Education Choice under Uncertainty: Implications for Public Policy*

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Abstract

We analyse how progressive taxation and education subsidies affect schooling decisions when the returns to education are stochastic. We use the theory of real options to solve the problem of education choice in a dynamic stochastic model. We show that education attainment will be an increasing function of the risk associated with education. Furthermore, this result holds regardless of the degree of risk aversion. We also show that progressive taxes will tend to reduce education attainment.

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1 Introduction

This paper examines the effect of public policy on individual education choices in a theoretical framework that allows both for intertemporal optimisation and uncertainty. In doing so we build upon a large literature that views education choice as an investment in human capital, to be thought of in much the same way as we think of investment in physical or financial capital (see Card, 2001, for a comprehensive survey). Nevertheless, the concept of risk – routinely included in theoretical and empirical discussions of other investment – is often absent from discussions of individual schooling choice.

This is a curious omission as the risk associated with education choices will likely be an important determinant of how individuals arrive at those choices. Dominitz and Manski (1996) show that individuals believe that education carries substantial risk. Carneiro et al. (2003a) show that only 9% of the variance of returns to a college education are forecastable by individuals at the time of making college choices and that over one third of college graduates experience negative returns to their college education.

Our approach to modelling risk in education is to view education choice as an option problem.[1] We think of an individual in school as possessing an option to leave at any time and take up work at a wage related (stochastically) to the time spent in school. This approach is a close approximation to reality – at least for formal schooling and initial college education. Most individuals stay in education full-time until they judge it optimal to leave, and after leaving, they do not return. Empirically, in the OECD as a whole, only 6.4% of those aged 25-29 years are still in education (full or part-time), while in the UK, over 90% of college students have come directly from school.[2]

Using our model, we show that risk interacts with the education decision in some unexpected ways. Firstly, higher risk encourages individuals to accumulate *more* human capital whereas we might have expected risk aversion to lead to less investment in a more risky asset. As we show below, this result stems from the option structure of the problem and does not depend on the degree of risk aversion. Individuals can avoid bad draws by staying in school but can leave to take advantage of the good draws. Thus higher uncertainty increases the upside payoff by more than the downside, making education more attractive.

We use our model to analyse the effects of tax and education policy. Specifically, we simulate the response of individuals to a variety of policy measures (fee reductions, tax

[1] The application of option theory to various economic problems has been analysed in Malliaris and Brock (1982), Kamien and Schwartz (1991) and Dixit (1993). For specific examples from financial investment see Merton (1971); for physical capital see Caballero and Engle (1999); for irreversible physical investment (so called "real options") see Dixit and Pindyck (1994).

[2] See www.hesa.ac.uk and and Table E3.1 of OECD (2001) a summary of which is available from www.oecd.org

increases, reduction in progressivity of the tax system etc.). We build upon a large literature including Trostel (1993) and Heckman et al. (1998) who examine the effects of tax policy in dynamic general equilibrium models under certainty; Eaton and Rosen (1980) and Altonji (1993) who examine policy effects in stochastic two period models; Keane and Wolpin (1997) who estimated an empirical dynamic model of education choice and Williams (1979) who adapted the portfolio choice model of Merton (1971) to allow for investment in human capital.

As in Keane and Wolpin (1997) we show that a tuition subsidy could increase graduation rates. In contrast to Trostel (1993) and in line with Eaton and Rosen (1980), we find that proportional tax increases can actually increase education attainment. We also show that increases in progressivity can reduce the education attainment.

The paper proceeds as follows. Section 2 presents an overview of the problem and clarifies exactly how we model stochastic returns. We also solve a model of education choice with uncertain returns. Section 3 considers the policy implications of the model. Section 4 discusses some extensions and section 5 concludes.

2 Education Choice

We start with a model of education choice similar to Card (2001) with the exception that we allow for stochastic returns to education. An individual chooses the number of years schooling (S) in order to maximise his or her expected discounted life time utility (1) subject to a budget constraint (2).

$$V_o = E\left\{ \int_0^S e^{-\rho t} \left\{ u(c_t) + \phi_t \right\} dt + \int_S^T e^{-\rho t} u(c_t) dt \right\} \tag{1}$$

$$\begin{aligned} c_t &= \bar{c} \qquad & t < S \\ c_t &= Y_t(s) \qquad & t \geq S \end{aligned} \tag{2}$$

Assuming that the minimum school leaving age is normalised to $t = 0$, lifetime utility is provided by consumption (c) throughout life (i.e. both during and after school) via u, the instantaneous utility function and also by the direct (dis)utility of education, ϕ, where u is an increasing concave function and ρ is the constant rate of time preference. In order to focus on risk and the education decision, we abstract from the intertemporal consumption decision by assuming that there is no borrowing or lending, so that $c_t = \bar{c} \;\; \forall \; t < S$ and $c_t = Y_t(S) \;\; \forall \; t \geq S$ where $\bar{c}$ is the level of consumption (net of fees) pre graduation. Another interpretation of this budget constraint is that the agent is allowed smooth consumption in pre-graduation period and, separately, in the post graduation period. But she is restricted from smoothing across these two periods i.e. expected future income from education cannot be used subsidize living standards in education.[3]

Education choice is an optimal stopping problem, because the individual faces a once and for all decision to leave school (i.e. choose S) and he or she cannot return at a later date.[4] Implicitly we are assuming that there are psychic and financial fixed costs of returning to education that are sufficiently large to prevent return for all practical values of the other parameters of the model. As we discuss in section 4, relaxing this assumption (so that an individual can return at some finite cost) does not change our fundamental results. Note also that as the problem is literally an option problem, it is best suited to analysing education choice after the end of compulsory education. To this end, we interpret S as being the time spent in *post-compulsory* education and ignore the case of those who leave school early in violation of the law.

2.1 Risky Education

We can think of $Y(S)$ as the starting wage after leaving education with S years completed (or as the shadow wage of staying in education beyond S). The actual wage may grow after graduation as experience and seniority are accumulated. We identify education with time spent in school and college and not necessarily with the accumulation of formal credentials. Of course the two are closely related, but there is empirical evidence of so-called "sheep-skin" effects i.e. non-linearities in earnings associated with school and college completion dates.[5] Allowing for these effects would complicate the analysis without shedding much light on the role of risk.

Consider staying on in school for κ more periods. The return to this extra schooling, $r(\kappa)$, will equal

$$r(\kappa) \equiv \frac{Y(S+\kappa) - Y(S)}{Y(S)} \quad \backsim \quad N(g\kappa, \kappa\sigma^2)$$

which we assume is distributed as a normal random variable with mean g and standard deviation σ when $\kappa = 1$. Thus, in this context, risk means that two otherwise identical individuals may end up with different lifetime income profiles, just because of a different

[3] For an example relaxing this constraint, see Hogan and Walker (2002).

[4] This is a close approximation to reality – at least for formal schooling and initial college education. Most individuals stay in education full-time until they judge it optimal to leave, and after leaving, they do not return. The alternative, treating education as occurring continuously and at the same time as work is better suited to the study of on the job training (see Williams, 1979).

[5] See Denny and Harmon (2001). Altonji (1993) presents a three period model of college attendance with stochastic returns (via uncertain graduation) and sheepskin effects.

draw from the distribution of returns to education. By taking limits, we can show that, in continuous time, the return to a infinitesimally small extra period in school ($r \equiv dY/Y$) will be distributed as $N(gdS, \sigma^2 dS)$ implying that the shadow wage, Y, follows a geometric Brownian motion

$$\frac{dY}{Y} = gdS + \sigma dz \tag{3}$$

where dz represents the increments of a standard Weiner process i.e. where each increment is drawn from $N(0, dS)$. Note that in the absence of uncertainty ($\sigma^2 = 0$) the shadow wage process (3) reduces to $Y(S) = \exp(gS)$. Thus (3) is essentially a continuous time stochastic version of the standard Mincer (1974) earnings equation. Following our interpretation of S as post-compulsory schooling, we interpret $Y(0)$ as being the income profile of an individual with only the minimum education required by law and not as income of those with absolutely no education.

Equation (3) states that for each instant that the individual remains in school her shadow wage trends up at rate g. In addition, at each instant, the shadow wage is subject to a (proportionate) shock that has zero mean and variance equal to σ^2. Therefore even if individuals start with the same (deterministic) $Y(0)$ they will end up with different $Y(S)$.[6]

We can generalise (3) slightly by modeling the expected return to education as linear function of Y as in (4). In the case of the Brownian motion, $g_1 = 0$ whereas if $g_1 > 0$ then (4) is to an Ornstein-Uhlenbeck process. We will use this process below to model diminishing expected returns to education.[7]

$$\frac{dY}{Y} = (g_0 - g_1 Y)ds + \sigma dz \tag{4}$$

Note an implications of the specification (4). The increments of the shadow wage are normally distributed with both the mean and variance growing linearly with schooling i.e. $var(r) = \sigma^2 dS \neq (\sigma dS)^2$.[8] The empirical evidence for this is mixed. Simple tabulations of data from the U.K. Labour Force Survey (available on request) show the variance of earnings rising with school in an approximately linear manner. The specification is also consistent with empirical results of Harmon et al. (2003). Furthermore, Carneiro et al. (2003b) estimated a model controlling for selection in to education, to show that the variance of

[6] We treat $Y(0)$ as being deterministic as it will be known to the agent by the time she comes to make her education decision.

[7] Note that we have specified the return to education to be a diminishing function of the shadow wage and not a of elapsed schooling time. We do this for analytical convenience so as to avoid getting a partial deferential equation with time as a state variable.

[8] Judd (1998) models education risk explicitly as an implication of moral hazard. In his formulation, risk acts like a fixed cost of entry to the initial level of education and does not impact on the marginal effect of education above that level i.e. in our notation $r \sim N(gdS, \sigma^2)$

college earnings was higher than the variance of high school earnings. In contrast Belzil and Hansen (2002) found the opposite and Chen (2003) found no difference between the two.

2.2 Solving The Model

In this section, we solve the model where education returns are given by the Ornstein-Uhlenbeck process (4). The solution for the Brownian Motion (3) can be derived as a special case of the general result when $g_1 = 0$. We assume that utility is CRRA (i.e. $u(c_t) = c^{1-\gamma}/(1-\gamma)$). In principle ϕ could be negative if education is intrinsically disliked. However, as education returns could be constant, we will assume that ϕ is constant through time and positive, in order to avoid the corner solution (leave school immediately). Similarly we also assume that $\rho > g_0$ (otherwise the agent would never leave school in absence of diminishing returns). Finally, we assume that individuals are infinitely lived ($T = \infty$) so that time is not a state variable.

The intuition of the option approach is straight-forward. At any point in time, while the individual is still in school, she has the option of leaving school. This option itself has value. If she exercises this option she will loose the value of the option (because he cannot return to school in the future) and will receive a life time income that is a function of accumulated schooling. If she chooses not to exercise the option, she will receive whatever in-school income/utility she has and will wait until next period when she will have the chance to exercise the option again. By this time the value of the option will have changed in a manner related to the underlying process for the shadow wage given by (4). The resulting capital gain or loss is uncertain when viewed from the previous period. So exercising (or not) the option involves taking a gamble.

More formally, V_t in (1) can be thought of as the value of the option to leave school and start earning income at time t. Assuming that we don't exercise the option (i.e. for $t \in [0..S]$) then we can write equation (5) to describe how V will change over time.

$$\rho V_t dt = (u(\bar{c}) + \phi)\, dt + E\{dV_t\} \tag{5}$$

This Bellman equation (5) can best be understood as an arbitrage equation.[9] The right hand side is the return from staying in school (i.e. holding the option) for length of time dt. It consists of the dividend received over the period (which in our case is the constant utility derived from education) and the expected capital gain or loss in the value of the option over the period. Along the optimal path, this return must be equal to the return

[9] We can also derive (5) from (1) rigorously using Bellman's Principle of Optimality (see Kamien and Schwartz, 1991, pp. 259-262 , for details).

from the alternative investment strategy of selling the asset and investing the proceeds at the discount rate.

The optimal time in school (S^*) will be a stochastic variable, so it is easier to express the control variable in terms of the level of the shadow wage at which it will be optimal to leave school. This variable, which we denote Y^* will be deterministic. Because Y follows an Ornstein-Uhlenbeck process, so does V and using Ito's lemma we can write the stochastic differential for V as

$$dV(Y) = \left\{ (g_0 - g_1 Y) V_Y Y + \frac{\sigma^2}{2} Y^2 V_{YY} \right\} dt + \{\sigma V_Y Y\} dz$$

Note that $E[dV]$ contains a term in the variance of Y. This has important implications for the effect of risk on decisions. On average shocks have no effect on Y i.e. $E[dY] = Yg$. However if $V_{YY} > 0$ they will have a positive effect on the change in the value of the option because the effect of a negative shock will be smaller in absolute terms than will the effect of positive shocks. The results is that V will trend up (down) over time due to repeated shocks to Y, if V_{YY} is positive (negative).

We can substitute dV into the Bellman equation, use the fact that $E[dz] = 0$ and divide by dt to get

$$\rho V(Y) = u(\bar{c}) + \phi + (g_0 - g_1 Y) V_Y Y + \frac{\sigma^2}{2} Y^2 V_{YY} \tag{6}$$

The equation is a second order non-homogenous ordinary differential equation. It has a free boundary given by Y^*, the threshold level of the shadow wage at which the agent will choose to leave school. We can verify by substitution that the general solution will be

$$V(Y) = B_1 Y^{\theta_1} H(Y, \theta_1) + B_2 Y^{\theta_2} H(Y, \theta_2) + \phi/\rho + u(\bar{c})/\rho \tag{7}$$

where θ_1 is the positive and θ_2 the negative root of the fundamental quadratic Q

$$Q = \frac{1}{2}\sigma^2\theta^2 + (g_0 - \frac{1}{2}\sigma^2)\theta - \rho \tag{8}$$

and $H(.)$ is the series representation of the confluent hypergeometric function.[10]

$$\begin{aligned} H &= 1+\frac{\theta}{b}x+\frac{\theta(\theta+1)}{b(b+1)}\frac{x^2}{2!}+\frac{\theta(\theta+1)(\theta+2)}{b(b+1)(b+2)}\frac{x^3}{3!}... \\ x &\equiv \frac{2g_1Y}{\sigma^2} \\ b &\equiv 2\theta+\frac{2g_0}{\sigma^2} \end{aligned} \tag{9}$$

Economic theory provides three conditions (10) that determine the two constants of integration and the free boundary.

$$\begin{aligned} \lim_{Y\to 0} V(Y) &= \frac{\phi}{\rho} \\ V(Y^*) &= \Omega(Y^*) \\ V_Y(Y^*) &= \Omega_Y(Y^*) \end{aligned} \tag{10}$$

The first states that as the shadow wage tends to zero the individual will never leave education and so the value of being in school will simply equal the present value of the direct utility of perpetual education (ϕ/p).[11] This implies that the negative root, θ_2, should have no influence on V, as Y tends to zero. If it did then the value of the option to leave school would tend to infinity. The only way of ensuring this is if $B_2 = 0$.

The second part of (10) is the "value matching" condition. When income reaches a certain threshold level (Y^*) the option is exercised, the individual leaves school and receives that income for life. The present value of the utility generated by this perpetual income stream is denoted by $\Omega(Y^*)$. Thus at time $t = S$, when the option is about to be exercised, its value will equal $\Omega(Y^*)$

The third condition, the "smooth pasting" condition, states that for the threshold level of income to be chosen optimally, the net gain to any small changes in Y^* must have only second order effects. If we stay in school now while the market wage is Y, then we can leave school sometime in the future and earn (possibly) an even higher wage. The value of this option to leave, when the current shadow wage is Y, is given by $V(Y)$. When we leave school we gain $\Omega(Y^*)$ but loose $V(Y)$. The net gain from leaving school when the (shadow) wage is Y is therefore $\Omega(Y^*) - V(Y)$, so the optimal choice of Y^* implies the

[10] See Dixit and Pindyk (1994) page 163 and the references cited therein. Note that H reduces to the exponential function when $b = \theta$ and reduces to $H = 1$ when $g_1 = 0$

[11] Without loss of generality we normalise utility such that $u(\bar{c}) = 0$ or equivalently interpret ϕ to be the utility of being in education comprised of the intrinsic psychic value of education and the utility of consumption less education fees.

smooth pasting condition.[12]

When the individual exercises her option and leaves school she will receive a certain salary which will generate a certain lifetime utility, Ω (i.e. the second integral in (1)). The exact value of of post school life-time utility, $\Omega(Y)$, depends on how wages evolve after leaving school. If the individual smooths consumption *after graduation*, we can calculate $\Omega(Y)$ by direct integration to get the familiar solution (11) where α is the growth of income after graduation (i.e. the "experience" term in the Mincer equation) and where we assume that the growth of income after graduation is unaffected by education i.e. $Y_{t>S}(S)\exp(\alpha(t-S)$.[13]

$$\Omega(Y) = \frac{1}{1-\gamma}\left(\frac{1}{p}\right)^{\gamma}\left(\frac{Y}{\rho-\alpha}\right)^{1-\gamma} \tag{11}$$

Thus the optimal shadow wage Y^* is implicitly defined by the following equations

$$\begin{aligned} B_1 Y^{\theta} H(Y,\theta) + \phi/\rho &= \Omega(Y) \\ B_1 \theta Y^{\theta-1} H(Y,\theta) + B_1 Y^{\theta} H_Y(Y,\theta) &= \Omega_Y(Y) \end{aligned} \tag{12}$$

where θ is the positive root of (8); Ω is defined by (11); H is given by (2) and subscripts denote the partial derivative.

In general there is no closed form solution to (12). However when expected returns to education are constant, the shadow wage follows a Brownian motion (i.e. $g_1 = 0$) and $H = 1$. In this case we can generate a closed form solution which we state as Proposition 1 below.

Proposition 1 *When (i) returns to education are normally distributed with mean g_0 and variance σ^2; (ii) preferences are $u(c) = c^{1-\gamma}/(1-\gamma)$, the threshold level of the shadow wage at which it is optimal to cease education is given by*

$$Y^* = \frac{\rho}{\rho-\alpha}\left[\frac{\phi\theta(1-\gamma)}{\theta-(1-\gamma)}\right]^{\frac{1}{1-\gamma}}$$

where θ is the positive *root of Q in (8), ρ is the discount rate and ϕ is intrinsic utility of education. Furthermore we have* $\frac{\partial Y^*}{\partial g_0} > 0$, $\frac{\partial Y^*}{\partial \rho} < 0$, $\frac{\partial Y^*}{\partial \sigma} > 0$, $\frac{\partial Y^*}{\partial \phi} > 0$.

[12] This justification of the smooth pasting condition is intuitive but simplistic. A more complete treatment can be found in Dixit and Pindyck (1994).

[13] Heckman et al. (2001) have cast doubt on the empirical relevance of this time separability assumption, providing evidence that in the US at least, earnings growth after leaving school is a function of the education level. But we continue to assume it here for analytical convenience.

Proof. The expression for Y^* follows directly from solving for Y^* from (12) given (11) and (8). The derivatives follow by application of the implicit function theorem to (8) and (2). ■

Sufficient conditions for $Y^* > 0$ are that $\phi > 0$ and $\rho > g$. If the latter were not the case, school would always provide a better return (on average) and it would be optimal to stay in school for ever. As we would expect, Y^* is an increasing function of g and a decreasing function of ρ. Thus high returns to education will cause individuals to stay in school longer whereas a high discount rate will induce them to leave earlier.

The threshold level of the shadow wage (Y^*) is also an increasing function of risk, so the threshold is higher than under certainty, which we can verify by direct integration of (1) when returns are certain. We can also show that Y^* becomes infinite as $\sigma^2 \to \infty$, implying that the agent will never leave school.

The fact that risk increases the amount of schooling is, perhaps, surprising. Using the investment analogy, one might have expected less investment in human capital as the risk associated with that investment rose. Our result is due to the fact that leaving school is an irreversible decision. Risk creates a value to waiting because if we stay in school we have the option to leave next period in order to take advantage of a good draw from the distribution of returns or to remain in education so as to avoid a bad draw. Uncertainty has an asymmetric effect, increasing the potential upside payoff from the option, but, because we will stay in school if the market wage turns out to be low, the downside payoff is unchanged. This effect becomes stronger as the riskiness of education increases. Indeed when risk becomes infinite, the agent will never want to exercise the option to leave.

This result is in line with what we would expect from financial option theory. Increased risk in the underlying security tends to increase the value of the option because increased variability implies that the option is more likely to be "in the money" at some point in the future.

Note also that risk has an effect on the education decision even if the agent is apparently risk neutral i.e. if $\gamma = 0$ then Y^* is still a function of σ via θ. In fact the risk aversion does not affect sign of any of the derivatives in Proposition 1; γ just acts as a scaling factor. Again the reason is that risk in the presence of an irreversibility creates a value to waiting — even for the risk neutral investor. Another way of seeing this is to note that while instantaneous utility is linear, lifetime utility, V, has $V_{YY} > 0$. In fact, it is easy to show that the coefficient of relative risk aversion for lifetime utility is negative for any $g_1 \geq 0$. It is as if the irreversibility has changed a risk neutral agent into a risk lover.

All this is intuitive, but note that it has the implication that the individual will accumulate more human capital when the risk associated with that investment is higher. This prediction contrasts with that of the portfolio model of Williams (1979). In his model, an increase in the risk of human capital (or any other asset), would cause the individual to accumulate less of it, other things being equal. The reason for this difference is the nature of the choice facing the agent. His approach treats education as occurring continuously and at the same time as work. There is no irreversibility, the agent can come in and out of education as she pleases for zero cost (other than forgone wages). Because there is no irreversibility there is no value to waiting.

Figure 1 illustrates the solution of the model for the simple case where $\gamma = \alpha = g_1 = 0$. The graph shows the function $V(Y)$, the value of the option to wait and the function $\Omega(Y) = Y/\rho$, the value of leaving education when the market wage is Y. At the optimal point, V and Ω are equal and meet as tangents. For shadow wages less than the optimal ($Y < Y^*$), the value of the option to wait (V) is greater than the life-time utility from leaving now (Ω), so the individual remains in education. When the shadow wage is zero, the optimal decision would be to say in school for ever, generating a life-time utility of ϕ/ρ. As the shadow wage increases, Ω, the gain from leaving also increases. But so does the cost of leaving i.e. the value of the option to leave at some point in the future. At the optimal threshold the two are equal. Note it may appear from the diagram that it is optimal to remain in school if $Y > Y^*$. This is not true. Because of the value matching condition, the value of lifetime income is given by $\Omega(Y)$ when $Y > Y^*$, so that the full function V is given by [abd]. The line segment [bc] is irrelevant.

2.3 Numerical Simulation

Proposition 1 applied only in the case of constant expected returns to education. When we allow for diminishing returns, the model (12) requires a numerical solution. In this section we present numerical simulations of the model to illustrate the effects of changes in various parameters. Table 1 presents the baseline values of the parameters used in the simulation. All are plausible, if conservative, values. For simplicity we simulate the model assuming $\gamma = 1$ i.e. log utility. We assume that the expected rate of return on education (g_0) is 7% per annum which is in line with OLS estimates but less than most IV estimates (see Card, 2001). The estimate of risk (σ) at 2% seems reasonable given our choice of g. It is also in line with estimates provided by Harmon et al. (2003) and Conneely and Uusitalo (1999) but more than estimated by Carneiro et al. (2003b) and less than estimated by Chen (2003). Unless otherwise indicated we will assume that $g_1 = 0$ so that expected returns are constant. The discount rate (ρ) is equal to 10%.

Together $Y(0)$ and ϕ act as numeraires for the problem. The parameter $Y(0)$ can be thought of as representing the income received by an individual who leaves school immediately after the end of compulsory education. Without loss of generality, $Y(0)$ is set equal to unity so that Y is expressed in terms of a multiple of the wage associated with minimum education. We set the intrinsic utility of school so as to ensure that, in the absence of uncertainty, an individual would optimally choose to leave after exactly 2 years of post compulsory schooling. Given these baseline parameters, $Y^* = 1.15$ implying an expectation of just over two years of post compulsory schooling.

Proposition 1 states that the effect of increases in the expected rate of return to education is unambiguously positive for the constant returns case ($g_1 = 0$). Figure 2 illustrates the point numerically for various values of g_0 and g_1. Unless the diminishing returns parameter is extremely large, higher expected returns induce the individual to stay in education longer. The effect of risk on education is also positive for essentially the same reason as before: an irreversible decision in the presence of risk creates an incentive to wait. Figure 3 illustrates this effect numerically, showing the interaction with expected returns (g_0). The effect is positive, but is much smaller than the effect of expected returns. The interaction between the three parameters σ, g_0 and g_1 is illustrated in table 2.

Because the evolution of income is stochastic, there is no expression for S as there would be in the certainty case. When returns are stochastic, S^* will be a random variable and the best we can do is to describe its distribution. We describe it numerically for the constant returns case.[14] Figure 4 and Table 3 present the results of this simulation for various different levels of risk. As can be seen, increasing risk leads to an increase in $E(S^*)$. This is to be expected given that Y^*, the target level of the shadow wage will have increased (more easily seen in Table 3). It is also clear that the variance of S^* will rise. Again this is intuitive: as the process for the shadow wage gets more uncertain, the time it takes for that process to reach any given level becomes more uncertain. What is more surprising is that the distribution of S^* becomes increasingly skewed at higher levels of risk. The reason is that direct effect of higher risk on the mean and variance of S^* makes higher values of S^* relatively more likely than lower values. This coupled with the fact that S^* is bounded at zero results in a skewed distribution.

[14] If the individual starts with income Y_0 how long will it take for income to reach the threshold value (Y^*) when it evolves according to (3)? The probability that an individual will still be in school at time t (so that S^* greater than t) is equal to the probability that the income process will not have reached the trigger level at time t (so $Y_t < Y^*$). This implies that $P(S^* \leq t) = 1 - P(Y_t < Y^*) = 1 - \Phi(Z_t^*)$ where $Z_t^* = (\ln Y^* - \ln Y_0 - \mu t) / \left(\sigma\sqrt{t}\right)$ and Φ is the c.d.f. of a standard normal random variable.

3 Policy Implications

In the section we use the model to examine the impact on individuals' education decisions of some simple stylised government policies. We can model the direct effect of an education subsidy as an increase in ϕ. We already know from Proposition 1 that the effect of an increase in ϕ is to increase the threshold shadow wages, and thus lead to an increase in schooling.

If the subsidy is financed from general taxation, then there will be no other effects on the individuals education choice. Many real world tuition finance programmes, however, require the student to pay back some of the tuition after graduation. For reasons of public policy, the government, unlike a private lender, is willing to overlook the moral hazard problem. Indeed by making re-payment conditional on graduation or some means test, the government is implicitly acknowledging that the option structure of individual education choice and even encouraging moral hazard behaviour.

We can think of three broad types of tuition payment plans. Firstly, tuition could be paid back in fixed installments as with a standard loan repayment. In the context of our model, this would be equivalent to levying a lump-sum tax on earnings after graduation. Alternatively, the repayments could be fixed as proportion of earnings. This is the equivalent of a proportional tax on labour income. Alternatively, a tuition payment plan could combine proportionate and lump sum elements equivalent to a progressive (or even regressive) wage income tax.

It turns out that we can easily accommodate the three different taxes in the model. The state variable is still Y, but now we interpret it now as being the (shadow) wage gross of taxes/repayments. We define a new variable $\omega = f(Y)$ which is the net wage received upon graduation.

$$\omega = f(Y) = Y - \tau Y^{\varepsilon} \tag{13}$$

The function $f(Y)$ summarizes the relevant parameters of the tax system. The parameter ε is equal to the ratio of the marginal tax rate to the average tax rate. It represents the extent to which the tax system is progressive or regressive. For lump-sum taxes $\varepsilon = 0$ (i.e. perfectly regressive) and we interpret τ as the amount of the lump sum tax. For proportional taxes, $\varepsilon = 1$ and we interpret τ as the proportionate tax rate. For regressive taxes, $\varepsilon \in [0, 1)$, the marginal tax rate is less than the average tax rate for all income. For a progressive tax system, $\varepsilon > 1$, marginal tax rates are higher than average tax rates for all incomes.[15]

[15] Note for simplicity we assume that capital gains and net interest payments are not taxable income.

The variable ω directly effects the problem only through Ω, the utility after graduation. The structure of the option is unaffected as is the form of the function V which must still solve the Bellman equation (6). The value matching and smooth pasting conditions will change to $V(Y) = \Omega(f(Y))$ and $V_Y(Y) = \Omega_\omega(f(Y)) * f_Y(Y)$ respectively.[16] This modification to the model allows us to state Proposition 2 for the case of constant returns.

Proposition 2 *When (i) returns to education are normally distributed with mean g_0 and variance σ^2; (ii) preferences are $u(c) = c^{1-\gamma}/(1-\gamma)$, the imposition of either a lump sum or a proportional tax will lead to an increase in Y^*. An increase in the degree of progressivity of the tax system could lead to an increase or decrease in Y^* depending on the degree of risk aversion and the degree of progressivity.*

Proof. See Appendix ■

At first glance this may seem a curious result. The tax reduces the benefit of schooling, so that the value of the option to wait falls. But the value of leaving school, Ω, falls by more. The net result is that school becomes relatively more attractive, and the individual stays for longer. Figure 5 illustrates this for the case of the simple model of proposition 1 with $\alpha = 0$, $g_1 = 0$ and $\gamma = 1$ (log utility). Following the imposition of a tax, the individual seeks to maintain living standards by boosting her gross wage. The only way to to this is to stay in school longer. In essence, we have an income effect without any associated substitution effect. There is no counteracting substitution effect because both a lump-sum and a proportionate tax will not change the risk and return associated with continuing to the next level of education. In fact, it is straight forward to show that if the tax revenue is returned to the individuals in a lump-sum, the income effect will be nullified, thus compensated changes in proportional taxes will have no effect on education attainment. This is a standard result in the literature (for example, see Heckman et al., 1998).

The situation can be different when taxes are progressive (or regressive). In that case, the after-tax risk and return to education will be different for different levels of education. For example, a progressive tax will levy a higher proportional charge on higher incomes, so that the risk and return associated with proceeding from a lower to a higher level of education will both be reduced. This in turn, will reduce the value of the option to wait. If large enough, this substitution effect can overcome the income effect and lead to fall in education. As we show in the appendix, a necessary (but not sufficient) condition for this

[16] Note that the actual value of the option will be affected via the smooth pasting and value matching conditions, leading to a different value for the constant B_1.

to occur is that $\varepsilon \ln Y^* < 1$. This condition illustrates how risk (via Y^*) interacts with the degree of progressivity of the tax system to determine the strength of the substitution effect. When higher risk (lower Y^*) is combined with higher progressivity, the condition will hold and progressivity can have a negative (uncompensated) impact on education choice.

We illustrate this in Figure 6, where the baseline parameters are from Table 1. The horizontal axis represents the parameter ε which goes from zero (representing a perfectly regressive lump sum tax) through to unity (representing a proportional tax) and beyond (representing progressive taxation). The fact that taxes (progressivity or otherwise) will increase education attainment raises the interesting possibility that of policy of education subsidy (i.e. higher ϕ) financed by taxation of education returns after graduation will raise education attainment both directly via the subsidy and indirectly via the tax. Thus a self-financing education loan or subsidy scheme would have a positive effect on education.

Our results differ with some of the rest of the literature, particularly Trostel (1993). He calibrates a dynamic general equilibrium model of human capital accumulation (without uncertainty) to show that a proportional (compensated) wage tax can have a negative impact on human capital accumulation. This result is generated in part by the assumption that labour supply is elastic. In this case, the imposition of the tax reduces labour supply, and hence also reduces the effective return to human capital. However, as Heckman (1993) shows, there is now something of a consensus that the labour supply elasticity of men (on intensive margin) is highly inelastic suggesting that the effect identified by Trostel (1993) is not empirically significant.

Lin (1998) shows that in a non-stochastic OLG model, an uncompensated increase in a (proportional) wage tax can reduce human capital accumulation. This result depends crucially on a capital market channel that is absent in our model. An increase in wage taxes can reduce savings, leading to a lower stock of physical capital. This in turn leads to higher interest rates which makes investment in human capital less attractive at the margin. The negative effect disappears if tax revenue is redistributed to tax payers. In this case their income and saving remain the same so interest rates remain unchanged.

Eaton and Rosen (1980) is one of the few papers to consider explicitly the effect of taxation in model of education choice with uncertainty.[17] They show that in a two period model, the imposition of a proportional (uncompensated) wage tax will have an ambiguous effect on education. However, when preferences exhibit constant relative risk aversion and initial wealth is sufficiently high, they show that an uncompensated proportional wage tax

[17] In their model uncertainty is multiplicative in income, so the marginal product of human capital is stochastic but the rate of return is deterministic i.e. $Y(s) = \lambda W(s)$ where λ is stochastic (mean one) and W is deterministic.

has a positive effect on human capital accumulation. Keane and Wolpin (1997) estimate the parameters of an empirical dynamic model of education choice. They show that their results imply that a tuition subsidy of \$2,000 would increase school and college graduation rates by 3.5 and 8.5 percentage points respectively.

4 Discussion and Extensions

The model we have presented was structured so that it would yield analytical solutions. In this section we argue that the results of section 2 are quite robust and that most (but not all) of the extensions that we might contemplate would not change the fundamental results at the cost of considerable complication in the analysis.

The most obvious extension to the model would be to account for finite life and education opportunities i.e. $S \leq T < \infty$. It turns out that it is very easy to accommodate this change. If we do not insist on a deterministic length of life, we can allow death/retirement to arrive according to a Poisson process with parameter λ. As is well known, this is equivalent to increasing the discount rate from ρ to $\rho + \lambda$ and keeping $T = \infty$. So the qualitative results will be exactly the same.

For a deterministic death/retirement date, time becomes the third state variable of the problem. A term involving V_t will appear in the Bellman equation (6) and the value matching and smooth pasting conditions will be $V(Y, t) = \Omega(Y, t)$ and $V_Y(Y, t) = \Omega_Y(Y, t)$ respectively. This free boundary problem will have to be solved numerically as there will be no closed form solutions for V or Ω. However, the basic results of Proposition 1 will not be affected as the structure of the problem is unchanged. There is a still an option. Its value still increases in uncertainty. It is this value of waiting that drives all the main results of the model. All that has changed is that the option now has a finite expiry date. In fact the problem is now very close to the Black-Scholes analysis of a financial call option.

Another extension is to include post schooling risk i.e. that the income process after graduation should be stochastic. Again, the overall structure of the option problem would not change. With irreversibility, there would still be a value to waiting due to the uncertainty regarding the shadow wage (i.e. uncertainty before graduation). The value matching and smooth pasting conditions that determine the value of the option would still be defined in terms of the same function V. The introduction of uncertainty post graduation leaves the structure of the problem unchanged. The only difference would be that the function $\Omega(Y)$ would not have an analytical representation and its value would be affected (negatively) by the variance of the wage process after graduation. This case has been analysed in detail by Hartog et al. (2004).

A more fundamental change would be to allow σ to vary with S. Our model assumed that the variance of the distribution of the returns to education was the same for all levels of education (so that the variance of the shadow wage rose linearly with education). This makes the problem tractable and, given the relative paucity of information on this issue, seems reasonable. But it is at least possible that risk could increase or decrease with education. If further education decreased risk, we might expect individuals to choose further education as a form of insurance. However, to be set against this is the fact that lower risk would decrease the value of the option to stay in school suggesting that it would be optimal to leave earlier. Analysing how these two effects interact would make for a interesting extension to our model.

Another useful extension could be made by explicitly considering "sheep-skin" effects i.e. the possibility that the mean and variance of education returns may be function not of time in school, but of qualifications attained. This change would generate a different stochastic process for the shadow wage. However, the option to leave school would still have value (one that was increasing in risk) and so our basic qualitative results would still hold.

Another obvious change that we could make is to relax the assumption that individuals cannot return to education. We could allow individuals to return to education for some finite cost. However, to the extent that return was not completely costless, there would still be some partial irreversibility. Our basic results would continue to hold for the same reason as before – irreversibility in the presence of uncertainty creates a value to waiting (albeit lower than the case of complete irreversibility). It is this value that generates our results.

5 Conclusions

In this paper we apply the techniques of option theory to the study the education decisions of individuals when the returns to education are uncertain. We view an individual in school as possessing an option to leave at any time and take up work at a wage related (stochastically) to the time spent in school. Once that option is exercised, the individual cannot return to school.

We show that high returns to education will cause individuals to stay in school longer whereas a high discount rate will induce them to leave earlier. Furthermore we also show that increasing risk will cause an individual to delay leaving school. This result is not dependent on the risk preferences of agents as it holds for risk neutral agents also. On the face of it, this is curious result, we would expect that higher risk would lead to less investment in human capital. The result stems from treating education as an option. Higher uncertainty, therefore provides an incentive to delay leaving so as to see if uncertainty may resolve itself favorably.

In contrast to many models of human capital accumulation, we show that increased labour income taxation would induce individuals to stay in school longer. An exception could occurr when high levels of progressivity are combined with high levels of risk. In this case further increases in progressivity may cause eduaction attainment to fall. Finally we argued that changes in the specification of the model (time limits, costly return to school etc.) would not affect the nature of the results as long as the basic option structure remained the same.

A Proof of Proposition 2

We can prove Proposition 2 for the case of constant expected returns to education ($g_1 = 0$). We extend the model to account for taxes, by specifying ω to be net income and ε to be parameter that models the progressivity of the tax system, as in (13). The function $f(Y)$ summarizes the three types of taxes. In general ε equals the ratio of marginal to average tax rates. For lump-sum taxes $\varepsilon = 0$ and we interpret τ as the amount of the lump sum tax. For proportional taxes, $\varepsilon = 1$ and we interpret τ as the proportionate tax rate. For a progressive tax system we have $\varepsilon > 1$. Again we can only prove the proposition analytically for constant expected returns ($g_1 = 0$).

We make use of the matrix version of the implicit function theorem. Re-write the (12) system of implicit equations that jointly determine Y^* and the constant of integration B_1 as (14) below.

$$\begin{aligned} G_1 &= \Omega_\omega(f(Y^*))f_Y(Y) - V_Y(Y^*) = 0 \\ G_2 &= \Omega(f(Y^*)) - V(Y^*) = 0 \end{aligned} \tag{14}$$

For constant returns to education $H \equiv 1$, therefore we have:

$$\begin{array}{ll} V_{B_1} = Y^{\theta_1} > 0 & \Omega_{B_1} = 0 \\ V_{YB_1} = \theta_1 Y^{\theta_1 - 1} > 0 & \Omega_{YB_1} = 0 \\ V_\phi > 0 & V_{Y\phi} = 0 \\ & V_{Y\theta} = B_1 Y^{\theta-1}(\ln\theta + 1) \\ V_\theta = B_1 Y^\theta \ln\theta & V_{YY} = B_1\theta(\theta-1)Y^{\theta-2} > 0 \end{array} \tag{15}$$

We can sign some more derivatives on the assumption that preferences are CRRA and $\gamma \geq 0$. Note that these derivatives would probably hold for any "well behaved" preferences i.e. $u_{cc}/u_c < 0$.

$$\Omega_{\omega\omega} \leq 0$$

Furthermore $f_{YY} \leq 0$ for $\varepsilon = 0$ and $\varepsilon \geq 1$. Thus

$$\begin{aligned} \Omega_Y &= \Omega_\omega f_Y > 0 \\ \Omega_{YY} &= \Omega_{\omega\omega} f_Y f_Y + \Omega_\omega f_{YY} \leq 0 \end{aligned}$$

The Jacobian of the system (14) is given by J. Its determinant, $|J| \neq 0$, so the implicit function theorem applies.

$$|J| = -V_{B_1}\left[\Omega_{xx} f_Y f_Y + f_{YY}\Omega_x - V_{YY}\right] > 0$$

Thus $|J| > 0$ and the distinction between net and gross income will not affect the sign of any of the derivatives in Proposition 1 when taxes are lump sum, proportional or progressive. Only in the case of a particular choice of parameters and for some particular values of ε that must be between zero and one, will the derivatives change sign.

Define J_x to be the matrix

$$J_x = \begin{bmatrix} -\frac{\partial G_1}{\partial x} & -V_{YB_1}(Y^*) \\ \\ -\frac{\partial G_2}{\partial x} & -V_{B_1}(Y^*) \end{bmatrix}$$

where all derivatives are evaluated in the neighbourhood of the optimum. Using the implicit function theorem we can state the following derivatives hold in the neighbourhood of Y^*:

$$\begin{aligned} \frac{\partial Y^*}{\partial \phi} &= \frac{|J_\phi|}{|J|} = \frac{V_{YB_1} V_\phi}{|J|} > 0 \\ \frac{\partial Y^*}{\partial \theta} &= \frac{|J_{\theta_1}|}{|J|} = \frac{-V_{B_1} V_{Y\theta} + V_{YB} V_\theta}{|J|} < 0 \end{aligned}$$

Because g and σ affect Y^* only through θ, itself determined implicitly by Q in (8), we have

$$\begin{aligned} \frac{\partial Y^*}{\partial g} &= \frac{\partial Y^*}{\partial \theta}\frac{\partial \theta}{\partial g} > 0 \\ \frac{\partial Y^*}{\partial \sigma} &= \frac{\partial Y^*}{\partial \theta}\frac{\partial \theta}{\partial \sigma} > 0 \end{aligned}$$

In order to prove Proposition 2 we calculate

$$\begin{aligned} |J_\tau| &= +V_{B_1} f_\tau f_Y \Omega_{xx} - V_{YB_1}\Omega_x f_\tau > 0 \\ \\ |J_\varepsilon| &= +V_{B_1}\left[f_\varepsilon f_Y \Omega_{xx} + f_{Y\varepsilon}\Omega_x\right] - V_{YB_1} f_\varepsilon \Omega_x \end{aligned}$$

For $\varepsilon = 0$ or $\varepsilon = 1$, we have

$$\frac{\partial Y^*}{\partial \tau} = \frac{|J_\tau|}{|J|} > 0$$

where τ can be interpreted as either a lump-sum or proportional tax rate, depending on ε.

For progressive taxes, the effect of changes in the degree of progressivity are more complex. For most values of the parameters $|J_\varepsilon| > 0$ and so

$$\frac{\partial Y^*}{\partial \varepsilon} = \frac{|J_\varepsilon|}{|J|} > 0$$

All the terms in $|J_\varepsilon|$ are can be signed unambiguously with the exception of $f_{Y\varepsilon}$. If $f_{Y\varepsilon} > 0$ then $|J_\varepsilon| > 0$, so a sufficient condition for $\frac{\partial Y^*}{\partial \varepsilon} > 0$ is that $\varepsilon \ln Y^* < 1$. Note that there is no simple condition sufficient to ensure that $|J_\varepsilon| < 0$.

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Table 1: Baseline Parameters for Simulation

Parameter	Definition	Value
γ	CRRA	1.0
Y_0	Wage with min. Schooling	1.0
ρ	Discount rate	0.1
ϕ	Intrinsic utility from Education	1.659
g_0	Mean Return to Education	0.07
g_1	Diminishing return	0.00
α	Growth of Y after graduation	0.00
σ	Stn. Dev. of Return to Education	0.02

Table 2: Threshold Income with Diminishing Returns to Education

	g_0	0.15	0.15	0.2	0.2
	g_1	0.025	0.05	0.02	0.01
Risk (σ)					
0.00		2.00	1.00	5.00	10.00
0.02		2.01	1.01	5.01	10.02
0.04		2.02	1.01	5.04	10.08
0.06		2.04	1.02	5.09	10.18
0.08		2.07	1.04	5.16	10.31
0.10		2.1	1.05	5.24	10.47
0.12		2.14	1.07	5.33	10.65
0.14		2.18	1.09	5.43	10.85
0.16		2.23	1.12	5.54	11.08
0.18		2.28	1.14	5.66	11.32
0.20		2.33	1.17	5.79	11.57

1. Simulation of basic model as in equation (12)
2. Key parameters: as table 1 but with $\phi = 0$

Table 3: Optimal School Leaving

Education Risk σ	Threshold Income Y^*	Time in School (S^*) $E(S^*)$	$Stn(S^*)$	$Skew$
0	1.1506526	2.0	-	-
0.01	1.1509341	2.02	0.20	0.30
0.02	1.1517757	2.07	0.42	0.60
0.03	1.1531684	2.16	0.66	0.90
0.04	1.1550980	2.28	0.93	1.15
0.05	1.1575448	2.42	1.20	1.21
0.06	1.1604849	2.57	1.44	1.13
0.07	1.1638910	2.68	1.63	1.03
0.08	1.1677324	2.76	1.76	0.94
0.09	1.1719768	2.80	1.86	0.88

1. Based on 10,000 draws from distribution of S*
2. Key parameters as in table 1

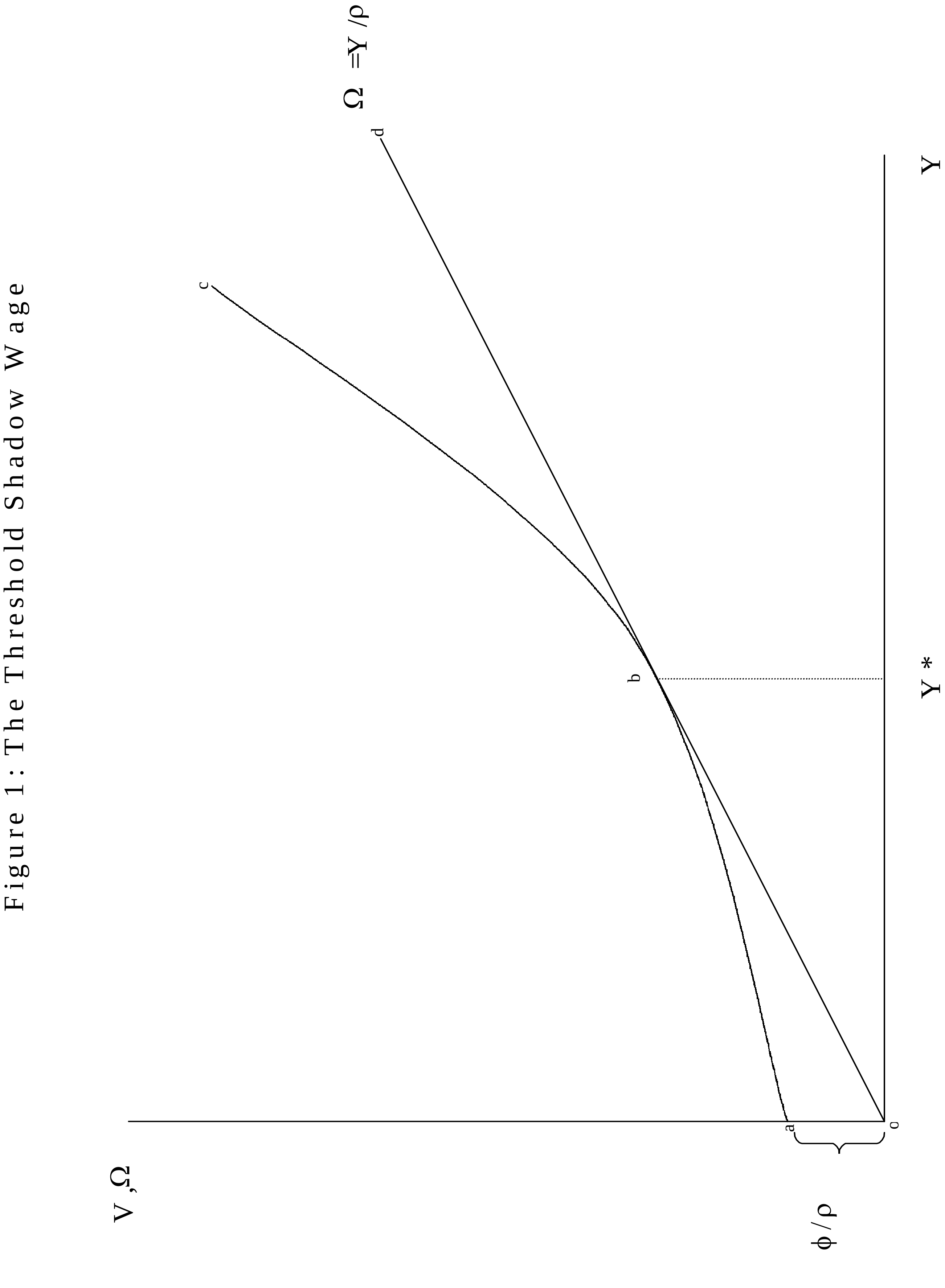

Figure 1:

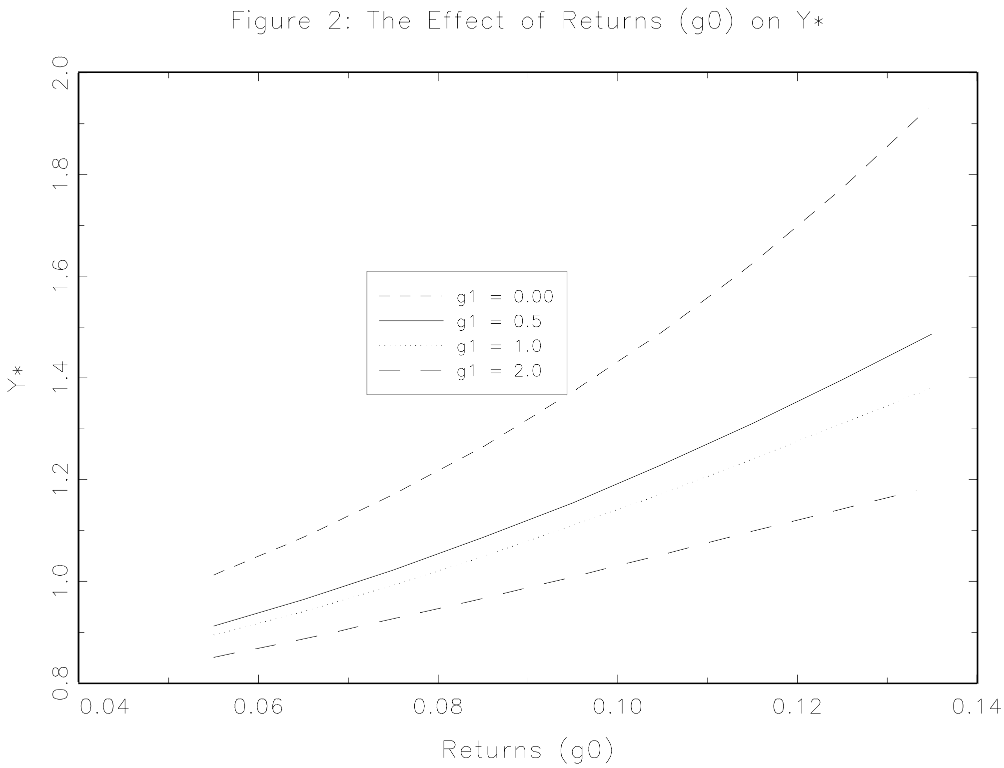
Figure 2: The Effect of Returns (g0) on Y*
g1 = 0.00
g1 = 0.5
g1 = 1.0
g1 = 2.0
Y*
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0.04
0.06
0.08
0.10
0.12
0.14
Returns (g0)

Figure 2:

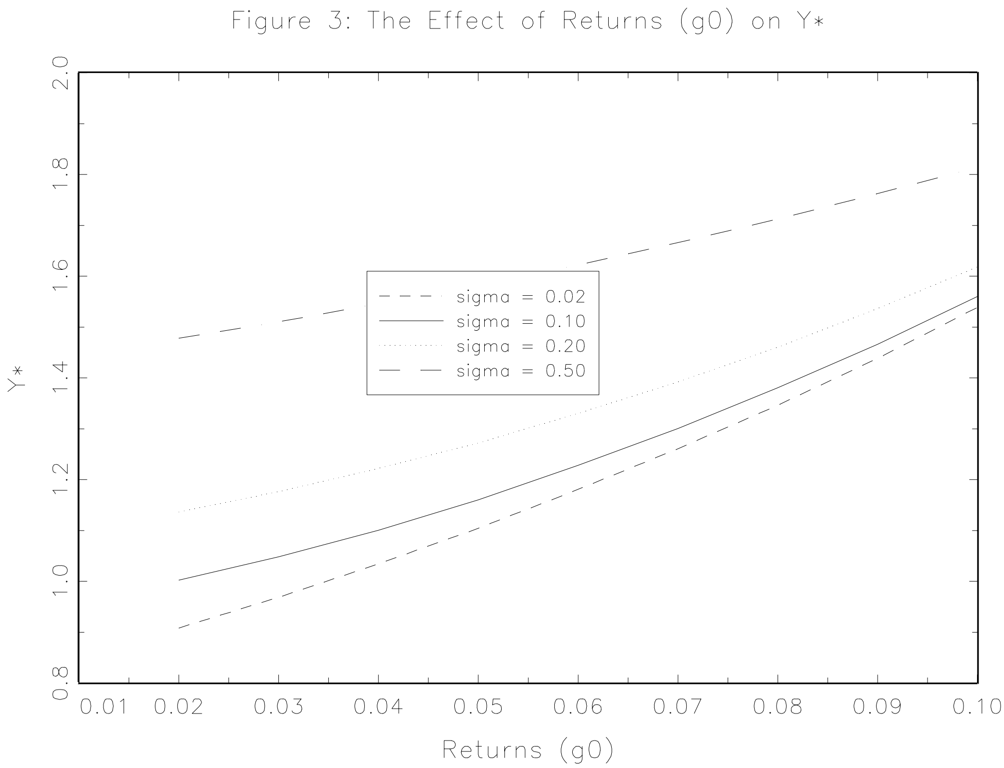
Figure 3: The Effect of Returns (g0) on Y*
sigma = 0.02
sigma = 0.10
sigma = 0.20
sigma = 0.50
Y*
0.8
1.0
1.2
1.4
1.6
1.8
2.0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
Returns (g0)

Figure 3:

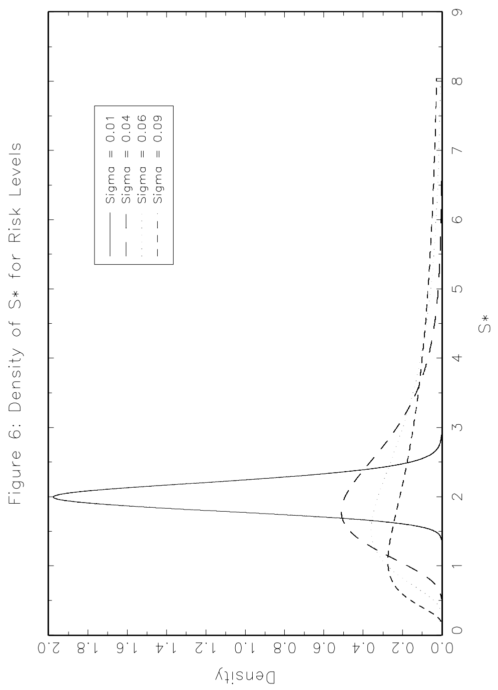
Figure 6: Density of S* for Risk Levels
Sigma = 0.01
Sigma = 0.04
Sigma = 0.06
Sigma = 0.09
Density
S*
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
0 1 2 3 4 5 6 7 8 9

Figure 4:

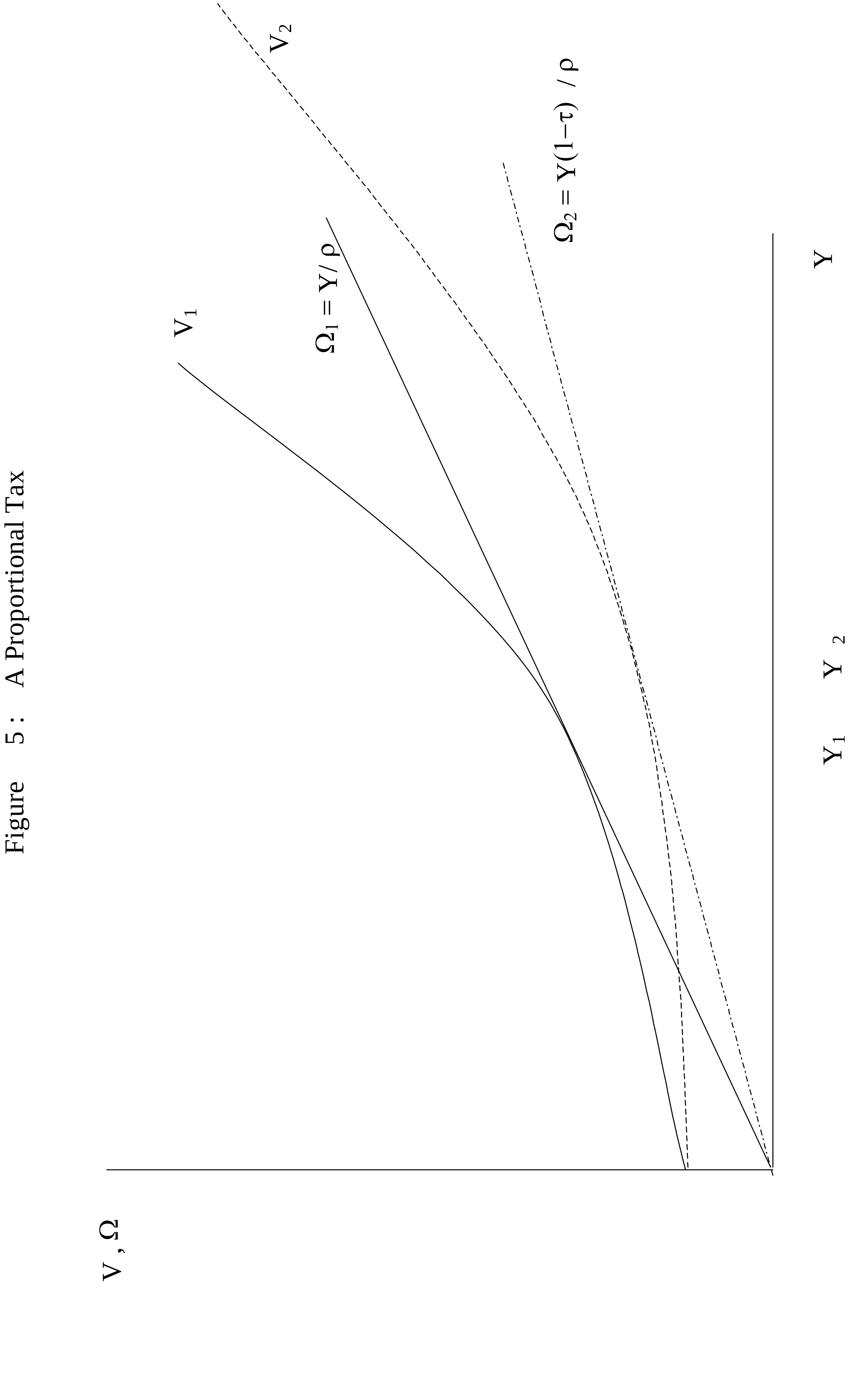

Figure 5:

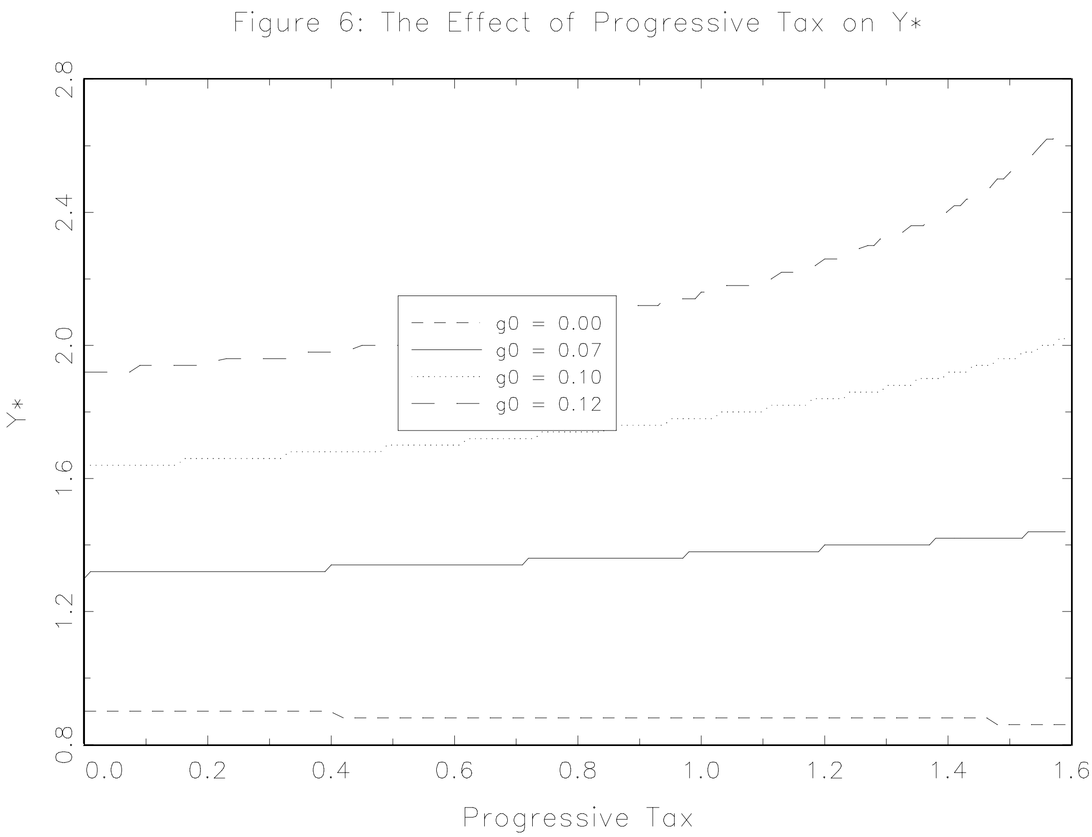

Figure 6: The Effect of Progressive Tax on Y*
2.8
2.4
2.0
1.6
1.2
0.8
Y*
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Progressive Tax
g0 = 0.00
g0 = 0.07
g0 = 0.10
g0 = 0.12

Figure 6: