Homogenous panel unit root tests under nonstationary volatility

Helmut Herwartz and Florian Siedenburg

Christian-Albrechts-University of Kiel

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Abstract Noting that many economic variables display occasional shifts in their second order moments, we investigate the performance of homogenous panel unit root tests in the presence of permanent volatility shifts. It is shown that in this case, panel unit root tests derived under time invariant variances lose control over actual significance levels while the test proposed by Herwartz and Siedenburg (2008) retains size control. The wild bootstrap is suggested as a general means to overcome the difficulties associated with both, cross sectional dependence and time varying volatility. As an empirical illustration, we reassess evidence on the Fisher hypothesis.

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Corresponding author: F. Siedenburg, Institute for Statistics and Econometrics, Christian-Albrechts-University of Kiel, Olshausenstr. 40-60, D–24118 Kiel, Germany.

Email: fsiedenb@economics.uni-kiel.de. Phone: +49-431-8802225.

1 Introduction

Panel unit root tests (PURTs) have become a standard tool in macroeconometric applications. Making use of the cross sectional dimension allows to overcome power deficiencies of univariate unit root tests and helps to avoid the multiple testing problem. Moreover, a number of macroeconomic models imply stationarity of some key variables as, for example, the purchasing power parity hypothesis implies stationarity of real exchange rates (see Taylor and Taylor, 2004 for a survey). First generation PURTs (e.g. Levin et al., 2002 or Im et al., 2003) rely on the assumption of cross sectionally independent error terms. Since the work of O'Connell (1998), however, it is widely recognized that a violation of this assumption leads to severe size distortions of first generation tests, and therefore, second generation tests relying on less restrictive assumptions have been suggested (see Hurlin and Mignon, 2007 and Breitung and Pesaran, 2008 for recent surveys). Two general directions of coping with the nuisance parameters invoked by the cross sectional dependence can be identified. On the one hand, approaches presuming a common factor structure for the error terms and, on the other hand, tests building on robust variance estimators.

Second order invariance of model disturbances is another assumption underlying PURTs. However, this assumption is also unlikely to generally hold in reality, as many macroeconomic and financial variables are characterized by structural shifts in their unconditional volatility. In fact, what has become known as the "Great Moderation" is a substantial decline in numerous key macroeconomic variables' volatility across the G7 economies since the mid 1980s (Kim and Nelson, 1999, McConnell, 2000 and van Dijk et al., 2002). The adverse effects of variance shifts on unit root tests for single time series have been studied by, for instance, Hamori and Tokihisa (1997), Kim et al. (2002), Cavaliere (2004), Cavaliere and Taylor (2007) and Cavaliere and Taylor (2008). The main findings are that the Augmented Dickey-Fuller (Dickey and Fuller, 1979, ADF henceforth) and other unit root tests are biased in the presence of variance shifts. The bias is invoked by nuisance parameters in the asymptotic distribution of the test statistic that depend on the variance profile. Generally, largest (positive) size biases are observed for early negative and late positive shifts in the level of the unconditional variance of the process. So far, there have been no attempts to generalize these results to the field of panel unit root testing. This paper attempts to fill in this gap, concentrating on the class of homogenous PURTs based on a pooled DF regression. We show that the second generation 'White-type' corrected PURT proposed in Herwartz and Siedenburg (2008) retains a standard normal limiting distribution under a discrete upward shift of the innovation variance. In contrast, the first generation test of Levin et al. (2002) and the second generation test of Breitung and Das (2005) do not converge to a nuisance free limiting distribution in this case. Moreover, the wild bootstrap is suggested as a way of robustyfing PURTs under general forms of non-spherical disturbances.

As an illustrative example, we reconsider PURT based evidence on the Fisher hypothesis in Crowder (2003). Postulating a one-to-one co-movement of nominal interest rates and expected rates of inflation, the Fisher hypothesis implies stationary real interest rates. The considered cross section of 9 developed economies over the period 1961Q2-2007Q2 mirrors core issues discussed in this paper, such as shifts in unconditional volatility and cross sectional dependence.

The remainder of the paper is organized as follows. A brief review over the effects of nonstationary volatility on unit root tests in the pure time series case is given in the next section. The panel model is introduced and asymptotic results for the considered PURTs are derived in Section 3. Section 4 provides the results of a Monte Carlo simulation study. The empirical illustration is presented in Section 5 and Section 6 concludes. Mathematical proofs are contained in the Appendix.

2 Effects of nonstationary volatility on univariate unit root tests

The effects of nonstationary volatility on unit root tests in the single time series case have been investigated by Hamori and Tokihisa (1997), Kim et al. (2002), Cavaliere (2004), Cavaliere and Taylor (2007) and Cavaliere and Taylor (2008). For an autoregressive model of order 1 (AR(1)) with a single upward shift in the innovation variance, Hamori and Tokihisa (1997) show that the DF t-statistic does not converge to the usual DF distribution under H_0. In particular, consider the following data generating process (DGP)

$$y_t = \rho y_{t-1} + e_t, \quad t = 1, ..., T. \tag{1}$$

In (1), the variance shift is modeled by means of the composite error term e_t, i.e.

$$e_t = \epsilon_t + \eta_t DU_t, \quad \epsilon_t \sim iid(0, \sigma_1^2), \quad \eta_t \sim iid(0, \sigma_2^2), \tag{2}$$

$$DU_t = \begin{cases} 1, & \text{if } t > T_B, \ (1 < T_B < T) \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Let $\lambda = T_B/T$ denote the ratio of pre-break to total sample period, then, as $T \to \infty$, the asymptotic distribution of the DF t-ratio of $\hat{\rho} - 1$ is given by

$$t_{DF} \xrightarrow{d} \frac{\frac{1}{2}[\{W(1)\}^2 - 1]}{\sqrt{\int_0^1 W^2 - \Xi \left[(1-\lambda) \int_0^\lambda W + \lambda \int_\lambda^1 \left(\frac{1-r}{r}\right) W\right]}}, \ \Xi = \frac{(\sigma_2/\sigma_1)^2}{1 + (\sigma_2/\sigma_1)^2 (1-\lambda)}, \tag{4}$$

where $\int_a^b W$ and $\int_a^b W^2$ are shorthand notations for $\int_a^b W(r)dr$ and $\int_a^b \{W(r)\}^2 dr$, and $W(r)$ denotes standard Brownian motion. It is easy to verify that the nuisance parameters in the limiting distribution depend on both the strength and the timing of the variance break. The standard DF case is covered by $\sigma_2 = 0$ and $\lambda = 0$ or $\lambda = 1$. Hamori and Tokihisa (1997) provide simulation evidence suggesting that a late positive variance shift leads to the largest (upward) bias of empirical type one errors. Kim et al. (2002) generalize the previous result to models with deterministic terms and propose an unbiased test for the unit root null hypothesis based on prior break date estimation. In a series of papers, Cavaliere (2004) and Cavaliere and Taylor (2007, 2008) further extend these results in three directions. First, they allow for a wider class of volatility processes, including multiple breaks and trending volatility. Second, they extend the analysis to the M class of unit root tests proposed by Perron and Ng (1996), Stock (1999) and Ng and Perron (2001). Finally, they propose simulation and resampling methods to obtain unbiased unit root tests under nonstationary volatility. Cavaliere and Taylor (2008) show that, by mimicking the heteroskedasticity of the original data, wild bootstrap resampling of unit root test statistics yields critical values, leading to unbiased inference under a wide range of volatility processes.

3 PURTs under nonstationary volatility

3.1 The autoregressive, heteroskedastic panel model

In the following, we study the effects of nonstationary volatility on homogenous PURTs. More specifically, the limiting distributions of alternative t-statistics obtained from pooled DF regressions are derived for a panel generalization of the Hamori and Tokihisa (1997) framework assuming cross sectional independence. Since deterministic terms, autocorrelated disturbances and cross sectional dependence are important features of most macroeconometric data sets, we later discuss potential modifications necessary to account for these departures from the most simple model design.

The heteroskedastic panel model is given by

$$y_{it} = \rho_i y_{i,t-1} + e_{it} \quad t = 1, ..., T, \quad i = 1, ..., N, \tag{5}$$

$$e_{it} = \epsilon_{it} + DU_t \eta_{it}, \quad \epsilon_{it} \sim iid(0, \sigma_1^2), \quad \eta_{it} \sim iid(0, \sigma_2^2) \tag{6}$$

$$DU_t = \begin{cases} 1, & \text{if } t > T_B, \ (1 < T_B < T) \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

Panel model (5)-(7) formalizes a pure panel autoregressive process of order one with a break in the innovation variance. The break point (T_B) is assumed to be the same across cross section members and individual innovation variances are assumed to be homogenous. Moreover, we define the average innovation variance as

$$\sigma_e^2 = \sigma_1^2 + (1 - \lambda)\sigma_2^2. \tag{8}$$

In addition to equations (5)-(7) we make the following assumptions which are assumed to hold throughout.

Assumption 1 ($\mathcal{A}_1$) *The autoregressive coefficients ρ_i satisfy either $\rho_i = 1$ or $\rho_i < 1$ for all i.*

Assumption 2 ($\mathcal{A}_2$) *The vector of initial values $\boldsymbol{y}_0 = (y_{i0}, ..., y_{N0})' = 0$.*

Assumption ($\mathcal{A}_1$) restricts the process to either contain a unit root for all cross sectional members (H_0) or to be stationary for all cross sectional members otherwise (H_1). In practice however, a rejection of H_0 not necessarily implies that the process is in fact stationary for all cross sectional units. Breitung and Pesaran (2008) point out that tests designed against the homogenous alternative defined in ($\mathcal{A}_1$) might also have power against the heterogenous alternative considered, for instance, in Im et al. (2003). Hence, a rejection of H_0 should be carefully interpreted as indicating stationarity for a significant fraction of the AR(1) processes in the cross section. The assumption of known initial values ($\mathcal{A}_2$) is largely standard in the literature on (P)URTs and the specific choice of $\boldsymbol{y}_0 = 0$ is made without loss of generality.

3.2 Asymptotic bias of homogenous PURTs

Consider the AR(1) panel model defined in (5)-(7). The unit root null hypothesis, $H_0 : \rho_i = \rho = 1$ can be tested by means of the OLS t-ratio of $\hat{\phi}$ from the pooled DF regression

$$\Delta \boldsymbol{y}_t = \phi \boldsymbol{y}_{t-1} + \boldsymbol{e}_t,$$

with $\Delta \boldsymbol{y}_t = (y_{1t} - y_{1,t-1}, ..., y_{Nt} - y_{N,t-1})'$, $\boldsymbol{y}_{t-1} = (y_{1,t-1}, ..., y_{N,t-1})'$ and $\boldsymbol{e}_t = (e_{1t}, ..., e_{Nt})'$. The test statistic is

$$t_{OLS} = \frac{\sum_{t=1}^{T} \boldsymbol{y}_{t-1}' \Delta \boldsymbol{y}_t}{\sqrt{\sigma_e^2 \sum_{t=1}^{T} \boldsymbol{y}_{t-1}' \boldsymbol{y}_{t-1}}}, \tag{9}$$

where, σ_e^2 is replaced by $\hat{\sigma}_e^2 = (NT)^{-1} \sum_{t=1}^{T} (\Delta \boldsymbol{y}_t - \boldsymbol{y}_{t-1}\hat{\phi})'(\Delta \boldsymbol{y}_t - \boldsymbol{y}_{t-1}\hat{\phi})$. The results in Levin et al. (2002) imply that under H_0 in (5) and cross sectionally independent, time homoskedastic error terms, t_{OLS} is asymptotically Gaussian as $T \to \infty$ followed by $N \to \infty$. Violations of the assumption of cross sectional independence can be overcome along the lines in Breitung and Das (2005) or Herwartz and Siedenburg

(2008), either by means of robust covariance estimation or resampling methods. To our knowledge, however, the effects of a break in the innovation variance on PURTs have not yet been studied. In the following, it is shown that in analogy to the univariate case, t_{OLS} does not converge to a nuisance free limiting distribution and, hence, loses control over the asymptotic size of the test.

Proposition 1 *Assume the panel DGP is given by (5)-(7) and $\sigma_2^2 > 0$. Then, under $H_0 : \rho_i = \rho = 1$ and for $T \to \infty$ followed by $N \to \infty$, $t_{OLS} \xrightarrow{d} N(0, \bar{\sigma}^2)$, $\bar{\sigma}^2 > 1$.*

The proof of Proposition 1 is deferred to the Appendix. The result directly shows that a positive and permanent shift of the innovation variance of $\boldsymbol{y}_t$ leads to a positive size bias of the PURT based on t_{OLS}, if critical values are taken from the Gaussian distribution. Moreover, the asymptotic bias is also shared by the cross sectional dependence robust statistic t_{Rob} suggested by Breitung and Das (2005). The statistic corrects for (weak form) cross sectional dependence by applying *panel corrected standard errors*, (Beck and Katz, 1995). It is given as

$$t_{Rob} = \frac{\sum_{t=1}^{T} \boldsymbol{y}_{t-1}' \Delta \boldsymbol{y}_t}{\sqrt{\sum_{t=1}^{T} \boldsymbol{y}_{t-1}' \hat{\Omega} \boldsymbol{y}_{t-1}}}, \quad \text{with} \quad \hat{\Omega} = \frac{1}{T} \sum_{t=1}^{T} \hat{\boldsymbol{e}}_t \hat{\boldsymbol{e}}_t'. \tag{10}$$

The covariance matrix $\hat{\Omega}$ is constructed such that its diagonal elements equal σ_e^2. Under cross sectional independence, Ω is a diagonal matrix and, hence, t_{Rob} and t_{OLS} are asymptotically identical.

3.3 A volatility-break robust test

Herwartz and Siedenburg (2008) propose a test statistic, which is based on a 'White-type' covariance estimator, making use of residuals obtained under H_0. The test statistic is given as

$$t_{HS} = \frac{\sum_{t=1}^{T} \boldsymbol{y}_{t-1}' \Delta \boldsymbol{y}_t}{\sqrt{\sum_{t=1}^{T} \boldsymbol{y}_{t-1}' \check{\boldsymbol{e}}_t \check{\boldsymbol{e}}_t' \boldsymbol{y}_{t-1}}}, \quad \check{\boldsymbol{e}}_t = \Delta \boldsymbol{y}_t = \boldsymbol{e}_t. \tag{11}$$

It is shown that under finite fourth order moments of $\boldsymbol{e}_t$, t_{HS} is asymptotically Gaussian under weak-form cross sectional dependence as defined in Breitung and

Das (2005). The statistic was originally proposed as an alternative to t_{Rob} in finite samples where the number of cross sectional units is relatively large compared to the time dimension. However, Herwartz and Siedenburg (2008) note that the proposed statistic might also be useful to obtain size control if there is some time variation in the second order moments of $\boldsymbol{e}_t$. Similarly, Hamori and Tokihisa (1997) suggest the White correction as a potential means to appropriately cope with the nuisance invoked by a variance shift.

The following Proposition states asymptotic Gaussianity of the statistic t_{HS} under a volatility break defined by (6)-(7).

Proposition 2 *Assume the DGP is given by (5)-(7) and the error components* ϵ_{it} *and* η_{it} *have finite fourth order moments,* $0 < E[\epsilon_{it}^4], E[\eta_{it}^4] < \infty$*. Then under* $H_0: \rho_i = \rho = 1$ *and for* $T \to \infty$ *followed by* $N \to \infty$*,* $t_{HS} \xrightarrow{d} N(0,1)$*.*

The proof of Proposition 2 is given in the Appendix. Even though this proof is based upon the presumption of cross sectional independence, in the light of the results in Herwartz and Siedenburg (2008), it is likely that t_{HS} retains asymptotic pivotalness even in the case of (weak form) cross sectional dependence. This conjecture rests on the idea of constructing a suitable decomposition of the covariance matrix, which could allow to separate the issue of robustifying the statistic against cross sectional dependence from the problem of coping with variance shifts.

Furthermore, innovation variances σ_1^2 and σ_2^2 might display some variation over the cross section. It appears sensible to assume that t_{HS} remains pivotal in this case, as the denominator converges to the average of the cross sectional innovation variances, correctly adjusted for the common volatility shift.

3.4 Wild bootstrap PURTs

As demonstrated in Cavaliere and Taylor (2008) basing test decisions on wild bootstrap critical values circumvents the problems of nonpivotalness of the original test statistics invoked by nonstationary innovation variances. Moreover, Herwartz and Siedenburg (2008) have proven that the wild bootstrap is efficient to immunize

PURTs against cross sectional dependence, even for a fixed cross sectional dimension. Wild resampling of homogenous PURTs proceeds as follows:

1. Calculate the PURT statistic, denoted ψ, for a given data set and obtain estimated OLS residuals, $\hat{\boldsymbol{e}}_t = \Delta \boldsymbol{y}_t - \boldsymbol{y}_{t-1}\hat{\phi}$.

2. Replicate sufficiently often the following steps:

   (i) draw bootstrap residuals $\boldsymbol{e}_t^b$ from $\hat{\boldsymbol{e}}_t$ as

   $$\boldsymbol{e}_t^b = (e_{1t}^b, \dots, e_{Nt}^b)' = \zeta_t(\hat{e}_{1t}, \dots, \hat{e}_{Nt})', \quad \zeta_t \sim iid(0,1),$$

   where $\zeta_t,\ t = 1, \dots, T$, is independent from the panel data;

   (ii) construct the bootstrap sample $\boldsymbol{y}_t^b$ according to the DGP presumed under H_0 as

   $$\boldsymbol{y}_t^b = \boldsymbol{y}_{t-1}^b + \boldsymbol{e}_t^b, \quad \boldsymbol{y}_0^b = \boldsymbol{y}_0;$$

   (iii) calculate the bootstrap version ψ^b of ψ.

3. Decision: Reject H_0 with significance α if $\psi < c_\alpha^b$, the α-quantile of ψ^b.

Several choices of ζ_t are available from the literature (Liu, 1988; Mammen, 1993). Davidson and Flachaire (2001) highlight the particular merits of the Rademacher distribution (Liu, 1988), where

$$\zeta_t = 1 \text{ with probability } 0.5 \text{ and } \zeta_t = -1 \text{ with probability } 0.5.$$

There are also alternative ways of constructing $\hat{\boldsymbol{e}}_t$. For the case of univariate unit root testing, Cavaliere and Taylor (2008) point out that $\hat{\boldsymbol{e}}_t$ obtained by imposing the null hypothesis, i.e. $\hat{\boldsymbol{e}}_t = \check{\boldsymbol{e}}_t = \Delta \boldsymbol{y}_t$ dominates other alternatives in finite samples in terms of empirical size and power properties.

3.5 Deterministic terms and serial correlation

3.5.1 Deterministic terms

If the DGP contains (individual specific) deterministic intercepts or trends, a pooled regression of (5) leads to biased tests. For these cases, Breitung (2000) discusses data transformations which retain asymptotic Gaussianity of test statistics based on the pooled regression.

Consider the case of deciding between a driftless random walk and a stationary process with individual specific intercept terms. The DGP can then be written as

$$y_{it} = (1 - \rho_i)\mu_i + \rho_i y_{i,t-1} + e_{it}, \tag{12}$$

where μ_i denotes individual specific intercepts. The effect of the intercept terms can be removed by subtracting the first observation from the data. Hence, the pooled test regression is based on the transformed data

$$\Delta \boldsymbol{y}_t = \phi \boldsymbol{y}^*_{t-1} + \boldsymbol{e}_t, \quad \text{with} \quad \boldsymbol{y}^*_{t-1} = \boldsymbol{y}_{t-1} - \boldsymbol{y}_0.$$

Breitung and Meyer (1994) point out that the power of tests based on a regression with the transformed data does not depend on the individual effects.

If the test is performed to discriminate a random walk with drift from a trend stationary process, the underlying DGP may be written as

$$y_{it} = \mu_i + (1 - \rho_i)\beta_i t + \rho_i y_{i,t-1} + e_{it}. \tag{13}$$

Breitung (2000) suggests the Helmert transformation to center the first differences of the data in a forward looking manner, i.e.

$$\begin{aligned} \Delta y^*_{it} &= s_t \left[\Delta y_{it} - \frac{1}{T-t}(\Delta y_{i,t+1} + ... + \Delta y_{iT}) \right], \text{and} \\ & s_t^2 = (T-t)/(T-t+1). \end{aligned} \tag{14}$$

Detrending of the the test regression's right hand side variable proceeds as

$$y^*_{it} = y_{it} - y_{i0} - \hat{\beta}_i t = y_{it} - y_{i0} - \frac{y_{iT} - y_{i0}}{T} t. \tag{15}$$

It can be shown that the detrending in (14) and (15) is such that $E(\Delta y^*_{it} y^*_{i,t-1}) = 0$. Breitung (2000) shows that the PURT statistics performed on the transformed data y^*_{it} remain asymptotically Gaussian. This result, however, relies on Δy^*_{it} being white noise with constant variance. As this assumption is violated in our setting, it is unclear if the proposed detrending scheme still obtains a pivotal PURT statistic. We address this issue by means of the Monte Carlo study in Section 4.

3.5.2 Short run dynamics

If the error terms of the DGP are serially correlated, Breitung and Das (2005) show that the pooled statistic in (9) remains asymptotically Gaussian if it is computed from prewhitened data. Prewhitening proceeds by running individual specific, ADF regressions under H_0, i.e.

$$\Delta y_{it} = \delta_i d_{it} + \sum_{j=1}^{p} c_{ij} \Delta y_{i,t-j} + e_{it},$$

where d_{it} is a vector of ones in the case of a random walk with drift and zero otherwise. The estimates $\hat{\boldsymbol{c}}_i = (\hat{c}_{i1}, ..., \hat{c}_{ip})$ are then used to obtain prewhitened data as

$$y_{it}^* = y_{it} - \hat{c}_{i1} y_{i,t-1} - ... - \hat{c}_{ip_i} y_{i,t-p_i} \tag{16}$$

and

$$\Delta y_{it}^* = \Delta y_{it} - \hat{c}_{i1} \Delta y_{i,t-1} - ... - \hat{c}_{ip_i} \Delta y_{i,t-p_i}. \tag{17}$$

The choice of lag lengths p_i can be based on any consistent lag-length selection criterion. If the DGP features both, short run dynamics and deterministic terms, the data is first prewhitened and then detrended as discussed in Section 3.5.1.

4 Monte Carlo study

4.1 The simulation design

We evaluate the finite sample properties of pooled PURTs under the following 3 scenarios

DGP1: $y_{it} = (1-\rho_i)\mu_i + \rho_i y_{i,t-1} + e_{it}, \quad i = 1, ..., N,\ t = -51, ..., T,$

DGP2: $y_{it} = \mu_i + (1-\rho_i)\beta_i t + \rho_i y_{i,t-1} + e_{it},$

DGP3: $y_{it} = (1-\rho_i)\mu_i + \rho_i y_{i,t-1} + u_{it}, \quad u_{it} = c_i u_{i,t-1} + e_{it}$

The first two DGPs formalize first order autoregressive models with serially uncorrelated errors, whereas the last one introduces AR(1) disturbances. DGPs 1 and

3 formalize the panel unit root against a panel stationary process with individual effects whereas DGP 2 distinguishes between a panel random walk with drift against trend stationary processes with individual effects. Rejection frequencies under H_0 are computed with $\rho_i = \rho = 1$ whereas empirical (size adjusted) power is computed against individual specific local alternatives, $\rho_i = 1 - \frac{a_i}{T\sqrt{N}}$, where $a_i \sim iidU(10, 20)$. Following Pesaran (2007) the deterministic terms are parameterized such that the processes display the same average trend properties under the null and alternative hypothesis. In particular, $\mu_i \sim iidU(0, 0.02)$, and $\beta_i \sim iidU(0, 0.02)$. The short run dynamics in DGP 3 are chosen as in Pesaran (2007), i.e. $c_i \sim iidU(0.2, 0.4)$.

Six distinct scenarios for the error term covariance matrix Ω_t are simulated for each DGP. With regard to contemporaneous correlation, cases of cross sectionally independent, as well as of contemporaneously correlated panels are considered. Three different scenarios are simulated with respect to volatility shifts: constant volatility, a late positive variance shift and an early negative variance break.

To formalize these scenarios, we make use of the decomposition $\Omega_t = {\Phi_t}^{1/2}\Psi{\Phi_t}^{1/2}$. In this formulation Ψ is the correlation matrix describing the time invariant pattern of cross sectional correlation and $\Phi_t^{1/2}$ is the diagonal matrix, capturing the (time varying) standard deviations of the error terms. Cross sectionally uncorrelated data is generated by setting $\Psi = I_N$ and $\Phi_t = I_N\sigma_t^2$, while a common factor structure is assumed for the case of contemporaneously correlated error terms. The latter is specified as

$$e_{it} = \gamma_i f_t + \varepsilon_{it}, \text{ with } f_t \sim iidN(0,1), \; \gamma_i \sim U(0.5, 1.5), \text{ and } \varepsilon_{it} \sim iidN(0,1),$$

where factor loadings γ_i measure the impact of the common factor f_t on cross section unit i. Let $\Gamma = (\gamma_1, ..., \gamma_N)'$ denote the vector of factor loadings, then Ψ is the correlation matrix associated with $\Gamma\Gamma' + I_N$ and $\Phi_t = \text{diag}(\Gamma\Gamma' + I_N)\sigma_t^2$. As mentioned above, three distinct cases of Φ_t and, hence, of σ_t^2 are simulated. Let $\sigma_{\lfloor sT \rfloor} = \sigma_1 \mathbb{I}(s \leq s_B) + \sigma_2 \mathbb{I}(s > s_B)$, where $s_B \in [0, 1]$ indicates the timing of the variance break and $\lfloor sT \rfloor$ denotes the integer part of sT. Then, in the homoskedastic case, we set $\sigma_t = \sigma_1$, with $\sigma_1 = 1$. The early negative break scenario is parameterized as $s_B = 0.2$ and $\sigma_2 = 1/3$, while the late positive break is given by $s_B = 0.8$ and $\sigma_2 = 3$.

Data is generated for all combinations of $N \in [10, 50]$ and $T \in [10, 50, 100, 250]$. A burn-in-phase of 51 presample values is generated throughout. To compute empirical rejection probabilities under H_0, we calculate each PURT statistic for the appropriately transformed data and the nominal size equals 5%. Reported estimates for local power are adjusted such that empirical type one errors equal 5%. Throughout, we use 5000 replications and 499 bootstrap replications.

4.2 Results

Table 1 documents empirical rejection frequencies obtained for DGP1. To economize on space, we refrain from reporting results for the case $N = 10$, $T = 250$ and local power estimates are only reported for $N = 50$, $T \in [100, 250]$.

The left hand side of Table 1 documents results obtained under cross sectional independence while entries on the right hand side refer to results obtained under a common factor structure. The upper panel reports empirical rejection frequencies under H_0 calculated at the nominal 5% level and the lower panel shows local power estimates. The first block in the upper left panel is the benchmark case, characterized by cross sectional independence and time invariant innovation variances. In this setting, all employed statistics have a standard normal limiting distribution and, hence, should display empirical rejection frequencies close to 5% as T and N become large. In fact, the documented results show some evidence of small sample biases. Empirical rejection frequencies obtained by t_{OLS} range around 7% for panels with $N = 10$, whereas application of t_{Rob} leads to undersizing for small values of T. Results obtained for the 'White-type' statistic t_{HS} display only small finite sample biases. The results based on wild bootstrap critical values confirm that faster convergence to nominal significance levels can be obtained by bootstrapping asymptotically pivotal test statistics (Horowitz, 2001). Throughout, Empirical rejection probabilities are very close to 5%.

The right hand side of the first block presents results for the common factor model with constant volatility. All three tests fail asymptotic Gaussianity under cross sectional dependence formalized by a common factor model. However, while rejection frequencies for t_{OLS} diverge as the sample grows larger, rejection rates for t_{Rob} and t_{HS} are only moderately oversized, a fact earlier noted by Jönsson (2005). The largest observed empirical type one errors for the latter two statistics are 8.1% and 8.5% respectively. Note that in the case of $N = 50$, $T = 10$, application of t_{Rob} obtains an empirical rejection probability of only 1.0%. As argued in Section 3.4, the wild bootstrap yields critical values allowing for unbiased inference even if the DGP is driven by a common factor model. Empirical rejection frequencies based on bootstrap critical values vary between 5.0% and 5.7% and, hence, are practically indistinguishable from the nominal level.

For the case of an early negative variance shift, results obtained for t_{OLS} and t_{Rob} indicate a tendency of undersizing in finite samples, where the downward bias of empirical rejection frequencies positively depends on the size of N. The finding of downward biased PURTs in the presence of an early variance reduction is in contrast to results for univariate unit root tests, where positive size biases are reported (e.g. Kim et al., 2002, Cavaliere and Taylor, 2007). Rejection frequencies obtained by the 'White-type' statistic t_{HS}, as well as all bootstrap variants display only minor deviations from the nominal significance level. Results obtained under a common factor model are quite similar to the case with constant variances. While results based on t_{OLS} are markedly oversized, rejection frequencies for t_{Rob} and t_{HS} are only moderately oversized for large T. However, while the former is undersized if $N \geq T$, the latter statistic yields empirical rejection frequencies in the range of 4.1% to 8.4%. As before, the bootstrap variants obtain empirical rejection probabilities close to the nominal level for all combinations of N and T.

If the innovation variance features an upward shift towards the end of the sample, empirical rejection frequencies for t_{OLS} are in the range of 11.3-13.6% for all combinations of N and T. Rejection frequencies for t_{Rob} depend on the relative length of the time dimension: for T large relative to N, H_0 is rejected significantly too often while for $N > T$, the undersizing observed in the previous experiments persists. The observed upward bias is quantitatively in line with results obtained in a similar setting for the univariate DF test (Hamori and Tokihisa, 1997). In contrast, empirical rejection frequencies close to the nominal level are obtained by t_{HS} if $N, T \geq 50$. Tests based on bootstrap critical values are not affected by shifts in the unconditional variance. Rejection frequencies under H_0 are accurately close to the nominal level. If the data is generated under a common factor model with positive variance shift, unbiased inference is only available based on the bootstrap critical values. Biases of empirical rejection frequencies observed for t_{OLS} and t_{Rob} are larger than in the case of a common factor model with constant variance. Finally, empirical rejection frequencies for t_{HS} are similar to the constant-variance case and, thus, indicate the robustness of this statistic against time varying volatilities.

Local power estimates show that all tests have power against near integrated local alternatives. Cavaliere and Taylor (2008) show that wild bootstrap versions of univariate unit root tests have the same asymptotic local power functions as the original tests. Our finite sample results confirm this finding as the bootstrapped statistics do not suffer from power loss. Moreover, local power results show that all considered tests are less powerful if the data is cross sectionally correlated. This finding can be explained by noting that the panel contains less independent information if cross sectional units are correlated (Hanck, 2008). Moreover, concentrating on the bootstrapped statistics under the common factor model yields that t^b_{OLS} is substantially more powerful than t^b_{Rob} and t^b_{HS}, a fact also observed in Herwartz and Siedenburg (2008).

Table 2 reports results for DGP2, with all tests computed for detrended data. For the benchmark scenario assuming constant variances and cross sectional independence, results are similar to those obtained for DGP1, with rejection frequencies for all statistics close to the nominal level. As before, a large T relative to N is required in order to obtain rejection probabilities close to 5% for t_{Rob}. If the data is generated by a common factor model (upper right block), bootstrap based inference is less precise compared to the case of cross sectional independence. In particular, the results obtained by t^b_{OLS} display substantial positive deviations from the 5% nominal level.

For both scenarios of variance shifts, all tests based on detrended data lose size control. If there is a reduction in the innovation variance, most tests show empirical rejection frequencies which increase with the sample size. In contrast, empirical rejection frequencies of all tests tend to zero in the case of a late positive variance shift.

Local power estimates for the constant variance case are markedly smaller (approximately by a factor of 3) than those obtained for centered data in Table 1. The latter fact has been studied by several authors as mentioned in Breitung and Pesaran (2008). We do not report local power results for the scenarios featuring variance shifts, as size features of the tests appear prohibitive for applied research.

Table 3 reports results for data featuring serially correlated disturbances. These results show a general tendency of the tests to overreject H_0 if $N = 50$ and $T = 10$ in the scenarios of constant volatility and a negative volatility break. The latter observation, however, does not apply to t_{Rob}, which stays downward biased for this panel dimension. Conditional on this finding, results obtained under H_0 are qualitatively similar to those obtained for DGP1. In particular, an early negative variance shift diminishes rejection probabilities under H_0, while a late positive shift leads to increased rejections of H_0. Moreover, t_{HS} and the bootstrapped statistics are robust against time varying volatility and, as before, application of t_{OLS} leads to markedly oversized rejection rates if the data is cross sectionally correlated.

Local power estimates are similar to those obtained for serially uncorrelated error terms (DGP1). The results moreover confirm the previous finding that all tests are asymptotically less powerful if the data is cross sectionally correlated by means of a common factor model.

An additional set of simulations has been conducted for trending data with serially correlated error terms. For the case of constant variances, the results indicate that a large time dimension is required to obtain reasonable empirical size properties. Otherwise, the results confirm the failure of the detrending scheme in the cases of discrete jumps in the volatility process. Hence, to conserve on space, the detailed results remain unreported but are available from the authors upon request.

4.3 Summary of simulation results

The main result obtained by the simulation study is that an early negative (late positive) variance shift leads to a downward (upward) bias of rejection frequencies for PURTs derived under the assumption of invariant second order moments. If the DGP formalizes a random walk without drift under H_0, rejection rates obtained by the 'White-type' statistic t_{HS} are not affected by variance breaks. Wild bootstrap critical values lead to unbiased inference for all three tests even under (strong-form) cross sectional dependence, time varying variances and a finite cross section dimension. If, on the other hand, the DGP formalizes a random walk with drift under H_0, the applied detrending scheme proposed by Breitung (2000) does not lead to unbiased inference if there is a break in the innovation variance. Prewhitening data to remove the effect of serially correlated error terms leaves the main findings unaffected, however, a larger time dimension is required for the empirical type one errors of the tests to come reasonably close to the nominal level.

5 Testing the Fisher hypothesis by means of PURTs

5.1 Economic background

The Fisher hypothesis (Fisher, 1930) postulates a stable one-to-one relationship between nominal interest rates and the expected rate of inflation. This hypothesis has been investigated in numerous empirical studies (see e.g. Rose, 1988, Crowder, 2003, or Cooray, 2003). In its simplest form, the Fisher hypothesis states that the nominal interest rate in country i at time t, R_{it}, consists of the sum of the ex-ante real interest rate, $E_{t-1}[r_{it}]$ and the ex-ante expected inflation rate, $E_{t-1}[\pi_{it}]$, i.e.

$$R_{it} = E_{t-1}[r_{it}] + E_{t-1}[\pi_{it}] + \upsilon_{it},$$

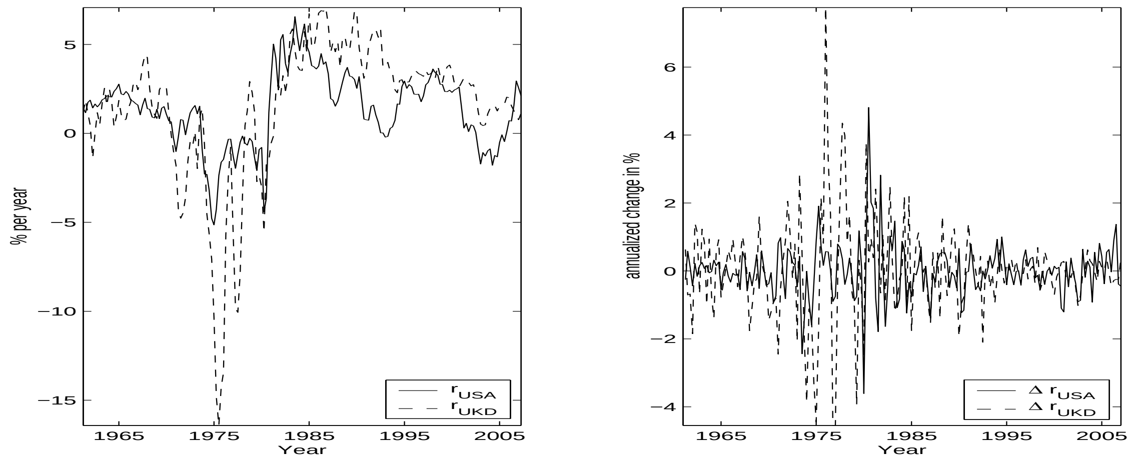
where υ_{it} denotes an uninformative forecast error. Under rational expectations, actual and expected values differ only by a white-noise error term, i.e. $\pi_{it} = E_{t-1}[\pi_{it}] + \nu_{it}^{(1)}$ and $r_{it} = E_{t-1}[r_{it}] + \nu_{it}^{(2)}$. Accordingly, the ex-post real interest rate can be expressed as

$$r_{it} = R_{it} - \pi_{it} + \nu_{it}, \tag{18}$$

with $\nu_{it} = \upsilon_{it} - \nu_{it}^{(1)} - \nu_{it}^{(2)}$. The representation in (18) is a starting point for empirical investigations of the Fisher hypothesis by means of unit root tests. If, for instance, inflation and nominal interest rates are found to be I(1) variables, the Fisher hypothesis would imply (1, -1) cointegration between the two, leading to an I(0) real interest rate. In contrast, a finding of nominal interest rates being I(1) and inflation being I(0) would contradict the Fisher hypothesis.

Prevalence of the Fisher hypothesis is still a question open to empirical research. Using univariate unit root tests on data for 18 economies, Rose (1988) obtains that nominal interest rates follow a unit root process while inflation rates are stationary. On the other hand, Rapach and Weber (2004) report evidence in favor of both variables being integrated of order one, albeit not forming a cointegration relationship. Evidence favorable for a stable long run relationship between inflation and nominal interest rates is reported in Herwartz and Reimers (2006) and Crowder (2003). However, assessments of the Fisher hypothesis based on first generation PURTs yield conflicting results. For instance, Crowder (2003) finds some evidence of stationary nominal interest rates based on the PURT of Levin et al. (2002) for a panel of 9 industrialized economies. In the latter case, it is argued that these results must be interpreted carefully, as first generation PURTs are generally upward biased through (neglected) cross sectional correlation. However, as highlighted by Kaliva (2008), analyses of the Fisher hypothesis must explicitly account for time-varying volatility as interest and inflation data displays marked discrete volatility shifts. Hence, as pointed out in the previous sections, accounting for cross sectional dependence could be insufficient as second generation PURTs might suffer from size distortions invoked by breaks in the innovation variances. In the following assessment of the Fisher hypothesis, we document the presence of volatility breaks and cross sectional dependence in inflation and interest rate panel data sets. Subsequently, the PURTs discussed above are applied to the data to compare the marginal impacts of accounting for both departures from the assumptions of first- (and second-) generation

Figure 1: Real interest rates, levels and 1st differences, US vs. UK



PURTs.

5.2 Data and preliminary analyses

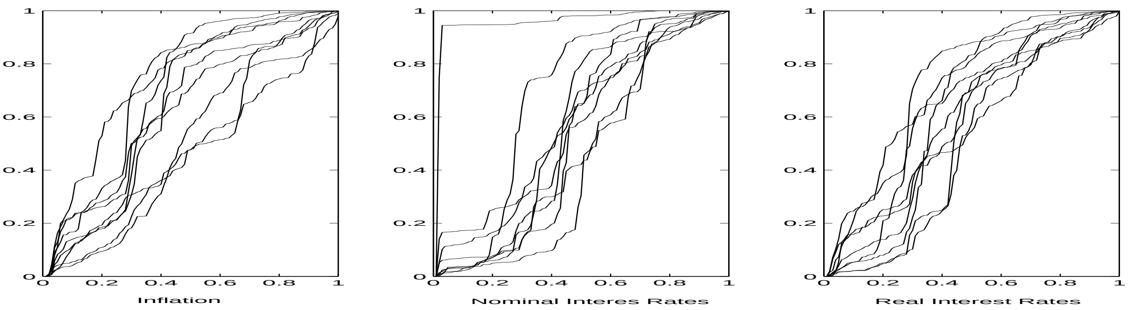
The empirical illustration is based upon the same sample of 9 developed economies considered in Crowder (2003).[1] Data is drawn from the International Financial Statistics of the IMF at the quarterly frequency, ranging from 1961Q2 to 2007Q2.[2] Inflation rates π_i are annual changes of the CPIs. Nominal interest rates, R_{it}, are selected depending on data availability and real interest rates, r_{it}, are obtained as $r_{it} = R_{it} - \pi_{it}$. Table 5 contains country specific definitions of interest rate data. Figure 3 displays the data and eyeball inspection reveals close accordance to the figures provided in Crowder (2003).

Figure 1 graphically illustrates the prevalence of cross sectional dependence and time varying volatility. The left hand side graph shows that the US and UK real interest rates evolved in a similar fashion over the sample period. This is not surprising, given that both economies are highly integrated in the world economy and

[1]These countries are: Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, United Kingdom and the United States.

[2]CPI data for the Netherlands is drawn from Dutch national statistics office as IFS data displays discretionary jumps, leading to inflation rates ranging between +30% and -17%.

Figure 2: Estimated variance profiles



face similar external shocks, as for instance, abrupt oil price swings. The right hand side graph displays the first differences of the two time series, illustrating a substantial reduction of volatility around 1985, ending roughly a decade of rather high fluctuations of real interest rates.

To get an impression of the volatility processes governing the example data, Figure 2 displays the estimated variance profiles $\hat{\vartheta}_i(s)$ for the three variables under investigation (see Cavaliere and Taylor, 2007 for details and alternative estimators of variance profiles). Variance profiles $\vartheta_i(s)$ are calculated as

$$\hat{\vartheta}_i(s) = \frac{\sum_{t=1}^{\lfloor sT \rfloor} \hat{e}_{it}^2 + (sT - \lfloor sT \rfloor)\hat{e}_{i\lfloor sT \rfloor+1}^2}{\sum_{t=1}^{T} \hat{e}_{it}^2}, \tag{19}$$

where the $\hat{e}_{it}$'s are residuals from the first order autoregression of the considered process. While a (perfectly) homoskedastic variance profile would be represented by the 45° line, time varying volatilities are characterized by marked deviations from the diagonal. Inspection of Figure 2 reveals that time-varying variances are rather the rule more than an exception for most of the countries in the sample. Moreover, it is obvious that estimated variance profiles differ across countries. However, focussing on the overall picture, there is some evidence of an upward shift followed by a downward shift in the first half of the sample period for all three variables and most of the economies considered.

In the following, we analyze to what extend previous evidence on the Fisher hypothesis obtained via first generation PURTs might have been distorted by both, cross sectional correlation and (unconditional) volatility shifts.

5.3 Results

The first step of the empirical analysis is to prewhiten the raw data. We use the SIC to determine individual specific lag lengths and subsequently apply the prewhitening procedure discussed in Section 3.5.2. In order to obtain a balanced panel, the maximum of the individual lag lengths is applied to all cross sectional units, hence prewhitening regressions for most cross sectional units are likely moderately overfitted. We use 12, 5 and 8 lags of the first differenced series for prewhitening inflation, nominal interest, and real interest rates, respectively. Assuming that inflation as well as interest rates most likely contain a non-zero mean under the stationary alternative, prewhitened data is centered by subtracting the first observations. All PURTs are then computed for the resulting balanced panels of prewhitened and centered data. Table 4 lists the results of the empirical application. Test statistics for the pooled PURTs are documented in columns 3-5, whereas values in columns 6-8 represent 5% bootstrap critical values. The numbers in parentheses are $p-$values obtained from the Gaussian CDF or the bootstrap quantiles, respectively. Results for the three variables are listed by rows, first the inflation rate, π, then the nominal interest rate, R and finally the real interest rate, r.

Using the statistic t_{OLS} to test the order of integration of the inflation rate yields a t-ratio of -3.52 and, hence, a rejection of the unit root null hypothesis at any conventional significance level. This result is in line with Crowder (2003), reporting a t-ratio -5.32 obtained via the Levin et al. (2002) procedure. Given that based on univariate tests, the unit root hypothesis is maintained for all sample economies, Crowder (2003) argues that the rejection of H_0 obtained by the PURT might be due to size bias, invoked by cross sectional dependence. Accordingly, we apply the robust statistic proposed by Breitung and Das (2005) given in (10). The resulting t-ratio of -2.45 is substantially smaller in absolute value, however, it still yields a rejection of the null hypothesis. The relative impact of time varying volatility of the sample data on pooled PURTs might be assessed by the results obtained for the volatility break robust statistic t_{HS}. The resulting t-ratio of -1.85 is larger than the $t-$ratios obtained by t_{OLS} and t_{Rob}. The corresponding marginal significance level is 3.2% and the bootstrap version t^b_{HS} obtains a p-value of 4.4%.

Qualitatively similar results are obtained for the nominal interest rate, R. The first generation test t_{OLS} obtains a t-ratio of -4.22, which is substantially smaller in absolute value than -7.57 reported in Crowder (2003), nevertheless, leads to a rejection of H_0. Again, application of the robust tests and the wild bootstrap increase marginal significance levels. The cross sectional dependence robust test statistic t_{Rob} still obtains a rejection of H_0 at the 1% level while its bootstrap counterpart yields a marginal significance level of 1.3%. In this instance, application of t_{HS} and t^b_{HS} might lead to a different test decision as $p-$values of 4.8% or 7%, respectively, are just below or above the widespread 5% threshold for a rejection of the PUR hypothesis.

Finally, we test for the panel unit root in the real interest rate. All tests yield results in support of panel stationarity of the real interest rate, and thus, of the Fisher hypothesis. Note however, that (at the conventional 5% significance level) even robust procedures do not completely rule out the possibility of inflation and nominal interest rates being likewise panel stationary variables. Accordingly, one should be careful in interpreting stationarity of real interest rates as a cointegration relationship, linking two nonstationary variables.

Concluding the empirical example, we note that it is important to properly account not only for cross sectional dependence but also for variance breaks in panel unit root testing. In our empirical example, this is illustrated by marked differences between the three test procedures, yielding highest marginal significance levels for that test statistic which is robust in both directions.

6 Conclusions

This paper investigates the effects of discrete jumps in the innovation variance on homogenous panel unit root tests. It is shown that size distortions documented in the literature on univariate unit root tests under time varying variances carry over to the panel case.

The limiting distribution of a first generation PURT test under a positive variance shift is derived and it is shown that the test statistic loses asymptotic pivotalness. Moreover, it is proved that the 'White-type' PURT statistic proposed in Herwartz and Siedenburg (2008) remains asymptotically Gaussian, even under a one time upward variance shift. By means of a Monte Carlo study we analyze a variety of possible model settings, including deterministic trends, autocorrelated disturbances and cross sectional correlation. The analysis is restricted to homogenous PURT procedures and wild bootstrap variants thereof. As an empirical illustration, recent evidence on the Fisher hypothesis in Crowder (2003) is reconsidered. Based on data for a cross section of 9 developed economies over the period 1961Q2 - 2007Q2, we test the order of integration of inflation rates as well as of nominal and real interest rates. The results illustrate the importance of robust panel unit root tests, accounting for nonstationary innovation variances and cross sectional dependence.

This paper offers a number of interesting issues for future research. It appears worthwile to derive the limiting distribution of the 'White-type' statistic under more complex forms of nonstationary volatility as, for instance, outlined for the single time series case in Cavaliere (2004). Moreover, it remains to be shown that weak form cross sectional dependence does not invalidate asymptotic Gaussianity of the 'White-type' statistic in the presence of a volatility break. Finally, noting that the detrending scheme proposed in Breitung (2000) is apparently not applicable under time varying innovation variances, it appears promising to study alternative detrending schemes.

A Appendix

A.1 Proof of Proposition 1

To derive the limiting distribution of t_{OLS} consider

$$t_{OLS} = \frac{N^{-0.5}T^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T} y_{i,t-1}\Delta y_{it}}{\sqrt{N^{-1}T^{-2}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\sigma}_e^2 y_{i,t-1}^2}} = \frac{a_{NT}}{\sqrt{b_{NT}}}$$

Under H_0, it follows that

$$a_{NT} = N^{-0.5}T^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T} y_{i,t-1}\Delta y_{it} = N^{-0.5}T^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T} y_{i,t-1}e_{it}.$$

As $T \to \infty$ the result in Hamori and Tokihisa (1997) directly yields

$$a_{NT} \xrightarrow{d} N^{-0.5}\sum_{i=1}^{N}\frac{1}{2}\{\sigma_1^2 + (1-\lambda)\sigma_2^2\}[\{W_i(1)\}^2 - 1].$$

As we assume cross sectional independence and noting that $W_i(1)^2$ is a χ^2 random variable with one degree of freedom, we obtain from the central limit theorem as $N \to \infty$ that

$$a_{NT} \xrightarrow{d} \{\sigma_1^2 + (1-\lambda)\sigma_2^2\}x, \quad x \sim N\left(0, \frac{1}{2}\right). \tag{20}$$

With respect to the denominator term, as $T \to \infty$, the results in Hamori and Tokihisa (1997) imply that

$$b_{NT} \quad \xrightarrow{d} \quad N^{-1}\sum_{i=1}^{N}\sigma_e^2\left\{\sigma_e^2\int_0^1 W_i^2 - \sigma_2^2\left[(1-\lambda)\int_0^\lambda W_i^2 + \lambda\int_\lambda^1\left(\frac{1-r}{r}\right)W_i^2\right]\right\}.$$

Substituting $\sigma_1^2 + (1-\lambda)\sigma_2^2$ for σ_e^2, it follows from the law of large numbers that for $N \to \infty$

$$\begin{aligned} b_{NT} \quad \xrightarrow{p} \quad & \left(\sigma_1^2 + (1-\lambda)\sigma_2^2\right)^2 \mathrm{E}\left[\int_0^1 W^2\right] - \sigma_2^2\left(\sigma_1^2 + (1-\lambda)\sigma_2^2\right) \times \\ & \left\{(1-\lambda)\mathrm{E}\left[\int_0^\lambda W^2\right] + \lambda\mathrm{E}\left[\int_\lambda^1\left(\frac{1-r}{r}\right)W^2\right]\right\}. \end{aligned} \tag{21}$$

Some algebra then yields that t_{OLS} has the following limiting distribution for $T \to \infty$ followed by $N \to \infty$

$$t_{OLS} \xrightarrow{d} \frac{x}{\sqrt{\mathrm{E}\left[\int_0^1 W^2\right] - \Xi\left\{(1-\lambda)E\left[\int_0^\lambda W^2\right] + \lambda E\left[\int_\lambda^1\left(\frac{1-r}{r}\right)W^2\right]\right\}}}, \tag{22}$$

with Ξ defined as in Section 2. Since $\text{Var}[x] = 1/2$ and $\text{E}\left[\int_0^1 W^2\right] = 1/2$, and

$$\Xi\left\{(1-\lambda)E\left[\int_0^\lambda W^2\right] + \lambda\text{E}\left[\int_\lambda^1 \left(\frac{1-r}{r}\right)W^2\right]\right\} > 0,$$

it follows that

$$t_{OLS} \xrightarrow{d} N(0, \bar{\sigma}^2), \quad \bar{\sigma}^2 > 1. \tag{23}$$

□

A.2 Proof of Proposition 2

As in the proof of Proposition 1, rewrite t_{HS} as

$$t_{HS} = \frac{N^{-0.5}T^{-1}\sum_{i=1}^N \sum_{t=1}^T y_{i,t-1}\Delta y_{it}}{\sqrt{N^{-1}T^{-2}\sum_{i=1}^N \sum_{t=1}^T y_{i,t-1}^2 e_{it}^2}} = \frac{a_{NT}^*}{\sqrt{b_{NT}^*}}.$$

It is immediate that $a_{NT}^* = a_{NT}$. Consequently, we have to prove that for $T \to \infty$ followed by $N \to \infty$

$$b_{NT}^* \xrightarrow{p} \frac{1}{2}\{\sigma_1^2 + (1-\lambda)\sigma_2^2\}^2,$$

to ensure asymptotic Gaussianity of t_{HS}. Owing to (6), b_{NT}^* may be rewritten as

$$\begin{aligned} b_{NT}^* &= N^{-1}T^{-2}\sum_{i=1}^N \sum_{t=1}^T y_{i,t-1}^2 e_{it}^2 \\ &= N^{-1}T^{-2}\sum_{i=1}^N \sum_{t=1}^T \left(\sum_{s=0}^{t-1}\epsilon_{is} + \sum_{s=0}^{t-1} DU_s\eta_{is}\right)^2 (\epsilon_{it} + DU_t\eta_{it})^2 . \end{aligned}$$

Define $y_{it} = \widetilde{y_{it}} + \widetilde{\widetilde{y_{it}}}$ where $\widetilde{y_{it}} = \sum_{s=1}^t \epsilon_{is}$ and $\widetilde{\widetilde{y_{it}}} = \sum_{s=1}^t DU_s\eta_{is} = \sum_{s=T_B+1}^t \eta_{is}$. Then $b_{i,T}^*$ can be rewritten as

$$b_{N,T}^* = N^{-1}\sum_{i=1}^N (A_i + B_i),$$

with

$$A_i = T^{-2}\left(\sum_{t=1}^T \widetilde{y_{i,t-1}}^2 \epsilon_{it}^2 + \sum_{t=1}^T \widetilde{\widetilde{y_{i,t-1}}}^2 \epsilon_{it}^2 + \sum_{t=1}^T \widetilde{y_{i,t-1}}^2 DU_t\eta_{it}^2 + \sum_{t=1}^T \widetilde{\widetilde{y_{i,t-1}}}^2 DU_t\eta_{it}^2\right),$$

and

$$B_i = T^{-2}\left(2\sum_{t=1}^T \left(\epsilon_{it}DU_t\eta_{it}(\widetilde{y_{i,t-1}}^2 + \widetilde{\widetilde{y_{i,t-1}}}^2)\right) + 2\sum_{t=1}^T \left(\widetilde{y_{i,t-1}}\widetilde{\widetilde{y_{i,t-1}}}(\epsilon_{it} + DU_t\eta_{it})^2\right)\right).$$

The following propositions state the asymptotic behavior of A_i and B_i for $T \to \infty$ and finite N.

Proposition A.1 *For $T \to \infty$,*

$$A_i \xrightarrow{d} \{\sigma_1^2 + (1-\lambda)\sigma_2^2\}^2 \int_0^1 W_i^2.$$

Proposition A.2 *For $T \to \infty$,*

$$B_i \sim (0, \sigma_{B_i}^2).$$

Proof of Proposition A.1

Note that for all $t \leq T_B$, $DU_t = 0$ and, hence, also $\widetilde{\widetilde{y_{it}}} = 0$. Thus

$$A_i = T^{-2}\sum_{t=1}^{T} \widetilde{y_{i,t-1}}^2 \epsilon_{it}^2 + T^{-2}\sum_{t=T_B+1}^{T} \widetilde{\widetilde{y_{i,t-1}}}^2 \epsilon_{it}^2 + T^{-2}\sum_{t=T_B+1}^{T} \widetilde{y_{i,t-1}}^2 \eta_{it}^2 + T^{-2}\sum_{t=T_B+1}^{T} \widetilde{\widetilde{y_{i,t-1}}}^2 \eta_{it}^2.$$

For $T \to \infty$, it follows from convergence results for Brownian motions with constant variances that

$$T^{-2}\sum_{t=1}^{T} \widetilde{y_{i,t-1}}^2 \epsilon_{it}^2 \xrightarrow{d} (\sigma_1^2)^2 \int_0^1 W_i^2. \tag{24}$$

Similarly,

$$\begin{aligned} T^{-2}\sum_{t=T_B+1}^{T} \widetilde{\widetilde{y_{i,t-1}}}^2 \eta_{it}^2 &= \frac{(T-T_B)^2}{T^2}(T-T_B)^{-1}\sum_{t=T_B+1}^{T}\left(\frac{\widetilde{\widetilde{y_{i,t-1}}}}{\sqrt{T-T_B}}\right)^2 \eta_{it}^2 \\ &\xrightarrow{d} (1-\lambda)^2(\sigma_2^2)^2 \int_0^1 W_i^2, \end{aligned} \tag{25}$$

$$\begin{aligned} T^{-2}\sum_{t=T_B+1}^{T} \widetilde{\widetilde{y_{i,t-1}}}^2 \epsilon_{it}^2 &= \frac{(T-T_B)^2}{T^2}(T-T_B)^{-1}\sum_{t=T_B+1}^{T}\left(\frac{\widetilde{\widetilde{y_{i,t-1}}}}{\sqrt{T-T_B}}\right)^2 \epsilon_{it}^2 \\ &\xrightarrow{d} (1-\lambda)^2\sigma_1^2\sigma_2^2 \int_0^1 W_i^2, \end{aligned} \tag{26}$$

$$\begin{aligned} T^{-2}\sum_{t=T_B+1}^{T} \widetilde{y_{i,t-1}}^2 \eta_{it}^2 &= T^{-1}\sum_{t=1}^{T}\left(\frac{\widetilde{y_{i,t-1}}}{\sqrt{T}}\right)^2 \eta_{it}^2 - \frac{T_B^2}{T^2}T_B^{-1}\sum_{t=1}^{T_B}\left(\frac{\widetilde{y_{i,t-1}}}{\sqrt{T_B}}\right)^2 \eta_{it}^2 \\ &\xrightarrow{d} \sigma_1^2\sigma_2^2\int_0^1 W_i^2 - \lambda^2\sigma_1^2\sigma_2^2\int_0^1 W_i^2. \end{aligned} \tag{27}$$

Simple algebra yields that (27) can be written as $(1-\lambda^2)\sigma_1^2\sigma_2^2\int_0^1 W_i^2$. Consequently, adding the terms in (26) and (27) obtains $2(1-\lambda)\sigma_1^2\sigma_2^2\int_0^1 W_i^2$, which, together with (24) and (25), establishes Proposition A.1.

Proof of Proposition A.2

To prove that $B_i \sim (0, \sigma_{Bi}^2)$, we decompose B_i as

$$\begin{aligned}
B_i^{(a)} &= T^{-2}\sum_{t=1}^{T} 2\epsilon_{it}DU_t\eta_{it}(\widetilde{y_{i,t-1}}^2 + \widetilde{\widetilde{y_{i,t-1}}}^2) \\
B_i^{(b1)} &= T^{-2}\sum_{t=1}^{T} 2\widetilde{y_{i,t-1}}\widetilde{\widetilde{y_{i,t-1}}}\epsilon_{it}^2 \\
B_i^{(b2)} &= T^{-2}\sum_{t=1}^{T} 4\widetilde{y_{i,t-1}}\widetilde{\widetilde{y_{i,t-1}}}\epsilon_{it}DU_t\eta_{it} \\
B_i^{(b3)} &= T^{-2}\sum_{t=1}^{T} 2\widetilde{y_{i,t-1}}\widetilde{\widetilde{y_{i,t-1}}}DU_t\eta_{it}^2.
\end{aligned}$$

Writing

$$B_i^{(a)} = \frac{(T-T_B)^2}{T^2}(T-T_B)^{-1}\sum_{t=T_B+1}^{T} 2\left(\widetilde{\zeta}_{it} + \widetilde{\widetilde{\zeta}}_{it}\right),$$

where $\widetilde{\zeta}_{it} = \epsilon_{it}\eta_{it}\left(\frac{\widetilde{y}_{i,t-1}}{\sqrt{T-T_B}}\right)^2$ and $\widetilde{\widetilde{\zeta}}_{it} = \epsilon_{it}\eta_{it}\left(\frac{\widetilde{\widetilde{y}}_{i,t-1}}{\sqrt{T-T_B}}\right)^2$ can be shown to be martingale difference sequences (MDS) with finite variances. Therefore, it follows immediately that $B_i^{(a)} = o_p(1)$ for $T \to \infty$. Similarly,

$$B_i^{(b2)} = \frac{(T-T_B)^2}{T^2}(T-T_B)^{-1}\sum_{t=T_B+1}^{T} 4\frac{\widetilde{y_{i,t-1}}}{\sqrt{T-T_B}}\frac{\widetilde{\widetilde{y_{i,t-1}}}}{\sqrt{T-T_B}}\epsilon_{it}\eta_{it} = o_p(1).$$

Next consider $B_i^{(b1)}$, noting that

$$\begin{aligned}
B_i^{(b1)} &= \frac{(T-T_B)^2}{T^2}(T-T_B)^{-1}\sum_{t=T_B+1}^{T} 2\frac{(\widetilde{y_{i,t-1}} - \widetilde{y_{iT_B}})}{\sqrt{T-T_B}}\frac{\widetilde{\widetilde{y_{i,t-1}}}}{\sqrt{T-T_B}}\epsilon_{it}^2 \\
&\quad + \frac{(T-T_B)^2}{T^2}(T-T_B)^{-0.5}(T-T_B)^{-1}\sum_{t=T_B+1}^{T} 2\widetilde{y_{iT_B}}\frac{\widetilde{\widetilde{y_{i,t-1}}}}{\sqrt{T-T_B}}\epsilon_{it}^2. \qquad (28)
\end{aligned}$$

Denoting the first term in (28) as $B_i^{(b1a)}$, one obtains for $T \to \infty$

$$B_i^{(b1a)} \xrightarrow{d} 2(1-\lambda)^2\sigma_1\sigma_1^2\sigma_2\int_0^1 W_{1i}W_{2i},$$

with[3]

$$\mathrm{E}\left[B_i^{(b1a)}\right] = 0 \text{ and } \mathrm{Var}\left[B_i^{(b1a)}\right] = 4(1-\lambda)^4(\sigma_1^2)^3\sigma_2^2 \mathrm{Var}\left[\int_0^1 W_{1i}W_{2i}\right] = \frac{2}{3}(1-\lambda)^4(\sigma_1^2)^3\sigma_2^2. \quad (29)$$

Similarly, denoting the second term in (28) as $B_i^{(b1b)}$, we have

$$\begin{aligned} B_i^{(b1b)} &\xrightarrow{d} 2(1-\lambda)^2(T-T_B)^{-0.5}\sigma_1^2 N\left(0, T_B\sigma_1^2\right)\sigma_2\int_0^1 W_{2i} \\ &= 2(1-\lambda)^2\sigma_1^2 N\left(0, \frac{T_B/T}{(T-T_B)/T}\sigma_1^2\right)\sigma_2\int_0^1 W_{2i} \\ &= 2(1-\lambda)^2\sigma_1^2 N\left(0, \frac{\lambda}{(1-\lambda)}\sigma_1^2\right)\sigma_2\int_0^1 W_{2i}. \quad (30) \end{aligned}$$

It follows directly that[4]

$$\mathrm{E}\left[B_i^{(b1b)}\right] = 0 \text{ and } \mathrm{Var}\left[B_i^{(b1b)}\right] = 4(1-\lambda)^3\lambda(\sigma_1^2)^3\sigma_2^2 \mathrm{Var}\left[\int_0^1 W_{2i}\right] = \frac{4}{3}(1-\lambda)^3\lambda(\sigma_1^2)^3\sigma_2^2. \quad (31)$$

Combining (29) and (31) yields

$$\mathrm{Var}\left[B_i^{(b1)}\right] = \frac{2}{3}(1-\lambda)^3(1+\lambda)(\sigma_1^2)^3\sigma_2^2.$$

Similar reasoning applies for $B_i^{(b3)}$, yielding

$$\mathrm{E}\left[B_i^{(b3)}\right] = 0 \text{ and } \mathrm{Var}\left[B_i^{(b3)}\right] = \frac{2}{3}(1-\lambda)^3(1+\lambda)\sigma_1^2(\sigma_2^2)^3.$$

Hence,

$$\sigma_B^2 = \sigma_{B_i}^2 = \mathrm{Var}\left[B_i^{(b1)}\right] + \mathrm{Var}\left[B_i^{(b3)}\right] + o(1) = \frac{2}{3}(1-\lambda)^3(1+\lambda)\sigma_1^2\sigma_2^2((\sigma_1^2)^2 + (\sigma_2^2)^2) < \infty,$$

which establishes Proposition A.2.

We are now in a position to conclude the proof of Proposition 2, namely

$$b_{NT}^* = N^{-1}\sum_{i=1}^N \left((\sigma_1^2 + (1-\lambda)\sigma_2^2)^2 \int_0^1 W_i^2 + B_i\right), \text{ where } B_i \sim iid(0, \sigma_B^2). \quad (32)$$

[3]The first and second order moments $\mathrm{E}[\int_0^1 W_{1i}W_{2i}] = 0$ and $\mathrm{Var}[\int_0^1 W_{1i}W_{2i}] = 1/6$ have been established by simulation.

[4]$\mathrm{Var}[\int_0^1 W_{2i}] = 1/3$ has been determined by simulation.

As $N \to \infty$, the law of large numbers yields

$$\begin{aligned} b^*_{NT} &\xrightarrow{p} (\sigma_1^2 + (1-\lambda)\sigma_2^2)^2 E\left[\int_0^1 W^2\right] + \mathrm{E}\left[B\right] \\ &= \frac{1}{2}(\sigma_1^2 + (1-\lambda)\sigma_2^2)^2. \end{aligned} \tag{33}$$

□

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Table 1: DGP1

N	T	CS independence *OLS*	*Rob*	*HS*	OLS^b	Rob^b	HS^b	Common factor model *OLS*	*Rob*	*HS*	OLS^b	Rob^b	HS^b
							Empirical rejection frequencies under H_0						
Constant variance													
10	10	6.9	1.6	5.7	5.2	5.2	5.2	21.1	3.4	5.5	5.3	5.4	5.2
10	50	7.1	5.3	6.9	5.4	5.4	5.5	20.7	7.0	7.2	5.4	5.2	5.2
10	100	6.8	5.8	6.3	5.0	5.0	5.2	21.6	7.6	7.8	5.2	5.4	5.3
50	10	5.7	0.0	5.5	5.2	5.3	5.3	42.7	1.0	5.2	5.1	5.1	5.1
50	50	5.9	1.1	5.6	5.2	5.3	5.1	47.0	5.8	8.0	5.1	5.2	5.4
50	100	5.7	2.6	5.6	4.9	4.8	5.0	47.2	7.1	7.8	5.0	5.3	5.2
50	250	5.7	4.3	5.7	5.2	5.1	5.1	48.1	8.1	8.5	5.7	5.2	5.1
Early negative variance shift													
10	10	2.9	0.2	5.1	5.5	5.7	5.4	12.3	1.3	4.3	5.1	5.3	5.2
10	50	4.2	2.5	6.6	5.2	5.0	4.9	15.8	5.6	7.1	5.0	5.0	5.1
10	100	4.3	3.6	6.2	4.8	5.1	5.0	16.1	6.9	7.4	4.9	5.1	5.1
50	10	1.7	0.0	5.2	5.4	5.5	5.3	29.2	0.2	4.1	5.4	5.3	5.4
50	50	3.1	0.1	5.6	5.2	5.3	5.0	38.9	3.3	6.5	4.9	5.0	4.5
50	100	3.0	0.3	5.5	5.2	4.9	5.0	40.5	5.6	7.5	4.8	4.7	5.3
50	250	3.0	1.1	5.6	5.1	4.9	5.1	41.1	8.3	8.4	5.5	5.4	5.3
Late positive variance shift													
10	10	13.6	4.4	4.0	5.4	5.3	5.5	28.0	7.5	4.0	5.3	5.2	5.3
10	50	13.5	11.0	5.6	4.7	4.6	4.6	33.8	12.7	6.2	4.7	4.9	5.0
10	100	13.3	11.7	6.3	4.8	5.0	4.9	32.1	12.5	6.9	4.7	4.7	5.0
50	10	11.3	0.1	3.0	5.1	4.9	4.5	43.3	3.9	3.4	4.4	4.1	4.7
50	50	12.9	4.7	5.5	5.3	5.3	5.3	51.2	10.4	6.1	5.2	4.6	4.8
50	100	12.8	7.2	5.7	4.9	4.9	5.2	53.6	13.3	7.7	5.1	5.7	5.3
50	250	12.7	10.5	5.9	5.2	5.1	5.1	53.1	13.7	7.9	5.2	5.1	5.3
							Local power						
Constant variance													
50	100	100.	100.	100.	100.	100.	100.	69.0	41.1	38.1	69.6	40.0	37.6
50	250	100.	100.	100.	100.	100.	100.	75.5	41.9	42.7	75.3	42.6	41.4
Early negative variance shift													
50	100	68.5	70.6	65.1	66.4	69.9	64.6	34.3	18.7	15.4	37.2	18.9	15.4
50	250	98.1	98.3	97.8	98.1	98.2	97.8	56.7	25.2	23.5	57.2	24.7	23.0
Late positive variance shift													
50	100	92.3	90.8	90.0	92.2	90.3	89.6	51.2	29.7	26.9	54.0	29.8	26.7
50	250	94.0	92.9	92.2	93.9	92.9	92.0	57.1	31.8	28.0	58.2	32.0	28.2

Notes: Data is generated according to DGP1 and all tests are computed on demeaned data. For further notes see Table 2.

Table 2: Empirical rejection frequencies, DGP2

		CS independence						Common factor model					
N	T	OLS	Rob	HS	OLS^b	Rob^b	HS^b	OLS	Rob	HS	OLS^b	Rob^b	HS^b
Empirical rejection frequencies under H_0													
Constant variance													
10	10	7.3	1.6	5.1	6.3	5.9	4.8	22.2	4.6	5.4	8.7	6.9	5.6
10	50	6.6	4.8	6.2	4.8	4.7	4.9	22.3	6.5	6.4	6.9	5.0	4.5
10	100	6.3	5.4	6.0	4.8	4.7	4.4	23.0	7.3	7.0	6.7	5.1	4.5
50	10	6.4	0.0	4.9	6.9	6.6	4.9	39.4	1.8	4.5	11.5	8.0	4.6
50	50	5.9	1.2	5.9	5.5	5.4	5.4	42.5	5.9	6.8	10.3	5.6	4.6
50	100	5.6	2.7	5.4	5.0	4.9	4.9	42.2	6.4	6.9	10.2	4.5	4.2
50	250	5.4	4.0	5.2	4.9	4.8	4.8	41.9	7.1	7.4	9.8	4.6	4.4
Early negative variance shift													
10	10	9.4	0.8	4.5	12.5	10.5	4.7	20.9	3.90	3.5	14.2	11.0	4.0
10	50	13.8	8.9	9.4	15.7	14.8	7.4	31.3	12.0	8.3	16.7	11.5	6.2
10	100	14.9	11.8	10.4	16.1	15.6	8.1	32.9	14.9	9.4	17.9	12.1	6.7
50	10	12.2	0.0	5.2	18.8	17.5	5.2	38.6	1.1	4.1	20.3	15.6	4.5
50	50	21.2	2.2	15.4	27.8	27.2	14.0	53.2	11.2	10.1	27.9	16.6	7.2
50	100	23.1	7.7	18.1	29.9	29.3	16.2	56.4	16.1	11.7	30.0	15.8	7.9
50	250	23.9	14.9	19.6	30.7	30.2	17.7	57.6	19.6	13.0	31.3	15.5	8.9
Late positive variance shift													
10	10	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0
10	50	0.1	0.1	0.1	0.0	0.0	0.1	1.5	0.4	0.2	0.1	0.1	0.1
10	100	0.1	0.1	0.0	0.0	0.0	0.0	1.3	0.3	0.1	0.0	0.1	0.1
50	10	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.0	0.0	0.0
50	50	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0
50	100	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.0	0.0
50	250	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
Local power													
Constant variance													
50	100	29.1	29.2	29.3	29.5	29.8	28.4	16.0	12.5	11.7	16.2	12.0	12.2
50	250	34.5	34.7	34.4	33.8	33.8	33.8	17.1	12.2	11.8	16.8	11.6	11.9

Notes: OLS, Rob and HS refer to the PURT statistics defined in (9), (10),(11) and OLS^b, Rob^b, HS^b denote the wild bootstrap versions, respectively. Results are based on 5000 replications and 499 bootstrap replications. Data is generated according to DGP2 and all tests are computed on detrended data.

Table 3: Empirical rejection frequencies, DGP3

		CS independence						Common factor model					
N	*T*	*OLS*	*Rob*	*HS*	OLS^b	Rob^b	HS^b	*OLS*	*Rob*	*HS*	OLS^b	Rob^b	HS^b
		Empirical rejection frequencies under H_0											
Constant variance													
10	10	6.9	1.0	4.9	4.7	4.6	4.5	18.6	1.1	3.5	3.8	3.2	3.4
10	50	6.5	4.8	6.3	4.8	4.8	4.8	21.0	5.5	6.2	4.8	3.9	4.0
10	100	7.4	6.4	7.1	5.4	5.4	5.2	22.5	7.2	7.2	5.5	4.8	4.6
50	10	15.1	0.0	12.3	12.3	12.2	11.8	42.5	0.2	3.6	6.9	4.5	3.4
50	50	7.0	1.6	6.8	6.2	6.0	6.0	45.3	4.6	6.5	6.1	4.1	4.4
50	100	6.2	3.1	6.3	5.7	5.5	5.4	46.0	7.0	7.8	6.0	4.6	4.8
50	250	6.2	4.5	6.0	5.4	5.6	5.4	48.5	7.7	8.2	5.6	5.1	5.1
Early negative variance shift													
10	10	5.4	0.5	8.2	8.3	8.3	7.5	14.9	1.2	6.3	7.3	6.6	6.1
10	50	4.3	2.5	6.6	5.3	5.0	5.2	15.3	4.4	5.2	5.1	4.1	3.6
10	100	4.3	3.4	6.8	5.1	4.9	5.3	16.1	6.2	6.5	5.3	4.6	4.4
50	10	7.9	0.0	11.2	10.5	10.4	11.1	35.9	0.3	5.7	9.1	7.7	6.1
50	50	4.5	0.1	8.2	8.2	8.2	7.5	39.4	2.9	5.4	7.1	4.5	3.5
50	100	3.9	0.5	7.2	6.6	6.9	6.5	39.2	5.6	6.5	6.8	5.3	4.2
50	250	3.2	1.3	6.1	5.2	5.3	5.5	41.3	7.4	7.8	6.0	4.8	4.8
Late positive variance shift													
10	10	6.1	0.9	3.5	2.6	2.7	3.9	14.7	0.7	2.5	1.3	1.3	2.8
10	50	13.0	10.0	6.0	4.3	4.3	4.9	29.1	10.6	4.7	4.3	3.7	3.2
10	100	13.8	12.3	6.4	5.1	5.0	4.9	31.0	12.5	6.7	5.5	4.3	4.5
50	10	8.7	0.0	4.4	4.0	3.9	5.2	31.4	0.1	2.3	1.5	1.3	2.5
50	50	18.9	6.2	7.8	7.5	7.5	7.2	51.7	9.7	5.3	7.1	4.7	3.9
50	100	15.7	9.1	6.9	6.4	6.4	6.1	52.2	12.0	6.4	6.8	4.4	4.2
50	250	14.5	11.9	6.7	5.9	5.8	6.0	52.9	12.7	7.4	6.1	5.1	4.4
		Local power											
Constant variance													
50	100	99.7	99.7	99.7	99.7	99.7	99.7	63.6	36.8	34.1	63.7	37.4	33.6
50	250	100.0	100.0	100.0	100.0	100.0	100.0	74.6	42.8	41.4	74.5	41.8	40.2
Early negative variance shift													
50	100	53.9	57.7	51.4	53.6	57.3	50.7	24.5	14.1	13.4	25.8	14.0	13.2
50	250	98.0	97.9	97.5	97.6	97.7	97.3	55.2	25.8	24.6	54.6	26.0	24.1
Late positive variance shift													
50	100	88.8	87.0	86.2	88.1	85.8	84.8	39.5	26.2	26.5	42.2	27.0	26.3
50	250	94.1	93.6	92.8	94.1	93.4	92.7	51.9	29.6	30.1	52.3	29.9	29.2

Notes: Data is generated according to DGP3 and all tests are computed on prewhitened and centered data. For further notes see Table 2.

Table 4: Empirical results

Variable	T	OLS	Rob	HS	OLS^b	Rob^b	HS^b
π	172	-3.52	-2.45	-1.85	-2.03	-1.80	-1.80
		(.000)	(.007)	(.032)	(.002)	(.012)	(.044)
R	179	-4.22	-2.60	-1.67	-2.43	-1.98	-1.81
		(.000)	(.005)	(.048)	(.000)	(.013)	(.070)
r	176	-4.69	-3.49	-2.83	-1.96	-1.81	-1.83
		(.000)	(.000)	(.002)	(.000)	(.002)	(.002)

Notes: $\widetilde{T}$ denotes the number of included time series observation in the balanced panels. OLS, Rob, and HS refer to the PURT statistics defined in (9), (10),(11) and OLS^b, Rob^b, HS^b denote 5% wild bootstrap critical values. Numbers in parentheses are $p-$values. Bootstrap critical values are obtained by 4999 bootstrap replications.

Table 5: Interest rates, definitions

Country	Label	Interest rate
Belgium	BEL	Treasury paper
Canada	CAN	Treasury Bill rate
France	FRA	Government Bond yield
Germany	GER	Call money rate
Italy	ITA	Government Bond yield medium-term
Japan	JAP	Lending rate
Netherlands	NED	Government Bond yield
United Kingdom	UKD	Treasury Bill rate
United States	USA	Treasury Bill Rate

Figure 3: Nominal Interest Rates and Inflation rates, 1961Q2 - 2007Q2