{\beta}_{GLS} &= \left(X'\Lambda_\varepsilon^{-1}X\right)^{-1}X'\Lambda_\varepsilon^{-1}y \\ &= \left(X^{*\prime}\Omega_u^{-1}X^*\right)^{-1}X^{*\prime}\Omega_u^{-1}y^* \end{aligned} \quad (22)$$

with typical element of the jth equation

$$X_j^* = \left(I_T \otimes H_{Nj}^{-1}\right)X_j^* \quad (23)$$

$$y_j^* = \left(I_T \otimes H_{Nj}^{-1}\right)y_j^* \quad (24)$$

The y_j^* and X_j^* can be viewed as the result of a spatial Cochrane-Orcutt type transformation of the original model. More specifically, premultiplication of (1) and (2) with $\left(I_T \otimes H_{Nj}^{-1}\right)$ yields

$$y_j^* = X_j^*\beta_j + u_j \quad (25)$$

since $\varepsilon_j = \left(I_T \otimes H_{Nj}\right)u_j$. Stacking the set of M equations, we get

$$y^* = X^*\beta + u \quad (26)$$

with $y^* = A^{-1}y$ and $X^* = A^{-1}X$. In light of the properties of (13), we can write

$$\Omega_u^{-1/2} = \Omega_1^{-1/2} \otimes Q_1 + \Omega_v^{-1/2} \otimes Q_2 \quad (27)$$

Guided by the classical error component literature, we note that a convenient way of computing the GLS estimator $\widehat{\beta}_{GLS}$ is to further transform the model in (26) by premultiplying it by $\Omega_u^{-1/2}$. The GLS estimator of β is then identical to the OLS estimator of β computed from the resulting transformed model. $\Omega_v^{-1/2}$ and $\Omega_1^{-1/2}$ can be obtained from a Cholesky decomposition of Ω_v and Ω_1, see Kinal and Lahiri (1990). We note that if $\rho_j = 0$ (resp. $\lambda_j = 0$), $\forall j = 1, 2, ..., M$, so that there is no spatial autocorrelation, then the GLS estimator reduces to that proposed by Avery (1977) and Baltagi (1980) for the SUR panel data model.

Let $\widehat{\rho}_j$ (resp. $\widehat{\lambda}_j$), $\widehat{\sigma}^2_{1_{jl}}$ and $\widehat{\sigma}^2_{v_{jl}}$ be estimators of ρ_j (resp. λ_j), $\sigma^2_{1_{jl}}$ and $\sigma^2_{v_{jl}}$. The corresponding feasible GLS estimator of β, say $\widehat{\beta}_{FGLS}$, is then obtained by replacing ρ_j (resp. λ_j), $\sigma^2_{1_{jl}}$ and $\sigma^2_{v_{jl}}$ by those estimators in the expression for the GLS estimator

$$\widehat{\beta}_{FGLS} = \left(\widehat{X}^{*\prime}\widehat{\Omega}_u^{-1}\widehat{X}^{*}\right)^{-1}\widehat{X}^{*\prime}\widehat{\Omega}_u^{-1}\widehat{y}^{*} \tag{28}$$

where $\widehat{X}^{*} = \widehat{A}^{-1}X$ and $\widehat{y}^{*} = \widehat{A}^{-1}y$. This estimator can be easily computed as an OLS estimator on a transformed system of equations described above. We propose a FGLS procedure that can be obtained in two steps :

- Estimate each equation with SAR-RE (resp. SMA-RE) process using the GM spatial panel data estimator proposed by Kapoor, et al. (2007) (resp. Fingleton (2007a)) to obtain consistent estimates of $\widehat{\rho}_j$ (resp. $\widehat{\lambda}_j$) for $j = 1, ..., M$. We can also estimate $\widehat{\rho}_j$ (resp. $\widehat{\lambda}_j$) using GM cross-section estimator proposed by Kelejian and Prucha (1999) (resp. Fingleton (2007b)). This estimates cross-sectional GM estimator for each equation with SAR disturbances (resp. SMA disturbances) for each time period and averages the estimates over time $\widehat{\widehat{\rho}}_j = 1/T\sum_{t=1}^{T}\widehat{\rho}_{jt}$ (resp. $\widehat{\widehat{\lambda}}_j = 1/T\sum_{t=1}^{T}\widehat{\lambda}_{jt}$).

- Knowing the true disturbances u_j, the analysis of variance estimates of Ω_v and Ω_1 are given by $\widehat{\Omega}_v = U'Q_2U/N(T-1)$ and $\widehat{\Omega}_1 = U'Q_1U/N$ where $U = [u_1, ..., u_M]$ is the $NT \times M$ matrix of disturbances for all M equations, see Avery (1977) and Baltagi (1980). Using the consistent estimates of the residuals from step 1, one obtains consistent estimates of Ω_v and Ω_1.

- Obtain $\widehat{\beta}_{FGLS}$ as in (28) using $\widehat{\rho}_j$ (resp. $\widehat{\lambda}_j$) from step 1 and $\widehat{\Omega}_v$ and $\widehat{\Omega}_1$ from step 2.

The GM estimation method is computationally simple and yields consistent estimates under mild conditions given in Kapoor, et al. (2007). This was suggested as an alternative to the standard MLE (under normality of the disturbances) which is computationally demanding even for the single equation case. Under normality of the distribances, the log-likelihood function is given by:

$$L = -\frac{1}{2}\ln|\Lambda_\varepsilon| - \frac{1}{2}(y - X\beta)'\Lambda_\varepsilon^{-1}(y - X\beta) \tag{29}$$

and basic mathematical manipulations result in the following:

$$L = \begin{cases} -\frac{N}{2}\ln|\Sigma_u| + T\sum_{j=1}^{M}\ln\left|H_{Nj}^{-1}\right| \\ -\frac{1}{2}(y - X\beta)'(A')^{-1}(\Sigma_u^{-1}\otimes I_N)A^{-1}(y - X\beta) \end{cases} \tag{30}$$

The parameters in (30) are intertwined, and the first order conditions of maximization are non-linear. However, the model can be estimated using a three-step method (see Wang and Kockelman (2007))[2] :

- First, β can be estimated using a feasible generalized least squares estimator (FGLS), conditional on Ω_μ, Ω_v and ρ (resp. λ), i.e., by maximizing the conditional likelihood $L(\beta/\rho, \Omega_\mu, \Omega_v)$.
- Second, Ω_μ and Ω_v can be estimated conditional on β and ρ (resp. λ), i.e., by maximizing the conditional likelihood $L(\Omega_\mu, \Omega_v/\beta, \rho)$. These two steps are iterated until the optimal Ω_μ, Ω_v and β are found (conditional on ρ (resp. λ)).
- Third, we maximize the concentrated log-likelihood function $L(\rho/\Omega_\mu, \Omega_v, \beta)$ over ρ (resp. λ). The optimized values of Ω_μ, Ω_v and β from the first two steps are plugged in the likelihood and the values of ρ are obtained by non-linear optimization. The estimated ρ (resp. λ) then re-enters the estimation of Ω_μ, Ω_v and β. This procedure is iterated until convergence.

4 Monte Carlo design

In this section, we consider the Monte Carlo design to study the small sample performance of several estimators of a SUR with spatial error components disturbances. The data generating process (DGP) considers two specifications on the remainder errors (2), namely SAR and SMA. We suppose that $M = 2$, then our spatial SUR specification is:

$$\begin{cases} y_1 = \beta_{0,1} + X_1\beta_{1,1} + \varepsilon_1 \\ y_2 = \beta_{0,2} + X_2\beta_{1,2} + \varepsilon_2 \end{cases} \tag{31}$$

[2] Wang and Kockelman (2007) consider only the SAR-RE. We provide the extension here for the SMA-RE specification.

or

$$y = X\beta + \varepsilon \tag{32}$$

where

$$X = \begin{pmatrix} \iota_{TN} & X_1 & 0 & 0 \\ 0 & 0 & \iota_{TN} & X_2 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} \beta_{0,1} \\ \beta_{1,1} \\ \beta_{0,2} \\ \beta_{1,2} \end{pmatrix}$$

with $\beta_{0,1} = \beta_{1,1} = \beta_{0,2} = \beta_{1,2} = 1$, ι_{TN} is an $(TN \times 1)$ vector of ones, (X_1, X_2) are two explanatory variables. The DGP of $x_{j,it}$, $j = 1, 2$, is defined by:

$$x_{j,it} = \delta_{j,i} + \omega_{j,it} \tag{33}$$

with $\delta_{j,i} \sim iid.U\,(-7.5, 7.5)$ and $\omega_{j,it} \sim iid.U\,(-5, 5)$. The $(2NT \times 1)$ spatial-RE vector of the disturbances ε is:

$$\varepsilon = A\,[\mu + v] \tag{34}$$

where the matrix A is defined by (19) with $W_1 = W_2 = W_N$ where W_N is the spatial weight matrix defined by Kelejian and Prucha (1999). We use two weight matrices which essentially differ in their degree of sparseness. The weight matrices are labelled as "s ahead and s behind" with the non-zero elements being $1/2s$, $s = 1$ and 5. We generate the error components term as:

$$(\mu + v) \;\sim\; N\left(0, \Sigma_u \otimes I_N\right), \; \Sigma_u = \Omega_\mu \otimes J_T + \Omega_v \otimes I_T \tag{35}$$

The variance-covariance matrices Ω_μ and Ω_v are defined by:

$$\Omega_\mu = \begin{pmatrix} \sigma^2_{\mu_1} & \rho_\mu \sigma_{\mu_1}\sigma_{\mu_2} \\ \rho_\mu \sigma_{\mu_1}\sigma_{\mu_2} & \sigma^2_{\mu_2} \end{pmatrix} \text{ and } \Omega_v = \begin{pmatrix} \sigma^2_{v_1} & \rho_v \sigma_{v_1}\sigma_{v_2} \\ \rho_v \sigma_{v_1}\sigma_{v_2} & \sigma^2_{v_2} \end{pmatrix}$$

with[3]

$$\sigma^2_{\mu_1} = \sigma^2_{\mu_2} = 1, \; \sigma^2_{v_1} = \sigma^2_{v_2} = 1, \; \rho_v = \rho_\mu = 0.5$$

[3] We consider other values for the variances but the results were qualitatively similar to those reported in our tables.

In order to generate the vector of disturbances $(\mu + v)$, we use the Cholesky decomposition[4]. We consider several individual and time dimensions $N = (50, 100)$, $T = (10, 20)$. For all experiments, 1000 replications are performed. For each experiment, we consider the following 18 estimators:

Homogeneous estimators (without spatial):

1. The pooled OLS equation by equation which ignores the individual heterogeneity, the spatial correlation and the correlation across equations.

2. The random effects (RE) estimator, equation by equation, which asssumes that the μ_i's are $iid(0, \sigma_\mu^2)$, and independent of the remainder disturbances v_{it}'s. This estimator accounts for random individual effects but does not take into account the spatial autocorrelation nor the correlation across equations.

3. The fixed-effects (FE) estimator, equation by equation, which accounts for fixed individual effects but does not take into account the spatial autocorrelation and correlation across equations.

4. Zellner's (1962) SUR-FGLS estimator which ignores the individual heterogeneity and spatial correlation.

5. The SUR fixed effects (FE) estimator which ignores spatial autocorrelation but takes into account the correlation across equations.

6. The SUR-ML random effects (RE) estimator which ignores spatial correlation.

[4] As $(\mu + v) \sim N(0, \Sigma_u \otimes I_N)$ and μ and v are uncorrelated, $\mu \sim N(0, (\Omega_\mu \otimes J_T \otimes I_N))$ and $v \sim N(0, (\Omega_v \otimes I_{NT}))$, then,

$$v \simeq (C_v \otimes I_{NT}) \begin{bmatrix} \widetilde{u}_1 \\ \widetilde{u}_2 \end{bmatrix} \text{ and } \mu \simeq (C_\mu \otimes \iota_T \otimes I_N) \begin{bmatrix} \widetilde{a}_1 \\ \widetilde{a}_2 \end{bmatrix}$$

where $((NT)\widetilde{u}_1, (NT)\widetilde{u}_2)$ and $((N)\widetilde{a}_1, (N)\widetilde{a}_2)$ are $IIN(0,1)$ random variables. C_μ (resp. C_v) is the lower triangular matrix defined by the decomposition: $\Omega_\mu = C_\mu C_\mu^0$ (resp. $\Omega_v = C_v C_v^0$) namely

$$C_\mu = \begin{pmatrix} \sigma_{\mu_1} & 0 \\ \rho_\mu \sigma_{\mu_2} & \sigma_{\mu_2}\sqrt{1-\rho_\mu^2} \end{pmatrix} \text{ and } C_v = \begin{pmatrix} \sigma_{v_1} & 0 \\ \rho_v \sigma_{v_2} & \sigma_{v_2}\sqrt{1-\rho_v^2} \end{pmatrix}$$

(see Anderson (1984)).

7. SUR-FGLS random effects (RE) estimator which ignores spatial correlation.

   Homogeneous estimators (with spatial):

8. The SUR-ML random effects (RE) estimator which takes into account the spatial autocorrelation of the SAR type.

9. The SUR-ML random effects (RE) estimator which takes into account the spatial autocorrelation of the SMA type.

10. The SUR-ML fixed effects (FE) estimator which takes into account the spatial autocorrelation of the SAR type.

11. The SUR-ML fixed effects (FE) estimator which takes into account the spatial autocorrelation of the SMA type.

12. The SUR-FGLS random effects (RE) estimator which takes into account the spatial autocorrelation of the SAR type, using the GM method. In the first step, we estimate each equation with SAR-RE process using the GM spatial panel data estimator proposed by Kapoor, et al. (2007) to obtain consistent estimates of $\widehat{\rho}_j$, $j = 1, 2$.

13. The SUR-FGLS random effects (RE) estimator which takes into account the spatial autocorrelation of the SMA type, using the GM method. In the first step, we estimate each equation with SMA-RE process using the GM spatial panel data estimator proposed by Fingleton (2007a) to obtain consistent estimates of $\widehat{\lambda}_j$, $j = 1, 2$.

    Heterogeneous estimator (without spatial):

14. The average heterogeneous OLS, equation by equation, to obtain a pooled estimator, see Pesaran and Smith (1995).

    Heterogeneous estimators (with spatial):

15. The average heterogeneous SUR assuming a SAR specification on the remainder disturbances using Kelejian and Prucha (1999) GM approach to estimate $\widehat{\rho}_{jt}$. This estimates cross-sectional GM-OLS with SAR disturbances for each time period and averages the estimates over time.

16. The average heterogeneous SUR assuming a SMA specification on the remainder disturbances using Fingleton (2007b) GM approach to estimate $\widehat{\lambda}_{jt}$. This estimates cross-sectional GM-OLS with SMA disturbances for each time period and averages the estimates over time.

17. The SUR-FGLS random effects (RE) estimator which takes into account the spatial autocorrelation of the SAR type, using GM-Average-within residuals. In the first step, we estimate $\widehat{\rho}_j$, $j = 1, 2$, using GM cross-section estimator proposed by Kelejian and Prucha (1999). This estimates cross-sectional GM estimator for each equation with SAR disturbances for each time period and averages the estimates over time $\widehat{\widehat{\rho}}_j = 1/T \sum_{t=1}^{T} \widehat{\rho}_{jt}$, $j = 1, 2$.

18. The SUR-FGLS random effects (RE) estimator which takes into account the spatial autocorrelation of the SMA type, using GM-Average-within residuals. In the first step, we estimate $\widehat{\lambda}_j$, $j = 1, 2$, using GM cross-section estimator proposed by Fingleton (2007b). This estimates cross-sectional GM estimator for each equation with SMA disturbances for each time period and averages the estimates over time $\widehat{\widehat{\lambda}}_j = 1/T \sum_{t=1}^{T} \widehat{\lambda}_{jt}$, $j = 1, 2$.

We focus on the estimates $\widehat{\beta}_{1,1}$, $\widehat{\beta}_{1,2}$, $\widehat{\rho}_1$, $\widehat{\lambda}_1$, $\widehat{\rho}_2$, $\widehat{\lambda}_2$, the standard errors $\widehat{\sigma}_{\widehat{\beta}_{1,1}}$, $\widehat{\sigma}_{\widehat{\beta}_{1,2}}$, and the variance components $\widehat{\sigma}^2_{\mu_1}$, $\widehat{\sigma}^2_{\mu_2}$, $\widehat{\sigma}^2_{v_1}$, $\widehat{\sigma}^2_{v_2}$, $\widehat{\sigma}^2_{\varepsilon_1}$, $\widehat{\sigma}^2_{\varepsilon_2}$. Following Kapoor, et al. (2007), we adopt a measure of dispersion which is closely related to the standard measure of root mean square error (RMSE) defined as follows:

$$\text{RMSE} = \left[bias^2 + \left(\frac{IQ}{1.35} \right)^2 \right]^{1/2} \tag{36}$$

where bias is the difference between the median and the true value of the parameter, and IQ is the interquantile range defined as $c_1 - c_2$ where c_1 is the 0.75 quantile and c_2 is the 0.25 quantile. Clearly, if the distribution is normal the median is the mean and, aside from a slight rounding error, IQ$/1.35$ is the standard deviation. In this case, the measure (36) reduces to the standard RMSE.

Moreover, we check the prediction-performance of the 18 alternative estimators considered. Here, we use the usual RMSE criterion and compute the out of sample forecast errors for each predictor associated with the 18 estimators. An average RMSE is calculated across the N individuals at different forecasts horizons.

5 Monte Carlo results

5.1 The Spatial Dependence Specification Effect

5.1.1 RMSE performance of the estimators

Table 1 gives the RMSE for the various estimators considered when the true DGP is a SUR panel model with SAR-RE remainder disturbances. The sample size is $(N, T) = (50, 10)$, the weight matrix is $W(1, 1)$, i.e., one neighbor behind and one neighbor ahead. The spatial coefficients are $(\rho_1, \rho_2) = (0.5, 0.3)$ with $\sigma^2_{\mu_1} = \sigma^2_{\mu_2} = 1$, $\sigma^2_{v_1} = \sigma^2_{v_2} = 1$ and $\rho_\mu = \rho_v = 0.5$. Focusing on the RMSE of the slope coefficient of the first equation ($\beta_{1,1}$), we observe the following results: Not surprisingly, OLS and average OLS perform the worst because they ignore the spatial correlation, the individual heterogeneity and the cross-equation correlation. Taking into account only the cross-equation correlation by performing Zellner's SUR estimation ignoring the spatial effects and the individual heterogeneity reduces the RMSE from 0.02546 for OLS to 0.02234 for Zellner's SUR. Interestingly, if one performed RE or FE ignoring the spatial effects and the cross-equation correlation, the reduction in RMSE would have been even more (0.01776 and 0.02019, respectively). Correcting for both individual heterogeneity and cross-equation correlation by performing SUR-FE and SUR-FGLS RE reduces the RMSE further to 0.01769 and 0.01577, respectively.

Note also that SUR-RE leads to similar RMSE for feasible GLS and MLE, respectively 0.01577 and 0.01593. Correcting for spatial correlation, individual heterogeneity and the cross-equation correlation by performing SUR–ML or SUR-FGLS SAR-RE yields the lowest RMSE of 0.01123 (for feasible GLS) and 0.01140 (for the corresponding ML). The RMSE for SUR SAR-FE using ML is 0.01308. If the wrong spatial structure was used in the estimation, i.e., SMA rather than SAR, the corresponding RMSE for SUR SMA-RE would be 0.01599 for ML and 0.01767 for the SUR-ML SMA-FE. Ignoring the individual effects but not the spatial correlation or the cross-equation correlation, by applying Average SUR SAR yield a RMSE of 0.01855. Interestingly, this RMSE remains almost the same had one misspecified the SAR process and performed Average SUR SMA. If we take into account the individual effects, the corresponding heterogeneous RMSE for SUR-FGLS SAR-RE (av.) is 0.01116 and 0.01213 for SUR-FGLS SMA-RE (av.).

Similar results are obtained had we focused on the slope coefficient of the second equation $\beta_{1,2}$. Only the magnitudes of the RMSEs would have been different. For example, the RMSE of OLS is 0.02655, that of FE is 0.01566, that of RE is 0.01464. Zellner's SUR is 0.02332. SUR-FE is 0.01477 and SUR-FGLS RE is 0.01417 and 0.01432 for SUR-ML RE. The lowest RMSE is obtained for SUR SAR-RE (0.01230) whether FGLS or ML. Misspecifying the SAR process by a SMA process yields a RMSE of 0.01336 for SUR SMA-RE by FGLS and 0.01248 by ML. The corresponding RMSE for SUR-ML SMA-FE is 0.01512. The heterogeneous estimators yield a RMSE of 0.2685 for average OLS, 0.02146 for Average SUR SAR and 0.02081 for Average SUR SMA. The corresponding heterogeneous RMSE for SUR-FGLS SAR-RE (av.) is 0.01250 and 0.01232 for SUR-FGLS SMA-RE (av.).

Table 2 gives the RMSE for the various estimators considered when the true DGP is a SUR panel model with SMA-RE remainder disturbances. The sample size is $N = 50$ and $T = 10$, the weight matrix is $W(1,1)$, i.e., one neighbor behind and one neighbor ahead. The spatial coefficients are $(\rho_1, \rho_2) = (0.5, 0.3)$ with $\sigma^2_{\mu_1} = \sigma^2_{\mu_2} = 1$, $\sigma^2_{v_1} = \sigma^2_{v_2} = 1$ and $\rho_\mu = \rho_v = 0.5$. Focusing on the RMSE of the slope coefficient of the first equation ($\beta_{1,1}$), we observe the following results: OLS and average OLS still perform the worst. Now, Zellner's SUR (0.02136) performs better in terms of RMSE than OLS (0.02488) but worse than RE (0.01475), FE (0.01747), SUR-FE (0.01564), SUR-ML RE (0.01336) and SUR-FGLS RE (0.01343). Correcting for spatial correlation, individual heterogeneity and the cross-equation correlation by performing SUR–ML or SUR-FGLS SAR-RE yields the lowest RMSE of 0.01044 and 0.01025, respectively. If the wrong spatial structure was used in the estimation, i.e., SAR rather than SMA, the corresponding RMSE for SUR SAR-RE would be 0.01038 for FGLS, 0.01052 for ML. Similar results are obtained for the slope coefficient of the second equation $\beta_{1,2}$.

5.1.2 Forecasts Accuracy

Table 3 gives the forecast RMSE results when the true DGP is a SUR panel model with SAR-RE remainder disturbances. The sample size is still $N = 50, T = 10$, and the weight matrix is $W(1,1)$. In general, for $(\rho_1, \rho_2) = (0.5, 0.3)$ with $\sigma^2_{\mu_1} = \sigma^2_{\mu_2} = 1$, $\sigma^2_{v_1} = \sigma^2_{v_2} = 1$ and $\rho_\mu = \rho_v = 0.5$, the lowest forecast RMSE is that of SUR SAR-RE (ML, FGLS and FGLS (av.)). This is followed closely by SMA-RE. Misspecifying the SAR by an SMA in an error component model does not seem to affect the forecast performance as long as it is taken into account. However, the magnitudes of the RMSE in Table 3 (where the true DGP is a SAR-RE process) are higher than those in Table 4 (where the true DGP is a SMA-RE process). Once again, the forecast RMSE based on ML and FGLS are quite similar. Pooled OLS, SUR-FGLS, average heterogeneous OLS, average SUR SAR and average SUR SMA perform worse in terms of forecast RMSE than spatial/panel homogeneous estimators. This forecast performance is robust whether we are predicting one period, two periods or 5 periods ahead and is also reflected in the average over the five years. Once again, the gain in forecast performance is substantial once we account for RE or FE and is only slightly improved by additionally accounting for spatial autocorrelation.

5.2 Sensivity Analysis

5.2.1 The spatial Weight Matrix effect

For the various estimators considered, Tables 5 and 6 report the RMSE results as Tables 1 and 2 except that the weight matrix is changed from a $W(1,1)$ to $W(5,5)$, i.e., five neighbors behind and five neighbors ahead. Except for the magnitudes of the RMSE, the same rankings in terms of RMSE performance are exhibited as before.

For forecasts accuracy, Tables 7 and 8 report the forecast RMSE results as Tables 3 and 4 except that the weight matrix is now $W(5,5)$ rather than $W(1,1)$. Except for the magnitudes of the forecast RMSE, the same rankings in terms of RMSE performance are exhibited as before. From our limited experiments, we conclude that our results are robust to the W matrices considered.

5.2.2 Stronger correlation across equations

In Table 9, we consider a set of experiments with higher correlation across equations. In particular, we let $\rho_\mu = \rho_v = 0.9$ rather than $\rho_\mu = \rho_v = 0.5$ as in Table 1. The sample size is still fixed at $(N,T) = (50,10)$, the weight matrix is $W(1,1)$, i.e., one neighbor behind and one neighbor ahead.; the spatial coefficients are $(\rho_1, \rho_2) = (0.5, 0.3)$ with $\sigma^2_{\mu_1} = \sigma^2_{\mu_2} = 1$, $\sigma^2_{v_1} = \sigma^2_{v_2} = 1$. Table

9 gives the RMSE for the various estimators considered when the true DGP is a SUR panel model with SAR-RE remainder disturbances. Focusing on the RMSE of the slope coefficient of the first equation ($\beta_{1,1}$), we observe that the estimators that correct for spatial correlation, individual heterogeneity and the cross-equation correlation continue to give the lowest RMSE. Comparing these results with those in Table 1, we find that the RMSE of the SUR-ML SAR-RE estimator is reduced from 0.01140 to 0.00614 , while that of OLS increased from 0.02546 to 0.03048. The former takes into account the stronger cross-equation correlation, while the latter does not. Similar results are obtained had we focused on the slope coefficient of the second equation $\beta_{1,2}$.

6 Conclusion

Our Monte Carlo study finds that when the true DGP is SUR with a SAR-RE or SMA-RE remainder disturbances, estimators and forecasts that ignore heterogeneity/spatial correlation and cross-equation correlation, perform badly in terms of the RMSE criteria. For our experiments, accounting for heterogeneity improves the RMSE forecast performance by a big margin, and accounting for spatial correlation improves the RMSE forecast performance, but by a smaller margin. Ignoring both leads to the worst forecasting performance. Heterogeneous estimators based on averaging perform worse than homogeneous estimators in forecasting performance. These Monte Carlo experiments confirm earlier empirical studies that report similar findings but now for multiple equations and SUR estimation setting.

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Table 1 - RMSE of coefficients, standard errors and variances - (ρ_1,ρ_2) = (0.5,0.3), (ρ_μ,ρ_ν) = (0.5,0.5), (N,T)=(50,10), SAR data generating process for ε, W(1,1), 1000 replications

	$\hat{\beta}_{1,1}$	$\hat{\sigma}_{\hat{\beta}_{1,1}}$	$\hat{\beta}_{1,2}$	$\hat{\sigma}_{\hat{\beta}_{1,2}}$	$\hat{\rho}_1$ $(\hat{\lambda}_1)$	$\hat{\rho}_2$ $(\hat{\lambda}_2)$	$\hat{\sigma}^2_{\mu_1}$	$\hat{\sigma}^2_{\mu_2}$	$\hat{\sigma}^2_{\nu_1}$	$\hat{\sigma}^2_{\nu_2}$
Homogeneous estimators										
(without spatial)										
OLS	0.02546	0.00214	0.02655	0.00109	—	—	—	—	—	—
RE	0.01776	0.00579	0.01464	0.00320	—	—	0.57852	0.27427	0.54896	0.16645
FE	0.02019	0.00832	0.01566	0.00492	—	—	—	—	0.55235	0.16874
SUR-FGLS	0.02234	0.00083	0.02332	0.00117	—	—	—	—	—	—
SUR-FE	0.01769	0.00481	0.01477	0.00194	—	—	—	—	0.39753	0.08118
SUR-ML RE	0,01593	0,00373	0,01432	0,00128	—	—	0,56557	0,28513	0,54844	0,16776
SUR-FGLS RE	0,01577	0,00377	0,01417	0,00131	—	—	0,57422	0,28732	0,54617	0,16729
Homogeneous estimators										
(with spatial)										
SUR-ML SAR-RE	0.01140	0.00051	0.01230	0.00041	0.03544	0.03895	0.21047	0.21461	0.06703	0.06530
SUR-ML SMA-RE	0.01599	0.00052	0.01248	0.00042	0.04065	0.03934	0.37615	0.23816	0.33543	0.11636
SUR-ML SAR-FE	0.01308	0.00127	0.01361	0.00059	0.03561	0.03983	—	—	0.12174	0.12376
SUR-ML SMA-FE	0.01767	0.00161	0.01512	0.00064	0.05470	0.03958	—	—	0.19288	0.06885
SUR-FGLS SAR-RE	0.01123	0.00047	0.01231	0.00042	0.03761	0.04425	0.21307	0.22302	0.06850	0.06661
SUR-FGLS SMA-RE	0.01155	0.00093	0.01336	0.00047	0.10315	0.05350	0.54627	0.24893	0.47524	0.12746
Heterogeneous estimator										
(without spatial)										
Av. Heterogeneous OLS	0.02602	0.00241	0.02685	0.00127	—	—	—	—	—	—
Heterogeneous estimators										
(with spatial)										
Average SUR SAR	0.01855	0.00314	0.02146	0.00214	0.06758	0.05327	—	—	—	—
Average SUR SMA	0.01895	0.00384	0.02081	0.00213	0.10453	0.06167	—	—	—	—
SUR-FGLS SAR-RE (av.)	0.01116	0.00055	0.01250	0.00047	(1)	(1)	0.21215	0.21548	0.07578	0.06757
SUR-FGLS SMA-RE (av.)	0.01213	0.00097	0.01232	0.00052	(2)	(2)	0.50194	0.27247	0.42749	0.14235

(1) We have used the average values $\hat{\bar{\rho}}_1$ and $\hat{\bar{\rho}}_2$ of average SUR SAR estimator. (2) We have used the average values $\hat{\bar{\lambda}}_1$ et $\hat{\bar{\lambda}}_2$ of average SUR SMA estimator.

Table 2 - RMSE of coefficients, standard errors and variances - (λ_1,λ_2) = (0.5,0.3), (ρ_μ,ρ_v) = (0.5,0.5), (N,T)=(50,10), SMA data generating process for ε, W(1,1), 1000 replications

	$\hat{\beta}_{1,1}$	$\hat{\sigma}_{\hat{\beta}_{1,1}}$	$\hat{\beta}_{1,2}$	$\hat{\sigma}_{\hat{\beta}_{1,2}}$	$\hat{\rho}_1 \left(\hat{\lambda}_1\right)$	$\hat{\rho}_2 \left(\hat{\lambda}_2\right)$	$\hat{\sigma}^2_{\mu_1}$	$\hat{\sigma}^2_{\mu_2}$	$\hat{\sigma}^2_{v_1}$	$\hat{\sigma}^2_{v_2}$
Homogeneous estimators										
(without spatial)										
OLS	0,02488	0,00258	0,02497	0,00133	—	—	—	—	—	—
RE	0,01475	0,00533	0,01532	0,00317	—	—	0,28950	0,24350	0,15034	0,08454
FE	0,01747	0,00727	0,01685	0,00468	—	—	—	—	0,15245	0,08576
SUR-FGLS	0,02136	0,00101	0,02124	0,00096	—	—	—	—	—	—
SUR-FE	0,01564	0,00416	0,01476	0,00182	—	—	—	—	0,08210	0,09430
SUR-ML RE	0,01336	0,00321	0,01369	0,00122	—	—	0,28952	0,24251	0,15174	0,08527
SUR-FGLS RE	0,01343	0,00324	0,01364	0,00124	—	—	0,28950	0,24350	0,15034	0,08454
Homogeneous estimators										
(with spatial)										
SUR-ML SAR-RE	0,01052	0,00053	0,01219	0,00043	0,04559	0,04016	0,30281	0,22340	0,22305	0,10913
SUR-ML SMA-RE	0,01044	0,00043	0,01225	0,00046	0,03427	0,03805	0,22293	0,21804	0,06961	0,06732
SUR-ML SAR-FE	0,01109	0,00114	0,01363	0,00076	0,04500	0,04064	—	—	0,29984	0,19095
SUR-ML SMA-FE	0,01065	0,00068	0,01351	0,00069	0,03590	0,04159	—	—	0,14357	0,12701
SUR-FGLS SAR-RE	0,01038	0,00052	0,01233	0,00044	0,04043	0,04654	0,25640	0,20979	0,23270	0,11318
SUR-FGLS SMA-RE	0,01025	0,00050	0,01224	0,00046	0,05005	0,04877	0,22611	0,22248	0,07623	0,06928
Heterogeneous estimator										
(without spatial)										
Av. Heterogeneous OLS	0,02513	0,00281	0,02586	0,00151	—	—	—	—	—	—
Heterogeneous estimators										
(with spatial)										
Average SUR SAR	0,01855	0,00169	0,02003	0,00221	0,12443	0,11145	—	—	—	—
Average SUR SMA	0,01809	0,00191	0,01991	0,00220	0,15609	0,15127	—	—	—	—
SUR-FGLS SAR-RE (av.)	0,01065	0,00066	0,01217	0,00045	(1)	(1)	0,25806	0,21525	0,21868	0,11204
SUR-FGLS SMA-RE (av.)	0,01057	0,00073	0,01238	0,00054	(2)	(2)	0,22717	0,21401	0,07641	0,06975

(1) We have used the average values $\hat{\bar{\rho}}_1$ and $\hat{\bar{\rho}}_2$ of average SUR SAR estimator. (2) We have used the average values $\hat{\bar{\lambda}}_1$ et $\hat{\bar{\lambda}}_2$ of average SUR SMA estimator.

Table 3 - Forecasts RMSE - (ρ_1,ρ_2)= (0.5,0.3), (ρ_μ,ρ_v)= (0.5,0.5), (N,T)=(50,10), SAR data generating process for ε, W(1,1), 1000 replications

	1st year		2sd year		3th year		4th year		5th year		Average	
	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2
Homogeneous estimators												
(without spatial)												
OLS	1.3768	1.1978	1.5050	1.3082	1.5534	1.3510	1.5791	1.3729	1.5959	1.3881	1.5220	1.3236
RE	1.0324	0.8936	1.1466	0.9924	1.1931	1.0320	1.2184	1.0536	1.2336	1.0675	1.1648	1.0078
FE	1.0367	0.8974	1.1512	0.9966	1.1980	1.0364	1.2234	1.0580	1.2385	1.0718	1.1696	1.0120
SUR-FGLS	1.3776	1.1987	1.5060	1.3089	1.5543	1.3517	1.5799	1.3735	1.5967	1.3886	1.5229	1.3243
SUR-FE	1.0366	0.8972	1.1510	0.9964	1.1977	1.0360	1.2232	1.0577	1.2382	1.0715	1.1693	1.0118
SUR-ML RE	1.0337	0.8954	1.1482	0.9943	1.1946	1.0338	1.2201	1.0555	1.2353	1.0695	1.1664	1.0097
SUR-FGLS RE	1.0324	0.8935	1.1466	0.9924	1.1931	1.0320	1.2184	1.0536	1.2336	1.0675	1.1648	1.0078
Homogeneous estimators												
(with spatial)												
SUR-ML SAR-RE	1.0319	0.8933	1.1462	0.9922	1.1927	1.0318	1.2180	1.0533	1.2331	1.0673	1.1644	1.0076
SUR-ML SMA-RE	1.0322	0.8935	1.1464	0.9923	1.1929	1.0319	1.2182	1.0535	1.2334	1.0674	1.1646	1.0077
SUR-ML SAR-FE	1.0362	0.8970	1.1507	0.9963	1.1974	1.0359	1.2229	1.0575	1.2378	1.0713	1.1690	1.0116
SUR-ML SMA-FE	1.0366	0.8972	1.1510	0.9964	1.1977	1.036	1.2232	1.0577	1.2382	1.0715	1.1693	1.0118
SUR-FGLS SAR-RE	1.0318	0.8933	1.1461	0.9922	1.1926	1.0317	1.2179	1.0533	1.2330	1.0672	1.1643	1.0075
SUR-FGLS SMA-RE	1.0322	0.8933	1.1465	0.9921	1.1929	1.0317	1.2182	1.0533	1.2333	1.0672	1.1646	1.0075
Heterogeneous estimator												
(without spatial)												
Av. Heterogeneous OLS	1.3768	1.1979	1.5051	1.3083	1.5535	1.3510	1.5792	1.3729	1.5960	1.3881	1.5221	1.3236
Heterogeneous estimators												
(with spatial)												
Average SUR SAR	1.3779	1.1992	1.5064	1.3096	1.5547	1.3524	1.5801	1.3743	1.5970	1.3893	1.5232	1.3249
Average SUR SMA	1.3785	1.1991	1.5068	1.3095	1.5554	1.3524	1.5807	1.3742	1.5976	1.3893	1.5238	1.3249
SUR-FGLS SAR-RE (av.)	1.0319	0.8933	1.1461	0.9921	1.1926	1.0317	1.2179	1.0533	1.2330	1.0672	1.1643	1.0075
SUR-FGLS SMA-RE (av.)	1.0320	0.8934	1.1463	0.9922	1.1927	1.0317	1.2180	1.0533	1.2331	1.0672	1.1644	1.0075

Table 4 - Forecasts RMSE - (λ_1,λ_2)= (0.5,0.3), (ρ_μ,ρ_v)= (0.5,0.5), (N,T)=(50,10), SMA data generating process for ε, W(1,1), 1000 replications

	1st year		2sd year		3th year		4th year		5th year		Average	
	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2
Homogeneous estimators (without spatial)												
OLS	1,1803	1,1438	1,2920	1,2537	1,3306	1,2934	1,3526	1,3126	1,3674	1,3259	1,3046	1,2659
RE	0,8820	0,8538	0,9806	0,9491	1,0186	0,9871	1,0398	1,0059	1,0526	1,0185	0,9947	0,9629
FE	0,8854	0,8569	0,9841	0,9525	1,0227	0,9907	1,0441	1,0097	1,0567	1,0223	0,9986	0,9664
SUR-FGLS	1,1812	1,1444	1,2928	1,2542	1,3312	1,2939	1,3531	1,3132	1,3680	1,3266	1,3053	1,2665
SUR-FE	0,8850	0,8568	0,9837	0,9524	1,0223	0,9905	1,0436	1,0095	1,0562	1,0221	0,9982	0,9663
SUR-ML RE	0,8819	0,8538	0,9805	0,9492	1,0185	0,9871	1,0396	1,0059	1,0525	1,0185	0,9946	0,9629
SUR-FGLS RE	0,8819	0,8538	0,9804	0,9492	1,0185	0,9871	1,0395	1,0058	1,0524	1,0185	0,9946	0,9629
Homogeneous estimators (with spatial)												
SUR-ML SAR-RE	0,8813	0,8535	0,9799	0,9490	1,0180	0,9869	1,0391	1,0057	1,0520	1,0183	0,9941	0,9627
SUR-ML SMA-RE	0,8812	0,8534	0,9798	0,9489	1,0180	0,9869	1,0391	1,0057	1,0520	1,0183	0,9940	0,9626
SUR-ML SAR-FE	0,8845	0,8567	0,9831	0,9522	1,0219	0,9904	1,0432	1,0093	1,0558	1,0219	0,9977	0,9661
SUR-ML SMA-FE	0,8844	0,8566	0,9830	0,9522	1,0219	0,9904	1,0432	1,0093	1,0558	1,0219	0,9976	0,9660
SUR-FGLS SAR-RE	0,8813	0,8535	0,9798	0,9489	1,0180	0,9869	1,0391	1,0056	1,0519	1,0182	0,9940	0,9626
SUR-FGLS SMA-RE	0,8812	0,8534	0,9797	0,9488	1,0180	0,9868	1,0391	1,0056	1,0519	1,0182	0,9940	0,9626
Heterogeneous estimator (without spatial)												
Av. Heterogeneous OLS	1,1803	1,1438	1,2921	1,2537	1,3306	1,2934	1,3526	1,3126	1,3675	1,3259	1,3046	1,2659
Heterogeneous estimators (with spatial)												
Average SUR SAR	1,1822	1,1447	1,2932	1,2546	1,3318	1,2942	1,3535	1,3135	1,3685	1,3269	1,3058	1,2668
Average SUR SMA	1,1823	1,1447	1,2934	1,2545	1,3320	1,2941	1,3536	1,3134	1,3686	1,3268	1,3060	1,2667
SUR-FGLS SAR-RE (av.)	0,8813	0,8535	0,9798	0,9489	1,0180	0,9869	1,0390	1,0056	1,0519	1,0182	0,9940	0,9626
SUR-FGLS SMA-RE (av.)	0,8813	0,8535	0,9798	0,9489	1,0180	0,9869	1,0391	1,0056	1,0520	1,0182	0,9940	0,9626

Table 5 - RMSE of coefficients, standard errors and variances - (ρ_1,ρ_2) = (0.5,0.3), (ρ_μ,ρ_v) = (0.5,0.5), (N,T)=(50,10), SAR data generating process for ε, W(5,5), 1000 replications

	$\hat{\beta}_{1,1}$	$\hat{\sigma}_{\hat{\beta}_{1,1}}$	$\hat{\beta}_{1,2}$	$\hat{\sigma}_{\hat{\beta}_{1,2}}$	$\hat{\rho}_1\ (\hat{\lambda}_1)$	$\hat{\rho}_2\ (\hat{\lambda}_2)$	$\hat{\sigma}^2_{\mu_1}$	$\hat{\sigma}^2_{\mu_2}$	$\hat{\sigma}^2_{v_1}$	$\hat{\sigma}^2_{v_2}$
Homogeneous estimators										
(without spatial)										
OLS	0,02568	0,00080	0,02282	0,00092	—	—	—	—	—	—
RE	0,01418	0,00299	0,01536	0,00230	—	—	0,25402	0,21979	0,17067	0,07611
FE	0,01582	0,00486	0,01693	0,00418	—	—	—	—	0,17290	0,07714
SUR-FGLS	0,02254	0,00154	0,02008	0,00225	—	—	—	—	—	—
SUR-FE	0,01358	0,00187	0,01478	0,00128	—	—	—	—	0,08980	0,09783
SUR-ML RE	0,01185	0,00102	0,01338	0,00052	—	—	0,25333	0,22248	0,17121	0,07682
SUR-FGLS RE	0,01195	0,00104	0,01327	0,00052	—	—	0,25402	0,21979	0,17067	0,07611
Homogeneous estimators										
(with spatial)										
SUR-ML SAR-RE	0,01093	0,00045	0,01325	0,00045	0,06424	0,08307	0,20917	0,21880	0,07061	0,06730
SUR-ML SMA-RE	0,01194	0,00046	0,01329	0,00045	0,19963	0,12298	0,22208	0,21788	0,10631	0,06807
SUR-ML SAR-FE	0,01231	0,00073	0,01430	0,00087	0,06177	0,08205	—	—	0,12443	0,12537
SUR-ML SMA-FE	0,01355	0,00092	0,01485	0,00092	0,06060	0,08089	—	—	0,08143	0,10947
SUR-FGLS SAR-RE	0,01110	0,00043	0,01314	0,00043	0,06974	0,08956	0,21846	0,21887	0,07044	0,06726
SUR-FGLS SMA-RE	0,01205	0,00049	0,01311	0,00048	0,36022	0,23681	0,23914	0,21366	0,11497	0,07473
Heterogeneous estimator										
(without spatial)										
Av. Heterogeneous OLS	0,02634	0,00081	0,02282	0,00088	—	—	—	—	—	—
Heterogeneous estimators										
(with spatial)										
Average SUR SAR	0,02143	0,00226	0,02029	0,00272	0,26700	0,11192	—	—	—	—
Average SUR SMA	0,02179	0,00228	0,02039	0,00267	0,47592	0,40932	—	—	—	—
SUR-FGLS SAR-RE (av.)	0,01081	0,00045	0,01325	0,00044	(1)	(1)	0,21326	0,21480	0,07674	0,06658
SUR-FGLS SMA-RE (av.)	0,01096	0,00052	0,01320	0,00050	(2)	(2)	0,23240	0,21811	0,10860	0,07259

(1) We have used the average values $\bar{\hat{\rho}}_1$ and $\bar{\hat{\rho}}_2$ of average SUR SAR estimator. (2) We have used the average values $\bar{\hat{\lambda}}_1$ et $\bar{\hat{\lambda}}_2$ of average SUR SMA estimator.

Table 6 - RMSE of coefficients, standard errors and variances - (λ_1,λ_2) = (0.5,0.3), (ρ_μ,ρ_v) = (0.5,0.5), (N,T)=(50,10), SMA data generating process for ε, W(5,5), 1000 replications

	$\hat{\beta}_{1,1}$	$\hat{\sigma}_{\hat{\beta}_{1,1}}$	$\hat{\beta}_{1,2}$	$\hat{\sigma}_{\hat{\beta}_{1,2}}$	$\hat{\rho}_1\ (\hat{\lambda}_1)$	$\hat{\rho}_2\ (\hat{\lambda}_2)$	$\hat{\sigma}^2_{\mu_1}$	$\hat{\sigma}^2_{\mu_2}$	$\hat{\sigma}^2_{v_1}$	$\hat{\sigma}^2_{v_2}$
Homogeneous estimators										
(without spatial)										
OLS	0,02422	0,00072	0,02374	0,00094	—	—	—	—	—	—
RE	0,01414	0,00240	0,01516	0,00219	—	—	0,24020	0,22444	0,07397	0,06911
FE	0,01554	0,00413	0,01653	0,00405	—	—	—	—	0,07485	0,06944
SUR-FGLS	0,02099	0,00187	0,02027	0,00231	—	—	—	—	—	—
SUR-FE	0,01340	0,00129	0,01401	0,00118	—	—	—	—	0,10170	0,11453
SUR-ML RE	0,01250	0,00059	0,01253	0,00049	—	—	0,24540	0,22703	0,07454	0,06980
SUR-FGLS RE	0,01249	0,00061	0,01263	0,00050	—	—	0,24020	0,22444	0,07397	0,06911
Homogeneous estimators										
(with spatial)										
SUR-ML SAR-RE	0,01246	0,00041	0,01239	0,00046	0,14452	0,09942	0,22648	0,21964	0,07437	0,07020
SUR-ML SMA-RE	0,01242	0,00041	0,01254	0,00047	0,12224	0,11156	0,22454	0,22175	0,06624	0,06683
SUR-ML SAR-FE	0,01311	0,00073	0,01389	0,00090	0,14090	0,09387	—	—	0,14845	0,13283
SUR-ML SMA-FE	0,01314	0,00070	0,01387	0,00088	0,12736	0,11480	—	—	0,11823	0,12111
SUR-FGLS SAR-RE	0,01234	0,00041	0,01231	0,00045	0,14528	0,08202	0,21945	0,21855	0,07578	0,07057
SUR-FGLS SMA-RE	0,01237	0,00042	0,01221	0,00047	0,14403	0,13633	0,22383	0,22322	0,06521	0,06814
Heterogeneous estimator										
(without spatial)										
Av. Heterogeneous OLS	0,02450	0,02380	0,00072	0,00088	—	—	—	—	—	—
Heterogeneous estimators										
(with spatial)										
Average SUR SAR	0,02080	0,00225	0,02046	0,00272	0,20462	0,06256	—	—	—	—
Average SUR SMA	0,02113	0,00227	0,02047	0,00265	0,34952	0,30770	—	—	—	—
SUR-FGLS SAR-RE (av.)	0,01252	0,00041	0,01236	0,00045	(1)	(1)	0,22012	0,21598	0,07297	0,06913
SUR-FGLS SMA-RE (av.)	0,01260	0,00047	0,01264	0,00049	(2)	(2)	0,23060	0,22370	0,06672	0,07143

(1) We have used the average values $\hat{\bar{\rho}}_1$ and $\hat{\bar{\rho}}_2$ of average SUR SAR estimator. (2) We have used the average values $\hat{\bar{\lambda}}_1$ et $\hat{\bar{\lambda}}_2$ of average SUR SMA estimator.

Table 7 - Forecasts RMSE - (ρ_1,ρ_2)= (0.5,0.3), (ρ_μ,ρ_v)= (0.5,0.5), (N,T)=(50,10), SAR data generating process for ε, W(5,5), 1000 replications

	1st year		2sd year		3th year		4th year		5th year		Average	
	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2
Homogeneous estimators												
(without spatial)												
OLS	1,1821	1,1379	1,2910	1,2430	1,3341	1,2835	1,3581	1,3050	1,3716	1,3178	1,3074	1,2574
RE	0,8861	0,8479	0,9858	0,9419	1,0260	0,9793	1,0488	0,9989	1,0630	1,0124	1,0019	0,9561
FE	0,8894	0,8509	0,9895	0,9453	1,0298	0,9828	1,0525	1,0023	1,0668	1,0159	1,0056	0,9594
SUR-FGLS	1,1830	1,1386	1,2918	1,2434	1,3350	1,2840	1,3588	1,3057	1,3722	1,3184	1,3082	1,2580
SUR-FE	0,8892	0,8506	0,9893	0,9449	1,0295	0,9824	1,0523	1,0020	1,0665	1,0156	1,0054	0,9591
SUR-ML RE	0,8861	0,8478	0,9857	0,9416	1,0260	0,9791	1,0488	0,9987	1,0629	1,0122	1,0019	0,9559
SUR-FGLS RE	0,8861	0,8478	0,9857	0,9416	1,0260	0,9791	1,0487	0,9987	1,0628	1,0122	1,0019	0,9559
Homogeneous estimators												
(with spatial)												
SUR-ML SAR-RE	0,8859	0,8478	0,9855	0,9416	1,0258	0,9790	1,0485	0,9987	1,0626	1,0122	1,0016	0,9559
SUR-ML SMA-RE	0,8861	0,8478	0,9857	0,9416	1,0259	0,9791	1,0487	0,9987	1,0627	1,0122	1,0018	0,9559
SUR-ML SAR-FE	0,8890	0,8506	0,9892	0,9449	1,0294	0,9824	1,0521	1,0019	1,0663	1,0155	1,0052	0,9591
SUR-ML SMA-FE	0,8892	0,8506	0,9893	0,9449	1,0295	0,9824	1,0523	1,0020	1,0665	1,0156	1,0054	0,9591
SUR-FGLS SAR-RE	0,8859	0,8477	0,9855	0,9416	1,0257	0,9790	1,0484	0,9986	1,0625	1,0121	1,0016	0,9558
SUR-FGLS SMA-RE	0,8859	0,8477	0,9854	0,9416	1,0257	0,9790	1,0484	0,9986	1,0625	1,0121	1,0016	0,9558
Heterogeneous estimator												
(without spatial)												
Av. Heterogeneous OLS	1,1821	1,1379	1,2910	1,2430	1,3342	1,2835	1,3582	1,3050	1,3716	1,3178	1,3074	1,2575
Heterogeneous estimators												
(with spatial)												
Average SUR SAR	1,1832	1,1385	1,2920	1,2433	1,3354	1,2839	1,3590	1,3056	1,3723	1,3184	1,3084	1,2580
Average SUR SMA	1,1831	1,1386	1,2920	1,2433	1,3353	1,2839	1,3590	1,3056	1,3723	1,3184	1,3083	1,2580
SUR-FGLS SAR-RE (av.)	0,8859	0,8477	0,9855	0,9416	1,0257	0,9790	1,0485	0,9986	1,0625	1,0121	1,0016	0,9558
SUR-FGLS SMA-RE (av.)	0,8859	0,8478	0,9855	0,9416	1,0258	0,9790	1,0485	0,9986	1,0626	1,0122	1,0626	1,0122

Table 8 - Forecasts RMSE - (λ_1,λ_2)= (0.5,0.3), (ρ_μ,ρ_v)= (0.5,0.5), (N,T)=(50,10), SMA data generating process for ε, W(5,5), 1000 replications

	1st year		2sd year		3th year		4th year		5th year		Average	
	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2	eq. 1	eq. 2
Homogeneous estimators												
(without spatial)												
OLS	1,1256	1,1231	1,2325	1,2300	1,2688	1,2693	1,2895	1,2884	1,3039	1,3015	1,2441	1,2425
RE	0,8418	0,8378	0,9369	0,9311	0,9737	0,9689	0,9937	0,9877	1,0059	1,0002	0,9504	0,9452
FE	0,8448	0,8410	0,9398	0,9345	0,9773	0,9724	0,9974	0,9914	1,0094	1,0039	0,9538	0,9486
SUR-FGLS	1,1265	1,1243	1,2333	1,2307	1,2695	1,2700	1,2901	1,2890	1,3045	1,3021	1,2448	1,2432
SUR-FE	0,8447	0,8407	0,9396	0,9342	0,9771	0,9721	0,9972	0,9912	1,0092	1,0037	0,9535	0,9484
SUR-ML RE	0,8419	0,8379	0,9370	0,9312	0,9737	0,9689	0,9937	0,9878	1,0059	1,0002	0,9504	0,9452
SUR-FGLS RE	0,8419	0,8378	0,9370	0,9311	0,9737	0,9689	0,9936	0,9877	1,0059	1,0002	0,9504	0,9452
Homogeneous estimators												
(with spatial)												
SUR-ML SAR-RE	0,8418	0,8378	0,9368	0,9311	0,9736	0,9689	0,9935	0,9877	1,0057	1,0002	0,9503	0,9451
SUR-ML SMA-RE	0,8418	0,8378	0,9368	0,9311	0,9736	0,9689	0,9935	0,9877	1,0057	1,0002	0,9503	0,9451
SUR-ML SAR-FE	0,8446	0,8407	0,9395	0,9341	0,9770	0,9721	0,9971	0,9912	1,0092	1,0037	0,9535	0,9484
SUR-ML SMA-FE	0,8446	0,8407	0,9395	0,9341	0,9770	0,9721	0,9971	0,9912	1,0092	1,0037	0,9535	0,9484
SUR-FGLS SAR-RE	0,8417	0,8377	0,9367	0,9310	0,9735	0,9688	0,9935	0,9877	1,0057	1,0001	0,9502	0,9451
SUR-FGLS SMA-RE	0,8417	0,8377	0,9367	0,9310	0,9735	0,9688	0,9935	0,9877	1,0057	1,0001	0,9502	0,9451
Heterogeneous estimator												
(without spatial)												
Av. Heterogeneous OLS	1,1255	1,1232	1,2325	1,2301	1,2688	1,2694	1,2895	1,2884	1,3039	1,3015	1,2440	1,2425
Heterogeneous estimators												
(with spatial)												
Average SUR SAR	1,1264	1,1242	1,2333	1,2306	1,2696	1,2699	1,2902	1,2890	1,3044	1,3021	1,2448	1,2432
Average SUR SMA	1,1263	1,1242	1,2332	1,2306	1,2695	1,2699	1,2901	1,2889	1,3044	1,3020	1,2447	1,2431
SUR-FGLS SAR-RE (av.)	0,8417	0,8377	0,9367	0,9310	0,9735	0,9688	0,9935	0,9877	1,0057	1,0001	0,9502	0,9451
SUR-FGLS SMA-RE (av.)	0,8418	0,8378	0,9369	0,9311	0,9736	0,9689	0,9935	0,9877	1,0058	1,0002	0,9503	0,9451

Table 9 - RMSE of coefficients, standard errors and variances - (ρ_1,ρ_2) = (0.5,0.3), (ρ_μ,ρ_ν) = (0.9,0.9), (N,T)=(50,10), SAR data generating process for ε, W(1,1), 1000 replications

	$\hat{\beta}_{1,1}$	$\hat{\sigma}_{\hat{\beta}_{1,1}}$	$\hat{\beta}_{1,2}$	$\hat{\sigma}_{\hat{\beta}_{1,2}}$	$\hat{\rho}_1$ $(\hat{\lambda}_1)$	$\hat{\rho}_2$ $(\hat{\lambda}_2)$	$\hat{\sigma}^2_{\mu_1}$	$\hat{\sigma}^2_{\mu_2}$	$\hat{\sigma}^2_{\nu_1}$	$\hat{\sigma}^2_{\nu_2}$
Homogeneous estimators										
(without spatial)										
OLS	0,03048	0,00948	0,02529	0,00652	—	—	—	—	—	—
RE	0,01758	0,01194	0,01577	0,00930	—	—	0,57098	0,26661	0,54502	0,16850
FE	0,01899	0,01380	0,01808	0,01123	—	—	—	—	0,54840	0,17083
SUR-FGLS	0,01396	0,00137	0,01168	0,00048	—	—	—	—	—	—
SUR-FE	0,00885	0,00272	0,00765	0,00142	—	—	—	—	0,39421	0,08077
SUR-ML RE	0,00860	0,00223	0,00681	0,00089	—	—	0,56050	0,26195	0,54721	0,17099
SUR-FGLS RE	0,00861	0,00233	0,00680	0,00097	—	—	0,57098	0,26661	0,54502	0,16850
Homogeneous estimators										
(with spatial)										
SUR-ML SAR-RE	0,00614	0,00022	0,00614	0,00025	0,02687	0,03251	0,22599	0,21840	0,06541	0,06684
SUR-ML SMA-RE	0,00618	0,00023	0,00637	0,00023	0,03845	0,03611	0,38920	0,24003	0,33120	0,11613
SUR-ML SAR-FE	0,00690	0,00024	0,00650	0,00043	0,02869	0,03429	—	—	0,11896	0,12038
SUR-ML SMA-FE	0,00665	0,00027	0,00661	0,00050	0,03850	0,03714	—	—	0,20541	0,06757
SUR-FGLS SAR-RE	0,00618	0,00024	0,00618	0,00025	0,04030	0,04226	0,23223	0,21482	0,06484	0,06779
SUR-FGLS SMA-RE	0,00657	0,00039	0,00650	0,00032	0,09695	0,05089	0,59845	0,26265	0,47097	0,12411
Heterogeneous estimator										
(without spatial)										
Av. Heterogeneous OLS	0,03048	0,00961	0,02565	0,00683	—	—	—	—	—	—
Heterogeneous estimators										
(with spatial)										
Average SUR SAR	0,01094	0,00043	0,01149	0,00087	0,07297	0,05234	—	—	—	—
Average SUR SMA	0,01125	0,00042	0,01175	0,00067	0,10214	0,09126	—	—	—	—
SUR-FGLS SAR-RE (av.)	0,00621	0,00033	0,00612	0,00026	(1)	(1)	0,22593	0,21481	0,07742	0,06687
SUR-FGLS SMA-RE (av.)	0,00668	0,00040	0,00655	0,00037	(2)	(2)	0,50394	0,24588	0,41528	0,12441

(1) We have used the average values $\hat{\hat{\rho}}_1$ and $\hat{\hat{\rho}}_2$ of average SUR SAR estimator. (2) We have used the average values $\hat{\hat{\lambda}}_1$ et $\hat{\hat{\lambda}}_2$ of average SUR SMA estimator.