Structural Breaks and Unit Root Tests for Short Panels

Elias Tzavalis*

Department of Economics

Athens University of Economics & Business

Athens 104 34, Greece

(email: E.Tzavalis@aueb.gr)

This version December 2008

Abstract

In this paper we suggest panel data unit root tests which allow for a potential structural break in the individual effects and/or the trends of each series of the panel, assuming that the time-dimension of the panel, T, is fixed. The proposed test statistics consider for the case that the break point is known and for the case that it is unknown. Monte Carlo evidence suggests that they have size which is very close to the nominal five percent level and power which is analogues to that of Harris and Tzavalis (1999) test statistics, which do not allow for a structural break.

JEL classification: C22, C23

Keywords: Panel data; Unit roots; Structural Breaks; Sequential Unit Root tests; Central Limit Theorem; economic convergence hypothesis

*This research was funded by the ESRC (UK) under grant R000239139

1 Introduction

The AR(1) model for short panels has been extensively used in the literature in studying the dynamic behaviour of many economic series across different cross section units, when the time dimension of the panel is considered as fixed (small) [see Baltazi and Kao (2000), and Arelano and Honore (2002), *inter alia*]. Of particular interest is to use the model in examining whether many economic series contain a unit root in their autoregressive component, across all the sectional units of the panel [see Levin, Lin and Chu (2002), for a recent survey of these type of tests]. Recent applications of unit roots tests for panel data include: the examination of the economic growth convergence hypothesis [see de la Fuente (1997), for a survey], the hypothesis that stock prices and dividends follow the random walk model [see Lo and MacKinlay (1995), *inter alia*] and, finally, the investigation of the validity of the purchasing power parity hypothesis [see Culver and Papell (1999), *inter alia*].

Motivated by recent studies suggesting that evidence of a unit root in the many economic series may be attributed to a structural break in the deterministic components of the series [see Perron (1999)], in this paper we introduce short panel data unit roots test statistics allowing for a potential structural break under the alternative hypothesis of stationary.[1]. The paper proposes test statistics for two cases: (i) when the break occurs in the individual effects and (ii) when it occurs in the deterministic trends of the panel, or in both the deterministic trends and the individual effects. For both cases, the timing of the structural break is assumed to be common for all the series of the panel. Regarding the dimensionality issue of the panel, the proposed test statistics are in line with Harris and Tzavalis' (1999) tests. They assume that the time dimension (T) of the panel is fixed, while the cross-section dimension (N) grows large. These

[1] As first pointed out by Perron (1989) for a single time series, ignoring a structural break in the deterministic component of a series is expected to bias the unit root tests towards falsely accepting the null hypothesis of a unit root. For panel data, this has been shown by Carrion-i-Silvestre, Barrio-Castro and Lopez-Bazo (2001), who considered the simpler, canonical case of the AR (1) panel data model with individual effects treating the time point of the break as known.

tests are appropriate for panels where T is smaller than N, as it is typical in many micro-economic (or finance) studies.

The proposed test statistics are based on the pooled least squares (LS) estimator of the autoregressive coefficient of the panel data model corrected for the inconsistency of the LS estimator [see Kiviet (1995), and Harris and Tzavalis (1999)]. This inconsistency arises from two sources: the presence of individual effects and/or trends in the AR(1) auxiliary panel data regression, and the allowance for a potential structural break in the deterministic components of the panel. Employing the corrected for its inconsistency LS estimator in drawing inference about unit roots in panel data models has the following two main advantages: first, it can lead to test statistics which are invariant to the initial conditions and/or the individual effects of the panel data model; second, it enables us to identify the autoregressive coefficient of the AR(1) panel data model under both the null and alternative hypothesis. In addition to the above, the only distribution assumption that is needed in deriving the limiting distribution of the test statistics based on the corrected for its inconsistency LS estimator is that the fourth-moment of the disturbance terms should exist.[2]

The paper considers two categories of tests: (i) when the break point is known and (ii) when it is unknown. The second category of the tests may be proved very useful in cases where it is difficult to single out any major exogenous event that could have caused a common change in any of the deterministic components of the series of the panel. When the time-point of the break is assumed to be known, we show that the limiting distribution of the proposed test statistics for panel data models with individual effects, and individual effects and/or trends is normal with variance which depends on the fraction the sample that the break occurs and the time dimension (T) of the panel. For this case, we employ the moments of the limiting distribution of the test statistic for

[2] This is in contrast to conditional Maximum Likelihood based panel data unit root inferece procedures suggested in the literature in order to circuvmnet the problem panel data initial observations. These procedures require the disturbance terms of the panel to be normally distributed.

the AR(1) model with individual effects in order to analyse the consequences of ignoring a potential structural break in the deterministic component of the series of the panel in drawing inference about unit roots. Our analysis shows that the tendency of the panel unit root tests to falsely accept the null hypothesis may be attributed to the smaller variance of the limiting distribution of the test statistics that ignore a potential break.

When the break-point is unknown, we suggest a sequential procedure to test for the null hypothesis of unit roots for the above panel data models. This is based on the minimum values of the one-sided test statistics for a known date break which are sequentially computed, for each point of the sample.[3] The distribution of the minimum values of the test statistics can be calculated by that of the minimum values of a fixed number of correlated normal variables. We derive the correlation matrix of these variables and provide critical values of the distribution of the minimum test statistics for both panel data models: with individual effects, and individual effects and trends by simulation methods. This is done for different values of T, after trimming the initial and final parts of the series of the panel, assuming that the disturbance terms are normally distributed.

To examine the small-N sample performance of the tests, we run some Monte Carlo simulation experiments. The results of these experiments indicate that both testing procedures, considering a known date break or not, have size which is very close to the 5% level and power which increases both with N and T, but grows faster with T. This performance of the tests is analogous to that of Harris and Tzavalis tests, which do not consider the case of a structural break.

The paper is organized as follows. The limiting distributions of the test statistics for known or unknown break-point are derived in Section 2. In Section 3, we present the results of the Monte Carlo study. In Section 4 we present an application of the sequential testing procedure in order to reexamine whether level of the real per capita income of each country is mean reverting to its steady

[3] Perron and Vogelsang (1992), and Zivot and Andrews (1992) adopted this procedure in testing for a unit root in single time series.

state level, as the economic growth convergence hypothesis asserts. Conclusions are summarized in Section 5.

2 The test statistics and their limiting distribution

2.1 The date of the break point is known

Consider the following first-order autoregressive panel data model, denoted AR(1),

$$y_i = \phi y_{i,-1} + X_i^{(\lambda)} \gamma_i^{(\lambda)} + u_i, \quad i = 1, 2, ..., N \tag{1}$$

where $y_i = (y_{i1}, ..., y_{iT})'$ is a $(TX1)$ dimension vector of time series observations of the dependent variable of each cross-section unit of the panel, i, $y_{i,-1} = (y_{i0}, ..., y_{iT-1})'$ is the vector y_i lagged one time period back, $u_i = (u_1, ..., u_T)$ is a $(TX1)$ dimension vector of disturbance terms and $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$ is a $(TX4)$ dimension matrix whose columns vectors are appropriately designed so that to capture a common, across i, structural break in the vector of coefficients $\gamma_i^{(\lambda)} = (a_i^{(\lambda)}, a_i^{(1-\lambda)}, \beta_i^{(\lambda)}, \beta_i^{(1-\lambda)})'$ of the deterministic components of the panel, i.e. the individual effects and trends, at the time point $T_0 = int[\lambda T]$, where $\lambda \in (0, 1)$ denotes the break fraction of T and *int* declares integer number. Specifically, the columns vectors of $X_i^{(\lambda)}$ are defined as follows: $e_t^{(\lambda)} = 1$ if $t \leq T_0$ and 0 otherwise, and $e_t^{(1-\lambda)} = 1$ if $t > T_0$ and 0 otherwise; $\tau^{(\lambda)} = t$ if $t \leq T_0$ and 0 otherwise, where t denotes the deterministic trend, while $\tau^{(1-\lambda)} = t$ if $t > T_0$ and 0 otherwise.

For model (1), the null hypothesis is that $\phi = 1$, i.e. there is a unit root in the autoregressive component of the model, and that there is no break in the deterministic components of the panel. Under the alternative hypothesis, it is assumed that $|\phi| < 1$ and there is a break in the individual effects ...

. The panel data auxiliary regression model given by equation (1) is appropriate to test the null hypothesis that each series of the panel follows a random walk with drift against the alternative hypothesis that each series is stationary around a broken trend. Note that under the null hypothesis the drift parameters can also have a break, i.e. $a_i^{(\lambda)} \neq a_i^{(1-\lambda)} \neq 0$. As we show in the appendix (see proof of Theorem 1), the inclusion of the deterministic trends in model (1) with the same break point as the individual effects renders the LS estimator of ϕ under the null hypothesis invariant to the individual effects, independently on whether they are subject to a structural break, or not. Finally, note that the auxiliary regression panel data model (1) can nest the model where $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$ and $\gamma_i^{(\lambda)} = (a_i^{(\lambda)}, a_i^{(1-\lambda)})'$. This model (1) is appropriate for testing for the null hypothesis that each series of the panel follows a driftless random walk against the alternative hypothesis that each series is stationary around a broken mean.

Test statistics of the null hypothesis $\phi = 1$ based on the above alternative specifications of the deterministic components of model (1), i.e. $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$ and $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$, can be derived based on the pooled LS estimator of the autoregressive coefficient, ϕ. The limiting distributions of these test statistics can be derived by noticing that the pooled LS estimator of ϕ, denoted $\hat{\phi}$, under the hypothesis $\phi = 1$ satisfies

$$\hat{\phi} - 1 = \left[\sum_{i=1}^{N} y'_{i,-1} Q^{(\lambda)} y_{i,-1}\right]^{-1} \left[\sum_{i=1}^{N} y'_{i,-1} Q^{(\lambda)} u_i\right], \tag{2}$$

where $Q^{(\lambda)} = \left[I - X^{(\lambda)} \left(X^{(\lambda)\prime} X^{(\lambda)}\right)^{-1} X^{(\lambda)\prime}\right]$ is the (TXT) "within" transformation matrix of the series of the panel [Baltagi (1995), *inter alia*]. Since $\hat{\phi}$ is an inconsistent estimator of $\phi = 1$, we need to correct the limiting distribution of $\hat{\phi} - 1$ for the inconsistency of the estimator in constructing test statistics for the hypothesis $\phi = 1$ based on LS estimators. The inconsistency of $\hat{\phi}$ arises from the within transformation of the data. This now allows for a potential break in the deterministic components of model (1).

The limiting distribution of $\hat{\phi} - 1$ corrected for the inconsistency of $\hat{\phi}$ can be derived by making the following assumption about the nature of the sequence of the disturbance terms $\{u_{it}\}$.

Assumption 1: $\{u_{it}\}$ is a sequence of independently and identically distributed (IID) random variables with $E(u_{it}) = 0$, $Var(u_{it}) = \sigma_u^2$, and $E(u_{it}^4) = k + 3\sigma_u^4$, $\forall\ i \in \{1, 2, ..., N\}$ and $\forall\ t \in \{1, 2, ..., T\}$, where $k < \infty$.

Assumption 1 enable us to derive the limiting distribution of the test statistic for the hypothesis $\phi = 1$ using classical (standard) asymptotic theory results, assuming that N goes to infinity and T is fixed. The condition $k < \infty$ of the assumption implies that the fourth moment of the disturbance terms, u_{it}, exists. This condition enables us to apply the law of large numbers (LLN) and the central limit theorem (CLT) in driving the limiting distribution of $\hat{\phi}-1$ corrected for the inconsistency of $\hat{\phi}$, without making any distributional assumptions about the sequence of the disturbance terms. In so doing, note that we do not need to make any assumption about the nature of the initial observations y_{io}, and the individual effects $a_i^{(\lambda)}$ and $a_i^{(1-\lambda)}$ of the panel. Under the null hypothesis, the test statistics that we propose are invariant to these nuisance parameters. This is achieved by including individual effects and trends in the auxiliary panel data regression model (1).[4]

The next theorem presents the limiting distribution of unit root test statistics for the alternative specifications of the deterministic components of panel data model (1), mentioned before.

Theorem 1 *Let the sequence $\{y_{it}\}$ be generated according to model (1), Assumption 1 hold and the date of the structural break, T_0, be known. Then, under the null hypothesis $\phi = 1$, as $N \to \infty$*

$$Z(\lambda, T) \equiv \sqrt{N}(\hat{\phi} - 1 - B(\lambda, T)) \xrightarrow{L} N(0, C(k, \sigma_u^2, \lambda, T)) \qquad (3)$$

where

[4] When $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$, the suggested test statistic renders invariant to the initial conditions of the panel by including only individual effects in the panel data auxiliary regression.

$$
\begin{aligned}
B(\lambda,T) &= \underset{N\rightarrow\infty}{p\lim}(\hat{\phi}-1) \\
&= tr[\Lambda' Q^{(\lambda)}]\{tr(\Lambda' Q^{(\lambda)}\Lambda)\}^{-1},
\end{aligned}
$$

and

$$
C(k,\sigma_u^2,\lambda,T)=\left\{k\sum_{j=1}^{T}a_{jj}^{(\lambda)^2}+2\sigma_u^4 tr(A^{(\lambda)^2})\right\}\left\{\sigma_u^2 tr(\Lambda' Q^{(\lambda)}\Lambda)\right\}^{-2},
$$

where Λ *is a* (TXT) *matrix defined as* $\Lambda_{r,c}=1$, *if* $r>c$ *and* 0 *otherwise,* $A^{(\lambda)}\equiv[a_{ij}]$ *is a* (TXT) *dimension symmetric matrix, defined as* $A^{(\lambda)}=\frac{1}{2}(\Lambda' Q^{(\lambda)}+Q^{(\lambda)}\Lambda)-B(\lambda,T)(\Lambda' Q^{(\lambda)}\Lambda)$ *and* '$\xrightarrow{L}$' *signifies convergence in distribution.*

The proof of the theorem is given in the appendix. Below, we make some remarks which highlight some interesting special cases of the results of the theorem and discuss how to implement the test statistics implied by the theorem.

Remark 2 *When* $X_i^{(\lambda)}=\left(e^{(\lambda)},e^{(1-\lambda)}\right)$, *Theorem 1 gives a test statistic which is appropriate for the special case where the panel model (1) under the null hypothesis consists of driftless random walks.*

Remark 3 *When* u_{it} *are* $NIID(0,\sigma_u^2)$, $k=0$. *Then, the variance of limiting distribution of* $Z(\lambda,T)$ *is given by* $\left\{2tr(A^{(\lambda)^2})\right\}\left\{tr(\Lambda' Q^{(\lambda)}\Lambda)\right\}^{-2}$.

The results of Theorem 1 show that the inconsistency of the pooled LS estimator $\hat{\phi}$, given by $B(\lambda,T)$, is a deterministic function of the fraction of the structural break of the sample, λ, and the time dimension of the panel, T. Subtracting $B(\lambda,T)$ from $\hat{\phi}-1$ leads to test statistics of the hypothesis $\phi=1$ for the cases of model (1) that $X_i^{(\lambda)}=\left(e^{(\lambda)},e^{(1-\lambda)}\right)$ and $X_i^{(\lambda)}=\left(e^{(\lambda)},e^{(1-\lambda)},\tau^{(\lambda)},\tau^{(1-\lambda)}\right)$. These statistics are based on a correction of the limiting distribution of $\hat{\phi}-1$ for the inconsistency of the LS estimator $\hat{\phi}$. Given consistent estimates of the nuisance parameters of the variance function $C(k,\sigma_u^2,\lambda,T)$, k and σ_u^2, the test statistics given by the theorem appropriately scaled by $C(k,\sigma_u^2,\lambda,T)$ can be readily used in practice to conduct inference about unit roots based on the tables of the standard normal distribution.[5] Remark 3 shows that the test statistics proposed by the theorem become invariant to the nuisance parameter σ_u^2 only under the normality assumption of the disturbance terms.

2.2 The effects of structural breaks on test for unit roots in dynamic panel data models

The results of Theorem 1 can be used to analyse the consequences of ignoring a structural break in the deterministic components of the panel on drawing inference about unit roots based on dynamic, autoregressive panel data models with deterministic components. The functional form of $Z(\lambda,T)$ shows that there will be two sources of biases of the panel unit root test statistics when ignoring a potential break. The first will come from the term correcting for the inconsistency of the LS estimator, i.e. $B(\lambda,T)$, while the second from the variance of the limiting distribution of the test statistic, given by $C(k,\sigma_u^2,\lambda,T)$. Both of these terms depend on the fraction of the structural break of the sample, λ. The effects of λ on the unit root tests can be rigorously studied by investigating the sensitivity of $B(\lambda,T)$ and $C(k,\sigma_u^2,\lambda,T)$ to changes in the values of λ. To this end, in the next corollary we derive analytic expressions of $B(\lambda,T)$ and $C(k,\sigma_u^2,\lambda,T)$ for the special, simpler case of model (1) where $X_i^{(\lambda)}=\left(e^{(\lambda)},e^{(1-\lambda)}\right)$ assuming that u_{it} are normally distributed.

Corollary 4 *Let the sequence of the disturbance terms,* $\{u_{it}\}$*, be normally distributed and the matrix of the deterministic components* $X_i^{(\lambda)}$ *be specified as* $X_i^{(\lambda)}=\left(e^{(\lambda)},e^{(1-\lambda)}\right)$*. Then, the limiting distribution of the test statistic given by Theorem 1 is given by*

$$z(\lambda,T)\equiv\sqrt{N}(\hat{\phi}-1-b(\lambda,T))\xrightarrow{L}N(0,c(\lambda,T)), \tag{4}$$

[5] Consistent estimates of k and σ_u^2 can be derived based on GMM estimates of the fourth and second moments of the first differences of the panel data y_{it}.

where

$$b(\lambda, T) = -3(T-2)\{\delta_1(\lambda)T^2 + \delta_0(\lambda)\}^{-1},$$

and

$$\begin{aligned} c(\lambda, T) &= \frac{3}{5}\{\pi_6(\lambda)T^6 + \pi_5(\lambda)T^5 + \pi_4(\lambda)T^4 + \pi_3(\lambda)T^3 + \pi_2(\lambda)T^2 \\ &\quad + \pi_1(\lambda)T + \pi_0(\lambda)\}\{\delta_1(\lambda)T^2 + \delta_0(\lambda)\}^{-4}, \end{aligned}$$

where the polynomial functions δ's and π's are defined in the appendix.

The proof of the corollary is given in the appendix. Some remarks on the results of the corollary are given below.

Remark 5 *For $\lambda = 0$, the test statistic given by Corollary 2 leads to the test statistic derived by Harris and Tzavalis (1999) for the case that $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$.*

Remark 6 *For sufficiently large T, Corollary 2 implies that*

$$z(\lambda) \equiv T\sqrt{N}\left(\hat{\phi} - 1 - \frac{-3}{\delta_1(\lambda)}\right) \xrightarrow{L} N\left(0, \frac{3\pi_6(\lambda)}{5\delta_1(\lambda)^4}\right). \tag{5}$$

The test statistic given by (5) is derived by taking limits of $b(\lambda, T)$ and $c(\lambda, T)$ for T going to infinity and scaling appropriately by T.

Figures 2.1 and 2.2 graphically present the values of the inconsistency correction term $b(\lambda, T)$ and the variance $c(\lambda, T)$ of the test statistic $z(\lambda, T)$, given by Corollary 4, with respect to T (see horizontal axis). This is done for the following set of values $\lambda = \{0.0, 0.5, 0.8\}$. To make interesting comparisons with the case that T is sufficiently large (see Remark 6), the values of $b(\lambda, T)$ and $c(\lambda, T)$ have been appropriately scaled by T.

The graphs lead to the following conclusions. First, both the values of the inconsistency correction term $b(\lambda, T)$ and the variance $c(\lambda, T)$ are smaller in

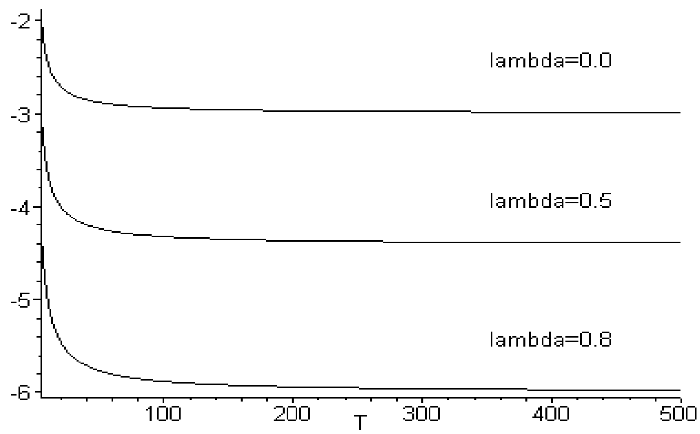


Figure 1: $Tb(\lambda, T)$

magnitude when the statistic allows for a break. This is true independently on whether T is large (see horizontal lines) or fixed. These results imply that there will be two counteracting effects on the size of the panel unit root test statistic $z(\lambda, T)$ which does not allow for a structural break. The smaller magnitude of $b(\lambda, T)$ will tend to increase the size of the test by shifting its whole distribution to the left of the empirical distribution of the test allowing for the break, while the smaller variance will tend to decrease the size of the test by drawing in the left tail of its distribution. If the first of these effects, referred to as mean effect, dominates the second, then the test will be oversized. If the second effect, referred to as variance effect, is dominant, then the test will be undersized, and thus will have low power to reject the null hypothesis against its alternative hypothesis with a potential break. The above analysis indicates that the failure of the panel unit root statistics to reject the null hypothesis when ignoring a potential break in the individual effects of the panel [see Carrion-i-Silvestre, Del Barrio-Castro and Lopez-Bazo (2001), for a simulation study] may be attributed to a downwards biased estimate of the variance of the test statistic which ignores a potential break.

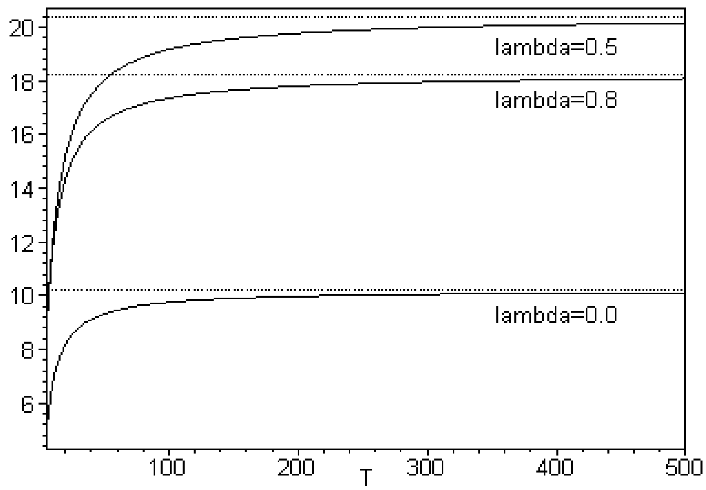


Figure 2: $T^2 c(\lambda, T)$ vs $\frac{3\pi_6(\lambda)}{5\delta_1(\lambda)^4}$ (horizontal lines)

The second conclusion which can be drawn from the graphs is that a substantial number of time series observations is required in order to apply the large-T test statistic implied by the corollary, denoted $z(\lambda)$, instead of the fixed-T test statistic $z(\lambda, T)$, in the presence of a break, otherwise serious size distortions will occur in the same way as in the case of ignoring a potential break, analysed above. Indeed, the graphs show that the number of the time series observations of the panel which are required in order $b(\lambda, T)$ and $c(\lambda, T)$ to reach their $T-$asymptotes, derived for T going to infinity, varies with λ. Note that this number reaches its maximum value when $\lambda = 0.5$, i.e. the break point is in the middle of the time dimension of the panel. For this case we need panels with very high time dimension (e.g. $T > 300$) for the inconsistency correction term $b(\lambda, T)$ and the variance of the limiting distribution of the statistic $c(\lambda, T)$ to reach their asymptotic limits, over T. This happens because, when $\lambda = 0.5$, the shift in the level of the mean of the series of the panel exhibits its longest persistence. The above analysis indicates that large-T based panel data unit root test statistics may require a substantial number of time series observations in order to be able to distinguish the persistency of the shift in the deterministic component from that of the disturbance terms.

2.3 The date of the break point is unknown

The results of Theorem 1 are based on the assumption that the break point is known. In this subsection, we relax this assumption and propose test statistics for model (1), with $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$ and $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$, which allow for a structural break at an unknown date. As in Perron and Vogelsang (1992), and Zivot and Andrews (1992), we view the selection of the break point as the outcome of minimizing the test statistics given by Theorem 1 over all possible break points of the sample, after trimming the initial and final parts of each series of the panel model (1).[6] That is, the minimum values of the test statistics $Z(\lambda, T)$, over all $\lambda \in (0,1)$, are chosen to give the least favorable result for the null hypothesis $\phi = 1$.

Let $\hat{\lambda}_{\inf}$ denote the break point at which the minimum value of $Z(\lambda, T)$, over all $\lambda \in (0,1)$, are obtained. Then, the null hypothesis will be rejected if

$$\inf_{\lambda \in (0,1)} Z(\lambda, T) < c_{\inf},$$

where $c_{\inf}$ denotes the size a left-tail critical value from the distribution of the minimum values of $Z(\lambda, T)$, denoted $\inf_{\lambda \in (0,1)} Z(\lambda, T)$. The following theorem enable us to calculate the distribution of $\inf_{\lambda \in (0,1)} Z(\lambda, T)$.

Theorem 7 *Let Assumption 1 hold and assume that the date of the break point be unknown. Then, as* $N \to \infty$*,*

$$\inf_{\lambda \in (0,1)} Z(\lambda, T) \xrightarrow{d} \inf_{\lambda \in (0,1)} N(0, R) \tag{6}$$

where $R \equiv [r_{\lambda s}]$ *is the correlation matrix of the test statistics* $Z(\lambda, T)$*, for all* $\lambda \in (0,1)$*, with elements given by* $r_{\lambda s}$

[6] For trimming the initial and final parts of the series of the panel, note that $\lambda \equiv \frac{T_0}{T}$ should range from $\frac{1}{T}$ to $\lambda = \frac{T-1}{T}$ for model (1) without the trends, while for model $M2$ it ranges from $\frac{2}{T}$ to $\lambda = \frac{T-2}{T}$ for model (1) with the trends. For the later model, note that the individual effects and the deterministic trends are not identified when $\lambda = \frac{1}{T}$ and $\lambda = \frac{T-1}{T}$.

$$r_{\lambda s} = \frac{k\sum_{j=1}^{T} a_{jj}^{(\lambda)} a_{jj}^{(s)} + 2\sigma_u^4 tr(A^{(\lambda)}A^{(s)})}{\left\{k\sum_{j=1}^{T} a_{jj}^{(\lambda)^2} + 2\sigma_u^4 tr(A^{(\lambda)^2})\right\}^{1/2} \left\{k\sum_{j=1}^{T} a_{jj}^{(s)^2} + 2\sigma_u^4 tr(A^{(s)^2})\right\}^{1/2}},$$

for $\lambda, s \in (0,1)$.

Proof: The result of Theorem 3 follows immediately from the result of Theorem 1 using the continuous mapping theorem. The functional form of the correlation coefficients $r_{\lambda s}$ of $Z(\lambda, T)$, for $\lambda, s \in (0,1)$, can be derived using $E(\xi_i^{(\lambda)}\xi_i^{(s)}) = k\sum_{j=1}^{T} a_{jj}^{(\lambda)} a_{jj}^{(s)} + 2\sigma_u^4 tr(A^{(\lambda)}A^{(s)})$, where $\xi_i^{(j)} = u_i' A^{(j)} u_i$ is defined in the Appendix (see proof of Theorem 1).

Remark 8 *When* u_{it} *is* $NIID(0, \sigma_u^2)$*, then* $k = 0$ *and* $r_{\lambda s}$ *are given by* $r_{\lambda s} = \frac{tr(A^{(\lambda)}A^{(s)})}{\left\{tr(A^{(\lambda)^2})\right\}^{1/2}\left\{tr(A^{(s)^2})\right\}^{1/2}}$.

The result of Theorem 7 shows that the critical values $c_{\inf}$ of the limiting distribution of the statistics $\inf_{\lambda \in (0,1)} Z(\lambda, T)$ can be calculated by those of the minimum values of a fixed number of correlated normal variables with correlation matrix R. In the case that u_{it} are normally distributed, Remark 8 shows that the critical values $c_{\inf}$ become invariant to the nuisance parameter σ_u^2.

Table 1: Critical Values of $\inf_{\lambda \in (0,1)} N(0, R)$

T	10	15	25	50	10	15	25	50
		A				B		
1%	−2.91	−2.95	−2.98	−3.05	−2.92	−2.97	−3.04	−3.10
5%	−2.15	−2.33	−2.37	−2.43	−2.31	−2.38	−2.43	−2.49
10%	−1.83	−2.00	−2.04	−2.10	−1.99	−2.07	−2.11	−2.16

Notes: Panel A of the table presents the critical values $c_{\inf}$ for the special case that model (1) contains only the individual effects in its deterministic components, i.e. $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$, while Panel B of the table presents the critical values for the full specification of the model, which contains both the individual effects and trends, i.e. $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$.

In Table 1, we present critical values of $\inf_{\lambda \in (0,1)} N(0, R)$ at 1%, 5% and 10% significance levels, and for different values of T, assuming that u_{it} is $NIID(0, \sigma_u^2)$.[7] These are calculated from 100000 Monte Carlo experiments as follows. For each replication, we generated a vector of observations from a multivariate normal distribution of dimension T minus the trimming points of the sample with correlation matrix R, defined in Remark 4. We then sorted the vector of observations in order and we selected the minimum value. The critical values reported in the table correspond to 1%, 5% and 10% percentile of the sorted vector of the 100000 replicated minimum values. Not surprisingly, these values are well below the left-tail critical values of the normal distribution, at the corresponding significance levels, and deviate more from them as T increases.

3 Simulation results

In this section we explore the finite sample performance of the test statistics suggested in the previous section by conducting 5000 Monte Carlo experiments. This is done for different combinations of N and T. In each experiment we assume that the disturbance terms, u_{it}, are generated as $u_{it} \sim NIID(0, 1)$.[8]

Tables 2(a)-(b) report the nominal size at a level of significance 5% and the size-adjusted power of the test statistics which consider the break point as known. In particular, Table 2(a) reports the results of the test statistic for the special case that model (1) contains only the individual effects in its deterministic part, i.e. $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$, while Table 2(b) for the full specification of the model (1), where $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$. To calculate the power of the test statistics, we generated the panel data according to the following two alternative hypotheses: $\phi = \{0.95, 0.90\}$. The initial observations of the panel, y_{i0}, are set equal to zero for both of the above specifications of model (1), as the test statistics are invariant to y_{i0}.

[7] A Rats programme calculating the critical values $c_{\inf}$ is available upon request.

[8] Note that we only consider cases of u_{it} and a_i with unit variance, as our test statistics under the null hypothesis are invariant to the variance σ_u^2 when $u_{it} \sim NIID$, and to the individual effects, a_i.

In the case that $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$, we assume that the structural break in the individual effects occurs both under the null and alternative hypotheses. In particular, we generated the deterministic components of the panel as: $a_i^{(\lambda)} = 0.0$ and $a_i^{(1-\lambda)} = 0.5 + \mathrm{a}_i$, where $\mathrm{a}_i \sim NIID(0,1)$, and $\beta_i^{(\lambda)} = 0$ and $\beta_i^{(1-\lambda)} = (1-\phi)a_i^{(1-\lambda)}$. This is done in order to investigate whether the test statistic has the power to distinguish between panels consisting of random walks with broken drift parameters and panels consisting of stationary series around broken trends and individual effects.

The results of Tables 2(a)-(b) clearly indicate that the test statistics given by Theorem 1 have a size very close to the 5% level in finite samples. This is true even for very small values of N, such as $N = 25$. The power performance of the tests is analogous to that of the fixed-T panel unit root tests of Harris and Tzavalis (1999), who considered the case of no structural break. The power increases as both N and T increases, and grows faster with T rather than N. Consistent also with the single time series tests, the power performance reduces when $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$, i.e. the panel includes both the individual effects and trends in its deterministic part. For this case, we found that one needs panels with $T > 25$ to achieve good performance of the test statistic, as for the case that $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$.

Table 3 reports the results of the size and the size adjusted powers of the sequential test statistics which treat the break point as unknown. Panel A of the table presents the results for the case that $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$,while Panel B for the case that $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$. The series of the panel for this simulation experiment were generated in the same way as with the previous one, which treats the break point as known. To calculate the power of the tests we assumed that under the alternative, there is a break at each possible time-point of the sample, sequentially searched for a break. To calculate the size of the tests we used the critical values of the tests at 5% significance level reported in Table 1.

The results of the table clearly show that the performance of the test statistics for the case that the break point is unknown is the same with that of the known-break case. This is true for both test statistics considered, i.e. for $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$ and $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$. These results support the view that Harris and Tzavalis' (1999) tests perform equally well for the case that they are adjusted for a structural break in any of the deterministic components of the series of the panel.

4 Empirical Application

As an empirical application of the tests, in this section we reexamine whether the level of the real per capita income of each country is mean reverting to its steady state, as the economic convergence hypothesis asserts [see Mankiew, Romer and Weil (1992), *inter alia*]. This implication of the economic convergence hypothesis is known as $\beta-$convergence. In particular, we employ our sequential test statistics for the specification of model (1) which contains individual effects in its deterministic part, i.e. $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$, in order investigate whether a unit root in per capita income, which implies economic divergence, can be rejected in favour of the alternative hypothesis of $\beta-$convergence. This specification of the panel data model (1) is often used in practice [see Islam (1995), and Caselli, Esquivel and Lefort (1996), *inter alia*] to test for the $\beta-$convergence hypothesis because it has more power to reject the divergence hypothesis against $\beta-$convergence, compared with the single time series based unit root tests [see Bernard and Durlauf (1994)] or the cross section based tests suggested by Barro and Sala-i-Martin (1995). This happens because the panel data based tests for unit roots have better power to distinguish between the null hypothesis of unit roots and its alternative of stationarity, as they can exploit both cross section and time series information of the data. Note that the sequential panel data unit root test statistic which we employ in this paper in order to reexamine the $\beta-$convergence hypothesis has also more power to reject the null hypothesis of divergence against its alternative of stationarity when there is a permanent shift in the level of the per capita income of each series of the panel, across all countries.

We carry out our sequential unit root test $\inf_{\lambda \in (0,1)} Z(\lambda, T)$, when $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$, for two different groups of countries: (i) the Non-oil countries ($N = 89$) and (ii) the OECD countries ($N = 22$). In order to mitigate the possible effects of cross section correlation on the results of the tests, time dummies have been included in the auxiliary regression model (1), as suggested by O'Connel (1998). To see if allowing for a structural break in the panel unit root tests makes any difference in drawing inference about the economic convergence/divergence hypothesis, we have also conducted Harris and Tzavalis' (1999) tests, which do not allow for a break. We found that these tests can not reject the economic divergence hypothesis for both groups of countries considered.[9]

The values of the sequential statistic, over all points of the sample, are graphically presented in Figures 3(a)-(b); Figure 3(a) presents the results of the statistic for the group of the Non-oil countries, while Figure 3(b) presents the results for the OECD countries. The graphs of the figures clearly indicate that the null hypothesis can be rejected only for the group of the OECD countries when considering for a break. For this group of countries, we found that the value of the test statistic is $\inf_{\lambda \in (0,1)} Z(\lambda, T) = -4.10$, which is clearly smaller than its critical value at 5% significance level, implied by the critical values of Table 1. The break point is found to occur in year 1978, just before the second oil-crisis in year 1979. For the group of the Non-oil countries, there is no evidence of convergence.

Summing up, the results of this section support the view that evidence of economic divergence found by many empirical economic growth studies even for groups of countries with the same level of economic convergence may be attributed to the existence of a structural break in the steady state of the per capita income, across countries. They also indicate that not all the groups of

[9]The values of Harris and Tzavalis' test statistics, denoted HT, are found to be: HT=4.22 for the group of the OECD countries and HT=8.99 for the group of the Noil-oil countries.

countries seems to convergence to their steady state real per capita income.

5 Conclusions

In this paper we proposed panel data unit root testing procedures that allow for a structural break in the individual effects and/or trends of the panel, assuming that the time dimension of the panel is fixed. The test statistics allow for a break point at either a known or unknown date. When the break point is considered as known, we show that the test statistics have normal limiting distributions whose variance depend on the fraction of the sample that the break occurs. When the break is considered as unknown, we suggested a sequential testing procedure of the null hypothesis of unit roots. This entails in computing the test statistics for known break point at each possible break point of the sample, and then selecting the test statistics with the minimum values to test the null hypothesis. The minimum values of the sequential test statistics have a distribution whose critical values can be tabulated by that of the minimum values of a fixed number of correlated normal variables, after trimming for the initial and final time-points of the sample. Critical values of these distributions have been tabulated based on Monte Carlo simulations.

To evaluate the finite sample performance of the tests statistics we run Monte Carlo simulations. We found that both categories of the test statistics, with known and/or unknown break point, have empirical size which is very close to the 5% and power which increases with both dimensions of the panel, but faster with the time-dimension. As an empirical illustration, we employed the test statistic for the panel data model with individual effects in its deterministic component under the alternative hypothesis in order to investigate whether evidence of economic divergence across countries can be attributed to a possible structural break in the steady state of the per capita income of each country, which is reflected in the individual effects of the panel. We found evidence of such a break in year 1978 for the group of the OECD countries.

A Appendix

In this appendix we present the proofs of the main theoretical results of the paper.

Proof of Theorem 1: To derive the limiting distribution of the test statistic, we proceed into stages. We first show that the pooled LS estimator, $\hat{\phi}$, is inconsistent, as $N \to \infty$. We then construct a normalised statistic based on $\hat{\phi}$ corrected for its inconsistency, and derive the limiting distribution under the null hypothesis of $\hat{\phi}$, as $N \to \infty$.

Decompose the vector $y_{i,-1}$ under the null hypothesis as

$$y_{i,-1} = ey_{i0} + \Lambda' e^{(\lambda)} a_i^{(\lambda)} + \Lambda' e^{(1-\lambda)} a_i^{(1-\lambda)} + \Lambda' u_i, \tag{7}$$

where e is the $(TX1)$ dimension vector of unities and the matrix Λ is defined in the theorem. Premultiplying equation (7) with the matrix $Q^{(\lambda)}$ yields

$$Q^{(\lambda)} y_{i,-1} = Q^{(\lambda)} \Lambda' u_i, \tag{8}$$

since $Q^{(\lambda)}(e, \Lambda' e^{(\lambda)}, \Lambda' e^{(1-\lambda)}) = (0, 0, ..., 0)$. Note that equation (8) also holds for the case that $a_i^{(\lambda)} = a_i^{(1-\lambda)} = a_i$ under the null hypothesis, i.e. there is no break. This happens because $Q^{(\lambda)}(\Lambda' e^{(\lambda)} a_i^{(\lambda)} + \Lambda' e^{(1-\lambda)} a_i^{(1-\lambda)}) = Q^{(\lambda)} \Lambda' e a_i = \mathbf{0}$.

Substituting (8) into (2) and noticing that $Q^{(\lambda)}$ is an indempotent and symmetric matrix yields

$$\hat{\phi} - 1 = \left[\sum_{i=1}^{N} u_i' \Lambda' Q^{(\lambda)} u_i \right] \left[\sum_{i=1}^{N} u_i' \Lambda' Q^{(\lambda)} \Lambda u_i \right]^{-1}. \tag{9}$$

Taking probability limits of equation (9) yields

$$
\begin{aligned}
B(\lambda, T) &= p \lim_{N\to\infty} (\hat{\phi} - 1) \\
&= E\left[u_i'\Lambda' Q^{(\lambda)} u_i\right] E\left[u_i'\Lambda' Q^{(\lambda)} \Lambda u_i\right]^{-1} \\
&= tr\left[\Lambda' Q^{(\lambda)}\right] \left\{tr\left[\Lambda' Q^{(\lambda)}\Lambda\right]\right\}^{-1}, \qquad (10)
\end{aligned}
$$

by the LLN.

Subtracting the term $B(\lambda, T)$ from (9) yields

$$
\begin{aligned}
&\hat{\phi} - 1 - B(\lambda, T) \\
= &\left\{\sum_{i=1}^{N}\left[u_i'\Lambda' Q^{(\lambda)} u_i - B(\lambda, T)(u_i'\Lambda' Q^{(\lambda)}\Lambda u_i)\right]\right\}\left\{\sum_{i=1}^{N} u_i'\Lambda' Q^{(\lambda)}\Lambda u\right\}^{-1} \\
= &\left\{\sum_{i=1}^{N}\xi_i^{(\lambda)}\right\}\left\{\sum_{i=1}^{N} u_i'\Lambda' Q^{(\lambda)}\Lambda u\right\}^{-1}, \qquad (11)
\end{aligned}
$$

where $\xi_i^{(\lambda)} = u_i'\Lambda' Q^{(\lambda)} u_i - B(\lambda, T)(u_i'\Lambda' Q^{(\lambda)}\Lambda u_i)$ is a random variable which has zero mean by construction and constant variance, denoted $Var(\xi_i^{(\lambda)})$, $\forall\ i$. Using standard results on quadratic forms, ξ_i can be written as

$$
\begin{aligned}
\xi_i^{(\lambda)} &= u_i'\frac{1}{2}\left(\Lambda' Q^{(\lambda)} + Q^{(\lambda)}\Lambda\right) u_i - B(\lambda, T)(u_i'\Lambda' Q^{(\lambda)}\Lambda u_i) \\
&= u_i'\left[\frac{1}{2}\left(\Lambda' Q^{(\lambda)} + Q^{(\lambda)}\Lambda\right) - B(\lambda, T)(\Lambda' Q^{(\lambda)}\Lambda)\right] u_i \\
&= u_i' A^{(\lambda)} u_i, \qquad (12)
\end{aligned}
$$

where $A^{(\lambda)} = \frac{1}{2}\left(\Lambda' Q^{(\lambda)} + Q^{(\lambda)}\Lambda\right) - B(\lambda, T)(\Lambda' Q^{(\lambda)}\Lambda)$ is a symmetric matrix, since $\frac{1}{2}\left(\Lambda' Q^{(\lambda)} + Q^{(\lambda)}\Lambda\right)$ and $(\Lambda' M^{(\lambda)}\Lambda)$ are symmetric matrices. Using results on quadratic forms for symmetric matrices, it can be seen that $Var(\xi_i^{(\lambda)})$ is given by

$$
\begin{aligned}
Var(\xi_i^{(\lambda)}) &= Var[u_i' A^{(\lambda)} u_i] \\
&= k \sum_{j=1}^{T} a_{jj}^{(\lambda)^2} + 2\sigma_u^4 tr \left(A^{(\lambda)^2} \right) \qquad (13)
\end{aligned}
$$

[see Anderson (1971)].

The result of the theorem can be proved by scaling (11) appropriately and using the following asymptotic results, for $N \to \infty$. This yields

$$
\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \xi_i^{(\lambda)} \xrightarrow{d} N(0, Var(\xi_i)) \qquad (14)
$$

by the CLT, and

$$
p \lim \frac{1}{N} \sum_{i=1}^{N} u_i' \Lambda' Q^{(\lambda)} \Lambda u_i = \sigma_u^2 tr \left[\Lambda' Q^{(\lambda)} \Lambda \right] \qquad (15)
$$

by the LLN, the Cramer-Wold lemma. These results hold under the conditions of Assumption 1. Note that the condition $k < \infty$ of the assumption guarantees that $Var(\xi_i^{(\lambda)})$ exists.

Proof of Corollary 2: To prove Corollary 2, first notice that under normality $k = 0$. To obtain the analytic results of the corollary, we will expand the inconsistency correction term $b(\lambda, T)$ and the variance $Var[\xi_i^{(\lambda)}]$ by replacing the matrix $Q^{(\lambda)}$ with $M^{(\lambda)} = \left[I - \frac{1}{\lambda T} e^{(\lambda)} e^{(\lambda)\prime} - \frac{1}{(1-\lambda)T} e^{(1-\lambda)} e^{(1-\lambda)\prime} \right]$, given for $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)} \right)$. Then, expanding $b(\lambda, T)$ yields

$$
\begin{aligned}
b(\lambda, T) &= tr \left[\Lambda' M^{(\lambda)} \right] \left\{ tr \left[\Lambda' M^{(\lambda)} \Lambda \right] \right\}^{-1} \\
&= -3(T-2) \left\{ \delta_1(\lambda) T^2 + \delta_0(\lambda) \right\}^{-1}, \qquad (16)
\end{aligned}
$$

where $\delta_1(\lambda) = (2\lambda^2 - 2\lambda + 1)$, and $\delta_0(\lambda) = -2$. The last result is derived by using the following two results $tr \left[\Lambda' M^{(\lambda)} \right] = -\frac{T-2}{2}$ and $tr \left[\Lambda' M^{(\lambda)} \Lambda \right] = \frac{1}{6} T^2 + \frac{1}{3} (\lambda T)^2 - \frac{1}{3} \lambda T^2$.

To derive an analytic form for $Var[\xi_i^{(\lambda)}]$, write

$$\begin{aligned} Var[\xi_i^{(\lambda)}] &= Var\left[u_i'\frac{1}{2}\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right)u_i\right] + b(\lambda,T)^2 Var\left[u_i'\Lambda' M^{(\lambda)}\Lambda u_i\right] \\ &\quad -2b(\lambda,T)Cov\left[u_i'\frac{1}{2}\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right)u_i; u_i'\Lambda' M^{(\lambda)}\Lambda u_i\right]. \quad (17) \end{aligned}$$

Expanding the component terms of $Var[\xi_i^{(\lambda)}]$ in the above expression yields the following results:

$$\begin{aligned} & Var\left[u_i'\frac{1}{2}\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right)u_i\right] \\ = & \frac{1}{2}\sigma_u^4 tr\left[\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right)^2\right] \\ = & \left[\frac{1}{12}(2\lambda^2 - 2\lambda + 1)T^2 + \frac{1}{2}T - \frac{7}{6}\right]\sigma_u^4, \end{aligned} \quad (18)$$

$$\begin{aligned} Var\left[u_i'\Lambda' M^{(\lambda)}\Lambda u_i\right] &= 2\sigma_u^4 tr\left[\left(\Lambda' M^{(\lambda)}\Lambda\right)^2\right] \\ &= \left[\frac{1}{45}(2\lambda^4 - 4\lambda^3 + 6\lambda^2 - 4\lambda + 1)T^4 \right. \\ &\quad \left. + \frac{1}{9}(\lambda^2 - \lambda + \frac{1}{2})T^2 - \frac{7}{45}\right]\sigma_u^4, \end{aligned} \quad (19)$$

and

$$\begin{aligned} & Cov\left[u_i'\frac{1}{2}\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right)u_i; u_i'\Lambda' M^{(\lambda)}\Lambda u_i\right] \\ = & 2tr\left[\frac{1}{2}\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right)\Lambda' M^{(\lambda)}\Lambda\right] \\ = & \left[\frac{1}{3}(-\lambda^2 + \lambda - \frac{1}{2})T^2 + \frac{1}{3}\right]\sigma_u^4 \end{aligned} \quad (20)$$

Substituting (18), (19) and (20) into (17) yields

$$
\begin{aligned}
& Var[\xi_i^{(\lambda)}] \\
= \quad & Var\left[u_i' \frac{1}{2}\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right) u_i\right] + B_{LSDV}(\lambda, T)^2 Var\left[u_i'\Lambda' M^{(\lambda)}\Lambda u_i\right] \\
& -2B_{LSDV}(\lambda, T) Cov\left[u_i' \frac{1}{2}\left(\Lambda' M^{(\lambda)} + M^{(\lambda)}\Lambda\right) u_i; u_i'\Lambda' M^{(\lambda)}\Lambda u_i\right] \\
= \quad & \frac{\sigma_u^4}{60}\left[\pi_6(\lambda)T^6 + \pi_5(\lambda)T^5 + \pi_4(\lambda)T^4 + \pi_3(\lambda)T^3 + \pi_2(\lambda)T^2\right. \\
& \left. + \, \pi_1(\lambda)T + \pi_0(\lambda)\right]\left[\delta_1(\lambda)T^2 + \delta_0(\lambda)\right]^{-2}, \qquad (21)
\end{aligned}
$$

where

$\pi_6(\lambda) = 40\lambda^6 - 120\lambda^5 + 204\lambda^4 - 208\lambda^3 + 162\lambda^2 - 78\lambda + 17,$

$\pi_5(\lambda) = -216\lambda^4 + 432\lambda^3 - 528\lambda^2 + 312\lambda - 78,$

$\pi_4(\lambda) = 216\lambda^4 - 432\lambda^3 + 588\lambda^2 - 372\lambda + 108,$

$\pi_3(\lambda) = 0,$

$\pi_2(\lambda) = -120\lambda^2 + 120\lambda - 144,$

$\pi_1(\lambda) = 216\lambda$, and

$\pi_0(\lambda) = -136.$

The results of the corollary follows immediately by substituting (16) and (21) into the test statistic $Z(\lambda, T)$, given by Theorem 1, and noticing that the $p\lim$ of the denominator of $\hat{\phi}$ is given by $\left\{tr\left[\Lambda' M^{(\lambda)}\Lambda\right]\right\} = \delta_1(\lambda)T^2 + \delta_0(\lambda)$ (see equation (16)).

References

[1] Arelano E. and B. Honoré (2002), Panel data models: Some recent developments, Handbook of Econometrics, edited by Heckman J. and E. Leamer editors, Vol 5, North Holland.

[2] Baltagi B.H. (1995), Econometric analysis of panel data, Wiley, Chichester.

[3] Baltagi B.H. (2000), Nonstationary panels, cointegration in panels and dynamic panels: A survey in advances in econometrics: nonstationaty panels, panel cointegration and dynamic panels, edited by Baltagi B.H, T.B. Fomby and R. C. Hill, 15, 7-51.

[4] Barro R.J. and X. Sala-i-Martin (1995), Economic growth, McCraw-Hill, New York.

[5] Bernard A.B. and S.N. Durlauf (1995), "Convergence of international output", *Journal of Applied Econometrics*, 10, 97-108.

[6] Carrion-i-Silvestre, T. Del Barrio-Castro, and E. Lopez-Bazo (2001), "Level shifts in a panel data based unit root test. An application to the rate of unemployment", mimeo, Department of Econometria, University of Barcelona, Spain.

[7] Casseli F., G. Esquivel and F. Lefort (1996), "Reopening the convergence debate: a new look at cross-country growth empirics", *Journal of Economic Growth, 1, 363-389.*

[8] Culver, S.E. and D.H. Papell (1999), Long-run power parity with short-run data: evidence with a null hypothesis of stationarity, *Journal of International Money and Finance*, 18, 751-768.

[9] de la Fuente (1997), The empirics of Growth and Convergence: A selective review, *Journal of Economic Dynamics and Control*, 21, 23-73.

[10] Harris R.D.F. and E. Tzavalis (1999), "Inference for unit roots in dynamic panels where the time dimension is fixed", *Journal of Econometrics*, 91, 201-226.

[11] Islam N. (1995), "Growth Empirics: a panel data approach", *Quarterly Journal of Economics*, 110, 1127-1170.

[12] Levin A., and C.F., Lin, and C-S. J, Chu (2002), "Unit root tests in panel data: asymptotic and finite-sample properties", *Journal of Econometrics*, 108, 1-24.

[13] Lo A.W. and A.C. MacKinlay (1995), A non-random walk down Wall Street, Princeton University Press.

[14] O'Connell P.G.J (1998), The overvaluation of purchasing power parity, *Journal of International Economics*, 44, 1-19.

[15] Perron P. (1989), "The great crash, the oil price shock, and the unit root hypothesis", *Econometrica*, 57, 1361-1401.

[16] Perron P. (1990), "Testing for a unit root in a time series with a changing mean", *Journal of Business & Economic Statistics*, 8, 153-162.

[17] Perron P. and T., Vogelsang (1992), "Nonstationarity and level shifts with an application to purchasing power parity", *Journal of Business & Economic Statistics, 10, 301-320.*

[18] Zivot E. and D.W.K. Andrews (1992), "Further evidence on the great crash, the oil price shock, and the unit-root hypothesis", *Journal of Business & Economic Statistics, 10, 251-270.*

Table 2(a): Monte Carlo Simulations for known break point

N	25	25	50	50	50	100	100	100	100
T	10	15	10	15	25	10	15	25	50
					$\lambda = 0.2$				
SIZE:	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.06
POWER: $\phi = 0.95$	0.21	0.30	0.35	0.53	0.80	0.56	0.80	0.97	1.00
$\phi = 0.90$	0.45	0.62	0.72	0.90	0.99	0.94	0.99	1.00	1.00
					$\lambda \approx 0.5$				
SIZE:	0.06	0.07	0.06	0.06	0.06	0.06	0.05	0.06	0.06
POWER: $\phi = 0.95$	0.19	0.25	0.28	0.40	0.66	0.42	0.66	0.90	1.00
$\phi = 0.90$	0.36	0.50	0.55	0.77	0.97	0.80	0.97	1.00	1.00
					$\lambda = 0.8$				
SIZE:	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05
POWER: $\phi = 0.95$	0.19	0.31	0.31	0.51	0.80	0.50	0.77	0.98	1.00
$\phi = 0.90$	0.43	0.66	0.69	0.91	1.00	0.92	1.00	1.00	1.00

Notes: This table presents the results of the Monte Carlo simulations for model (1) with individual effects, i.e. $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$.

Table 2(b): Monte Carlo Simulations for known break point

N	25	25	50	50	50	100	100	100	100
T	10	15	10	15	25	10	15	25	50
					$\lambda = 0.2$				
SIZE:	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.06	0.06
POWER: $\phi = 0.95$	0.05	0.06	0.05	0.07	0.09	0.06	0.07	0.11	0.44
$\phi = 0.90$	0.06	0.08	0.07	0.11	0.24	0.08	0.15	0.39	1.00
					$\lambda \approx 0.5$				
SIZE:	0.05	0.05	0.05	0.05	0.06	0.05	0.05	0.05	0.05
POWER: $\phi = 0.95$	0.05	0.05	0.05	0.06	0.07	0.06	0.06	0.10	0.22
$\phi = 0.90$	0.06	0.08	0.06	0.08	0.16	0.07	0.10	0.38	0.90
					$\lambda = 0.8$				
SIZE:	0.05	0.06	0.05	0.06	0.06	0.05	0.06	0.06	0.06
POWER: $\phi = 0.95$	0.05	0.06	0.06	0.06	0.09	0.05	0.07	0.11	0.44
$\phi = 0.90$	0.06	0.09	0.07	0.10	0.26	0.08	0.13	0.44	1.00

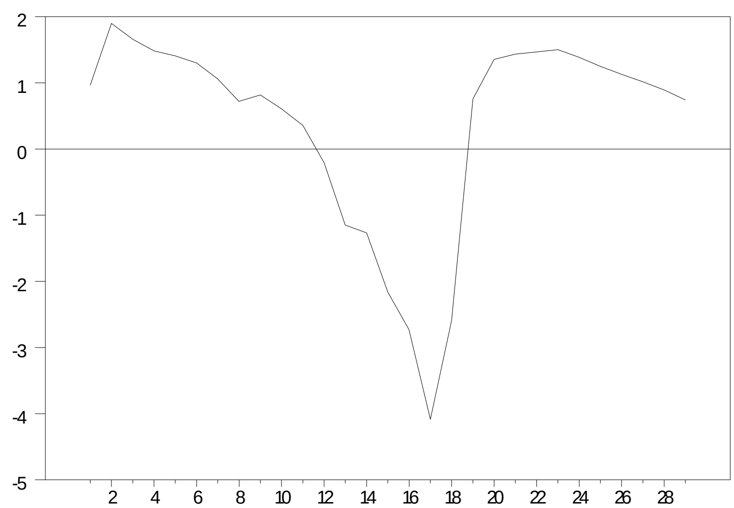
Notes:This table presents the results of the Monte Carlo simulations for model (1) with individual effects and trends, i.e. $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$.

Table 3: Monte Carlo Simulations for unknown break point

N	25	25	50	50	50	100	100	100	100
T	10	15	10	15	25	10	15	25	50
					A				
SIZE:	0.08	0.09	0.07	0.08	0.07	0.06	0.07	0.07	0.09
POWER: $\phi = 0.95$	0.22	0.32	0.36	0.55	0.87	0.62	0.84	0.99	1.00
$\phi = 0.90$	0.49	0.70	0.76	0.94	0.99	0.97	1.00	1.00	1.00
					B				
SIZE:	0.06	0.06	0.05	0.06	0.07	0.05	0.06	0.07	0.06
POWER: $\phi = 0.95$	0.05	0.06	0.05	0.06	0.09	0.06	0.07	0.09	0.49
$\phi = 0.90$	0.06	0.09	0.07	0.11	0.27	0.08	0.14	0.27	1.00

Notes: Panel A of the table presents the results of the Monte Carlo simulations for the sequential test statistics for the cases that $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}\right)$ (see Panel A) and $X_i^{(\lambda)} = \left(e^{(\lambda)}, e^{(1-\lambda)}, \tau^{(\lambda)}, \tau^{(1-\lambda)}\right)$ (see Panel B).

Figure 3: Sequential test statistic for the Non-oil countries



Sequential test statistic for the OECD countries.