Identifying and Estimating the Distributions of *Ex Post* and *Ex Ante* Returns to Schooling: A Survey of Recent Developments*

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Abstract

This paper surveys a recent body of research by Carneiro, Hansen, and Heckman (2001, 2003), Cunha and Heckman (2006), Cunha, Heckman, and Navarro (2005a,b, 2006), Heckman and Navarro (2006) and Navarro (2004) that estimates and identifies the *ex post* distribution of returns to schooling and determines *ex ante* distributions of returns on which agents base their schooling choices. We discuss methods and evidence, and state a fundamental identification problem concerning the separation of preferences, market structures and agent information sets. For a variety of market structures and preference specifications, we estimate that over 50% of the *ex post* variance in returns to college are forecastable at the time agents make their schooling choices.

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1 Introduction

In computing *ex ante* returns to schooling, it is necessary to characterize what is in the agent's information set at the time schooling decisions are made. To do so, the recent literature exploits the key idea that if agents know something and use that information in making their schooling decisions, it will affect their schooling choices. With panel data on earnings we can measure realized outcomes and assess what components of those outcomes were known at the time schooling choices were made.[1]

The literature on panel data earnings dynamics (e.g. Lillard and Willis, 1978; MaCurdy, 1982) is not designed to estimate what is in agent information sets. It estimates earnings equations of the following type:

$$Y_{i,t} = X_{i,t}\beta + S_i\tau + U_{i,t}, \tag{1}$$

where $Y_{i,t}, X_{i,t}, S_i, U_{i,t}$ denote (for person i at time t) the realized earnings, observable characteristics, educational attainment, and unobservable characteristics, respectively, from the point of view of the observing economist. The variables generating outcomes realized at time t may or may not have been known to the agents at the time they made their schooling decisions.

The error term $U_{i,t}$ is usually decomposed into two or more components. For example, it is common to specify that

$$U_{i,t} = \phi_i + \delta_{i,t}. \tag{2}$$

The term ϕ_i is a person-specific effect. The error term $\delta_{i,t}$ is often assumed to follow an ARMA (p, q) process (see Hause, 1980; MaCurdy, 1982) such as $\delta_{i,t} = \rho\delta_{i,t-1} + m_{i,t}$, where $m_{i,t}$ is a mean zero innovation independent of $X_{i,t}$ and the other error components. The components $X_{i,t}, \phi_i$, and $\delta_{i,t}$ all contribute to measured *ex post* variability across persons. However, the literature is silent about the difference between heterogeneity and uncertainty, the unforecastable part of earnings as of a given age. The literature on income mobility and on inequality measures all variability *ex post* as in Chiswick (1974), Mincer (1974) and Chiswick and Mincer (1972).

[1] An alternative approach summarized by Manski (2004) is to use survey methods to elicit expectations. We do not survey that literature in this paper.

An alternative specification of the error process postulates a factor structure for earnings,

$$U_{i,t} = \theta_i \alpha_t + \varepsilon_{i,t}, \tag{3}$$

where θ_i is a vector of skills (e.g., ability, initial human capital, motivation, and the like), α_t is a vector of skill prices, and the $\varepsilon_{i,t}$ are mutually independent mean zero shocks independent of θ_i. Hause (1980) and Heckman and Scheinkman (1987) analyze such earnings models. Any process in the form of equation (2) can be written in terms of (3). The latter specification is more directly interpretable as a pricing equation than (2).

Depending on the available market arrangements for coping with risk, the predictable components of $U_{i,t}$ will have a different effect on choices and economic welfare than the unpredictable components, if people are risk averse and cannot fully insure against uncertainty. Statistical decompositions based on (1), (2), and (3) or versions of them describe *ex post* variability but tell us nothing about which components of (1) or (3) are forecastable by agents *ex ante*. Is ϕ_i unknown to the agent? $\delta_{i,t}$? Or $\phi_i + \delta_{i,t}$? Or $m_{i,t}$? In representation (3), the entire vector θ_i, components of the θ_i, the $\varepsilon_{i,t}$, or all of these may or may not be known to the agent at the time schooling choices are made.

The methodology developed in Carneiro, Hansen, and Heckman (2003), Cunha, Heckman, and Navarro (2005a,b) and Cunha and Heckman (2006) provides a framework within which it is possible to identify components of life cycle outcomes that are forecastable and acted on at the time decisions are taken from ones that are not. In order to choose between high school and college, agents forecast future earnings (and other returns and costs) for each schooling level. Using information about educational choices at the time the choice is made, together with the *ex post* realization of earnings and costs that are observed at later ages, it is possible to estimate and test which components of future earnings and costs are forecast by the agent. This can be done provided we know, or can estimate, the earnings of agents under both schooling choices and provided we specify the market environment under which they operate as well as their preferences over outcomes.

For certain market environments where separation theorems are valid, so that consumption decisions are made independently of wealth maximizing decisions, it is not necessary to know agent preferences to decompose realized earnings outcomes in this fashion. Carneiro, Hansen, and Heckman (2003), Cunha, Heckman, and Navarro (2005a,b) and Cunha and Heckman (2006) use choice information to extract *ex ante* or forecast components of earnings and to distinguish them from realized earnings under different market environments. The difference between forecast and realized earnings allows them to identify the distributions of the components of uncertainty facing agents at the time they make their schooling decisions.

2 A Generalized Roy Model

To state these issues more precisely, consider a version of the generalized Roy (1951) economy with two sectors.[2] Let S_i denote different schooling levels. $S_i = 0$ denotes choice of the high school sector for person i, and $S_i = 1$ denotes choice of the college sector. Each person chooses to be in one or the other sector but cannot be in both. Let the two potential outcomes be represented by the pair $(Y_{0,i}, Y_{1,i})$, only one of which is observed by the analyst for any agent. Denote by C_i the direct cost of choosing sector 1, which is associated with choosing the college sector (e.g., tuition and non-pecuniary costs of attending college expressed in monetary values).

$Y_{1,i}$ is the *ex post* present value of earnings in the college sector, discounted over horizon T for a person choosing at a fixed age, assumed for convenience to be zero,

$$Y_{1,i} = \sum_{t=0}^{T} \frac{Y_{1,i,t}}{(1+r)^t},$$

and $Y_{0,i}$ is the *ex post* present value of earnings in the high school sector at age zero,

$$Y_{0,i} = \sum_{t=0}^{T} \frac{Y_{0,i,t}}{(1+r)^t},$$

where r is the one-period risk-free interest rate. $Y_{1,i}$ and $Y_{0,i}$ can be constructed from time se-

[2]See Heckman (1990) and Heckman and Smith (1998) for discussions of the generalized Roy model. In this paper we assume only two schooling levels for expositional simplicity, although our methods apply more generally.

ries of *ex post* potential earnings streams in the two states: $(Y_{0,i,0},\ldots,Y_{0,i,T})$ for high school and $(Y_{1,i,0},\ldots,Y_{1,i,T})$ for college. A practical problem is that we only observe one or the other of these streams. This partial observability creates a fundamental identification problem which can be solved using the methods described in Heckman, Lochner, and Todd (2006) and the references they cite.

The variables $Y_{1,i}, Y_{0,i}$, and C_i are *ex post* realizations of returns and costs, respectively. At the time agents make their schooling choices, these may be only partially known to the agent, if at all. Let $\mathcal{I}_{i,0}$ denote the information set of agent i at the time the schooling choice is made, which is time period $t=0$ in our notation. Under a complete markets assumption with all risks diversifiable (so that there is risk-neutral pricing) or under a perfect foresight model with unrestricted borrowing or lending but full repayment, the decision rule governing sectoral choices at decision time '0' is

$$S_i = \begin{cases} 1, & \text{if } E\left(Y_{1,i} - Y_{0,i} - C_i \mid \mathcal{I}_{i,0}\right) \geq 0 \\ 0, & \text{otherwise.}^3 \end{cases} \tag{4}$$

Under perfect foresight, the postulated information set would include $Y_{1,i}, Y_{0,i}$, and C_i. Under either model of information, the decision rule is simple: one attends school if the expected gains from schooling are greater than or equal to the expected costs. Thus under either set of assumptions, a separation theorem governs choices. Agents maximize expected wealth independently of their consumption decisions over time.

The decision rule is more complicated in the absence of full risk diversifiability and depends on the curvature of utility functions, the availability of markets to spread risk, and possibilities for storage. (See Cunha and Heckman, 2006, and Navarro, 2004, for a more extensive discussion.) In these more realistic economic settings, the components of earnings and costs required to forecast the gain to schooling depend on higher moments than the mean. In this paper we use a model with a simple market setting to motivate the identification analysis of a more general environment analyzed elsewhere (Carneiro, Hansen, and Heckman, 2003, and Cunha, Heckman, and Navarro, 2005a).

Suppose that we seek to determine $\mathcal{I}_{i,0}$. This is a difficult task. Typically we can only partially

[3] If there are aggregate sources of risk, full insurance would require a linear utility function.

identify $\mathcal{I}_{i,0}$ and generate a list of candidate variables that belong in the information set. We can usually only estimate the distributions of the unobservables in $\mathcal{I}_{i,0}$ (from the standpoint of the econometrician) and not individual person-specific information sets. Before describing their analysis, we consider how this question might be addressed in the linear-in-the-parameters Card (2001) model.

3 Identifying Information Sets in Card's Model of Schooling

We seek to decompose the "returns" coefficient or the gross gains from schooling in an earnings-schooling model into components that are known at the time schooling choices are made and components that are not known. For simplicity assume that, for person i, returns are the same at all levels of schooling. Write discounted lifetime earnings of person i as

$$Y_i = \alpha + \rho_i S_i + U_i, \tag{5}$$

where ρ_i is the person-specific *ex post* return, S_i is years of schooling, and U_i is a mean zero unobservable. We seek to decompose ρ_i into two components $\rho_i = \eta_i + \nu_i$, where η_i is a component known to the agent when he/she makes schooling decisions and ν_i is revealed after the choice is made. Schooling choices are assumed to depend on what is known to the agent at the time decisions are made, $S_i = \lambda(\eta_i, Z_i, \tau_i)$, where the Z_i are other observed determinants of schooling and τ_i represents additional factors unobserved by the analyst but known to the agent. Both of these variables are in the agent's information set at the time schooling choices are made. We seek to determine what components of *ex post* lifetime earnings Y_i enter the schooling choice equation.

If η_i is known to the agent and acted on, it enters the schooling choice equation. Otherwise it does not. Component ν_i and any measurement errors in $Y_{1,i}$ or $Y_{0,i}$ should not be determinants of schooling choices. Neither should future skill prices that are unknown at the time agents make their decisions. If agents do not use η_i in making their schooling choices, even if they know it, η_i

would not enter the schooling choice equation. Determining the correlation between realized Y_i and schooling choices based on *ex ante* forecasts enables economists to identify components known to agents and acted on in making their schooling decisions. Even if we cannot identify ρ_i, η_i, or ν_i for each person, under conditions specified in this paper we can identify their distributions.

If we correctly specify the X and the Z that are known to the agent at the time schooling choices are made, local instrumental variable estimates of the *MTE* identify *ex ante* gross gains. Any dependence between the variables in the schooling equation and $Y_1 - Y_0$ arises from information known to the agent at the time schooling choices are made. If the conditioning set is misspecified by using information on X and Z that accumulates after schooling choices are made and that predicts realized earnings (but not *ex ante* earnings), the estimated return is an *ex post* return relative to that information set. Thus, it is important to specify the conditioning set correctly to obtain the appropriate *ex ante* return. The question is how to pick the information set? To exposit these ideas we review the Card model.

3.1 The Card Model

A random coefficients model of the economic return to schooling has been an integral part of the human capital literature since the papers by Becker and Chiswick (1966), Chiswick (1974), Chiswick and Mincer (1972) and Mincer (1974). In its most stripped-down form and ignoring work experience terms, the Mincer model writes log earnings for person i with schooling level S_i as

$$\ln y_i = \alpha_i + \rho_i S_i, \tag{6}$$

where the "rate of return" ρ_i varies among persons as does the intercept, α_i. For the purposes of this discussion think of y_i as an annualized flow of lifetime earnings. Unless the only costs of schooling are earnings foregone, and markets are perfect, ρ_i is a percentage growth rate in earnings with schooling and not a rate of return to schooling. Let $\alpha_i = \bar{\alpha} + \varepsilon_{\alpha_i}$ and $\rho_i = \bar{\rho} + \varepsilon_{\rho_i}$ where $\bar{\alpha}$ and $\bar{\rho}$ are the means of α_i and ρ_i. Thus the means of ε_{α_i} and ε_{ρ_i} are zero. Earnings equation (6) can be written as

$$\ln y_i = \bar{\alpha} + \bar{\rho} S_i + \{\varepsilon_{\alpha_i} + \varepsilon_{\rho_i} S_i\}. \tag{7}$$

Equations (6) and (7) are the basis for a human capital analysis of wage inequality in which the variance of log earnings is decomposed into components due to the variance in S_i and components due to the variation in the growth rate of earnings with schooling (the variance in $\bar{\rho}$), the mean growth rate across regions or time ($\bar{\rho}$), and mean schooling levels ($\bar{S}$). (See, e.g. Mincer, 1974, and Willis, 1986.)

Given that the growth rate ρ_i is a random variable, it has become conventional to summarize the distribution of growth rates by the mean, although many other summary measures of the distribution are possible. For the prototypical distribution of ρ_i, the conventional measure is the "average growth rate" $E(\rho_i)$ or $E(\rho_i|X)$, where the latter conditions on X, the observed characteristics of individuals. Other means are possible such as the mean growth rates for persons who attain a given level of schooling.

The original Mincer model assumed that the growth rate of earnings with schooling, ρ_i, is uncorrelated with or is independent of S_i. This assumption is convenient but is not implied by economic theory. It is plausible that the growth rate of earnings with schooling declines with the level of schooling. It is also plausible that there are unmeasured ability or motivational factors that affect the growth rate of earnings with schooling and are also correlated with the level of schooling. Rosen (1977) discusses this problem in some detail within the context of hedonic models of schooling and earnings. A similar problem arises in analyses of the impact of unionism on relative wages and is discussed in Lewis (1963).

Allowing for correlated random coefficients (so S_i is correlated with ε_{ρ_i}) raises substantial problems that are just beginning to be addressed in a systematic fashion in the recent literature. Card's (2001) random coefficient model of the growth rate of earnings with schooling is derived from economic theory and is based on the explicit analysis of Becker's model by Rosen (1977).[4] We consider

[4] Random coefficient models with coefficients correlated with the regressors are systematically analyzed in Heckman and Robb (1985, 1986). See also Heckman and Vytlacil (1998). They originate in labor economics with the work of Lewis (1963). Heckman and Robb analyze training programs but their analysis applies to estimating the returns to schooling.

conditions under which it is possible to estimate the effect of schooling on the schooled in his model.

In Card's (1999, 2001) model, the preferences of a person over income (y) and schooling (S) are

$$U(y,S)=\ln y\left(S\right)-\varphi(S) \qquad \varphi'(S)>0 \qquad \text{and} \qquad \varphi''(S)>0.$$

The schooling-earnings relationship is $y=g(S)$. This is a hedonic model of schooling, where $g(S)$ reveals how schooling is priced out in the labor market. This specification is written in terms of annualized earnings and abstracts from work experience.[5] It assumes perfect certainty and abstracts from the sequential resolution of uncertainty that is central to the modern literature. In this formulation, discounting of future earnings is kept implicit. The first order condition for optimal determination of schooling is

$$\frac{g'(S)}{g(S)}=\varphi'(S). \tag{8}$$

The term $\frac{g'(s)}{g(s)}$ is the percentage change of earnings with schooling or the "growth rate" at level s. Card's model reproduces Rosen's (1977) model if r is the common interest rate at which agents can freely lend or borrow and if the only costs are S years of foregone earnings. In Rosen's setup an agent with an infinite lifetime maximizes $\frac{1}{r}e^{-rS}g(S)$ so $\varphi(S)=rS+\ln r$, and $\frac{g'(S)}{g(S)}=r$.

Linearizing the model, we obtain

$$\begin{aligned}
\frac{g'(S_i)}{g(S_i)} &= \beta_i(S_i)=\rho_i-k_1S_i, \qquad k_1\geq 0,\\
\varphi'(S_i) &= \delta_i(S_i)=r_i+k_2S_i, \qquad k_2\geq 0.
\end{aligned}$$

Substituting these expressions into the first order condition (8), we obtain that the optimal level of schooling is $S_i=\dfrac{(\rho_i-r_i)}{k}$, where $k=k_1+k_2$. Observe that if both the growth rate and the returns are independent of S_i, $(k_1=0,k_2=0)$, then $k=0$ and if $\rho_i=r_i$, there is no determinate level of schooling at the individual level. This is the original Mincer (1958) model.[6]

One source of heterogeneity among persons in the model is ρ_i, the way S_i is transformed into

[5] Adding work experience in a multiplicatively separable way produces the Mincer model.

[6] In that model, aggregate allocations of persons to schooling are determined by an arbitrage condition that returns must be equalized across choices.

earnings. (School quality may operate through the ρ_i for example, as in Behrman and Birdsall, 1983, and ρ_i may also differ due to inherent ability differences.) A second source of heterogeneity is r_i, the "opportunity cost" (cost of schooling) or "cost of funds." Higher ability leads to higher levels of schooling. Higher costs of schooling results in lower levels of schooling.

We integrate the first order condition (8) to obtain the following hedonic model of earnings,

$$\ln y_i = \alpha_i + \rho_i S_i - \frac{1}{2} k_1 S_i^2. \tag{9}$$

To achieve the familiar looking Mincer equation, assume $k_1 = 0$.[7] This assumption rules out diminishing "returns" to schooling in terms of years of schooling. Even under this assumption, ρ_i is the percentage growth rate in earnings with schooling, but is not in general an internal rate of return to schooling. It would be a rate of return if there were no direct costs of schooling and everyone faces a constant borrowing rate. This is a version of the Mincer (1958) model, where $k_2 = 0$, and r_i is constant for everyone but not necessarily the same constant. If $\rho_i > r_i$, person i takes the maximum amount of schooling. If $\rho_i < r_i$, person i takes no schooling and if $\rho_i = r_i$, schooling is indeterminate. In the Card model, ρ_i is the person-specific growth rate of earnings and overstates the true rate of return if there are direct and psychic costs of schooling.

This simple model is useful in showing the sources of endogeneity in the schooling earnings model. Since schooling depends on ρ_i and r_i, any covariance between $\rho_i - r_i$ (in the schooling equation) and ρ_i (in the earnings function) produces a random coefficient model. Least squares will not estimate the mean growth rate of earnings with schooling unless, $COV(\rho_i, \rho_i - r_i) = 0$.

Suppose that the model for schooling can be written in linear in parameters form, as in Card (2001):

$$S_i = \lambda_0 + \lambda_1 \eta_i + \lambda_2 \nu_i + \lambda_3 Z_i + \tau_i, \tag{10}$$

where τ_i has mean zero and is assumed to be independent of Z_i. The Z_i and the τ_i proxy costs and

[7]The Card model (1999) produces a Mincer-like model where ρ_i is the Mincer return for individual i. The mean return in the population is $E(\rho_i)$. It is an *ex post* return derived under the assumption that log earnings are linear in schooling, contrary to the literature, previously discussed, that shows pronounced nonlinearities and sheepskin effects.

may also be correlated with U_i and η_i and ν_i in (5). In this framework, the goal of the analysis is to determine the η_i and ν_i components. By definition, $\lambda_2 = 0$ if ν_i is not known when agents make their schooling choices.

As a simple example, suppose we observe the cost of funds, r_i, and assume $r_i \perp\!\!\!\perp (\rho_i, \alpha_i)$. This assumes that the costs of schooling are independent of the "return" ρ_i and the payment to raw ability, α_i.

Suppose that agents do not know ρ_i at the time they make their schooling decisions but instead know $E(\rho_i) = \bar{\rho}$.[8] If agents act on this expected return to schooling, decisions are given by

$$S_i = \frac{\bar{\rho} - r_i}{k}$$

and *ex post* earnings observed after schooling are

$$Y_i = \bar{\alpha} + \bar{\rho} S_i + \{(\alpha_i - \bar{\alpha}) + (\rho_i - \bar{\rho}) S_i\}.$$

In the notation introduced in the Card model, $\eta_i = \bar{\rho}$ and $\nu_i = \rho_i - \bar{\rho}$.

In this case,

$$COV(Y, S) = \bar{\rho} Var(S)$$

because $(\rho_i - \bar{\rho})$ is independent of S_i. Note further that $(\bar{\alpha}, \bar{\rho})$ can be identified by least squares because $S_i \perp\!\!\!\perp [(\alpha_i - \bar{\alpha}), (\rho_i - \bar{\rho}) S_i]$ where "$\perp\!\!\!\perp$" denotes independence.

If, on the other hand, agents know ρ_i at the time they make their schooling decisions, *OLS* breaks down for identifying $\bar{\rho}$ because ρ_i is correlated with S_i. We can identify $\bar{\rho}$ and the distribution of ρ_i using the method of instrumental variables. Under our assumptions, r_i is a valid instrument for S_i.

In this case

$$COV(Y_i, S) = \bar{\rho} Var(S) + COV(S, (\rho - \bar{\rho}) S).$$

Since we observe S, can identify $\bar{\rho}$ and can construct $(\rho - \bar{\rho})$ for each S, we can form both terms

[8] This is a rational expectations assumption. Under rational expectations with the currently specified information set, the mean *ex ante* return is the same as the mean *ex post* return, but the distributions of these returns may be very different.

on the right hand side. Under the assumption that agents do not know ρ but forecast it by $\bar{\rho}$, ρ is independent of S so we can test for independence directly. In this case the second term on the right hand side is zero and does not contribute to the explanation of $COV(\ln y, S)$. Note further that a Durbin (1954) – Wu (1973) – Hausman (1978) test can be used to compare the OLS and IV estimates, which should be the same under the model that assumes that ρ_i is not known at the time schooling decisions are made and that agents base their choice of schooling on $E(\rho_i) = \bar{\rho}$. If the economist does not observe r_i, but instead observes determinants of r_i that are exogenous, we can still conduct the Durbin-Wu-Hausman test to discriminate between the two hypotheses, but we cannot form $COV(\rho, S)$ directly.

If we add selection bias to the Card model (so $E(\alpha \mid S)$ depends on S), we can identify $\bar{\rho}$ by IV (Heckman and Vytlacil, 1998) but OLS is no longer consistent even if, in making their schooling decisions, agents forecast ρ_i using $\bar{\rho}$. Selection bias can occur, for example, if fellowship aid is given on the basis of raw ability. In this case the Durbin-Wu-Hausman test is not helpful in assessing what is in the agent's information set.

Even ignoring selection bias, if we misspecify the information set, in the case where r_i is not observed, the proposed testing approach based on the Durbin-Wu-Hausman test breaks down. Thus if we include the predictors of r_i that predict *ex post* gains $(\rho_i - \bar{\rho})$ and are correlated with S_i, we do not identify $\bar{\rho}$. The Durbin-Wu-Hausman test is not informative on the stated question. For example, if local labor market variables proxy the opportunity cost of school (the r_i), and also predict the evolution of *ex post* earnings $(\rho_i - \bar{\rho})$, they are invalid. The question of determining the appropriate information set is front and center and cannot in general be inferred using IV methods.

The method developed by Cunha, Heckman, and Navarro (2005a,b) and Cunha and Heckman (2006) exploits the covariance between S and the realized Y_t to determine which components of Y_t are known at the time schooling decisions are made. It explicitly models selection bias and allows for measurement error in earnings. It does not rely on linearity of the schooling relationship in terms of $\rho - r$. Their method recognizes the discrete nature of the schooling decision.

4 The Method of Cunha, Heckman and Navarro

Cunha, Heckman, and Navarro (2005a,b, henceforth CHN) and Cunha and Heckman (2006), exploit covariances between schooling and realized earnings that arise under different information structures to test which information structure characterizes the data. They build on the analysis of Carneiro, Hansen, and Heckman (2003). To see how the method works, simplify the model to two schooling levels. Heckman and Navarro (2006) extend this analysis to multiple schooling levels.

Suppose, contrary to what is possible, that the analyst observes $Y_{0,i}$, $Y_{1,i}$, and C_i. Such information would come from an ideal data set in which we could observe two different lifetime earnings streams for the same person in high school and in college as well as the costs they pay for attending college. From such information, we could construct $Y_{1,i} - Y_{0,i} - C_i$. If we knew the information set $\mathcal{I}_{i,0}$ of the agent that governs schooling choices, we could also construct $E\left(Y_{1,i} - Y_{0,i} - C_i \mid \mathcal{I}_{i,0}\right)$. Under the correct model of expectations, we could form the residual

$$V_{\mathcal{I}_{i,0}} = \left(Y_{1,i} - Y_{0,i} - C_i\right) - E\left(Y_{1,i} - Y_{0,i} - C_i \mid \mathcal{I}_{i,0}\right),$$

and from the *ex ante* college choice decision, we could determine whether S_i depends on $V_{\mathcal{I}_{i,0}}$. It should not if we have specified $\mathcal{I}_{i,0}$ correctly. In terms of the model of equations (5) and (10), if there are no direct costs of schooling, $E\left(Y_{1,i} - Y_{0,i} \mid \mathcal{I}_{i,0}\right) = \eta_i$, and $V_{\mathcal{I}_{i,0}} = \nu_i$.

A test for correct specification of candidate information set $\widetilde{\mathcal{I}}_{i,0}$ is a test of whether S_i depends on $V_{\widetilde{\mathcal{I}}_{i,0}}$, where $V_{\widetilde{\mathcal{I}}_{i,0}} = \left(Y_{1,i} - Y_{0,i} - C_i\right) - E\left(Y_{1,i} - Y_{0,i} - C_i \mid \widetilde{\mathcal{I}}_{i,0}\right)$. More precisely, the information set is valid if $S_i \perp\!\!\!\perp V_{\widetilde{\mathcal{I}}_{i,0}} \mid \widetilde{\mathcal{I}}_{i,0}$. ($A \perp\!\!\!\perp B \mid C$ means A is independent of B given C). In terms of the simple linear schooling model of equations (5) and (10), this condition says that ν_i should not enter the schooling choice equation ($\lambda_2 = 0$). A test of misspecification of $\widetilde{\mathcal{I}}_{i,0}$ is a test of whether the coefficient of $V_{\widetilde{\mathcal{I}}_{i,0}}$ is statistically significantly different from zero in the schooling choice equation.

More generally, $\widetilde{\mathcal{I}}_{i,0}$ is the correct information set if $V_{\widetilde{\mathcal{I}}_{i,0}}$ does not help to predict schooling. One can search among candidate information sets $\widetilde{\mathcal{I}}_{i,0}$ to determine which ones satisfy the requirement that the generated $V_{\widetilde{\mathcal{I}}_{i,0}}$ does not predict S_i and what components of $Y_{1,i} - Y_{0,i} - C_i$ (and $Y_{1,i} - Y_{0,i}$)

are predictable at the age schooling decisions are made for the specified information set.[9] There may be several information sets that satisfy this property.[10] For a properly specified $\widetilde{\mathcal{I}}_{i,0}$, $V_{\widetilde{\mathcal{I}}_{i,0}}$ should not cause (predict) schooling choices. The components of $V_{\widetilde{\mathcal{I}}_{i,0}}$ that are unpredictable are called intrinsic components of uncertainty, as defined in this paper.

It is difficult to determine the exact content of $\mathcal{I}_{i,0}$ known to each agent. If we could, we would perfectly predict S_i given our decision rule. More realistically, we might find variables that proxy $\mathcal{I}_{i,0}$ or their distribution. Thus, in the example of equations (5) and (10) we would seek to determine the distribution of ν_i and the allocation of the variance of ρ_i to η_i and ν_i rather than trying to estimate ρ_i, η_i, or ν_i for each person. This strategy is pursued in Cunha, Heckman, and Navarro (2005b, 2006) for a two-choice model of schooling, and generalized by Cunha and Heckman (2006) and Heckman and Navarro (2006).

4.1 An Approach Based on Factor Structures

Consider the following linear in parameters model for T periods. Write earnings in each counterfactual state as

$$\begin{aligned} Y_{0,i,t} &= X_{i,t}\beta_{0,t} + U_{0,i,t}, \\ Y_{1,i,t} &= X_{i,t}\beta_{1,t} + U_{1,i,t}, \qquad t = 0, \ldots, T. \end{aligned}$$

We let costs of college be defined as

$$C_i = Z_i\gamma + U_{i,C}.$$

Assume that the life cycle of the agent ends after period T. Linearity of outcomes in terms of parameters is convenient but not essential to the method of CHN.

Suppose that there exists a vector of factors $\theta_i = (\theta_{i,1}, \theta_{i,2}, \ldots, \theta_{i,L})$ such that $\theta_{i,k}$ and $\theta_{i,j}$ are mutually independent random variables for $k, j = 1, \ldots, L, k \neq j$. They represent the error term in

[9] This procedure is a Sims (1972) version of a Wiener-Granger causality test.

[10] Thus different combinations of variables may contain the same information. The issue of the existence of a smallest information set is a technical one concerning a minimum σ-algebra that satisfies the condition on $\mathcal{I}_{i,0}$.

earnings at age t for agent i in the following manner:

$$\begin{aligned} U_{0,i,t} &= \theta_i \alpha_{0,t} + \varepsilon_{0,i,t}, \\ U_{1,i,t} &= \theta_i \alpha_{1,t} + \varepsilon_{1,i,t}, \end{aligned}$$

where $\alpha_{0,t}$ and $\alpha_{1,t}$ are vectors and θ_i is a vector distributed independently across persons. The $\varepsilon_{0,i,t}$ and $\varepsilon_{1,i,t}$ are mutually independent of each other and independent of the θ_i. We can also decompose the cost function C_i in a similar fashion:

$$C_i = Z_i \gamma + \theta_i \alpha_C + \varepsilon_{i,C}.$$

All of the statistical dependence across potential outcomes and costs is generated by θ, X, and Z. Thus, if we could match on θ_i (as well as X and Z), we could use matching to infer the distribution of counterfactuals and capture all of the dependence across the counterfactual states through the θ_i. However, in general, CHN allow for the possibility that not all of the required elements of θ_i are observed.

The parameters α_C and $\alpha_{s,t}$ for $s = 0, 1$, and $t = 0, \ldots, T$ are the factor loadings. $\varepsilon_{i,C}$ is independent of the θ_i and the other ε components. In this notation, the choice equation can be written as:

$$\begin{aligned} S_i^* &= E\left(\sum_{t=0}^{T} \frac{\left(X_{i,t}\beta_{1,t} + \theta_i \alpha_{1,t} + \varepsilon_{1,i,t}\right) - \left(X_{i,t}\beta_{0,t} + \theta_i \alpha_{0,t} + \varepsilon_{0,i,t}\right)}{(1+r)^t} - \left(Z_i \gamma + \theta_i \alpha_C + \varepsilon_{iC}\right) \;\middle|\; \mathcal{I}_{i,0} \right) \\ S_i &= 1 \text{ if } S^* \geq 0; \;\; S_i = 0 \text{ otherwise.} \end{aligned} \tag{11}$$

The term inside the parentheses is the discounted earnings of agent i in college minus the discounted earnings of the agent in high school. The second term is the cost of college.

Constructing (11) entails making a counterfactual comparison. Even if the earnings of one schooling level are observed over the lifetime using panel data, the earnings in the counterfactual state are not. After the schooling choice is made, some components of the $X_{i,t}$, the θ_i, and the $\varepsilon_{i,t}$ may be revealed (e.g., unemployment rates, macro shocks) to both the observing economist and the agent, although different components may be revealed to each and at different times. For this reason, application of IV even in the linear schooling model is problematic. If the wrong information set is used, the IV method will not identify the true *ex ante* returns.

Examining alternative information sets, one can determine which ones produce models for outcomes that fit the data best in terms of producing a model that predicts date $t = 0$ schooling choices and at the same time passes the CHN test for misspecification of predicted earnings and costs. Some components of the error terms may be known or not known at the date schooling choices are made. The unforecastable components are intrinsic uncertainty as CHN define it. The forecastable information is called heterogeneity.[11]

To formally characterize the CHN empirical procedure, it is useful to introduce some additional notation. Let $\odot$ denote the Hadamard product ($a \odot b = (a_1 b_1, \ldots, a_L b_L)$) for vectors a and b of length L. This is a componentwise multiplication of vectors to produce a vector. Let Δ_{X_t}, $t = 0, \ldots, T$, Δ_Z, Δ_θ, Δ_{ε_t}, Δ_{ε_C}, denote coefficient vectors associated with the X_t, $t = 0, \ldots, T$, the Z, the θ, the $\varepsilon_{1,t} - \varepsilon_{0,t}$, and the ε_C, respectively. These coefficients will be estimated to be nonzero in a schooling choice equation if a proposed information set is not the actual information set used by agents. For a proposed information set $\widetilde{\mathcal{I}}_{i,0}$ which may or may not be the true information set on which agents act, CHN define the proposed choice index $\tilde{S}_i^*$ in the following way:

$$
\begin{aligned}
\tilde{S}_i^* \;=\; & \sum_{t=0}^{T} \frac{E\left(X_{i,t} \mid \widetilde{\mathcal{I}}_{i,0}\right)}{(1+r)^t}\left(\beta_{1,t} - \beta_{0,t}\right) + \sum_{t=0}^{T} \frac{\left[X_{i,t} - E\left(X_{i,t} \mid \widetilde{\mathcal{I}}_{i,0}\right)\right]}{(1+r)^t}\left(\beta_{1,t} - \beta_{0,t}\right) \odot \Delta_{X_t} \qquad (12) \\
& + E(\theta_i \mid \widetilde{\mathcal{I}}_{i,0}) \left[\sum_{t=0}^{T} \frac{(\alpha_{1,t} - \alpha_{0,t})}{(1+r)^t} - \alpha_C\right] + \left[\theta_i - E\left(\theta_i \mid \widetilde{\mathcal{I}}_{i,0}\right)\right] \left\{\left[\sum_{t=0}^{T} \frac{(\alpha_{1,t} - \alpha_{0,t})}{(1+r)^t} - \alpha_C\right] \odot \Delta_\theta\right\} \\
& + \sum_{t=0}^{T} \frac{E\left(\varepsilon_{1,i,t} - \varepsilon_{0,i,t} \mid \widetilde{\mathcal{I}}_{i,0}\right)}{(1+r)^t} + \sum_{t=0}^{T} \frac{\left[(\varepsilon_{1,i,t} - \varepsilon_{0,i,t}) - E\left(\varepsilon_{1,i,t} - \varepsilon_{0,i,t} \mid \widetilde{\mathcal{I}}_{i,0}\right)\right]}{(1+r)^t} \Delta_{\varepsilon_t} \\
& - E\left(Z_i \mid \widetilde{\mathcal{I}}_{i,0}\right)\gamma - \left[Z_i - E\left(Z_i \mid \widetilde{\mathcal{I}}_{i,0}\right)\right]\gamma \odot \Delta_Z - E\left(\varepsilon_{iC} \mid \widetilde{\mathcal{I}}_{i,0}\right) - \left[\varepsilon_{iC} - E\left(\varepsilon_{iC} \mid \widetilde{\mathcal{I}}_{i,0}\right)\right]\Delta_{\varepsilon_C}.
\end{aligned}
$$

To conduct their test, CHN fit a schooling choice model based on the proposed model (12). They

[11] The term 'heterogeneity' is somewhat unfortunate. Under this term, CHN include trends common across all people (e.g., macrotrends). The real distinction they are making is between components of realized earnings forecastable by agents at the time they make their schooling choices vs. components that are not forecastable.

estimate the parameters of the model including the Δ parameters. This decomposition for $\widetilde{S}_i^*$ assumes that agents know the β, the γ, and the α.[12] If it is not correct, the presence of additional unforecastable components due to unknown coefficients affects the interpretation of the estimates. A test of no misspecification of information set $\widetilde{\mathcal{I}}_{i,0}$ is a joint test of the hypothesis that the Δ are all zero. That is, when $\widetilde{\mathcal{I}}_{i,0} = \mathcal{I}_{i,0}$ then the proposed choice index $\widetilde{S}_i^* = S_i^*$.

In a correctly specified model, the components associated with zero Δ_j are the unforecastable elements or the elements which, even if known to the agent, are not acted on in making schooling choices. To illustrate the application of the method of CHN, assume for simplicity that the $X_{i,t}$, the Z_i, the $\varepsilon_{i,C}$, the $\beta_{1,t}, \beta_{0,t}$, the $\alpha_{1,t}, \alpha_{0,t}$, and α_C are known to the agent, and the $\varepsilon_{j,i,t}$ are unknown and are set at their mean zero values. We can infer which components of the θ_i are known and acted on in making schooling decisions if we postulate that some components of θ_i are known perfectly at date $t = 0$ while others are not known at all, and their forecast values have mean zero given $\mathcal{I}_{i,0}$.

If there is an element of the vector θ_i, say $\theta_{i,2}$ (factor 2), that has nonzero loadings (coefficients) in the schooling choice equation and a nonzero loading on one or more potential future earnings, then one can say that at the time the schooling choice is made, the agent knows the unobservable captured by factor 2 that affects future earnings. If $\theta_{i,2}$ does not enter the choice equation but explains future earnings, then $\theta_{i,2}$ is unknown (not predictable by the agent) at the age schooling decisions are made. An alternative interpretation is that the second component of $\left[\sum_{t=0}^{T} \frac{(\alpha_{1,t}-\alpha_{0,t})}{(1+r)^t} - \alpha_C\right]$ is zero, i.e., that even if the component is known, it is not acted on. CHN can only test for what the agent knows and acts on.

One plausible scenario case is that for their model $\varepsilon_{i,C}$ is known (since schooling costs are incurred up front), but the future $\varepsilon_{1,i,t}$ and $\varepsilon_{0,i,t}$ are not, have mean zero, and are insurable. If there are components of the $\varepsilon_{j,i,t}$ that are predictable at age $t = 0$, they will induce additional dependence between S_i and future earnings that will pick up additional factors beyond those initially specified. The CHN procedure can be generalized to consider all components of (12). With it, the analyst can test the predictive power of each subset of the overall possible information set at the date the

[12] CHN and Cunha and Heckman (2006) relax this assumption.

schooling decision is being made.[13,14]

In the context of the factor structure representation for earnings and costs, the contrast between the CHN approach to identifying components of intrinsic uncertainty and the approach followed in the literature is as follows. The traditional approach, as exemplified by Keane and Wolpin (1997), assumes that the θ_i are known to the agent while the $\{\varepsilon_{0,i,t}, \varepsilon_{1,i,t}\}_{t=0}^{T}$ are not.[15] The CHN approach allows the analyst to determine which components of θ_i and $\{\varepsilon_{0,i,t}, \varepsilon_{1,i,t}\}_{t=0}^{T}$ are known and acted on at the time schooling decisions are made.

Statistical decompositions do not tell us which components of (3) are known at the time agents make their schooling decisions. A model of expectations and schooling is needed. If some of the components of $\{\varepsilon_{0,i,t}, \varepsilon_{1,i,t}\}_{t=0}^{T}$ are known to the agent at the date schooling decisions are made and enter (12), then additional dependence between S_i and future $Y_{1,i} - Y_{0,i}$ due to the $\{\varepsilon_{0,i,t}, \varepsilon_{1,i,t}\}_{t=0}^{T}$, beyond that due to θ_i, would be estimated.

It is helpful to contrast the dependence between S_i and future $Y_{0,i,t}, Y_{1,i,t}$ arising from θ_i and the dependence between S_i and the $\{\varepsilon_{0,i,t}, \varepsilon_{1,i,t}\}_{t=0}^{T}$. Some of the θ_i in the *ex post* earnings equation may not appear in the choice equation. Under other information sets, some additional dependence between S_i and $\{\varepsilon_{0,i,t}, \varepsilon_{1,i,t}\}_{t=0}^{T}$ may arise. The contrast between the sources generating realized earnings outcomes and the sources generating dependence between S_i and realized earnings is the essential idea in the analysis of CHN. The method can be generalized to deal with nonlinear prefer-

[13] This test has been extended to a nonlinear setting, allowing for credit constraints, preferences for risk, and the like. See Cunha, Heckman, and Navarro (2005a) and Navarro (2004).

[14] A similar but distinct idea motivates the Flavin (1981) test of the permanent income hypothesis and her measurement of unforecastable income innovations. She picks a particular information set $\widetilde{\mathcal{I}}_{i,0}$ (permanent income constructed from an assumed ARMA (p, q) time series process for income, where she estimates the coefficients given a specified order of the AR and MA components) and tests if $V_{\widetilde{\mathcal{I}}_{i,0}}$ (our notation) predicts consumption. Her test of 'excess sensitivity' can be interpreted as a test of the correct specification of the ARMA process that she assumes generates $\widetilde{\mathcal{I}}_{i,0}$ which is unobserved (by the economist), although she does not state it that way. Blundell and Preston (1998) and Blundell, Pistaferri, and Preston (2004) extend her analysis but, like her, maintain an *a priori* specification of the stochastic process generating $\mathcal{I}_{i,0}$. Blundell, Pistaferri, and Preston (2004) claim to test for 'partial insurance.' In fact their procedure can be viewed as a test of their specification of the stochastic process generating the agent's information set. More closely related to the analysis of CHN is the analysis of Pistaferri (2001), who uses the distinction between expected starting wages (to measure expected returns) and realized wages (to measure innovations) in a consumption analysis.

[15] Keane and Wolpin assume one factor where the θ is a discrete variable and they assume all factor loadings are identical across periods. However, their specification of the uniquenesses or innovations is more general than that used in factor analysis. The analysis of Hartog and Vijverberg (2002) is another example and uses variances of *ex post* income to proxy *ex ante* variability, removing "fixed effects" (person specific θ).

ences and imperfect market environments.[16] A central issue, discussed next, is how far one can go in identifying income information processes without specifying preferences, insurance, and market environments.

5 More general preferences and market settings

To focus on the main ideas in the literature, we have used the simple market structures of complete contingent claims markets. What can be identified in more general environments? In the absence of perfect certainty or perfect risk sharing, preferences and market environments also determine schooling choices. The separation theorem allowing consumption and schooling decisions to be analyzed in isolation that has been used thus far breaks down.

If we postulate information processes *a priori*, and assume that preferences are known up to some unknown parameters as in Flavin (1981), Blundell and Preston (1998) and Blundell, Pistaferri, and Preston (2004), we can identify departures from specified market structures. Cunha, Heckman, and Navarro (2005a) postulate an Aiyagari (1994) – Laitner (1992) economy with one asset and parametric preferences to identify the information processes in the agent's information set. They take a parametric position on preferences and a nonparametric position on the economic environment and the information set.

An open question, not yet fully resolved in the literature, is how far one can go in nonparamet-

[16] In a model with complete autarky with preferences Ψ, ignoring costs,

$$I_i = \sum_{t=0}^{T} E\left[\frac{\Psi\left(X_{i,t}\beta_{1,t}+\theta_i\alpha_{1,t}+\varepsilon_{1,i,t}\right)-\Psi\left(X_{i,t}\beta_{0,t}+\theta_i\alpha_{0,t}+\varepsilon_{0,i,t}\right)}{(1+\rho)^t}\,\middle|\,\widetilde{\mathcal{I}}_{i,0}\right],$$

where ρ is the time rate of discount, we can make a similar decomposition but it is more complicated given the nonlinearity in Ψ. For this model we could do a Sims noncausality test where

$$\begin{aligned} V_{\widetilde{\mathcal{I}}_{i,0}} &= \sum_{t=0}^{T}\frac{\Psi\left(X_{i,t}\beta_{1,t}+\theta_i\alpha_{1,t}+\varepsilon_{1,i,t}\right)-\Psi\left(X_{i,t}\beta_{0,t}+\theta_i\alpha_{0,t}+\varepsilon_{0,i,t}\right)}{(1+\rho)^t} \\ &\quad -\sum_{t=0}^{T} E\left[\frac{\Psi\left(X_{i,t}\beta_{1,t}+\theta_i\alpha_{1,t}+\varepsilon_{1,i,t}\right)-\Psi\left(X_{i,t}\beta_{0,t}+\theta_i\alpha_{0,t}+\varepsilon_{0,i,t}\right)}{(1+\rho)^t}\,\middle|\,\widetilde{\mathcal{I}}_{i,0}\right]. \end{aligned}$$

This requires some specification of Ψ. See Carneiro, Hansen, and Heckman (2003), who assume $\Psi(Y)=\ln Y$ and that the equation for $\ln Y$ is linear in parameters. Cunha, Heckman, and Navarro (2005a) and Navarro (2004) generalize that framework to a model with imperfect capital markets where some lending and borrowing is possible.

rically jointly identifying preferences, market structures and information sets. Cunha, Heckman, and Navarro (2005a) add consumption data to the schooling choice and earnings data to secure identification of risk preference parameters (within a parametric family) and information sets, and to test among alternative models for market environments. Alternative assumptions about what analysts know produce different interpretations of the same evidence. The lack of full insurance interpretation given to the empirical results by Flavin (1981) and Blundell, Pistaferri, and Preston (2004) may be a consequence of their misspecification of the agent's information set generating process. We now present some evidence on *ex ante* vs. *ex post* returns based on the analysis of Cunha and Heckman (2006).

6 Evidence on Uncertainty and Heterogeneity of Returns

Few data sets contain the full life cycle of earnings along with the test scores and schooling choices needed to directly estimate the CHN model and extract components of uncertainty. It is necessary to pool data sets. See CHN who combine NLSY and PSID data sets. We summarize the analysis of Cunha and Heckman (2006) in this subsection. See their paper for their exclusions and identification conditions.

Following the preceding theoretical analysis, they consider only two schooling choices: high school and college graduation.[17] For simplicity and familiarity, we focus on their results based on complete contingent claims markets. Because they assume that all shocks are idiosyncratic and the operation of complete markets, schooling choices are made on the basis of expected present value income maximization. Carneiro, Hansen, and Heckman (2003) assume the absence of any credit markets or insurance. Cunha, Heckman, and Navarro (2005a) check whether their empirical findings about components of income inequality are robust to different assumptions about the availability of credit markets and insurance markets. They estimate an Aiyagari-Laitner economy with a single asset and borrowing constraints and discuss risk aversion and the relative importance of uncertainty. We summarize the evidence from alternative assumptions about market structures below.

[17] Heckman and Navarro (2006) present a model with multiple schooling levels.

6.1 Identifying Joint Distributions of Counterfactuals and the Role of Costs and Ability as Determinants of Schooling

Suppose that the error term for $Y_{s,t}$ is generated by a two factor model,

$$Y_{s,t} = X\beta_{s,t} + \theta_1\alpha_{s,t,1} + \theta_2\alpha_{s,t,2} + \varepsilon_{s,t}. \tag{13}$$

We omit the "i" subscripts to eliminate notational burden. Cunha and Heckman (2006) report that two factors are all that is required to fit the data.

They use a test score system of K ability tests:

$$T_{jT} = X_T\omega_{jT} + \theta_1\alpha_{jT} + \varepsilon_{jT}, \qquad j = 1, \ldots, K \tag{14}$$

Thus factor 1 is identified as an ability component. The cost function C is specified by:

$$C = Z\gamma + \theta_1\alpha_{C,1} + \theta_2\alpha_{C,2} + \varepsilon_C. \tag{15}$$

They assume that agents know the coefficients of the model and X, Z, ε_C and some, but not necessarily all, components of θ. Let the components known to the agent be $\bar{\theta}$. The decision rule for attending college is based on:

$$\begin{aligned} S^* &= E\left(\sum_{t=0}^{T} \frac{Y_{1,t} - Y_{0,t}}{(1+r)^t} \mid X, \bar{\theta}\right) - E\left(C \mid Z, X, \bar{\theta}, \varepsilon_C\right) \\ S &= \mathbf{1}\left(S^* \geq 0\right). \end{aligned} \tag{16}$$

Cunha and Heckman (2006) report evidence that the estimated factors are highly nonnormal.[18]

[18] They assume that each factor $k \in \{1, 2\}$ is generated by a mixture of J_k normal distributions,

$$\theta_k \sim \sum_{j=1}^{J_k} p_{k,j}\phi\left(f_k \mid \mu_{k,j}, \tau_{k,j}\right),$$

where the $p_{k,j}$ are the weights on the normal components.

Table 1 presents the conditional distribution of *ex post* potential college earnings given *ex post* potential high school earnings, decile by decile, as reported by Cunha and Heckman (2006). The table displays a positive dependence between the relative positions of individuals in the two distributions. However, the dependence is far from perfect. For example, about 10% of those who are at the first decile of the high school distribution would be in the fourth decile of the college distribution. Note that this comparison is not being made in terms of positions in the overall distribution of earnings. CHN can determine where individuals are located in the distribution of population potential high school earnings and the distribution of potential college earnings although in the data we only observe them in either one or the other state. Their evidence shows that the assumption of perfect dependence across components of counterfactual distributions that is maintained in much of the recent literature (e.g. Juhn, Murphy, and Pierce, 1993) is far too strong.

Figure 1 presents the marginal densities of predicted (actual) earnings for high school students and counterfactual college earnings for actual high school students. When we compare the densities of present value of earnings in the college sector for persons who choose college against the counterfactual densities of college earnings for high school graduates we can see that many high school graduates would earn more as college graduates. In Figure 2 we repeat the exercise, this time for college graduates.

Table 2 from Cunha and Heckman (2006) reports the fitted and counterfactual present value of earnings for agents who choose high school. The typical high school student would earn $968.51 thousand dollars over the life cycle. She would earn $1,125.78 thousand if she had chosen to be a college graduate.[19] This implies a return of 20% to a college education over the whole life cycle (i.e., a monetary gain of $157.28 thousand dollars). In table 3, from Cunha and Heckman (2006), the typical college graduate earns $1,390.32 thousand dollars (above the counterfactual earnings of what a typical high school student would earn in college), and would make only $1,033.72 thousand dollars over her lifetime if she chose to be a high school graduate instead. The returns to college education for the typical college graduate (which in the literature on program evaluation is referred to as the effect of Treatment on the Treated) is around 38% above that of the return for a high

[19] These numbers may appear to be large but are a consequence of using a 3% discount rate.

school graduate.

Figure 3 plots the density of *ex post* gross returns to education excluding direct costs and psychic costs for agents who are high school graduates (the solid curve), and the density of returns to education for agents who are college graduates (the dashed curve). In reporting our estimated returns, CHN follow conventions in the literature on the returns to schooling and present growth rates in terms of present values, and not true rates of return. Thus they insure option values. These figures thus report the growth rates in present values $\left(\frac{PV(1)-PV(0)}{PV(0)}\right)$ where "1" and "0" refer to college and high school and all present values are discounted to a common benchmark level. Tuition and psychic costs are ignored. College graduates have returns distributed somewhat to the right of high school graduates, so the difference is not only a difference for the mean individual but is actually present over the entire distribution. An economic interpretation of figure 3 is that agents who choose a college education are the ones who tend to gain more from it.

With their methodology, CHN can also determine returns to the marginal student. Table 4 reveals that the average individual who is just indifferent between a college education and a high school diploma earns \$976.04 thousand dollars as a high school graduate or \$1,208.26 thousand dollars as a college graduate. This implies a return of 28%. The returns to people at the margin are above those of the typical high school graduate, but below those for the typical college graduate. Since persons at the margin are more likely to be affected by a policy that encourages college attendance, their returns are the ones that should be used in order to compute the marginal benefit of policies that induce people into schooling.

6.2 *Ex ante* and *Ex post* Returns: Heterogeneity versus Uncertainty

Figures 4 through 6, from Cunha and Heckman (2006) separate the effect of heterogeneity from uncertainty in earnings. The figures plot the distribution of *ex ante* and *ex post* outcomes. The information set of the agent is $\mathcal{I} = \{X, Z, X_T, \varepsilon_C, \Theta\}$, Θ contains some or all of the factors. In their papers, the various information sets consist of different components of θ. We first consider figure 4. It presents results for a variety of information sets. First assume that agents do not know their factors; consequently, $\Theta = \varnothing$. This is the case in which all of the unobservables are treated as unknown by the agent, and, as a result, the density has a large variance. If we assume that the agents know factor 1 but not factor 2,[20] so that $\Theta = \{\theta_1\}$, there is a reduction in the forecast variance, but it is small. Factor 1, which is associated with cognitive ability, is important for forecasting educational choices, but does not do a very good job in forecasting earnings. The third case is the one in which the agent knows both factors, which is the case we test and cannot reject. The agent is able to substantially reduce the forecast variance of earnings in high school. Note the the variance in this case is much smaller than in the other two cases. Figure 6 reveals much the same story about the college earnings distribution.

Table 5 presents the variance of potential earnings in each state, and returns under different information sets available to the agent. We conduct this exercise for lifetime earnings, and report baseline variances and covariances without conditioning and state the remaining uncertainty as a fraction of the baseline no-information state variance when different components of θ are known to the agents. CHN—who estimate a three factor model—show that both θ_1 and θ_2 are known to the agents at the time they make their schooling decisions, but not θ_3.

This discussion sheds light on the issue of distinguishing predictable heterogeneity from uncertainty. CHN demonstrate that there is a large dispersion in the distribution of the present value of earnings. This dispersion is largely due to heterogeneity, which is forecastable by the agents at the time they are making their schooling choices. CHN provide tests that determine that agents know θ_1 and θ_2. The remaining dispersion is due to luck, or uncertainty or unforecastable factors as of age 17. Its contribution is smaller.

It is interesting to note that knowledge of the factors enables agents to make better forecasts. Figure 6 presents an exercise for returns to college $(Y_1 - Y_0)$ similar to that presented in figures 4 and 5 regarding information sets available to the agent. Knowledge of factor 2 also greatly improves the forecastability of returns. 56% of the variability in returns is forecastable at age 18. The levels also show high predictability (65% for high school; 56% for college). Most variability across people is due to heterogeneity and not uncertainty. However there is still a lot of variability in agent earnings after accounting for what is in the agent's information set. This is intrinsic uncertainty at

[20] As opposed to the econometrician who never gets to observe either θ_1 or θ_2.

the time agents make their schooling choices.

6.3 *Ex Ante* versus *Ex Post*

Once the distinction between heterogeneity and uncertainty is made, it is possible to be precise about the distinction between *ex ante* and *ex post* decision making. From their analysis, CHN conclude that, at the time agents pick their schooling, the ε's in their earnings equations are unknown to them. These are the components that correspond to "luck." It is clear that decision making would be different, at least for some individuals, if the agent knew these chance components when choosing schooling levels, since the decision rule would be based on (4) where all components of $Y_{1,i}$, $Y_{0,i}$, and C_i are known, and no expectation need be taken.

If individuals could pick their schooling level using their *ex post* information (i.e., after learning their luck components in earnings) 13.81% of high school graduates would rather be college graduates and 17.15% of college graduates would have stopped their schooling at the high school level. Using the estimated counterfactual distributions, it is possible to consider a variety of policy counterfactuals on distributions of outcomes locating persons in pre- and post-policy distributions. They analyze for example, how tuition subsidies move people from one quantile of a Y_0 distribution to another quantile of a Y_1 distribution. See Carneiro, Hansen, and Heckman (2001, 2003), Cunha, Heckman, and Navarro (2005b, 2006) and Cunha and Heckman (2006) for examples of this work.

7 Extensions and Alternative Specifications

Carneiro, Hansen, and Heckman (2003) estimate a version of the model just presented for an environment of complete autarky. Individuals have to live within their means each period. Cunha, Heckman, and Navarro (2005a) estimate a version of this model with restriction on intertemporal trade as in the Aiyagari-Laitner economy. Different assumptions about credit markets and preferences produce a range of estimates of the proportion of the total variability of returns to schooling that are unforecastable, ranging from 37% (Carneiro, Hansen, and Heckman, 2003) for a model with complete autarky and log preferences, to 53% (CHN) for complete markets, to 44% (Cunha and Heckman, 2006) for another complete market economy.

This line of work has just begun. It shows what is possible with rich panel data. The empirical evidence on the importance of uncertainty is not yet settled. Yet most of the papers developed within this research program suggest a substantial role for uncertainty in producing returns. Accounting for uncertainty and psychic costs may help to explain the sluggish response of schooling enrollment rates to rising returns to schooling that is documented in Ellwood and Kane (2000) and Card and Lemieux (2001) because of the wedge between utility and money returns.

8 Models with Sequential Updating of Information

We have thus far discussed one shot models of schooling choice. In truth, schooling is a sequential decision process made with increasingly richer information sets at later stages of the choice process. Keane and Wolpin (1997) and Eckstein and Wolpin (1999) pioneered the estimation of dynamic discrete choice models for analyzing schooling choices. They assume a complete market environment and do not consider the range of possible alternative market structures facing agents. In the notation of this paper, they assume a one (discrete) factor model with factor loadings that are different across different counterfactual states, but are constant over time ($\alpha_{s,t} = \alpha_s$, $s = 1, \ldots, \bar{S}$ where there are $\bar{S}$ states).[21] At a point in time, t, $\varepsilon_{s,t}$, $s = 1, \ldots, \bar{S}$ are assumed to be multivariate normal random variables. Over time the $\varepsilon_t = (\varepsilon_{1,t}, \ldots, \varepsilon_{s,t})$ are assumed to be independent and identically distributed. They assume agents know θ but not the ε_t, $t = 0, \ldots, T$. The unobservables are thus equicorrelated over time (age) because the factor loadings are assumed equal over time and ε_t is independent and identically distributed over time. They make parametric normality assumptions in estimating their models. They impose and do not test the particular information structure that they use.[22] In their model, about 90% of the variance in lifetime returns is predictable at age 16. Their estimate is at the extreme boundary of the estimates produced from the CHN analysis.

[21] Thus instead of assuming that θ is continuous, as do CHN, they impose that θ is a discrete-valued random variable that assumes a finite, known number of values.

[22] Keane and Wolpin (1997) impose their discrete factor in the schooling choice and outcome equations rather than testing for whether or not the factor appears in both sets of equations in the fashion of CHN as previously described.

Heckman and Navarro (2006) formulate and identify semiparametric sequential schooling models based on the factor structures exposited in this paper. Like Keane and Wolpin, they assume that a complete contingent claims market governs the data. They report substantial effects on empirical estimates from relaxing normality assumptions. Using their framework, it is possible to test among alternative information structures about the arrival of information on the components of vector θ at different stages of the life cycle. Preliminary analysis based on their model supports the analysis of CHN in finding a sizable role for heterogeneity (predictable variability) in accounting for measured variability. They estimate the sequential reduction of uncertainty as information is acquired. Their analysis shows that estimated option values for attending college are relatively small (at most 2% of the total return). This is consistent with a lot of information known about future returns at the time schooling decisions are being made. If their results hold up in subsequent analyses and replications, they imply that the theoretical possibilities that arise from accounting for option values may be empirically unimportant in computing rates of return.[23]

9 Summary and Conclusions

This paper surveys the main models and methods developed in Carneiro, Hansen, and Heckman (2003), Cunha, Heckman, and Navarro (2005a,b, 2006), Cunha and Heckman (2006) and Heckman and Navarro (2006) for estimating models of heterogeneity and uncertainty in the returns to schooling. The goal of this work is to separate variability from uncertainty and to estimate the distributions of *ex ante* and *ex post* returns to schooling. The key idea in the recent literature is to exploit the relationship between realized earnings and schooling choice equations to determine which components of realized earnings are in agent information sets at the time they make their schooling decisions. For a variety of market environments and assumptions about preferences, a robust empirical regularity is that over 50% of the *ex post* variance in the returns to schooling are forecastable at the time students make their college choices.

[23] Assuming that initial conditions are known by agents, the estimates of Keane and Wolpin (1997) are consistent with small option values although they do not report the option values implicit in their estimates.

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Table 1: Ex-Ante Conditional Distributions for the NLSY/1979 (College Earnings Conditional on High School Earnings)
Pr(d_i<Yc<d_i+1 | d_j<Yh<d_j+1) where d_i is the ith decile of the College Lifetime Ex-Ante Earnings Distribution and d_j is the jth decile of the High School Ex-Ante Lifetime Earnings Distribution
Individual fixes known θ at their means, so Information Set={θ_1=0, θ_2=0}
Corrrelation(Y_C,Y_H) = 0.4083

	College									
High School	1	2	3	4	5	6	7	8	9	10
1	0.1833	0.1631	0.1330	0.1066	0.0928	0.0758	0.0675	0.0630	0.0615	0.0535
2	0.1217	0.1525	0.1262	0.1139	0.1044	0.0979	0.0857	0.0796	0.0683	0.0498
3	0.1102	0.1263	0.1224	0.1198	0.1124	0.0970	0.0931	0.0907	0.0775	0.0506
4	0.0796	0.1083	0.1142	0.1168	0.1045	0.1034	0.1121	0.1006	0.0953	0.0652
5	0.0701	0.0993	0.1003	0.1027	0.1104	0.1165	0.1086	0.1112	0.1043	0.0768
6	0.0573	0.0932	0.1079	0.1023	0.1110	0.1166	0.1130	0.1102	0.1059	0.0825
7	0.0495	0.0810	0.0950	0.1021	0.1101	0.1162	0.1202	0.1174	0.1134	0.0950
8	0.0511	0.0754	0.0770	0.1006	0.1006	0.1053	0.1244	0.1212	0.1297	0.1147
9	0.0411	0.0651	0.0841	0.0914	0.1039	0.1117	0.1162	0.1216	0.1442	0.1206
10	0.0590	0.0599	0.0622	0.0645	0.0697	0.0782	0.0770	0.1028	0.1181	0.3087

Table 2
Average present value of ex post earnings[1] for high school graduates
Fitted and Counterfactual[2]
White males from NLSY79

	High School (Fitted)	College (Counterfactual)	Returns[3]
Average	968.5100	1125.7870	0.2055
Std. Err.	*7.9137*	*9.4583*	*0.0113*

[1] Thousands of dollars. Discounted using a 3% interest rate.

[2] The counterfactual is constructed using the estimated college outcome equation applied to the population of persons selecting high school.

[3] As a fraction of the base state, ie (PVearnings(Col)-PVearnings(HS))/PVearnings(HS).

Table 3
Average present value of ex post earnings[1] for college graduates
Fitted and Counterfactual[2]
White males from NLSY79

	High School (Counterfactual)	College (fitted)	Returns[3]
Average	1033.721	1390.321	0.374
Std. Err.	*14.665*	*30.218*	*0.280*

[1] Thousands of dollars. Discounted using a 3% interest rate.

[2] The counterfactual is constructed using the estimated high school outcome equation applied to the population of persons selecting college

[3] As a fraction of the base state, ie (PVearnings(Col)-PVearnings(HS))/PVearnings(HS).

Table 4

Average present value of ex post earnings[1] for individuals at margin

Fitted and Counterfactual[2]

White males from NLSY79

	High School	College	Returns[3]
Average	976.04	1208.26	0.2828
Std. Err.	*21.503*	*33.613*	*0.0457*

[1] Thousands of dollars. Discounted using a 3% interest rate.

[2] 2 It defines the result of taking a person at random from the population regardless of his schooling choice.

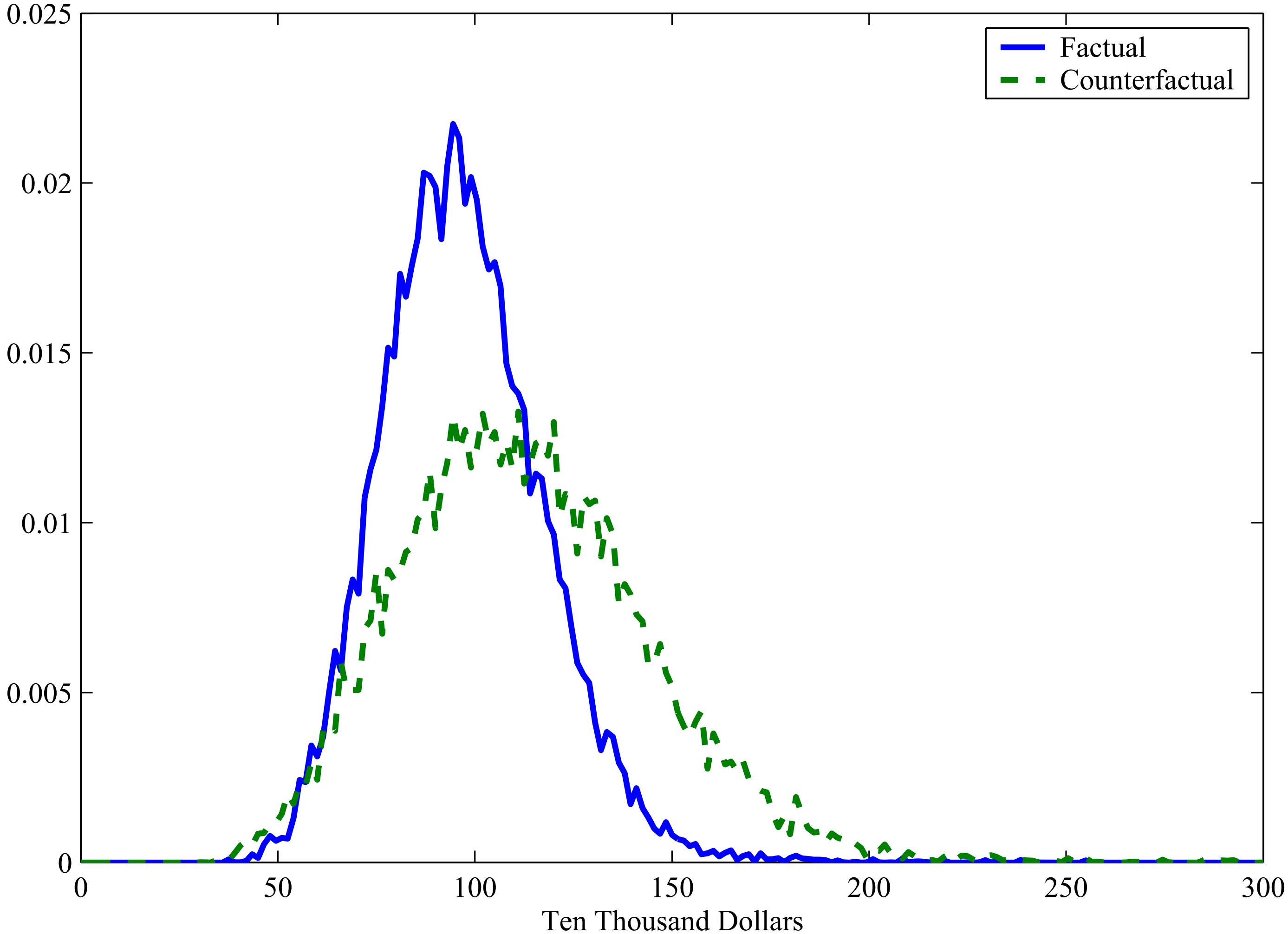
[3] As a fraction of the base state, ie (PVearnings(Col)-PVearnings(HS))/PVearnings(HS).

Table 5
Agent's Forecast Variance of Present Value of Earnings*
Under Different Information Sets - NLSY/1979

	Var(Y_c)	Var (Y_h)	Var(Y_c-Y_h)
For lifetime:			
Total Residual Variance	290.84	103.13	334.02
Share of Total Variance due to Forecastable Components	65.13%	55.94%	56.04%
Share of Total Variance due to Unforecastable Components	34.87%	44.06%	43.94%

*We use a discount rate ρ of 3% to calculate the present value of earnings.

Figure 1
Densities of present value of lifetime earnings for High School Graduates
Factual and Counterfactual NLSY/1979 Sample

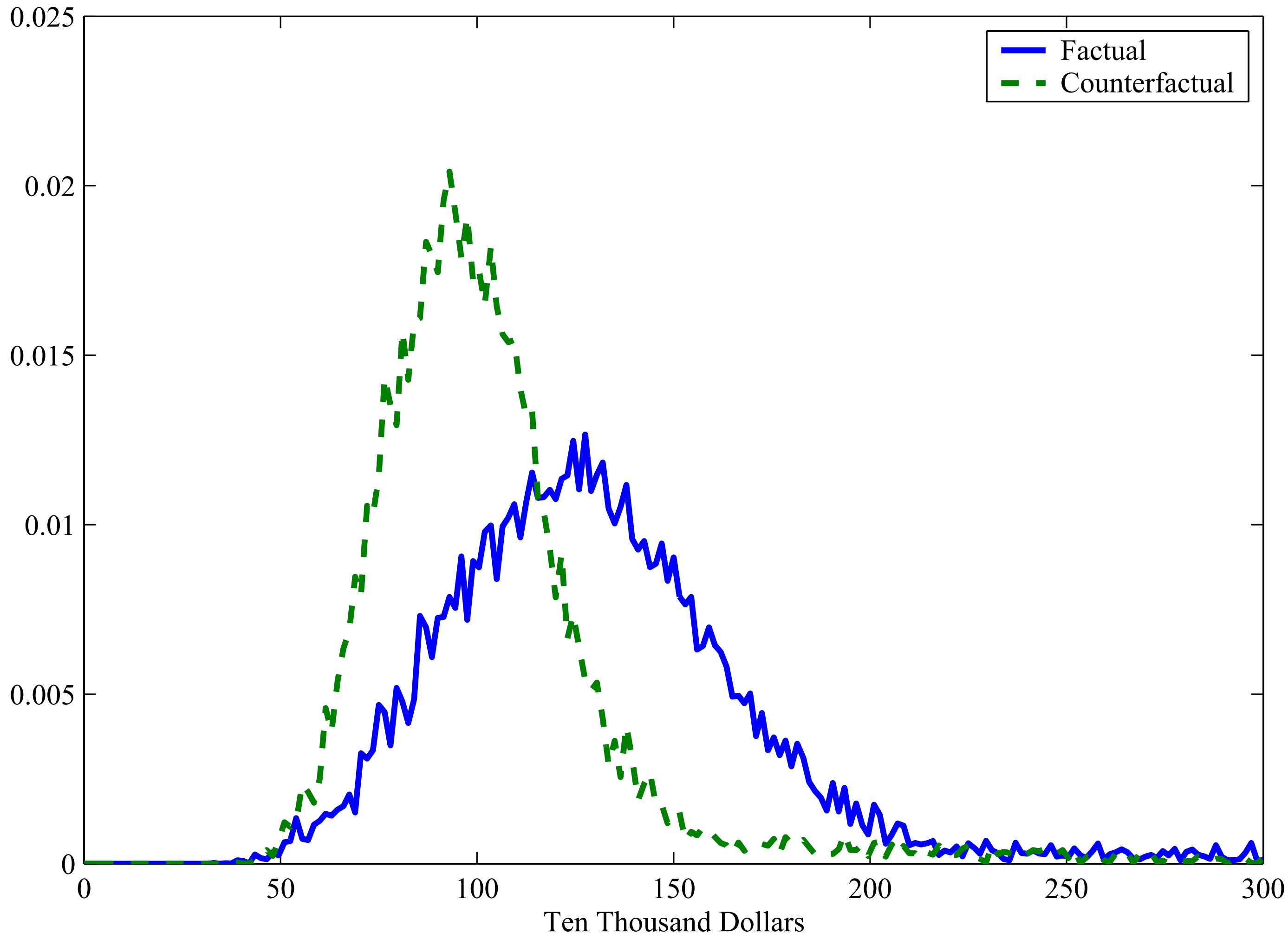


Present Value of Lifetime Earnings from age 18 to 65 for high school graduates using a discount rate of 3%. Let Y_0 denote present value of earnings in high school sector. Let Y_1 denote present value of earnings in college sector. In this graph we plot the factual density function $f(y_0 \mid S=0)$ (the solid line), against the counterfactual density function $f(y_1 \mid S=0)$. We use kernel density estimation to smooth these functions.

Figure 2

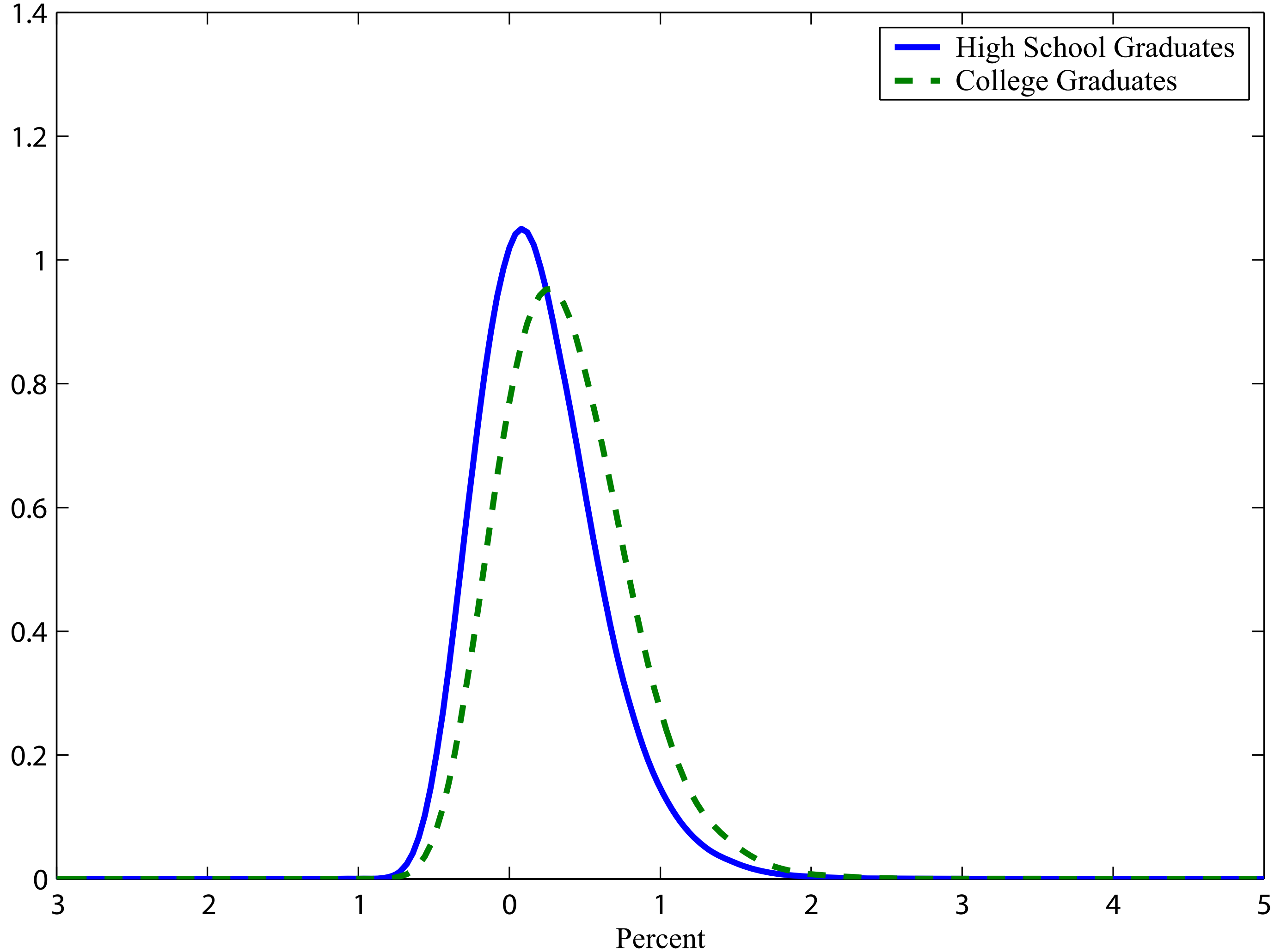
Densities of present value of earnings for College Graduates

Factual and Counterfactual NLSY/1979 Sample



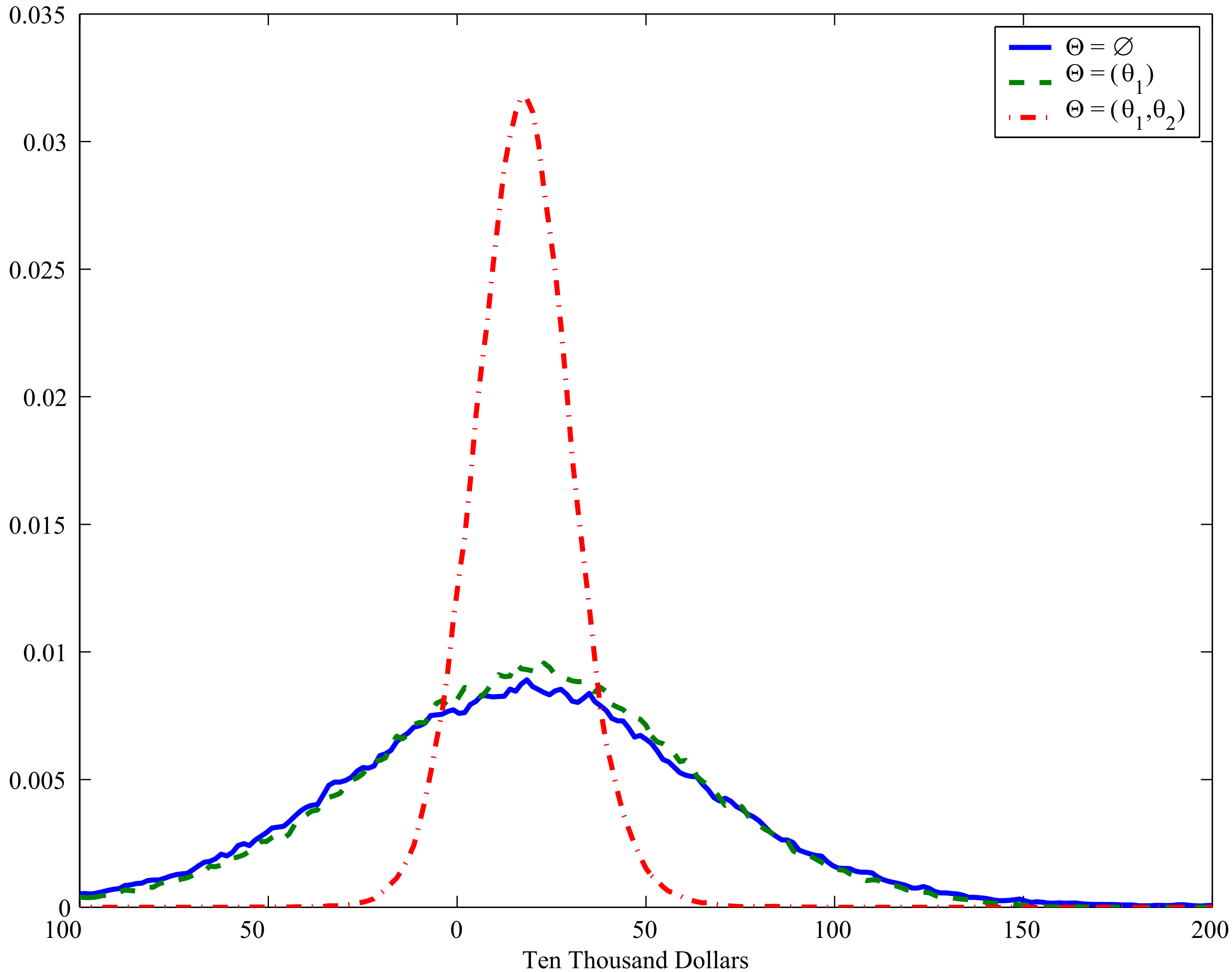
Present Value of Lifetime Earnings from age 18 to 65 for high school graduates using a discount rate of 3%. Let Y_0 denote present value of earnings in high school sector. Let Y_1 denote present value of earnings in college sector. In this graph we plot the counterfactual density function $f(y_1 \mid S=0)$ (the dashed line), against the factual density function $f(y_1 \mid S=0)$. We use kernel density estimation to smooth these functions.

Figure 3
Densities of Returns to College
NLSY/1979 Sample



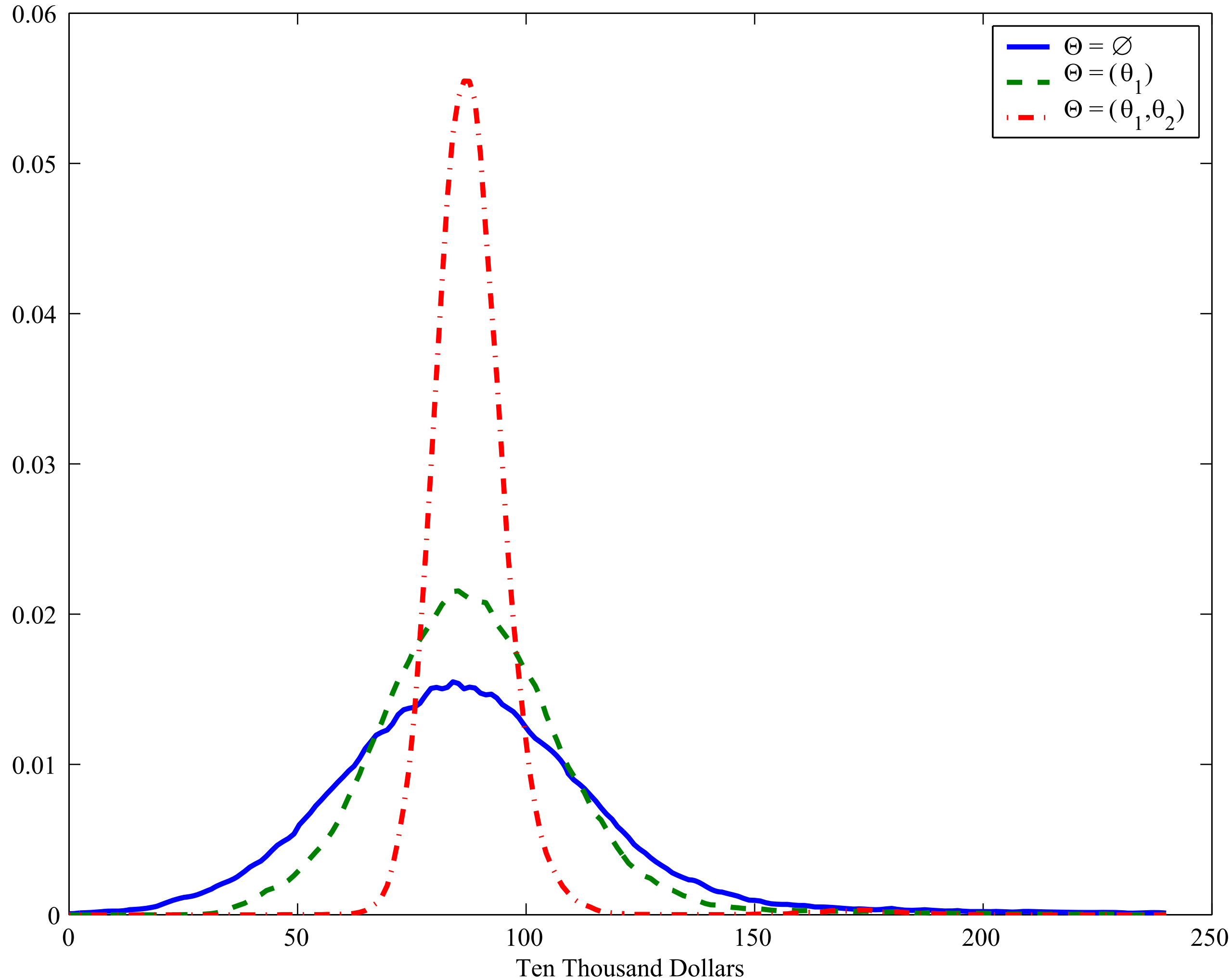
Let Y_0 denote present value of earnings in high school sector. Let Y_1 denote present value of earnings in college sector. Let $R = (Y_1 - Y_0)/Y_0$ denote the gross rate of return to college. In this graph we plot the density function of the returns to college conditional on being a high school graduate, $f(r \mid S=0)$ (the solid line), against the density function of returns to college conditional on being a college graduate, $f(r \mid S=1)$. We use kernel density estimation to smooth these functions.

Figure 4
densities of present value of returns - NLSY/1979
under different information sets for the agent calculated
for the entire population irregardless of schooling choice



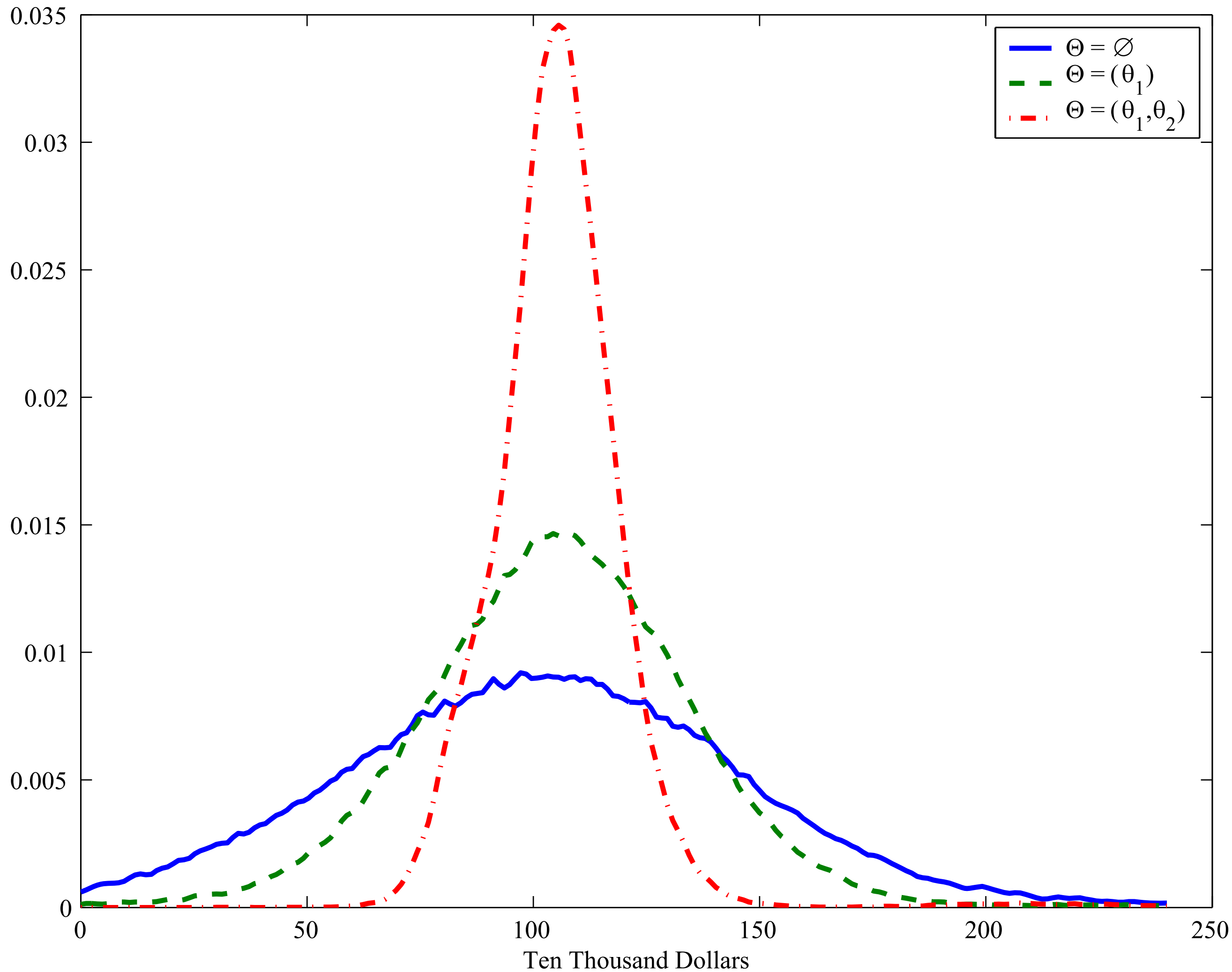
Let Θ denote the information set of the agent. Let Y_0 denote the present value of returns (discounted at a 3% interest rate). Let $f(y_0|\Theta)$ denote the density of Y_0 conditional on information set Θ. The solid line plots the density of Y_0 when $\Theta=\phi$. The dashed line plots the density of Y_0 when $\Theta=\{\theta_1\}$.The dotted and dashed line plots the density of of Y_0 when $\Theta=\{\theta_1,\theta_2\}$. The X variables are in the information set of the agent. The factors θ, when known, are evaluated at their mean, which is zero.

Figure 5
densities of present value of high school earnings - NLSY/1979
under different information sets for the agent calculated
for the entire population irregardless of schooling choice



Let Θ denote the information set of the agent. Let Y_0 denote the present value of earnings in the high school sector (discounted at a 3% interest rate). Let $f(y_0|\Theta)$ denote the density of Y_0 conditional on information set Θ. The solid line plots the density of Y_0 when $\Theta=\phi$. The dashed line plots the density of Y_0 when $\Theta=\{\theta_1\}$. The dotted and dashed line plots the density of of Y_0 when $\Theta=\{\theta_1,\theta_2\}$. The X variables are in the information set of the agent. The factors θ, when known, are evaluated at their mean, which is zero.

Figure 6
densities of present value of college earnings - NLSY/1979
under different information sets for the agent calculated
for the entire population irregardless of schooling choice



Let Θ denote the information set of the agent. Let Y_0 denote the present value of earnings in the college sector (discounted at a 3% interest rate). Let $f(y_0|\Theta)$ denote the density of Y_0 conditional on information set Θ. The solid line plots the density of Y_0 when $\Theta=\phi$. The dashed line plots the density of Y_0 when $\Theta=\{\theta_1\}$. The dotted and dashed line plots the density of of Y_0 when $\Theta=\{\theta_1,\theta_2\}$. The X variables are in the information set of the agent. The factors θ, when known, are evaluated at their mean, which is zero.