Real Options and Human Capital Investment*

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Abstract

This paper extends the standard human capital model with real options. Real options affect investment behavior when risky investments in human capital are irreversible and individuals can affect the timing of the investment. Option values make individuals more reluctant to invest in human capital and required returns on the investment increase. Higher tax rates (or lower subsidies) depress human capital investments, but to a lesser extent than in the standard human capital model. A flat income tax remains to be neutral if education expenditures are fully deductible. Real options may explain a large human capital premium, small responsiveness of human capital investments to financial incentives, large sensitivity of investment behavior to low-return outcomes and more delayed investment in human capital even when returns are high.

Keywords: human capital, risk, irreversible investment, real options, progressive taxation, education subsidies.
JEL codes: G1, H2, I2, J2.

1 Introduction

The human capital model as developed by Mincer (1958, 1962), Schultz (1963) and Becker (1964) is by now the mainstream framework to analyze education and training decisions. According to the human capital theory, individuals will maximize their life-time utility by choosing their investments in education and training optimally. In the absence of income risks and capital market failures, separation between consumption decisions and investments in human capital holds. An optimizing individual would then require that the marginal return to his investment in human capital is at least equal to the marginal costs of making the investments. At the optimum, the *rate* of return to human capital investment should be equal to the real safe rate of interest. In practice, however, this is not the case. Observed returns are typically larger than the risk-free rate as Palacios-Huerta (2004) has shown in a novel finance approach. The high return is also consistently found in the empirical labor literature. See for excellent overviews Card (1999), Ashenfelter et al. (1999), and Harmon et al. (2003). This begs the question why the private returns to education are so high.

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Standard arguments to explain the high private return to education are often based on financial market failures. Capital markets may not make sufficient borrowing available due to enforcement and information problems. Human capital is regarded as poor collateral and a non-liquid asset as argued by Friedman (1962). Asymmetric information may also cause capital market failures because of adverse selection and moral hazard issues (Stiglitz and Weiss, 1981). With capital market problems, consumption and investment decisions cannot be separated and credit constrained individuals will invest sub-optimally in human capital. The rates of return on human investments are not equalized with the rate of return on safe assets. The empirical plausibility of capital market failures to explain high returns to human capital investments is rather weak, however. Carneiro and Heckman (2003), Cameron and Taber (2004), Plug and Vijverberg (2004) and others find that liquidity constraints have only a slight impact on college enrolment which seems to be insufficient to explain high rates of return to education.

Income risk may also justify a high rate of return. In the absence of well functioning insurance markets, uninsurable risks in human capital returns break the separation between consumption and investment in human capital as well. Levhari and Weiss (1973) and Eaton and Rosen (1980) have shown that the standard optimality condition for investments in human capital should include a risk-premium when the human capital returns are uncertain. This is the analogue to the CAPM-model used in finance. Risk averse individuals want to be compensated for income risks with a higher expected return. Indeed, many papers find evidence for risk compensation in wages, see the overview by Hartog (2005). Nevertheless, the high return on human capital is suggestive of a human capital premium puzzle, just like in the finance literature (see e.g., Mehra and Prescott, 2003). Palacios-Huerta (2004) has shown that risk alone cannot explain the difference between the real return on human capital and the risk-free interest rate. Only implausibly large coefficients of relative risk aversion generate a high risk-premium on human capital investments. Judd (2000) argues that, if idiosyncratic income risks are so important, governments or markets would look for institutions to insure these risks. Apparently, neither is the case. Both private and public insurance may not emerge because moral hazard makes the income risks endogenous rather than idiosyncratic, see also Judd (2000) and Sinn (1998). Another empirical puzzle is that educated people typically face fewer income risks than uneducated people, i.e., the covariance between earnings and education seems to be negative due to lower unemployment rates, and a lower incidence of sickness or disability to work, see also Gould et al. (2000) and Palacios-Huerta (2003). This implies that human capital is an insurance device and the standard risk premium on the marginal human capital investment would be negative, cf. Levhari and Weiss (1973). These empirical findings go against the presence of a substantial risk premium on educational investments.

Neither capital market failures, nor uninsurable income risks appear to have sufficient explanatory power to explain the high return to human capital. This paper argues that real options provides an alternative explanation why returns are so high. Real options are present in irreversible and risky investments where there is a possibility to affect the timing of the investment. Investments in human capital are generally completely irreversible because it is virtually impossible to recover forgone earnings and tuition expenses after the investment has been made. Therefore investments in human capital are sunk. Individuals can affect the timing of the decision to invest in risky education, especially higher education. This implies that individuals have an option to wait for better information regarding the returns (or costs) of the investment. Individuals who invest in (higher) education may have to give up a valuable option to wait. This is only beneficial if the returns with immediate investments are sufficiently large. Therefore, the option value of postponing the investment drives up returns to investment in human capital and may explain why returns are high. Moreover, this high return is an equilibrium outcome in the presence of perfect capital and insurance markets. Failures in financial markets are therefore not necessary to get high returns on human capital investments.

This paper also analyzes the effects of (progressive) income taxes and education subsidies when options are important. If all costs of education are tax deductible and there is a flat tax, there is no effect of the tax system on option values to wait with investment. The neutrality of flat income taxes with full deductibility of investments in the standard human capital framework (cf. Heckman, 1976) therefore carries over to the real option model. With imperfect deductibility, higher income taxes do indeed discourage human capital investments, but to a lesser extent than in the human capital model because higher flat taxes reduce the value of option to wait. Similarly, education subsidies increase investments in human capital. Again, investments in education are less sensitive to subsidies due to the reduced option value compared to the standard human capital model.

The real option approach pursued in this paper may not only give an explanation why the human capital returns are high, but may also resolve a number of other empirical puzzles that the standard human capital model fails to address.[1] First, education decisions seem to be largely insensitive to returns, tax and education policy, whereas standard human capital theory would suggest that these variables are crucial in predicting human capital investments. This paper shows that positive option values reduce the sensitivity of the investment decision with respect to the returns.

Second, students seem to be much more concerned with possible low return outcomes than the average returns to higher education. The standard human capital model would predict that students should care for the average return, not the sizes or probabilities of downward risks. The option approach shows that low-return outcomes and their probabilities are the crucial determinants of the option to wait, not average returns.

Third, the returns to education are much larger in Anglo-Saxon countries, notably the UK and the US, than in continental European countries, see also Harmon et al. (2003). At the same time, going to college after a couple of years working is much more common in Anglo-Saxon countries. The question is why students do not go to college directly after secondary education when the returns are high. The human capital model appears to fail here as well. Options may again explain this phenomenon. In more turbulent labor markets the option value to wait is larger and students may want to enrol at later ages.

This paper is organized as follows. Section 2 briefly describes some earlier papers and how this paper relates to the literature. Section 3 describes the model of irreversible investment in higher education. Section 4 derives the main results and performs comparative static analysis. Section 5 presents some solutions to empirical puzzles and Section 6 concludes.

2 Earlier literature

Comay et al. (1973) and Hogan and Walker (2002) are to my knowledge the only papers who have earlier pursued the option approach in a human capital framework. Comay et al.

[1] Dixit and Pindyck (1994) criticize the neo-classical and q-theories of investment on the same grounds.

(1973) focus on the option values generated in a rather mechanical multi-stage investment process. They analyze the optimal decision to stop learning (and start working) or to continue with learning in the next stage of the education process. The uncertainty at each stage is generated by an exogenous probability to drop-out while there are no income risks. Consequently, continuation is more likely if the option value of entering the next stage is large. Although not explicitly discussed, this paper would predict a lower rate of return to higher education investments, because the option value to continue in education gives individuals incentives to keep learning even if the return on this investment is negative. Therefore, the human capital premium puzzle would become more puzzling.

Hogan and Walker (2002) also assume that the decision to work is the irreversible decision at stake. By deciding to stop learning and starting to work, the student gives up the option to enter the labor market at a later age. This decision is irreversible because students cannot re-enter education. The costs of going to work consist of the forgone utility benefits of staying in education for a longer time. For a number of reasons this approach seems to be less convincing. First, in practice students can re-enter education at a later age which suggests that the option to go to work is not really irreversible. Second, the sunk costs of deciding to work are the forgone utility benefits of staying in education longer. Utility benefits could be relevant, but opportunity and direct costs are probably much more important sunk costs. Third, Hogan and Walker (2002) derive counter intuitive results. For instance, higher human capital risk boosts investment in education. The option value to stay in higher education predicts lower returns, not higher returns. And, higher income taxes encourage investment in education and education subsidies discourage investment in education. These results appear to be theoretically and empirically less plausible.

In contrast to these earlier papers, this paper starts from the premise that the decision to start with learning is the irreversible decision, not the decision to start working. The returns (or costs) of the investment in human capital are uncertain. Waiting to enroll in higher education has the benefit of gaining more information on the returns (or costs) of the investment. The option value stems from the fact that one could wait to enroll and only do so when the returns are sufficiently large to compensate for the lost option value. The sunk cost of the investment is not the lost (utility-)benefit of staying in school, but the forgone labor earnings and tuition costs.

Which one of the two approaches is more adequate is an open empirical question. One could argue that education is an experience good whose costs and returns will only become known after enrolling in education. However, the approach of this paper appears to be more satisfactory from a theoretical perspective and its potential empirical content. We turn back to these issues in the conclusions.

3 Model

The model of Dixit and Pindyck (1994, ch.2) forms the basis of this paper. A risk neutral individual considers the discrete decision to make a risky investment in higher education. Capital and insurance markets are perfect, although the latter assumption is not needed with risk neutral individuals. The investment in higher education is completely irreversible. Investments in higher education mainly consist of forgone labor earnings and tuition costs. It is quite natural to assume that there is not a way of getting your money back once working time is forgone and tuition fees have been paid. The investment in education is therefore a sunk cost.

The returns of this investment are uncertain due to input and output uncertainty (Levhari and Weiss, 1973). Input uncertainty arises when the individual does not (fully) know his ability to complete the investment and to finish his study (in time). Failure to complete (in time) lowers the returns to the investment.[2] Alternatively, the individual faces uncertainty as regards his capacities to fully capture the returns to his human capital when choosing a particular type of study. Individuals may unintendedly choose the wrong study if it turns out that their individual capacities do not match with the job-types they can apply for.

Output uncertainty relates to uncertainty in the returns to human capital investments due to changing labor market conditions after graduation, for example the probability to find a job, the risks of sectoral shifts and business cycles. All these may cause losses of human capital and thereby lower the returns of the investment.

The individual may decide to go to college directly or he may postpone the investment for one year and work in the labor market during this year. Therefore, the individual has the option to wait in the presence of uncertainty. After one year, all uncertainty is revealed and there is no longer an option to wait. The model is probably more suited to describe the effects of input uncertainty because it is less likely that all output uncertainty is fully revealed after one year. This is the simplest possible set-up to analyze the consequences of options in human capital investments. All qualitative results will nevertheless carry over to the more general continuous time cases where uncertainty is never fully revealed and the individual can always decide to invest in higher education at later dates (see also Dixit and Pindyck, 1994).

The option to postpone the investment is analogous to a financial call option. The student has the right but not the obligation to buy an asset, i.e., human capital, at some future date. When the student decides to enroll immediately, he exercises his option to buy the asset and gives up the opportunity to wait and see whether the returns to the investment has improved.

The total investment consists of forgone labor earnings and the direct costs of higher education while enrolled. Forgone earnings are gross earnings per year net of taxes $(1-\tau)w$ where τ is the tax rate and w is the gross yearly wage. Taxes reduce the opportunity costs of enrolling in higher education. There are also direct costs such as tuition fees, books, materials and computers. Direct costs are $(1-s)k$ where k is the monetary cost of one year of higher education and s is the flat subsidy rate. Both forgone labor earnings and tuition costs are not time-varying for simplicity. It takes T years to graduate, hence the present value of total costs I of investing in higher education at the date of graduation, $t=0$, equals

$$I \equiv \sum_{t=-T}^{t=0} \frac{(1-\tau)w + (1-s)k}{(1+r)^t} = (1-\tau)\omega + (1-s)\kappa, \tag{1}$$

where r is the real interest rate and $\omega \equiv \sum_{t=-T}^{t=0} \frac{w}{(1+r)^t}$ and $\kappa \equiv \sum_{t=-T}^{t=0} \frac{k}{(1+r)^t}$ denote the present value of gross forgone earnings and direct costs of education, respectively. Perfect capital markets are assumed so that the individual can always borrow at rate r to finance investments in education I.

[2] One could also interpret this uncertainty as cost uncertainty, but that will not qualitatively affect the results of this paper, see also Dixit and Pindyck (1994, ch.2).

The time-horizon for the individual is assumed to be infinite for analytical simplicity. The qualitative results readily extend to the case with a finite horizon, however. When the individual invests with graduation at time $t = 0$ the expected return to the investment in higher education is R_0 each year from $t = 0$ to $t = \infty$. The future return is uncertain. At time $t = 1$ the return either increases to $R_1 \equiv (1 + \upsilon)R_0$ with probability q and the expected return decreases to $R_1 \equiv (1 - \delta)R_0$ with probability $(1 - q)$. After $t = 1$ all uncertainty is revealed and the return remains fixed at R_1 from $t = 1$ to $t = \infty$. υ is the upward swing and δ is the downward swing in the returns on the investment. The return on the investment is taxed at rate θ which may be higher than the marginal rate τ. In that case, marginal tax rates are increasing with income because returns will be larger than forgone labor earnings if the investment is actually made.

If it is only possible to invest immediately and with graduation at $t = 0$, the prospective student invests in higher education if the present value of labor earnings V_0 is larger than the total costs of the investment in higher education I. Otherwise, the student does not go to college. The net-present value rule is equivalent to the standard human capital approach.

The present value of labor earnings of investing with graduation at time $t = 0$ is

$$\begin{aligned} V_0 &\equiv (1-\theta)R_0 + (1-\theta)\left(q(1+\upsilon)R_0 + (1-q)(1-\delta)R_0\right)\sum_{t=1}^{\infty}\frac{1}{(1+r)^t} \\ &= \frac{(1-\theta)R_0(1+r+q(\upsilon+\delta)-\delta)}{r}. \end{aligned} \tag{2}$$

Ω_0 equals the net pay-off from the investment in higher education. Ω_0 equals the net present value of the investment if the investment is undertaken or zero if the net present value is negative and the investment is not undertaken. Formally, it is written as

$$\begin{aligned} \Omega_0 &\equiv \max\{V_0 - I, 0\} \\ &= \max\left\{\frac{(1-\theta)R_0(1+r+q(\upsilon+\delta)-\delta)}{r} - (1-\tau)\omega - (1-s)\kappa, 0\right\}. \end{aligned} \tag{3}$$

If investment with graduation at $t = 1$ is also possible the individual has the option to wait one year and see whether returns have gone up or down because uncertainty is revealed. Waiting one year, and thereby foregoing a one-year return, only makes sense if one can avoid a bad outcome which generates a lower net present value when the investment is made.

If the investment is postponed and human capital earns a high return at graduation date $t = 1$, the net present value of the investment in education with at graduation at $t = 1$ will be equal to

$$\begin{aligned} \overline{F}_1 &\equiv \max\{\overline{V}_1 - I, 0\} \\ &= \max\left\{\frac{(1-\theta)(1+\upsilon)R_0(1+r)}{r} - (1-\tau)\omega - (1-s)\kappa, 0\right\}, \end{aligned} \tag{4}$$

where the upper bar denotes a high return outcome. If the investment is postponed at and human capital earns a low return at graduation date $t = 1$, the net present value of the investment in education with graduation at $t = 1$ will be

$$\begin{aligned} \underline{F}_1 &\equiv \max\{\underline{V}_1 - I, 0\} \\ &= \max\left\{\frac{(1-\theta)(1-\delta)R_0(1+r)}{r} - (1-\tau)\omega - (1-s)\kappa, 0\right\}, \end{aligned} \tag{5}$$

where the lower bars denote outcomes when the return is low.

Now, the whole investment opportunity, i.e., investing either now or tomorrow has a value F_0 which is equal to the maximum return of the investment when the individual invests directly or when the individual postpones the investment and gets the discounted value of the returns when waiting:

$$\begin{aligned} F_0 &\equiv \max\left\{\Omega_0, \frac{q\overline{F}_1 + (1-q)\underline{F}_1}{1+r}\right\} \quad (6) \\ &= \max\left\{\max\{V_0 - I, 0\}, q\frac{\max\left\{\overline{V}_1 - I, 0\right\}}{1+r} + (1-q)\frac{\max\{\underline{V}_1 - I, 0\}}{1+r}\right\}, \end{aligned}$$

where

$$V_0 - I = \frac{(1-\theta)R_0(1+r+q(\upsilon+\delta)-\delta)}{r} - (1-\tau)\omega - (1-s)\kappa, \quad (7)$$

$$\frac{\overline{V}_1 - I}{1+r} = \frac{(1-\theta)(1+\upsilon)R_0}{r} - \frac{(1-\tau)\omega + (1-s)\kappa}{1+r}, \quad (8)$$

and

$$\frac{\underline{V}_1 - I}{1+r} = \frac{(1-\theta)(1-\delta)R_0}{r} - \frac{(1-\tau)\omega + (1-s)\kappa}{1+r}. \quad (9)$$

The value of the option to wait W is the difference between the value of the investment opportunity F_0 which covers the choice between investing now or next year and the net present value of directly investing Ω_0:

$$\begin{aligned} W &\equiv F_0 - \Omega_0 \quad (10) \\ &= \max\left\{\max\{V_0 - I, 0\}, q\frac{\max\left\{\overline{V}_1 - I, 0\right\}}{1+r} + (1-q)\frac{\max\{\underline{V}_1 - I, 0\}}{1+r}\right\} \\ &\quad - \max\{V_0 - I, 0\}. \end{aligned}$$

To derive simple analytical results, the formal analysis is restricted to the case where immediate investment has a positive present value, i.e., $\max\{V_0 - I, 0\} = V_0 - I > 0$, the good outcome yields a positive present value, so that $\max\left\{\overline{V}_1 - I, 0\right\} = \overline{V}_1 - I$ and the bad outcome yields a negative present value, and $\max\{\underline{V}_1 - I, 0\} = 0$.[3] With these restrictions, the option to wait has value

$$W = \max\left\{V_0 - I, q\left(\frac{\overline{V}_1 - I}{1+r}\right)\right\} - (V_0 - I). \quad (11)$$

As long as $V_0 - I > q\left(\frac{\overline{V}_1 - I}{1+r}\right)$ the option to wait is of no value and $W = 0$. In that case, it is optimal to go to college directly if the present value of direct investment is positive. If, however, $V_0 - I < q\left(\frac{\overline{V}_1 - I}{1+r}\right)$ the option to wait generates sufficient value and the investment will be postponed for one year. In that case, the individual will only invest if the returns are high and the individual will not invest at all if the returns are low.

[3]Little generality is lost by imposing these restrictions. All analytical results can be shown to be qualitatively robust for all plausible parameter values of the model under consideration using graphical analysis of the general model without restrictions.

4 To invest or not to invest in higher education?

This section derives the comparative statics of changing the parameters of the model on the willingness to invest in higher education and the willingness to postpone investment in higher education. The value of the option (W) depends on the probability of an upswing (q), the costs (I), the interest rate (r) and the gross present values of the investment in higher education now or tomorrow (V_0 and V_1). These costs and present values are, in turn, determined by the return on the investment (R_0), the sizes of the up and downswings in the returns (υ and δ), the costs of forgone earnings and tuition (ω and κ), the interest rates, the taxes (τ, θ) and the subsidy (s).

4.1 Options values and returns

R_0^* is the critical value of the average return to the investment in higher education at which individuals are indifferent between directly going to college or postponing one year and only going to college if the return turns out to be high:

$$R_0^* = \frac{(1+r-q)\,r}{(1+r)\,(r+(1-q)(1-\delta))} \frac{[(1-\tau)\omega + (1-s)\kappa]}{(1-\theta)}. \tag{12}$$

There is also a critical lower bound on the return $\underline{R}_0$ below which the individual will never consider to enroll in college, not even if the individual has the option to wait. This return is given by

$$\underline{R}_0 = \frac{r}{(1+r)(1+\upsilon)} \frac{[(1-\tau)\omega + (1-s)\kappa]}{(1-\theta)}. \tag{13}$$

The required return at which the student will never invest is lower than return at which the student postpones the investment: $\underline{R}_0 < R_0^*$.

The minimum required return $\underline{R}_0$ can be compared with the required return $\hat{R}_0$ that would make the net-present value of investing higher education non-negative if the option to delay the investment was not available:

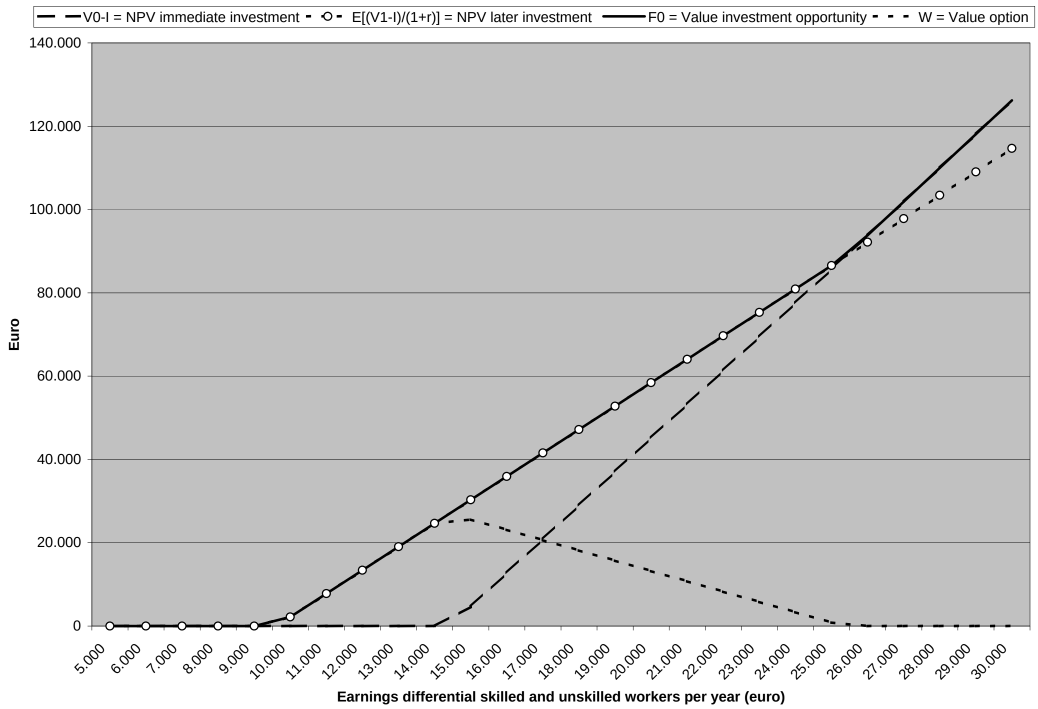
$$\hat{R}_0 = \frac{r}{(1+r+q(\upsilon+\delta)-\delta)} \frac{[(1-\tau)\omega + (1-s)\kappa]}{(1-\theta)}. \tag{14}$$

This return is larger than the return if the option to wait is available, i.e., $\hat{R}_0 > \underline{R}_0$. Therefore, having the option to wait has positive value because it reduces the required return to consider the investment in higher education.

At the same time, the return at which individuals invest directly can be shown to be larger than the return which would induce investments when options are not available: $R_0^* > \hat{R}_0$. This means that individuals are more reluctant to invest directly in the option model than in the standard human capital model.

Figure 1 graphs the value of the option to wait to go to college as a function of the average return on the investment. The parameter values underlying the graphs have been chosen in such a way to illustrate the potential relevance of options. However, the parameters are not completely unrealistic either. A more elaborate discussion follows shortly. Below $\underline{R}_0 = 9.613$ euro the individual will not invest at all. Between $\underline{R}_0$ and $R_0^* = 25.343$ euro the individual waits for one year and does not invest directly. The individual only invests at $t = 1$ if the return turns out to be high. When the return is above R_0^* the individual will always invest directly and gives up the option to wait.

Figure 1: Option values and returns to human capital investment



Baseline parameters: $r = 8\%$, $q = \upsilon = \delta = 0.5$, $\theta = \tau = s = 0.4$, $\omega = 145.998$ and $\kappa = 48.666$.

The value of the opportunity to go to college is a piecewise-linear function of the average return to the investment. If we subtract the value of immediate investment without the option $\Omega_0 \equiv \max\{V_0 - I, 0\}$ we get the line that gives the value W of the option to wait. The value of the option first increases and then declines when the return to college education increases. Note that the increasing part of the option is in the range where direct investment yields a negative present value. Therefore, the option to wait increases in value when the returns increase. The reason is that there are no opportunity costs of waiting to invest if the direct investment yields a negative present value and the individual would not invest anyhow. However, when the present value of direct investment becomes positive, the option becomes less valuable as returns increase because the individual looses positive returns of immediate investment. Therefore, higher returns R_0 reduce the value of the option and investing directly becomes relatively more profitable. From figure 1 can be read that the slope of the line of immediate investment $(V_0 - I)$ is steeper than the slope of the investment opportunity F_0 as long as the option W has positive value. As the return passes the critical level R_0^* immediate investment takes place and the option to wait is given up.

Note that the value of the option W is positively related to the required return R_0^* to make immediate investment optimal (if $W > 0$). The option value increases if the return of the whole investment opportunity F_0 increases (and the F_0-locus shifts outwards), or of the return of direct investment decreases (and the Ω_0-locus shifts inwards). In both cases, the required return R_0^* to make immediate investment optimal increases. In other words, if the required return to make immediate investment in human capital optimal increases, the option value of the investment increases. This property will be useful later on.

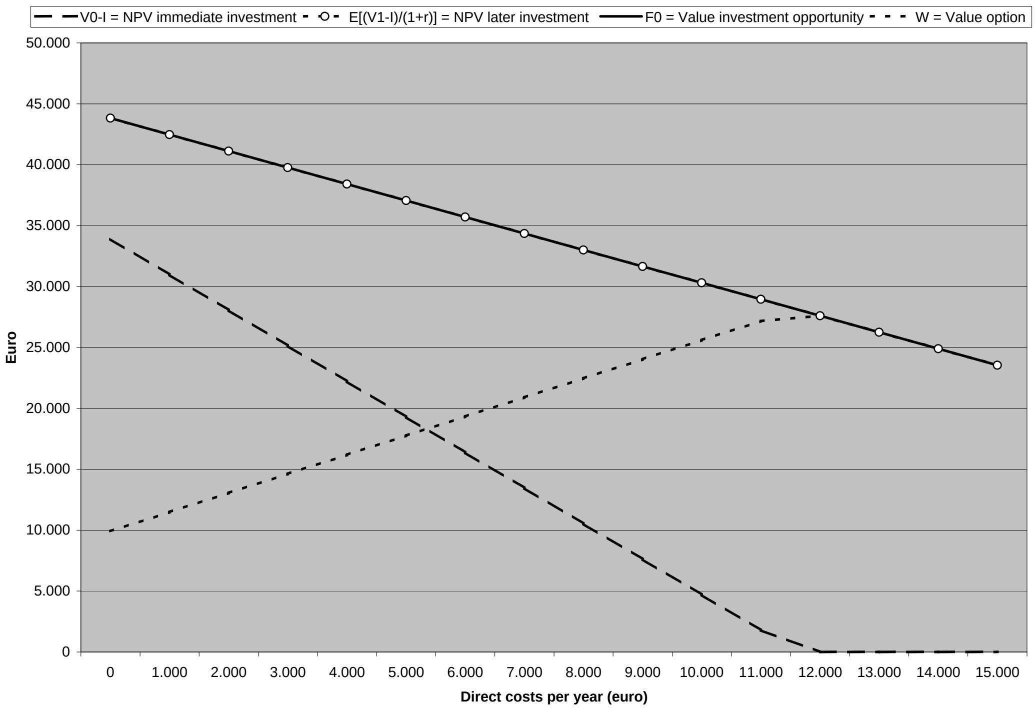
As a result of the option value of waiting to go to college the required return at which students do want to enroll increases. In other words, students want to be compensated with a larger return on their human capital investment if options are important. This is an equilibrium outcome which does not require failures in financial markets. An example is constructed with the purpose to illustrate the potential relevance of options for investments in higher education. However, the parameters used in the example are in line with empirical estimates.

Assume that the interest rate equals $r = 8\%$. This somewhat large value is chosen to correct for a potential bias of the infinite horizon in the model and the possibility of risk-aversion in the presence of non-insurable income risks which may drive up required risk-adjusted rates of return. Let the investment in higher education give a yearly skill premium of 15.000 euro per year. That is, skilled workers earn $R_0 = 15.000$ euro more than unskilled workers who are assumed to earn 30.000 euro per year on average. This corresponds to a Mincer return of 12,5% per year of education where we assume that higher education takes four years. As a result, the present value of life-time returns to higher education equal on average 187.500 euro. Suppose that with a 50% probability the return R_0 is 50% lower or higher, i.e., $q = \upsilon = \delta = 0.5$. That is, the skill-premium R_1 equals either 22.500 euro or 7.500 euro per year. This assumption is not backed by empirical estimates as the stylized two-outcome model has no empirical counterpart. Assume that taxes are flat at 40% and that education subsidies are such that all costs are effectively tax deductible, i.e., $\theta = \tau = s = 0.4$. Most countries have tax systems with increasing marginal tax rates and subsidies educational costs higher than 40%. Both the lower top rate and education subsidy counter each other. Finally, assume that forgone wages equal 30.000 euro per year which is equal to the unskilled wage. These costs may be a bit high but may also entail psychic and effort costs while enrolled in college. The direct costs of education are set to 10.000 euro per year. These figures give a present value of forgone earnings equal to $\omega = 145.998$ euro and the present value of the direct costs equal $\kappa = 48.666$ euro. All these numbers underlie figure 1.

In this case, we can derive that the net present value of direct investment equals 4.702 euro. Investment in higher education is optimal according to the standard human capital model. However, the value of the option to postpone investment is more than five times larger and equals 25.600 euro. Therefore, the investment should not be undertaken immediately. The reason is that when the individual invests immediately, he gives up his option to wait for a better outcome which is of positive value. A negative present value of the total investment opportunity results if the costs of giving up the option are subtracted from the positive present value of immediate investment. The real present value of the immediate investment is not 4.702 but $4.702 - 25.600 = -20.898$ euro instead.

Note that if the option to wait is valuable ($W > 0$), the value of the investment opportunity F_0 increases at a slower rate with the return R_0 than the return to the investment when there is no option value Ω_0. In terms of figure 1, the slope of the F_0 line is less steep than the Ω_0 line. This implies that an increase in the financial rewards of the investment will induce smaller effects on the decision to invest when options are present than without. In other words, the option approach predicts smaller sensitivity of human capital investments with respect to the returns than the standard human capital model.

Figure 2: Option values and yearly direct costs of investment k



Baseline parameters: $r = 8\%$, $R_0 = 15.000$ $q = \upsilon = \delta = 0.5$, $\theta = \tau = s = 0.4$, $\omega = 145.998$.

4.2 Options values and the cost of investment

Higher costs of the investment in higher education $I \equiv (1-\tau)\omega + (1-s)\kappa$ raise the critical return to induce immediate investment R_0^*. From equation (12) follows that

$$\frac{\partial R_0^*}{\partial I} = \frac{(1+r-q)\,r}{(1+r)\,(r+(1-q)(1-\delta))}\frac{1}{(1-\theta)} > 0. \tag{15}$$

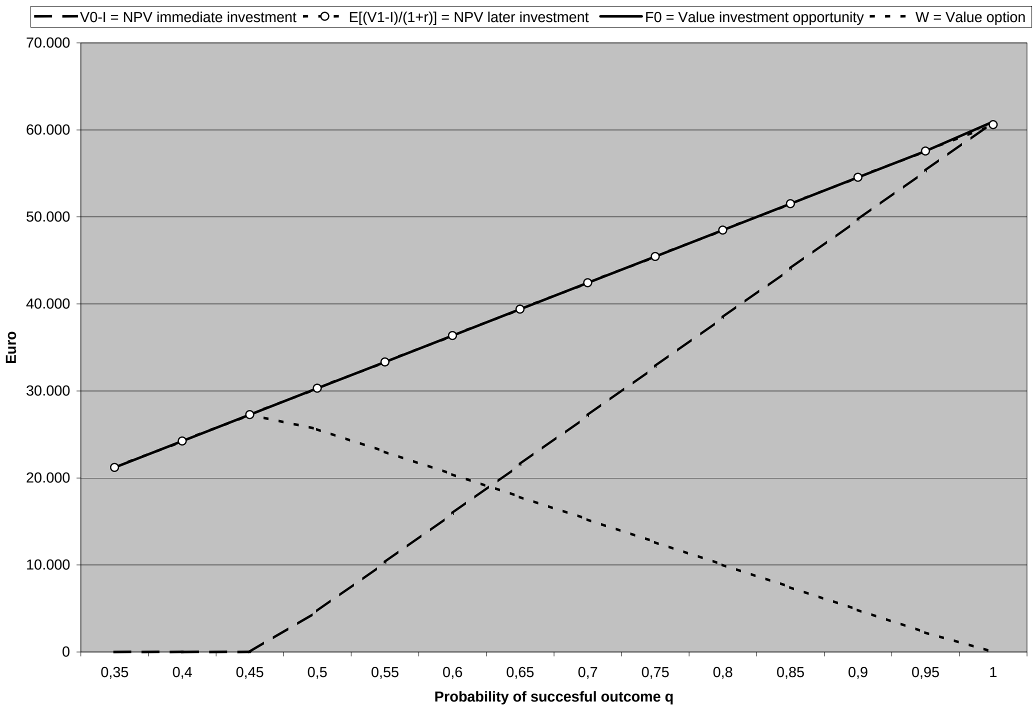
The intuition is that when the costs of investment increase, the opportunity costs of waiting (having a one year extra return) decrease more than the benefit of waiting (only invest if the return is high). Higher costs of education therefore increase the option value of waiting. See also figure 2 where the option value W increases as long as the net present value of immediate investment in human capital remains positive (as was assumed in the derivation of the last result).

Higher costs also raise the critical value of the return $\underline{R}_0$ below which the investment is not considered at all which follows from differentiating (13):

$$\frac{\partial \underline{R}_0}{\partial I} = \frac{r}{(1+r)(1+\upsilon)}\frac{1}{(1-\theta)} > 0. \tag{16}$$

See also figure 2. This result is analogous to the impact of higher costs on a higher required threshold return in the standard human capital model. Therefore, higher costs of investment result in less total investment in human capital, both because individuals would not consider the investment and because they tend to be more reluctant with immediate investment since the option value of waiting increases.

Figure 3: Option values and probability of high returns q



Baseline parameters: $r = 8\%$, $R_0 = 15.000$, $\upsilon = \delta = 0.5$, $\theta = \tau = s = 0.4$, $\omega = 145.998$ and $\kappa = 48.666$.

4.3 Option values and the probability of high returns

Differentiation of the required return to invest immediately (12) with respect to the probability of success q gives

$$\frac{\partial R_0^*}{\partial q} = -\frac{[(1-\tau)\omega + (1-s)\kappa]}{(1-\theta)(1+r)} \frac{r^2\delta}{[(r + (1-q)(1-\delta))]^2} < 0. \tag{17}$$

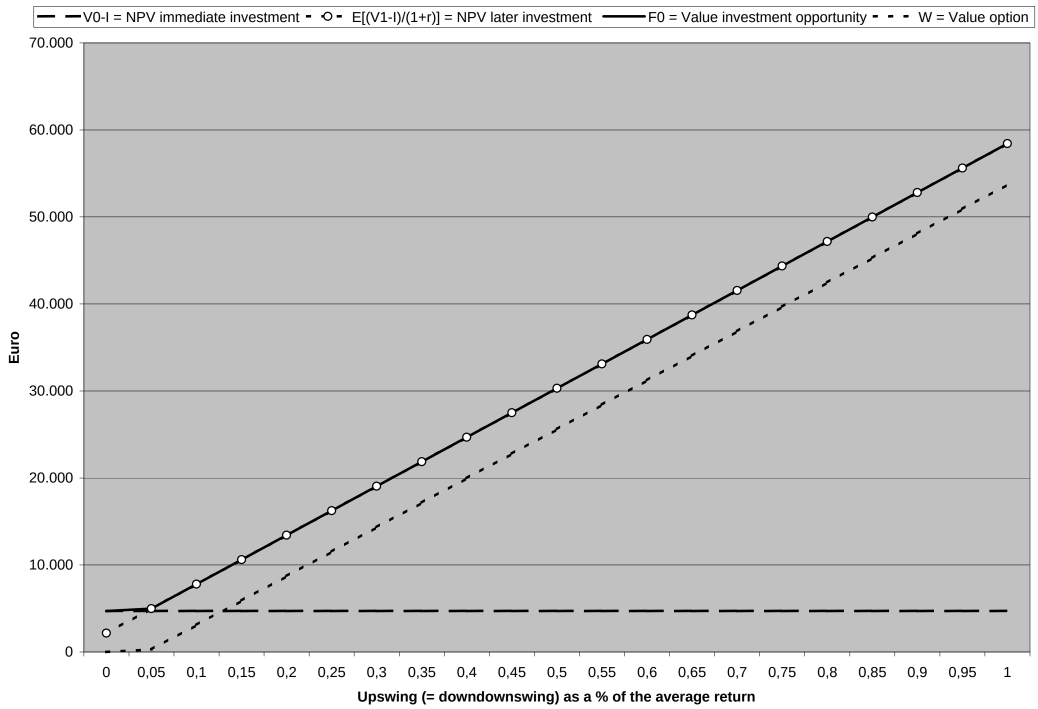
Therefore, if the probability of success increases, the opportunity costs of waiting in terms of missed returns increase. At the same time, the option value of waiting diminishes because the benefit of avoiding the low return outcome is lower because the low return outcome is less likely to occur. Therefore, the option value of waiting W decreases if the probability of a success increase. See also figure 3.

A higher probability of success has no effect on the minimum required return to consider the whole investment opportunity $\frac{\partial \underline{R}_0}{\partial q} = 0$. The intuition is that option values are only determined by the probability of a downswing, not an upswing, see also below.

4.4 Option values and riskiness in returns

Increasing the probability q of a good outcome both increases the return and reduces the risk of the investment at the same time. To isolate the effect of larger risks without changing the expected return, mean preserving increases in the spread of returns are considered. To keep mean returns fixed, the downswing and the upswing are linearly related, i.e., $q(1+\upsilon) = -(1-q)(1-\delta)$.

Figure 4: Option values and risk $\upsilon = \delta$



Baseline parameters: $r = 8\%$, $R_0 = 15.000$, $q = 0.5$, $\theta = \tau = s = 0.4$, $\omega = 145.998$ and $\kappa = 48.666$.

Note that (12) only depends on the downswing δ. Differentiation gives the effect of larger risk on the required return to induce immediate investment in human capital:

$$\frac{\partial R_0^*}{\partial \delta} = \frac{(1-\tau)\omega + (1-s)\kappa}{(1-\theta)(1+r)} \frac{(1-q)}{[(r+(1-q)(1-\delta))]^2} > 0. \tag{18}$$

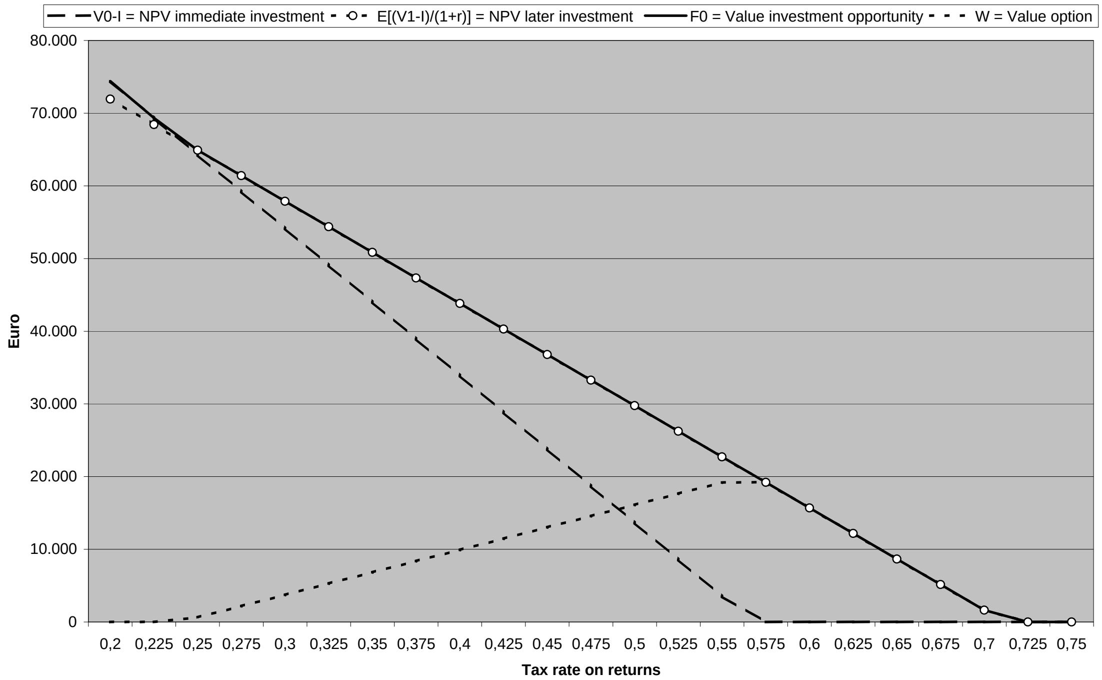
Therefore, increasing risk increases the return at which individuals want to invest directly. The intuition is that the option value of waiting W increases when the spread increases. The individual can reap the benefits of a higher potential upswing while avoiding the larger downward risks by not investing in that case. See also figure 4.

The required return for immediate investment (12) does not depend on the upward swing υ or the probability of a successful outcome q. Only the size of the bad outcome δ and the probability of the bad outcome $1-q$ determine whether individuals want to invest directly or not. This is the so called 'bad news principle', see also Dixit and Pindyck (1994). The option to wait is only valuable because it allows the individual to avoid the consequences of bad news.

Eaton and Rosen (1980) also find that higher income risks have a negative effect on investments in human capital. In that paper, however, individuals are risk averse and therefore want a risk-premium on their investments in human capital. In the current set-up with risk neutral individuals, the individuals require a premium on the return to give up a valuable option.

A higher risk has a negative effect on the minimum required return to consider the

Figure 5: Option values and top rate income tax θ



Baseline parameters: $r = 8\%$, $R_0 = 15.000$, $q = \upsilon = \delta = 0.5$, $\tau = 0.4$, $\omega = 145.998$ and $\kappa = 0$.

whole investment opportunity:

$$\frac{\partial \underline{R}_0}{\partial \upsilon} = -\frac{r}{(1+r)(1+\upsilon)^2} \frac{[(1-\tau)\omega + (1-s)\kappa]}{(1-\theta)} < 0. \tag{19}$$

The intuition is that with a mean-preserving spread, the larger option value lowers the critical return to consider the investment.

4.5 Option values and income taxes

The effects of taxes on returns and forgone earnings on the required return to induce immediate investments follow from differentiation of (12) with respect to θ and τ. The effect of a higher tax rate on future returns, the 'top rate', is given by

$$\frac{\partial R_0^*}{\partial \theta} = \frac{(1+r-q)\, r}{(1+r)\,(r+(1-q)(1-\delta))} \frac{[(1-\tau)\omega + (1-s)\kappa]}{(1-\theta)^2} > 0, \tag{20}$$

A higher tax top rate θ makes students less willing to invest directly. A higher tax on future earnings increases the option value W because the returns on immediate investments decrease faster than the returns on postponed investments. See figure 5. Note that we set the direct costs to zero for illustrative purposes ($k = 0$).

As in the standard human capital model, the top rate increases the value of the return below which individuals do not want to consider the investment opportunity:

$$\frac{\partial \underline{R}_0}{\partial \theta} = \frac{r}{(1+r)(1+\upsilon)} \frac{[(1-\tau)\omega + (1-s)\kappa]}{(1-\theta)^2} > 0. \tag{21}$$

A higher tax rate on forgone labor earnings τ decreases the threshold return above which immediate investments take place:

$$\frac{\partial R_0^*}{\partial \tau} = -\frac{(1+r-q)\,r}{(1+r)\,(r+(1-q)(1-\delta))}\frac{\omega}{(1-\theta)} < 0. \tag{22}$$

The intuition is the same as with lowering the costs of the investment I. Higher taxes on forgone earnings increase the opportunity costs of waiting more than than the benefit of waiting. The option value of waiting W decreases and the critical threshold for immediate investments R_0^* decreases. See also figure 2.

The required return to consider the investment in human capital also decreases as can be expected from the standard human capital model:

$$\underline{R}_0 = -\frac{r}{(1+r)(1+\upsilon)}\frac{\omega}{(1-\theta)} < 0. \tag{23}$$

Suppose that there is a flat income tax ($\tau = \theta$) and all costs of education are effectively tax deductible ($s = \tau$). In the absence of option values, the tax system is neutral with respect to investments in human capital because all costs and returns of the investment are reduced at the same rate (see e.g. Heckman, 1976; Bovenberg and Jacobs, 2005). With option values this neutrality still holds because $\left.\frac{\partial R_0^*}{\partial \tau}\right|_{\tau=\theta=s} = 0$. Therefore, the classical neutrality of flat tax rates with complete deductibility of investment costs on human capital investments carries over to the current set-up with option values. This contrasts with the Eaton and Rosen (1980) result where distorting proportional taxes are optimally positive with risk-averse investors. A progressive tax system mimics lacking income insurance and thereby reduces the required risk-premium on human capital investments.

If costs of education are not the same as the flat rate income tax ($s \neq \tau$) then we can find that a higher flat tax ($\theta = \tau$) increases the required return to induce immediate investment

$$\left.\frac{\partial R_0^*}{\partial \tau}\right|_{\tau=\theta} = \frac{(1+r-q)\,r}{(1+r)\,(r+(1-q)(1-\delta))}\frac{(1-s)\kappa}{(1-\tau)^2} > 0. \tag{24}$$

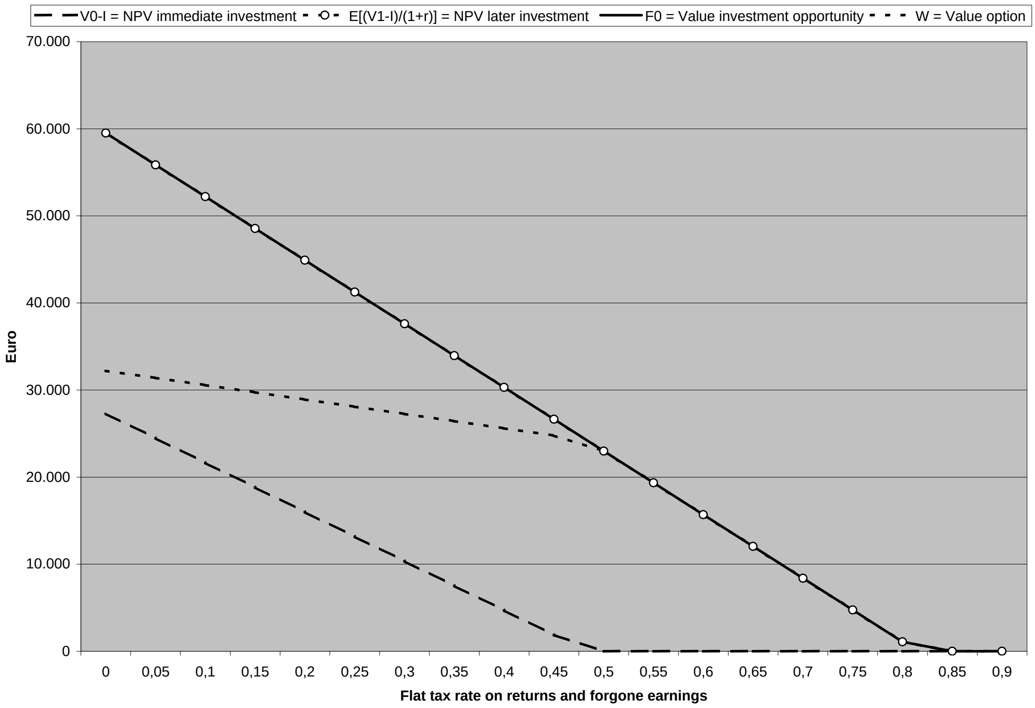
An increase in the flat tax rate has a negative impact on the marginal return required to induce immediate investment as expected. See also figure 6.

$$\left.\frac{\partial \underline{R}_0}{\partial \tau}\right|_{\tau=\theta} = \frac{r}{(1+r)(1+\upsilon)}\frac{(1-s)\kappa}{(1-\tau)^2} > 0. \tag{25}$$

With a flat tax and subsidies on education that are not equal ($s \neq \tau$), options do matter for the total impact of taxes on investments. The total effect of a flat tax on human capital investment consists of both a *positive* and the standard negative impact which dominates. First, the option value of postponing the investment decreases. From equations (20) and (22) can be seen that a higher tax on future benefits has a bigger impact on the required return than the higher tax on forgone earnings. Therefore, the impact of higher taxes on the returns dominate the impact of higher taxes on forgone earnings and option values will increase with an increase in the flat tax rate.

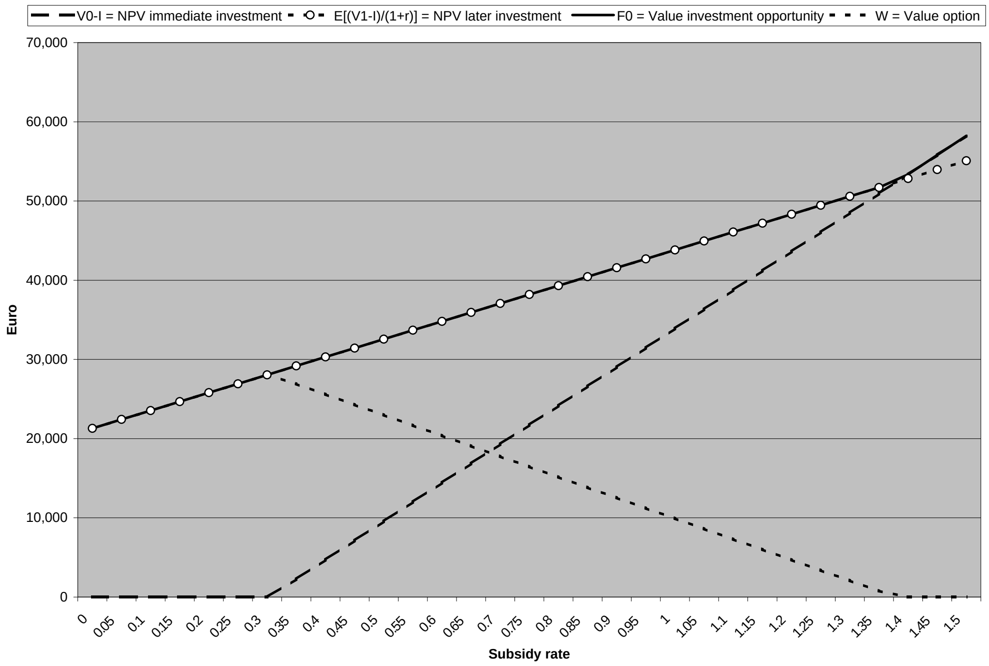
Second, the costs of the investment are reduced less by a higher flat tax rate than the benefits because the direct costs are not reduced by the tax rate. This is the standard effect in human capital models without options (Bovenberg and Jacobs, 2005). Due to the lower option values, the impact of flat taxes on human capital investments is typically smaller with option values than without. See also figure 6 where it is shown that a higher flat tax reduces the option value of investment in human capital.

Figure 6: Option values and flat rate income tax $\theta = \tau$



Baseline parameters: $r = 8\%$, $R_0 = 15.000$, $q = \upsilon = \delta = 0.5$, $s = 0.4$, $\omega = 145.998$ and $\kappa = 48.666$.

Figure 7: Option values and education subsidies s



Baseline parameters: $r = 8\%$, $R_0 = 15.000$, $q = \upsilon = \delta = 0.5$, $\tau = \theta = 0.4$, $\omega = 145.998$ and $\kappa = 48.666$.

4.6 Option values and subsidies on education

As a final exercise, the effects of larger education subsidies are analyzed. From differentiation of (12) follows that the marginal return to induce immediate investments diminishes with the subsidy:

$$\frac{\partial R_0^*}{\partial s} = -\frac{(1+r-q)\,r}{(1+r)\,(r+(1-q)(1-\delta))}\frac{\kappa}{(1-\theta)} < 0. \tag{26}$$

Higher subsidies lower the costs of the investments. The marginal benefits of investing directly increase more than the marginal benefits of postponing the investments. Therefore, the option value of waiting W decreases and the return at which immediate investments are optimal decreases. See also figure 7.

Note that higher education subsidies reduce the option value W of waiting to invest in human capital, cf. figure 7. This implies that the impact of education subsidies on individuals' decisions to consider the investment in human capital is partially off-set by lower option values. Therefore, the full impact of education subsidies on educational investment is again smaller than in the standard human capital model.

The impact of lower costs on the investment is that individuals with lower $\underline{R}_0$ consider the investment:

$$\frac{\partial \underline{R}_0}{\partial s} = -\frac{r}{(1+r)(1+\upsilon)}\frac{\kappa}{(1-\theta)} < 0. \tag{27}$$

Again, this is the standard human capital result.

5 Options and some empirical puzzles

5.1 Human capital equity premium

Options can offer an explanation as to why estimated Mincer returns and skill-premia for higher educated workers are so high. Returns should be high because they compensate the individual for the lost option value of waiting once he makes the irreversible investment in education. Therefore, there may not be a human capital premium puzzle at all. The discount rate would have to be higher if the net present value rule is adjusted so as to capture the value of the option.

Going back to the example of the previous section, the rate at which original net present value calculations should be discounted to get the correct decision, including the option value, is the rate of return r^* that would give a net-present value of -20.898 rather than 4.702. This discount rate satisfies

$$r^* = \left(\frac{1}{r} - \frac{W}{(1 + q(\upsilon + \delta) - \delta)(1 - \theta) R_0} \right)^{-1}. \tag{28}$$

Required rates of return r^* increase with the option value W. With baseline parameters, the critical rate of return r^* which gives the correct decision to invest in higher education equals 10.4%. In other words, the required rate of return to make the correct investment decision would be 2,4% points larger. In this specific example, the option value which is associated with irreversible investment in higher education can explain a significant part of the human capital premium.

The empirical content of the option model to explain actual skill premia can be substantial. Palacios-Huerta (2003, p.4) claims that about two-thirds of the observed skill-premium may be the result of illiquidities and irreversibilities. The work by Carneiro and Heckman (2003), Cameron and Taber (2004) and Plug and Vijverberg (2004), and others suggests that liquidity constraints are quantitatively of minor importance. This may leave the option model as a natural candidate to explain a large remaining part of human capital returns.

5.2 Small sensitivity of human capital investment to financial incentives

The analysis of the previous section has shown that option values tend to make human capital investments less responsive to the net returns of the investments compared to the standard human capital model. The standard human capital model also over-estimates the effects of taxes and education subsidies when options are important. Table 1 gives a summary of some recent estimates of the price responsiveness of tuition and the picture appears to be that doubling tuition costs will decrease enrolment rates with roughly 5-10%-points after controlling for selection. The presence of real options to wait may explain empirical findings that enrolment does not appear to be very price responsive. In the absence of options, price responsiveness of enrolment could be higher.

5.3 Debt-aversion and sensitivity to perceived low return outcomes

Students are often reported to be very debt averse. Many examples can be found of students that are reluctant to borrow for higher education investments even if the average

Table 1: Enrollment responses to increases in tuition costs and corresponding quasi elasticities

Study	Data	Type*	Control	Selection	Sign.?$^\diamond$	$-dq$	dp/p	e
Kane (1994)	US	CS+TS	Yes	No	Yes	.05	.83	.06
Kane (1995)	US	CS+TS	Yes	No	Yes	.035	1.33	.03
Hilmer (1998)	US	M	Yes	No	Yes	.01	.028	.36
Dynarksi (1999)	US	CS	Yes	Yes	Yes	.036	.10	.35 (.03)$^{\#}$
Heckman *et al.* (1998)	US	SM, P	Yes	Yes	Yes	.08	.80†	.07
Card and Lemieux (2000)	US	CS+TS	Yes	No	Yes	-	-	.01 – .04
Cameron and Heckman(2001)	US	P	Yes	Yes	Yes	.03 – .06	.80†	.02 – .05

Notes: 'Control' indicates whether estimations are done when controlling for background characteristics, IQ, and other individual characteristics. 'Selection' indicates whether corrections are made for the selectivity of individuals enrolling in college (non-observed heterogeneity). *CS=Cross-section; TS=Time-series; P=Panel; M=Meta analysis; SM=Structual model. $^\diamond$Indicates significance at the 5% level of the estimated coefficient for tuition. $^{\#}$Price change relative to all costs of college *including tuition, room and board.* In parenthesis we show elasticity evaluated at average tuition rates used by Cameron and Heckman (2001). †Price changes taken relative to an approximated weighted mean of 2 and 4 years tuition costs for Blacks, Hispanics and Whites in Cameron and Heckman (2001) ($1250).

returns on investment higher education are large and increasing. Moreover, the covariance between earnings and education is typically negative, see also Gould et al. (2000) and Palacios-Huerta (2004). This implies that higher levels of education serve as an insurance device and the risk premium on human capital should be lower with the level of education (Levhari and Weiss, 1973). Students should therefore be willing accept lower returns on education than the risk-free rate.

Options may also matter here. We have shown that students should be rightly worried about the probabilities of low return outcomes. The value of the option to wait is only determined by the probability of a low return outcome and the size of the downswing of the outcome, not the average returns. The larger the probability of a bad outcome or the downswing the more reluctant students should be to invest in higher education directly and they should optimally wait in order to reap the benefits of the waiting option, i.e., invest only when the returns are sufficiently large.

The previous analysis assumed that students used objective probabilities on the likelihood of good and bad outcomes. The results change of course with more subjective 'behavioral' or 'non-expected utility' approaches. Suppose that students use subjective probabilities and attach a significant lower probability to high return outcomes and a higher probability to a low return outcome. The perceived value of the option to wait will then be higher than objective probabilities would suggest. Even with risk neutral students, more subjective approaches to economic behavior can therefore strengthen the findings of this paper.

5.4 High returns and immediate enrolment

Perhaps it is not surprising that countries with more turbulent labor markets have high returns on education and more students returning to higher education at a later age. Think of the Anglo-Saxon countries such as the US and the UK. Without option values one can hardly explain high returns if one accepts that the risk-premium is not capable of explaining the differences. High returns could be the result of large option values due to risky environments. If option values are large, it makes sense to return to higher education only when one has more information about one's own capacities and future earnings prospects. In countries with less risky environments, the opposite pattern is typically observed. Individuals normally go to college immediately after secondary education and one would not expect them to enter higher education after a few years of working.

Subsidies also increase the willingness of students to invest in human capital, not only because the costs are lower, but also because subsidies reduce the option value of the investment. In the absence of subsidies, students would be more reluctant to take up the irreversible investment in higher education because they face uncertainty with respect to their capacities to generate a large return on the investment. Too high subsidies result in efficiently low option values, which induce too much risky investment, too high drop out rates and too low returns to higher education.[4] Anglo-Saxon countries tend to give much lower subsidies on higher education which results in larger option values of investment in education. Again, this may explain why more students drop out less, work harder and have high returns on their investments.

[4] In the model of the previous section, efficient subsidies would neutralize the impact of the tax system only. Higher subsidies would then give over-investment.

6 Conclusion

This paper analyzed the consequences of real options in human capital investments. Human capital investments are both risky and largely irreversible. It is generally impossible to recover forgone labor earnings and paid tuition fees. If individuals can affect the timing of the investment, i.e., decide to go to college now or later, option values will affect investment behavior. This paper has shown that with perfect financial markets, the option to postpone investment could explain why returns to education are high, why investment in human capital is not very sensitive to returns, taxes and subsidies, and why students are concerned with low return outcomes.

In future research, the model of this paper could be cast in a continuous time framework as in Hogan and Walker (2002). This would allow for a much better analysis of the potential explanatory powers of the option model than the very stylized two-period, two-outcome model of this paper. One could then allow for a variety of stochastic processes describing the returns to the investment. A more in depth treatment of cost uncertainty could also be analyzed. This paper assumed that costs where exogenous and not time-varying. Cost uncertainty is equivalent to what Levhari and Weiss (1974) call 'input-uncertainty', i.e., the uncertainty about individual capacities. If cost uncertainty decreases when the project is undertaken, there may be reasons to start investing immediately even if the net present value is negative, see also Dixit and Pindyck (1994). In that case, one would be get to the type of models as used by Hogan and Walker (2002). Another interesting avenue is to reconsider the life-cycle model of savings and investments in human capital when the returns to financial and human investments are both stochastic. Investments in human capital could be sensitive to interest rate uncertainty, see also Dixit and Pindyck (1994). Modifications to the basic framework can also be interesting. Examples include the effects of option values in a setting with sequential investments like in Comay et al. (1973). Further, the impact of options on the distribution of wages would be interesting to investigate in a general equilibrium setting where wages of skilled and unskilled workers are endogenously determined. Finally, interactions of options in human capital investment with various types of capital and insurance market failures may be a promising avenue for future research.

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